Implicit Functions

$$\theta := atan\left(cot(\varphi) - \frac{M}{sin(\varphi)}\right) \qquad \beta := atan\left(\frac{8 \cdot \frac{sin(\varphi)}{M} + 4 \cdot sin(\theta)}{8\, cos(\theta)^2}\right) \qquad OR := \text{ORIGIN}$$

$$\Lambda(\varphi, M) := \frac{1-\cos(\beta)}{2\cos(\beta)} + \frac{1-\cos(\theta)}{2\cos(\theta)} \xrightarrow{simplify} \frac{\sqrt{\left(\cot(\varphi) - \frac{M}{\sin(\varphi)}\right)^2 + 1}}{2} + \frac{\sqrt{\left(\frac{\left(\cot(\varphi) - \frac{M}{\sin(\varphi)}\right)^2}{8} + \frac{1}{8}\right)^2 \cdot \left(\frac{8 \cdot \sin(\varphi)}{M} + \frac{4 \cdot \cot(\varphi) - \frac{4 \cdot M}{\sin(\varphi)}}{\sqrt{\left(\cot(\varphi) - \frac{M}{\sin(\varphi)}\right)^2 + 1}}\right)^2 + 1}}{2} - 1$$

$$\lambda_{total}(x, y) := \frac{\sqrt{\left(\cot(x) - \frac{y}{\sin(x)}\right)^2 + 1}}{2} + \frac{\sqrt{\left(\frac{\left(\cot(x) - \frac{y}{\sin(x)}\right)^2}{8} + \frac{1}{8}\right)^2 \cdot \left(\frac{8 \cdot \sin(x)}{y} + \frac{4 \cdot \cot(x) - \frac{4 \cdot y}{\sin(x)}}{\sqrt{\left(\cot(x) - \frac{y}{\sin(x)}\right)^2 + 1}}\right)^2 + 1}}{2} - 1$$

$xmin := 10^{-8}$ $\quad xmax := 2$ $\quad ymin := 0.3$ $\quad ymax := 2.2$

$$coords := \begin{bmatrix} xmax & ymax \\ xmin & ymin \end{bmatrix} \qquad [nx \;\; ny] := [100 \;\; 100] \qquad grids := [nx \;\; ny]^T$$

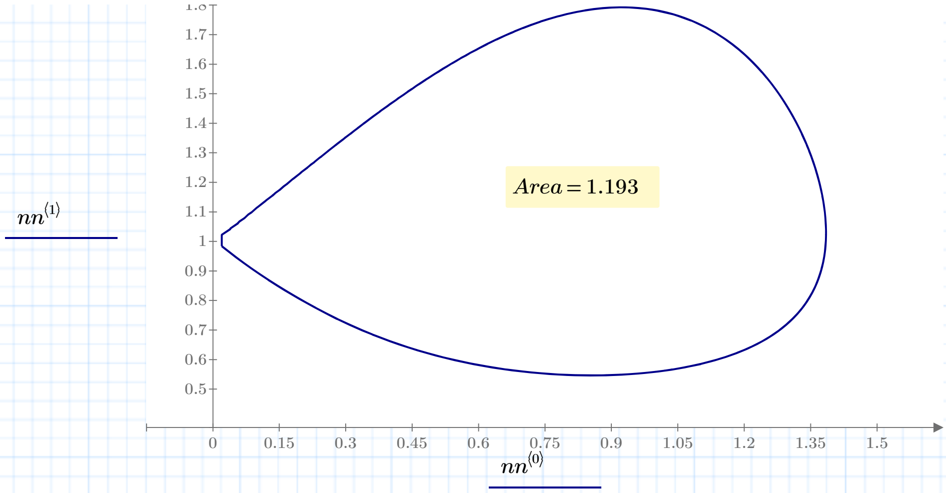
```
plot(λ) := | f(x, y) ← λtotal(x, y) − λ
           | implicitplot2d(f, coords, grids)
```

$H := plot(0.4)$

```
H2 := | "Get unique points"
      | k ← OR
      | for i ∈ OR .. last(H^⟨0⟩)
      |   | if mod(i, 3) = 0
      |   |   | R3^⟨k⟩ ← H^⟨i⟩
      |   |   | k ← k + 1
      | return R3
```

```
nn := | "Nearest neighbor sorting"
      | xyd ← H2
      | xyd^⟨OR+2⟩ ← 0
      | for i ∈ OR + 1 .. last(H2^⟨OR⟩)
      |   | final ← submatrix(xyd, OR, i − 1, OR, OR + 2)
      |   | H ← submatrix(xyd, i, last(H2^⟨OR⟩), OR, OR + 2)
      |   | for j ∈ OR .. last(H2^⟨OR⟩) − i
      |   |   | H[j, OR+2] ← sqrt((H[j, OR] − final[i−1, OR])^2 + (H[j, OR+1] − final[i−1, OR+1])^2)
      |   | H ← csort(H, OR + 2)
      |   | xyd ← stack(final, H)
      | xyd ← stack(xyd, xyd^⟨OR⟩)
      | return xyd
```

```
Area := | "Trapezoidal rule"
        | x ← nn^⟨OR⟩
        | y ← nn^⟨OR+1⟩
        | A ← 0
        | for i ∈ OR .. last(x) − 1
        |   | A ← A + (y[i+1] + y[i]) / 2 · (x[i+1] − x[i])
        | return A
```



Add a B-Spline

$x := nn^{\langle OR \rangle}$ $\quad y := nn^{\langle OR+1 \rangle}$

$t := 0..\text{last}(x) = \begin{bmatrix} 0 \\ 1 \\ \vdots \end{bmatrix}$

$n := 3$ $\quad N_{div} := 1000$ $\quad range := min(t), min(t) + \dfrac{\max(t) - min(t)}{N_{div}}..\max(t) = \begin{bmatrix} 0 \\ \vdots \end{bmatrix}$

Fit x vs t

$bx := \text{Spline2}(t, x, n)$

$spline1 := \text{Binterp}(range, bx)^{T}$

$f_{cubicx}(a) := \text{interp}\left(\text{cspline}\left(range, spline1^{\langle OR \rangle}\right), range, spline1^{\langle OR \rangle}, a\right)$

Fit y vs t

$by := \text{Spline2}(t, y, n)$

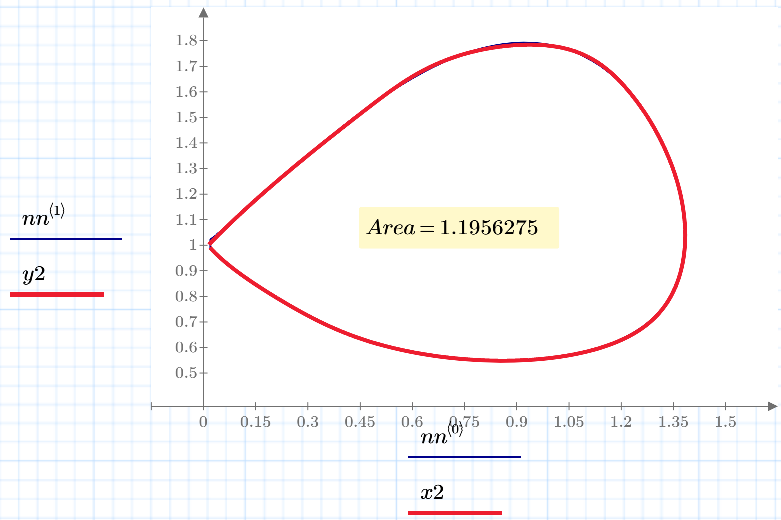
$spline2 := \text{Binterp}(range, by)^{T}$

$f_{cubicy}(a) := \text{interp}\left(\text{cspline}\left(range, spline2^{\langle OR \rangle}\right), range, spline2^{\langle OR \rangle}, a\right)$

$tt := 1, 1.1..\text{last}(x) + 1 = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

$x2 := f_{cubicx}(tt)$ $\quad y2 := f_{cubicy}(tt)$ $\quad \text{last}(tt) = 2.7 \cdot 10^{3}$

$Area :=$
"Trapezoidal rule"
$x \leftarrow x2$
$y \leftarrow y2$
$A \leftarrow 0$
for $i \in OR..\text{last}(x) - 1$
$\quad A \leftarrow A + \dfrac{y_{i+1} + y_i}{2} \cdot \left(x_{i+1} - x_i\right)$
return A



Monte Carlo

$N := 5000$ Total number of trials $x_{max} := 1.5$ $y_{max} := 2.0$

$k := 0..N-1$ $trials_k := k+1$

$x := \mathrm{runif}(N, 0, x_{max})$ $y := \mathrm{runif}(N, 0, y_{max})$

$$\lambda := \overrightarrow{\frac{\sqrt{\left(\cot(x)-\frac{y}{\sin(x)}\right)^2+1}}{2}+\frac{\sqrt{\left(\frac{\left(\cot(x)-\frac{y}{\sin(x)}\right)^2}{8}+\frac{1}{8}\right)^2\cdot\left(\frac{8\cdot\sin(x)}{y}+\frac{4\cdot\cot(x)-\frac{4\cdot y}{\sin(x)}}{\sqrt{\left(\cot(x)-\frac{y}{\sin(x)}\right)^2+1}}\right)^2+1}}{2}-1}$$

$hits := \lambda \le 0.4$

$$cumhits := \left\| \begin{array}{l} cum_0 \leftarrow hits_0 \\ \text{for } k \in 1..N-1 \\ \quad \| \; cum_k \leftarrow hits_k + cum_{k-1} \\ cum \end{array} \right.$$

$$AREA_k := \frac{cumhits_k}{trials_k}\cdot(x_{max}\cdot y_{max})$$

$AREA_{\mathrm{last}(AREA)} = 1.194$