functions

data :=

..\data.tx

$data1 := submatrix(data, 1, rows(data) - 1, 0, 1)$ remove header row

$t := data1^{\langle 0 \rangle}$ time

$p := data1^{\langle 1 \rangle}$ position

$N := rows(t)$ $N = 19703$ number of samples

$k1 := 0 .. N - 2$

$dt_{k1} := t_{k1+1} - t_{k1}$ the set of delta t

$max(dt) = 0.004$

$min(dt) = 0.003$ Time values in vector are not uniformly spaced, but can ignore for now.

$Tspan := max(t) - min(t)$ $Tspan = 60.250942$

$Ts := \frac{Tspan}{N - 1}$ $Ts = 0.003058$ average sampling inteval (N-1 intervals between N points)

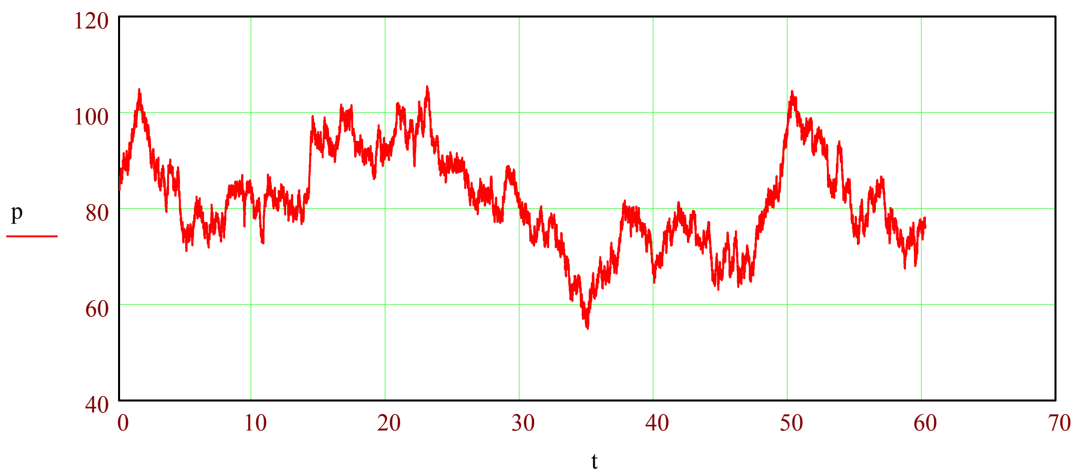
$fs := \frac{1}{Ts}$ $fs = 326.999037$ sampling frequency

The DC component:

$pavg := mean(p)$ $pavg^2 = 6727.000312$ DC power

The remaining AC component:

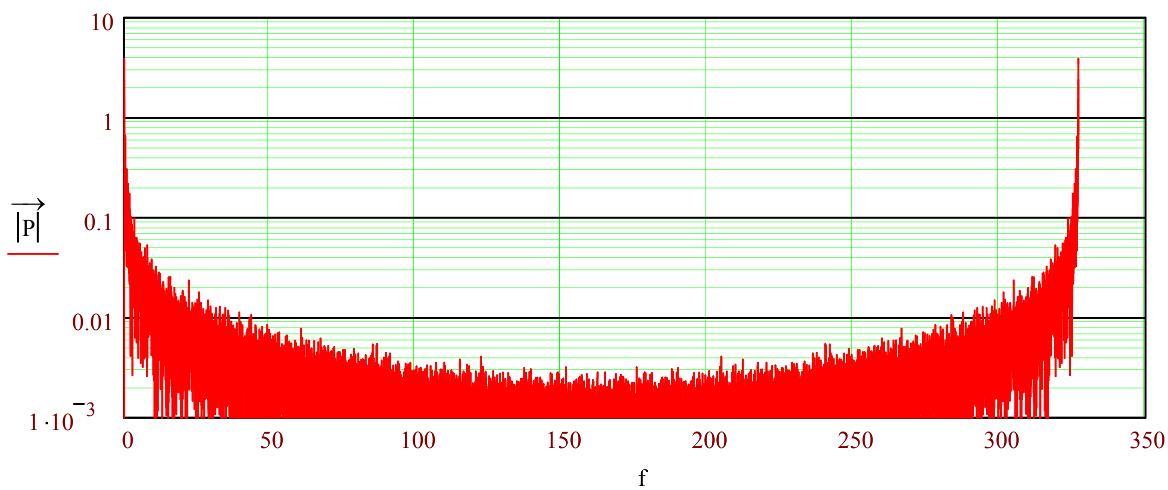
$p0 := p - pavg$ $var(p0) = 100.635957$ total AC power



$P := CFFT(p0)$ DFT of vector p0, without DC term

$f := \text{linvector}\left(0, \frac{N-1}{N} \cdot fs, N-1\right)$ f vector for full spectrum P

$\Delta f := \frac{fs}{N}$ width of one frequency bin



Compute Sp1, the single-sided PSD (f = 0 to fNyquist). The factor N/fs gives Sp(f) in physical units, e.g., V^2/Hz or mm^2/Hz, depending on units of input data p.

$Np := \text{floor}\left(\frac{N}{2}\right)$ $Np = 9851$ highest index need for single-sided output power density spectrum; the max index at or below fNyquist.

$fp := \text{submatrix}(f, 0, Np, 0, 0)$ corresponding f vector.

$fp_0 := 10^{-6}$ fp_0 = 0, so redefine fp_0 as a small, nonzero qty to allow log scale frequency plots.

$Sp1_0 := \frac{N}{fs} \cdot \left(\left|P_0\right|\right)^2$ the DC term has no mirror term for negative frequency, so it is not doubled.

$k := 1 .. Np - 1$

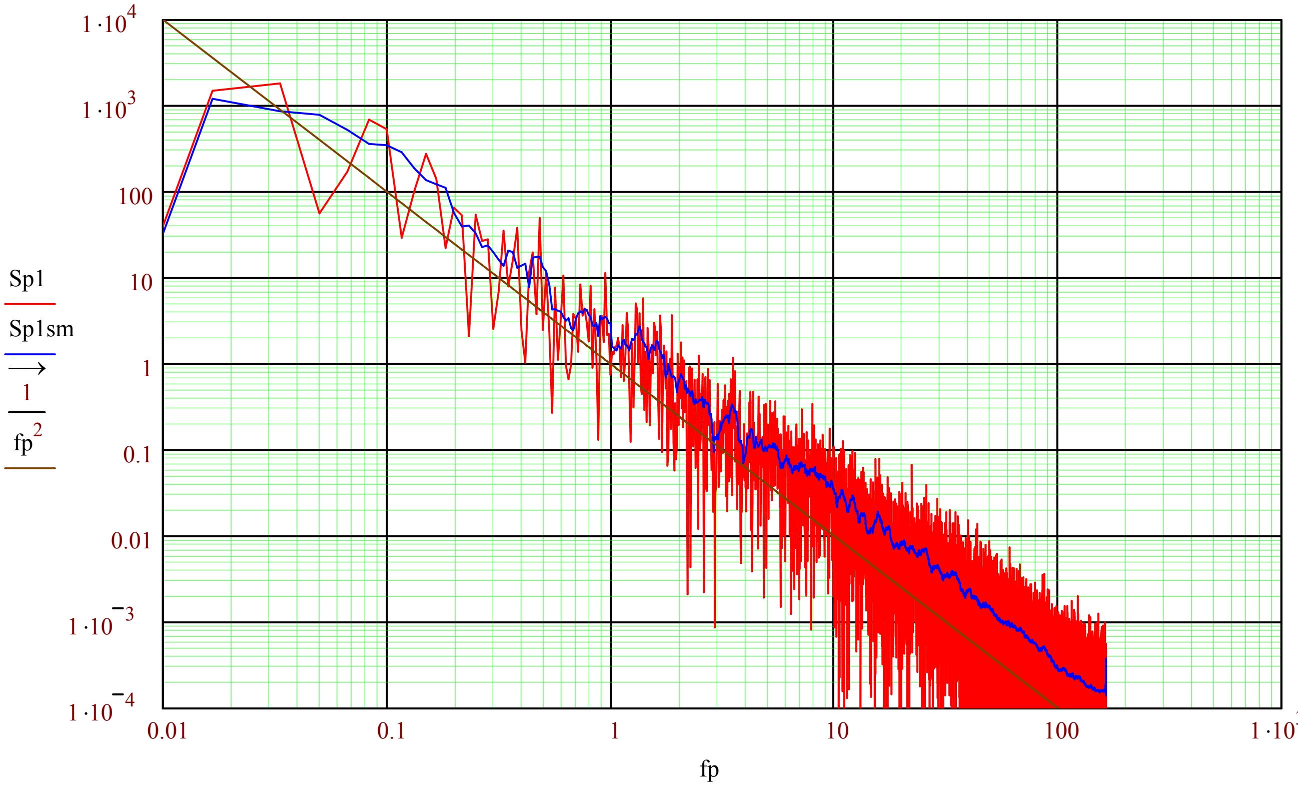
$Sp1_k := 2 \cdot \frac{N}{fs} \cdot \left(\left|P_k\right|\right)^2$ single-sided PSD has twice the value of the double-sided-sided PSD.

$Sp1_{Np} := \frac{2}{2 - \text{mod}(Np, 2)} \cdot \frac{N}{fs} \cdot \left(\left|P_{Np}\right|\right)^2$ If Np is even, then this term is exactly at Nyquist, and should not be doubled. If Np is odd, then the term is not at Nyquist and has a mirror term, hence is doubled.

total power of single-sided PSD, evaluated in frequency domain over f=0 to f=fs/2.

$$\sum_{k=0}^{Np} \left(Sp1_k \cdot \Delta f\right) = 100.635957$$

$Sp1sm := \text{smooth}(Sp1, 0, 4, 500)$ smoothed version of Sp1



power spectral density of p in counts^2/Hz

red: PSD, blue: smoothed PSD, brown: $1/f^2$