Packaging problem.

P. Watté, piet.watte@skynet.be

.

1. Statistical tolerancing in a packaging problem.

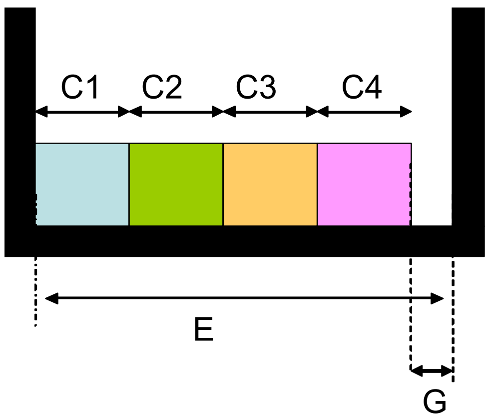
Hereafter, a well known design tolerancing problem is discussed, often cited in the six sigma literature [1]. It is the following: suppose we have a box with an inner opening of length E. We want to stack a set of 4 cubes in this box. In order to allow the removal of the cubes afterwards it is required to have a small gap G in the box.

Suppose we source the four boxes from four different suppliers. Each supplier furnishes us with cubes having a side belonging to a specific statistical distribution function. The latter does not necessarily have to be normal.

In the case each side of the boxes obeys normality, the calculation of the variation on the gap distance is rather straightforward: it is equal to

$s^2gap = s^2E + s^2Cube1 + s^2Cube2 + s^2cube3 + s^2cube4$ (Eq 1)

Figure 1: cubes in a box problem. 4 Cubes with sides C1, C2, C3 and C4 packed in a box with an inner opening of E. The resulting gap is G.



However, the complexity of the computation enhances considerably if the side of the cubes can belong to no matter what distribution function. In that case, no analytical solution exists and the computation of the range has to be done in a numerical way. This can be done by means of a Monte Carlo simulation.

3. Simulation of the gap distance, by making use of the Monte Carlo method.

Define here the number of iterations to be done, m

$m := 5000$

Then, it is common practice to set the number of classes of the histograms equal to

$bin := round(\sqrt{m})$

The distributions for the various sides of the cubes can be modeled as follows:

Suppose that the side of the cube 1 is normally distributed with mean 40 and standard deviation 0.4

Cube 1 $Cube1 := rnorm(m, 40, 0.4)$

The same assumption applies for cube 2, but now the standard deviation is somewhat lower, namely 0.15

Cube 2 $Cube2 := rnorm(m, 40, 0.15)$

The side of cube 3 is uniformly distributed between 38 and 43

Cube 3 $Cube3 := runif(m, 38, 43)$

Whereas, the side of cube 4 is log normally distributed

Cube4 $Cube4 := rlnorm(m, 3.689, 0.15)$

The inner length of the box is equal to

E $E := rnorm(m, 200, 2)$

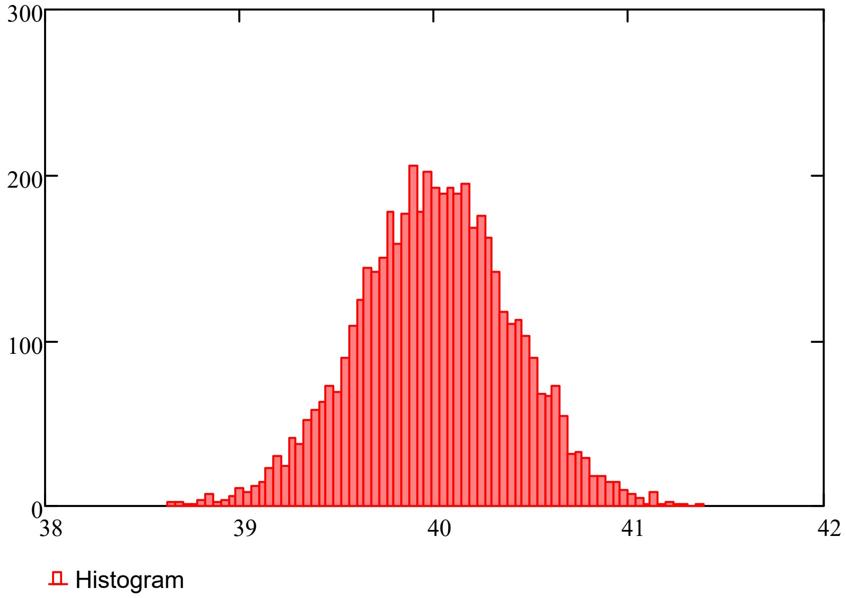
The transfer function for the gap distance is equal to

$$G = E - cube1 - cube2 - cube3 - cube4 \qquad \text{(Eq 2)}$$

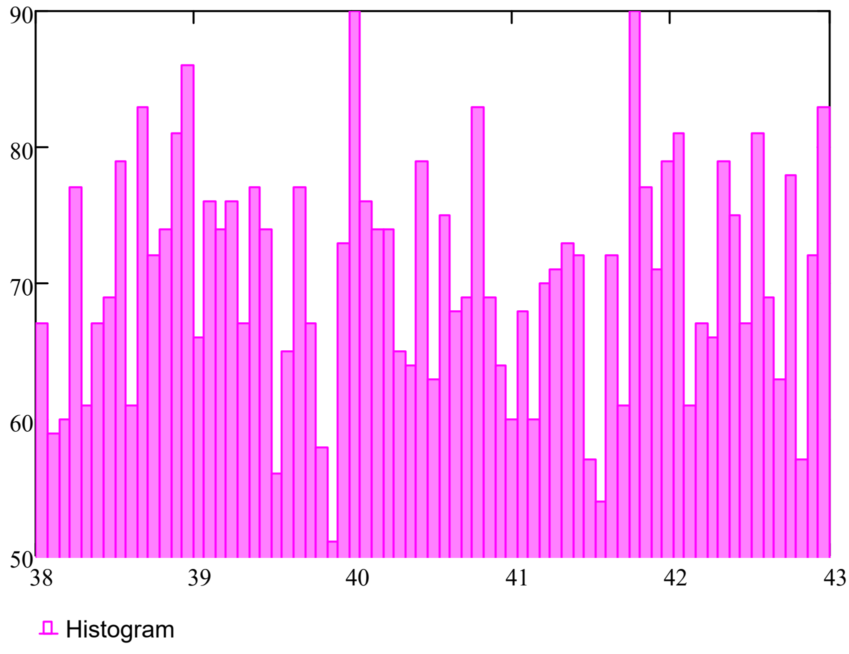
$j := 0 .. (m - 1)$

$$gap_j := E_j - Cube1_j - Cube2_j - Cube3_j - Cube4_j \qquad \text{(Eq 3)}$$

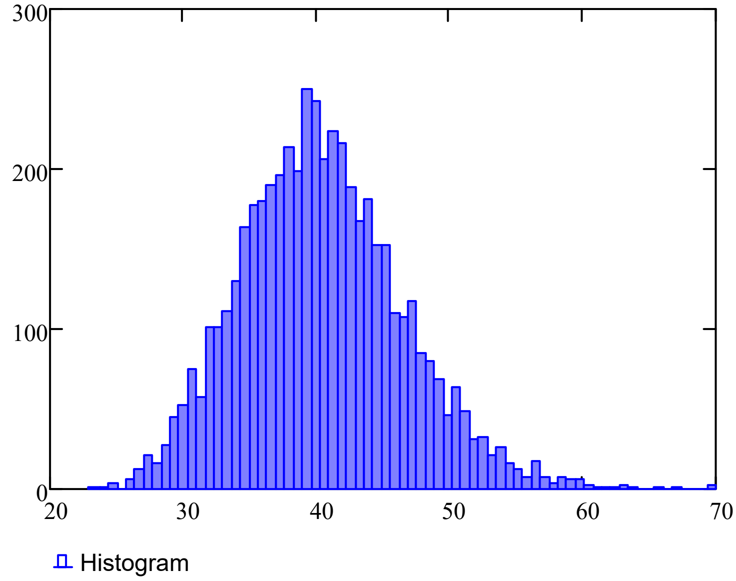
$F := \text{histogram}(\text{bin}, \text{Cube1})$ is normally distributed



$G := \text{histogram}(\text{bin}, \text{Cube3})$ is uniform distributed

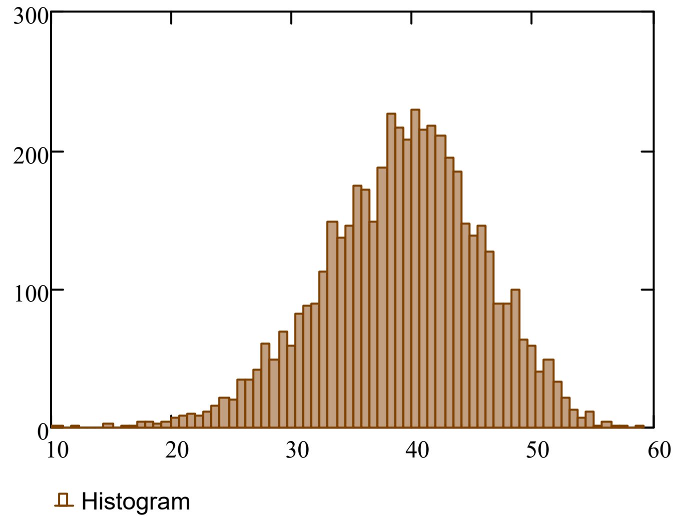


$H := \text{histogram}(\text{bin}, \text{Cube4})$ obeys a lognormal distribution



The resulting histogram of the gap distance is depicted below. As can be seen the gap distance is not equal to 200-3x40=80. It ranges between 6.891 and 57.845. The associated probability density can be derived from the histogram, that is also log normally distributed

$P := \text{histogram}(\text{bin}, \text{gap})$



$\text{median}(\text{gap}) = 39.53$ $\max(\text{gap}) = 59.188$ $\min(\text{gap}) = 10.233$

$\text{range} := \max(\text{gap}) - \min(\text{gap})$ $\text{range} = 48.955$

We calculated the range because we do not know how the gap is distributed.

3. References

[1] M. Harry and R. Stewart, "Six sigma mechanical design tolerancing", Motorola University Press, (1988), p3

[2] P. Watté, proceedings of the 4th European Six sigma conference, Malmö, Sweden, (2002), not published