quicksheets

Gradient, Divergence, and Curl

This QuickSheet illustrates how to define and evaluate the vector operators gradient, divergence, and curl.

Gradient

The gradient of a scalar-valued function

$$f(x) = f\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$$

is defined by

$$\nabla_x f(x) = \begin{pmatrix} \frac{\partial}{\partial x_0} f(x) \\ \frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x) \end{pmatrix}$$

You can compute the gradient using the Mathcad gradient operator. For example, suppose you define the following scalar-valued function:

$$f(x) := (x_0)^2 \cdot x_1 \cdot (x_2)^3$$

To insert the gradient operator, press **[Ctrl] [Shift] G:**

$$\nabla_{\blacksquare} \blacksquare$$

Type the variable vector x in the lower placeholder, and type the function f(x) in the upper placeholder.

$$\nabla_x f(x)$$

Finally, type **[Ctrl] [.]** to insert the symbolic equal sign and press **[Enter]**.

$$\nabla_x f(x) \rightarrow \begin{bmatrix} 2 \cdot x_0 \cdot x_1 \cdot (x_2)^3 \\ (x_0)^2 \cdot (x_2)^3 \\ 3 \cdot (x_0)^2 \cdot x_1 \cdot (x_2)^2 \end{bmatrix}$$

If you define the vector x numerically, the gradient is a numerical vector:

$$x := \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\nabla_x f(x) = \begin{pmatrix} -6 \\ -1 \\ 9 \end{pmatrix}$$

Divergence

The divergence of a vector-valued function

$$A(x,y,z) = (A(x,y,z)_0, A(x,y,z)_1, A(x,y,z)_2)$$

is defined by

$$\mathrm{div}(A,x,y,z) := \frac{d}{dx}A(x,y,z)_0 + \frac{d}{dy}A(x,y,z)_1 + \frac{d}{dz}A(x,y,z)_2$$

This can also be written as

$$\text{div} = \nabla \cdot A$$

where $\nabla = \left(\frac{d}{dx} \quad \frac{d}{dy} \quad \frac{d}{dz} \right)$ is the "del" operator.

For example, if you define

$$x := x$$

$$A(x,y,z) := \begin{pmatrix} x \cdot z \\ -y^2 \\ 2 \cdot x^2 \cdot y \end{pmatrix}$$

you can evaluate the divergence symbolically by:

$$\text{div}(A,x,y,z) \rightarrow z - 2 \cdot y$$

After substituting values for x, y, and z, you can evaluate the divergence numerically:

$$\text{div}(A,1,1,1) = -1$$

Curl

The curl of a vector-valued function A is defined by

$$\text{curl}(A,x,y,z) := \begin{pmatrix} \frac{d}{dy} A(x,y,z)_2 - \frac{d}{dz} A(x,y,z)_1 \\ \frac{d}{dz} A(x,y,z)_0 - \frac{d}{dx} A(x,y,z)_2 \\ \frac{d}{dx} A(x,y,z)_1 - \frac{d}{dy} A(x,y,z)_0 \end{pmatrix}$$

In terms of the del operator, this can be written as

$$\text{curl} = \nabla \times A$$

To symbolically evaluate the curl of the function A defined previously, use the symbolic equal sign:

$$\mathrm{curl}(A,x,y,z) \rightarrow \begin{pmatrix} 2\cdot x^2 \\ x-4\cdot x\cdot y \\ 0 \end{pmatrix}$$

Or, you can define values for x, y, and z, and evaluate the curl numerically:

$$\mathrm{curl}(A,1,1,1) = \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix}$$

Here are some more examples:

$$f(x,y,z) := x^2\cdot y\cdot z^3$$

$$B(x,y,z) := f(x,y,z)\cdot A(x,y,z)$$

$$B(x,y,z) \rightarrow \begin{pmatrix} x^3\cdot y\cdot z^4 \\ -x^2\cdot y^3\cdot z^3 \\ 2\cdot x^4\cdot y^2\cdot z^3 \end{pmatrix}$$

$$\mathrm{div}(B,x,y,z) \rightarrow 6\cdot x^4\cdot y^2\cdot z^2 - 3\cdot x^2\cdot y^2\cdot z^3 + 3\cdot x^2\cdot y\cdot z^4$$

$$\mathrm{curl}(B,x,y,z) \rightarrow \begin{pmatrix} 4\cdot x^4\cdot y\cdot z^3 + 3\cdot x^2\cdot y^3\cdot z^2 \\ 4\cdot x^3\cdot y\cdot z^3 - 8\cdot x^3\cdot y^2\cdot z^3 \\ -x^3\cdot z^4 - 2\cdot x\cdot y^3\cdot z^3 \end{pmatrix}$$