PSD matrix

$$S(f) = \begin{pmatrix} S_{xx}(f) & S_{xx.yy}(f) & S_{xx.xy}(f) \\ \overline{S_{xx.yy}(f)} & S_{yy}(f) & S_{yy.xy}(f) \\ \overline{S_{xx.xy}(f)} & \overline{S_{yy.xy}(f)} & S_{xy}(f) \end{pmatrix}$$

S(f) two-sided PSD matrix of x(t)
G(f) onr-sided PSD matrix of x(t) G = 2S

Covariance matrix

$$C = \begin{pmatrix} C_{xx} & C_{xx.yy} & C_{xx.xy} \\ C_{xx.yy} & C_{yy} & C_{yy.xy} \\ C_{xy.xx} & C_{xy.yy} & C_{xy} \end{pmatrix}$$

A matrix

$$A \equiv \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{2\cdot\sqrt{3}} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$f_c := 30$ $f_{min} := 15$ $f_{max} := 45$

$h_{xx} := .1$

$$G_{xx}(f) := \begin{cases} h_{xx} & \text{if } f \ge f_{min} \wedge f \le f_{max} \\ 0 & \text{otherwise} \end{cases}$$

$h_{yy} := .1$

$$G_{yy}(f) := \begin{cases} h_{yy} & \text{if } f \ge f_{min} \wedge f \le f_{max} \\ 0 & \text{otherwise} \end{cases}$$

$h_{xy} := \frac{1}{30}$

$$G_{xy}(f) := \begin{cases} h_{xy} & \text{if } f \ge f_{min} \wedge f \le f_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$G_{xx.yy}(f) := \sqrt{G_{xx}(f) \cdot G_{yy}(f)}$$

spectral moments

$$\lambda(n, G) := \int_0^{\infty} f^n \cdot G(f)\, df$$

zero-order spectral moments

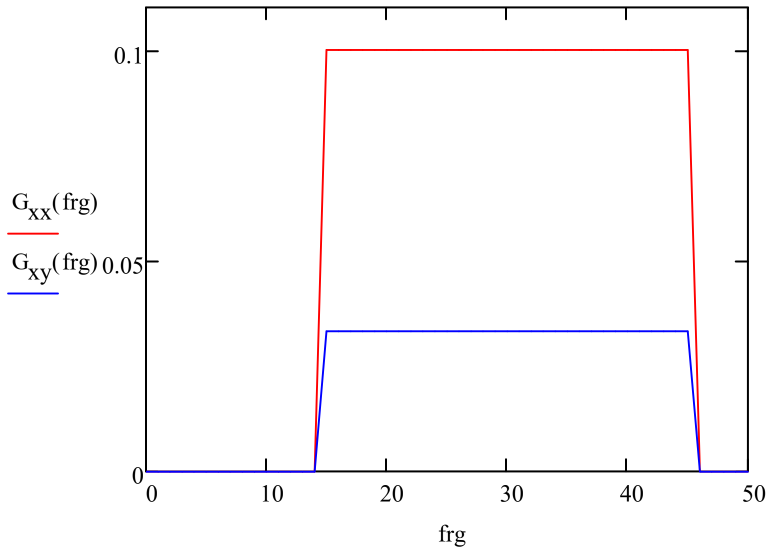
for xx $\lambda_{0.xx} := \lambda(0, G_{xx})$ $\lambda_{0.xx} = 3$ a check $ck1 := h_{xx} \cdot (f_{max} - f_{min}) = 3$

for yy $\lambda_{0.yy} := \lambda(0, G_{yy})$ $\lambda_{0.yy} = 3$

for xy $\lambda_{0.xy} := \lambda(0, G_{xy})$ $\lambda_{0.xy} = 1$

$\lambda_{0.xx.yy} := \lambda(0, G_{xx.yy})$ $\lambda_{0.xx.yy} = 3$

$frg := 0, 1 .. 50$



$$\underset{\sim}{S}(f) := \begin{pmatrix} G_{xx}(f) & 0 & 0 \\ 0 & G_{yy}(f) & 0 \\ 0 & 0 & G_{xy}(f) \end{pmatrix}$$

$$S'(f) := A \cdot S(f) \cdot A^T$$

$$\underset{\sim}{C} := \begin{pmatrix} \lambda_{0.xx} & 0 & 0 \\ 0 & \lambda_{0.yy} & 0 \\ 0 & 0 & \lambda_{0.xy} \end{pmatrix} \qquad C = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C' := A \cdot C \cdot A^T \qquad C' = \begin{pmatrix} 1.25 & -0.433 & 0 \\ -0.433 & 0.75 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda := \mathrm{sort}\left(\mathrm{eigenvals}\left(U^T \cdot C' \cdot U\right)\right) \qquad \lambda = \blacksquare$$

The PSD matrix allows the covariance matrix **C** (symmetric) to be computed:

$$\mathbf{C} = \begin{bmatrix} C_{xx} & C_{xx,yy} & C_{xx,xy} \\ C_{yy,xx} & C_{yy} & C_{yy,xy} \\ C_{xy,xx} & C_{xy,yy} & C_{xy} \end{bmatrix}$$

$$\mathbf{C}' = E\left[\mathbf{s}(t)\cdot\mathbf{s}(t)^T\right] = \mathbf{A}\cdot\mathbf{C}\cdot\mathbf{A}^T \quad\rightarrow\quad \mathbf{C}' = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} \\ & C'_{22} & C'_{23} \\ \text{sym} & & C'_{33} \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In general, matrix **C'** is not diagonal, i.e. some correlation exists between the stress components of $\mathbf{s}(t)$. If this happens, it is necessary to compute a new covariance matrix $\mathbf{C}'_p$ in which all cross-correlations (out-of-diagonal elements) are zero. This outcome yields by solving the following eigenvalue/eigenvector problem for **C'**:

$$\mathbf{C}'_p = \mathbf{U}^T\cdot\mathbf{C}'\cdot\mathbf{U} \qquad\rightarrow\qquad \mathbf{C}'_p = \begin{bmatrix} C'_{p11} & 0 & 0 \\ 0 & C'_{p22} & 0 \\ 0 & 0 & C'_{p33} \end{bmatrix}$$

The eigenvalues are in the main diagonal of $\mathbf{C}'_p$, the eigenvectors are the columns of **U**.