Telescope Visual Limiting Magnitude

"How low can you go?"

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INTRODUCTION {1}

"What is the faintest object I can see in my telescope?" is a common question asked by amateur astronomers. The answer depends on many variables. For instance, is the object a star (point source) or a nebula or galaxy (non-point source)? How experienced is the observer and how good are his/her eyes? What are the seeing conditions? Does the telescope have a clear aperture (e.g. refractor) or does it have a central obstruction (e.g. Newtonian, Schmidt-Cassegrain, etc.)? Regardless, with all other things being equal, the visual limiting magnitude of a telescope is most directly related to the size (area) of the objective lens or mirror. Over the years, various professional and amateur astronomers have published simple empirical equations to estimate the visual limiting magnitude for point sources.

The purpose of this worksheet is to compare five of these equations. For the record, the best I have done with my Celestron CPC-800 SCT (8" diameter, f/10, Schmidt-Cassegrain telescope) is detecting magnitude 15.4 stars during three successive nights observing the dwarf planet Pluto (then M13.9) as it slowly traversed a star field in western Sagittarius in 2009. I accomplished this feat under pristine and steady skies at Courtright Reservoir (elevation 8200 feet) in the Sierra Nevada east of Fresno, California. This site is technically Bortle Class 2, but mountain weather sometimes makes it worse.

DATA

Scope & Eyepiece Combos	**Scope "A"**	**Scope "B"**	{2}
Name:	**Celestron CPC-800**	**Celestron C5+**	
Aperture:	$D_A := 203\text{mm}$	$D_B := 125\text{mm}$	
Focal length:	$F_A := 2032\text{mm}$	$F_B := 1250\text{mm}$	
Focal reducer:	$R_A := 0.63\text{times}$	$R_B := 0.63\text{times}$	
Total light transmission:	$T_A := 85\%$	$T_B := 85\%$	

Available eyepieces:

Table variables:
f = focal length of eyepiece
$AFOV$ = apparent field of view

$$\text{Name}_A := \begin{pmatrix} \text{"TV Delos"} \\ \text{"TV Ethos"} \\ \text{"Celestron Plossl"} \\ \text{"TV Panoptic"} \\ \text{"TMB Paragon"} \end{pmatrix} \qquad f_A := \begin{pmatrix} 8 \\ 13 \\ 17 \\ 27 \\ 40 \end{pmatrix} \text{mm} \qquad AFOV_A := \begin{pmatrix} 72 \\ 100 \\ 52 \\ 68 \\ 69 \end{pmatrix} \text{deg}$$

$$\text{Name}_B := \begin{pmatrix} \text{"TV Delos"} \\ \text{"Vixen Lanthanum"} \\ \text{"Celestron Plossl"} \\ \text{"Celestron Plossl"} \\ \text{"Celestron Plossl"} \end{pmatrix} \qquad f_B := \begin{pmatrix} 8 \\ 10 \\ 17 \\ 25 \\ 32 \end{pmatrix} \text{mm} \qquad AFOV_B := \begin{pmatrix} 72 \\ 50 \\ 52 \\ 52 \\ 52 \end{pmatrix} \text{deg}$$

Observer & Seeing Conditions

Diameter of "dark-adapted" eye pupil: $d := 6\text{mm}$ {3}

"Naked-eye" limiting magnitude: $M_{NE} := 7.3$ (mean value for Bortle Class 2) {4}

TELESCOPE & EYEPIECE CALCULATIONS

Focal Ratio: $FR(F,D) := \frac{F}{D}$ Reduced focal length: $F_R(F,R) := F \cdot R$

Magnification (power): $P(F,f) := \frac{F}{f}$ True field of view: $TFOV(AFOV,P) := \frac{AFOV}{P}$

Scope "A": $D_A = 203 \cdot \text{mm}$ $F_A = 2032 \cdot \text{mm}$ $FR_A := FR(F_A, D_A) = 10.01$

$P_A := P(F_A, f_A)$ $TFOV_A := TFOV(AFOV_A, P_A)$

$$f_A = \begin{pmatrix} 8 \\ 13 \\ 17 \\ 27 \\ 40 \end{pmatrix} \cdot \text{mm} \qquad P_A = \begin{pmatrix} 254 \\ 156 \\ 120 \\ 75 \\ 51 \end{pmatrix} \cdot \text{times} \qquad TFOV_A = \begin{pmatrix} 0.28 \\ 0.64 \\ 0.44 \\ 0.9 \\ 1.36 \end{pmatrix} \cdot \text{deg} \qquad TFOV_A = \begin{pmatrix} 17 \\ 38 \\ 26 \\ 54 \\ 81 \end{pmatrix} \cdot \text{arcmin}$$

Scope "A": (reduced) $D_A = 203 \cdot \text{mm}$ $F_{AR} := F_R(F_A, R_A) = 1280 \cdot \text{mm}$ $FR_{AR} := FR(F_{AR}, D_A) = 6.31$

$P_{AR} := P(F_{AR}, f_A)$ $TFOV_{AR} := TFOV(AFOV_A, P_{AR})$

$$f_A = \begin{pmatrix} 8 \\ 13 \\ 17 \\ 27 \\ 40 \end{pmatrix} \cdot \text{mm} \qquad P_{AR} = \begin{pmatrix} 160 \\ 98 \\ 75 \\ 47 \\ 32 \end{pmatrix} \cdot \text{times} \qquad TFOV_{AR} = \begin{pmatrix} 0.45 \\ 1.02 \\ 0.69 \\ 1.43 \\ 2.16 \end{pmatrix} \cdot \text{deg} \qquad TFOV_{AR} = \begin{pmatrix} 27 \\ 61 \\ 41 \\ 86 \\ 129 \end{pmatrix} \cdot \text{arcmin}$$

Scope "B": $D_B = 125 \cdot \text{mm}$ $F_B = 1250 \cdot \text{mm}$ $FR_B := FR(F_B, D_B) = 10$

$P_B := P(F_B, f_B)$ $TFOV_B := TFOV(AFOV_B, P_B)$

$$f_B = \begin{pmatrix} 8 \\ 10 \\ 17 \\ 25 \\ 32 \end{pmatrix} \cdot \text{mm} \qquad P_B = \begin{pmatrix} 156 \\ 125 \\ 74 \\ 50 \\ 39 \end{pmatrix} \cdot \text{times} \qquad TFOV_B = \begin{pmatrix} 0.46 \\ 0.4 \\ 0.71 \\ 1.04 \\ 1.33 \end{pmatrix} \cdot \text{deg} \qquad TFOV_B = \begin{pmatrix} 28 \\ 24 \\ 42 \\ 62 \\ 80 \end{pmatrix} \cdot \text{arcmin}$$

Scope "B":
(reduced)

$D_B = 125 \cdot mm$ $\quad F_{BR} := F_R(F_B, R_B) = 788 \cdot mm$ $\quad FR_{BR} := FR(F_{BR}, D_B) = 6.3$

$P_{BR} := P(F_{BR}, f_B)$ $\quad TFOV_{BR} := TFOV(AFOV_B, P_{BR})$

$$f_B = \begin{pmatrix} 8 \\ 10 \\ 17 \\ 25 \\ 32 \end{pmatrix} \cdot mm \qquad P_{BR} = \begin{pmatrix} 98 \\ 79 \\ 46 \\ 31 \\ 25 \end{pmatrix} \cdot times \qquad TFOV_{BR} = \begin{pmatrix} 0.73 \\ 0.63 \\ 1.12 \\ 1.65 \\ 2.11 \end{pmatrix} \cdot deg \qquad TFOV_{BR} = \begin{pmatrix} 44 \\ 38 \\ 67 \\ 99 \\ 127 \end{pmatrix} \cdot arcmin$$

VISUAL LIMITING MAGNITUDE FORMULAS {5}

From www.celestron.com, et al:

$$M_{L\_Celestron}(D) := 7.7 + 5.0 \cdot \log\left(\frac{D}{cm}\right)$$

From "Visual Astronomy for the Deep Sky," by Roger Clark:

$$M_{L\_Clark}(D) := 3.7 + 2.5 \cdot \log\left[\left(\frac{D}{mm}\right)^2\right]$$

From "The Observational Amateur Astronomer," by Sir Patrick Moore:

$$M_{L\_Moore}(D) := 9.5 + 5.0 \cdot \log\left(\frac{D}{in}\right)$$

From http://en.wikipedia.org/wiki/Limiting_magnitude:

"First approximation": $M_{L\_W1}(D, d, M_{NE}) := M_{NE} + 5.0 \cdot \log\left(\frac{D}{d}\right)$

Derived from Robert Hodart's Telescope Limiting Magnitude Calculator (http://www.cruxis.com/scope/limitingmagnitude.htm):

$$M_{L\_W2}(D, F, f, T, M_{NE}) := M_{NE} - 2.0 + 2.5 \cdot \log\left[\overrightarrow{\left(\frac{D}{mm} \cdot \frac{F}{f} \cdot T\right)}\right]$$

VISUAL LIMITING MAGNITUDE CALCULATIONS {6}

Scope "A", Aperture-only estimates:

$$M_{LA\_A} := \begin{pmatrix} M_{L\_Celestron}(D_A) \\ M_{L\_Clark}(D_A) \\ M_{L\_Moore}(D_A) \\ M_{L\_W1}(D_A, d, M_{NE}) \end{pmatrix} = \begin{pmatrix} 14.24 \\ 15.24 \\ 14.01 \\ 14.95 \end{pmatrix}$$

Scope "B", Aperture-only estimates:

$$M_{LA\_B} := \begin{pmatrix} M_{L\_Celestron}(D_B) \\ M_{L\_Clark}(D_B) \\ M_{L\_Moore}(D_B) \\ M_{L\_W1}(D_B, d, M_{NE}) \end{pmatrix} = \begin{pmatrix} 13.18 \\ 14.18 \\ 12.96 \\ 13.89 \end{pmatrix}$$

W2~Hodart's Formula (Scope "A", top; Scope "B", bottom):

$$f_A = \begin{pmatrix} 8 \\ 13 \\ 17 \\ 27 \\ 40 \end{pmatrix} \cdot \mathrm{mm} \quad M_{LH\_A} := M_{L\_W2}\left(D_A, F_A, f_A, T_A, M_{NE}\right) = \begin{pmatrix} 16.9 \\ 16.38 \\ 16.09 \\ 15.58 \\ 15.16 \end{pmatrix} \qquad M_{LH\_AR} := M_{L\_W2}\left(D_A, F_{AR}, f_A, T_A, M_{NE}\right) = \begin{pmatrix} 16.4 \\ 15.88 \\ 15.58 \\ 15.08 \\ 14.66 \end{pmatrix}$$

$$f_B = \begin{pmatrix} 8 \\ 10 \\ 17 \\ 25 \\ 32 \end{pmatrix} \cdot \mathrm{mm} \quad M_{LH\_B} := M_{L\_W2}\left(D_B, F_B, f_B, T_B, M_{NE}\right) = \begin{pmatrix} 15.85 \\ 15.61 \\ 15.03 \\ 14.61 \\ 14.35 \end{pmatrix} \qquad M_{LH\_BR} := M_{L\_W2}\left(D_B, F_{BR}, f_B, T_B, M_{NE}\right) = \begin{pmatrix} 15.35 \\ 15.11 \\ 14.53 \\ 14.11 \\ 13.84 \end{pmatrix}$$

My own experience is that the Celestron and Moore formulas are too conservative for the best conditions but are probably fine for Bortle Class 3 or maybe Class 4 skies. Clark and W1 are probably about right for Bortle Class 2 skies, with Clark requiring perfect steadiness and transparency. I have not had a chance yet to test Hodart's Formula under varying sky conditions, but I suspect it's a bit optimistic.

PLOT EQUATIONS (NEW DATA)

{7}

The Celestron, Clark, and Moore equations require only two variables (D and M_L), while W1 and W2 require additional data as shown below. This data can be modified as required. In the following table, the diameter and focal ratio (FR) are used to calculate the focal length for the W2 equation. Equations W1 and W2 are linearly sensitive to the "naked-eye" limiting magnitude.

Apertures for first four formulas: $D_p := 2\mathrm{in}, 4\mathrm{in} .. 32\mathrm{in}$

Additional data for W1:

Diameter of "dark-adapted" eye pupil: $d_p := 6\mathrm{mm}$

"Naked-eye" limiting magnitude: $M_{NEp} := 7.3$ (mean value for Bortle Class 2)

Additional data for W2:

Focal lengths of eyepieces: $f_{p1} := 8\mathrm{mm}$ $f_{p2} := 17\mathrm{mm}$ $f_{p3} := 27\mathrm{mm}$

Refractors:

$$D_{pr} := \begin{pmatrix} 50 \\ 75 \\ 100 \\ 125 \\ 150 \end{pmatrix} \mathrm{mm} \qquad FR_{pr} := \begin{pmatrix} 8 \\ 6.5 \\ 6.5 \\ 6.5 \\ 6.5 \end{pmatrix} \qquad T_{pr} := \begin{pmatrix} 85 \\ 85 \\ 85 \\ 85 \\ 85 \end{pmatrix} \%$$

Based on various manufacturers and models.

Schmidt-Cassegrains:

$$D_{ps} := \begin{pmatrix} 5 \\ 6 \\ 8 \\ 9.25 \\ 11 \\ 14 \end{pmatrix} \text{in} \qquad FR_{ps} := \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 11 \end{pmatrix} \qquad T_{ps} := \begin{pmatrix} 85 \\ 85 \\ 85 \\ 85 \\ 85 \\ 85 \end{pmatrix} \%$$

Based on Celestron (f/11 is correct for the 14").

Newtonians:

$$D_{pn} := \begin{pmatrix} 4.5 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \\ 25 \\ 28 \\ 30 \\ 32 \end{pmatrix} \text{in} \qquad FR_{pn} := \begin{pmatrix} 7.9 \\ 8 \\ 5.9 \\ 4.7 \\ 4.9 \\ 4.6 \\ 4.4 \\ 4.2 \\ 4 \\ 4 \\ 3.6 \\ 3.6 \\ 3.6 \end{pmatrix} \qquad T_{pn} := \begin{pmatrix} 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \\ 88 \end{pmatrix} \%$$

Based on Orion Telescopes (4.5"16" SkyQuest), Obsession Telescope (18"25" Classics) and Webster Telescopes (28"32").

$$M_{L\_W2\_Ref\_fp1} := M_{L\_W2}\left[D_{pr}, \overrightarrow{\left(D_{pr}\cdot FR_{pr}\right)}, f_{p1}, T_{pr}, M_{NEp}\right]$$

$$M_{L\_W2\_Ref\_fp2} := M_{L\_W2}\left[D_{pr}, \overrightarrow{\left(D_{pr}\cdot FR_{pr}\right)}, f_{p2}, T_{pr}, M_{NEp}\right]$$

$$M_{L\_W2\_Ref\_fp3} := M_{L\_W2}\left[D_{pr}, \overrightarrow{\left(D_{pr}\cdot FR_{pr}\right)}, f_{p3}, T_{pr}, M_{NEp}\right]$$

$$M_{L\_W2\_SCT\_fp1} := M_{L\_W2}\left[D_{ps}, \overrightarrow{\left(D_{ps}\cdot FR_{ps}\right)}, f_{p1}, T_{ps}, M_{NEp}\right]$$

$$M_{L\_W2\_SCT\_fp2} := M_{L\_W2}\left[D_{ps}, \overrightarrow{\left(D_{ps}\cdot FR_{ps}\right)}, f_{p2}, T_{ps}, M_{NEp}\right]$$

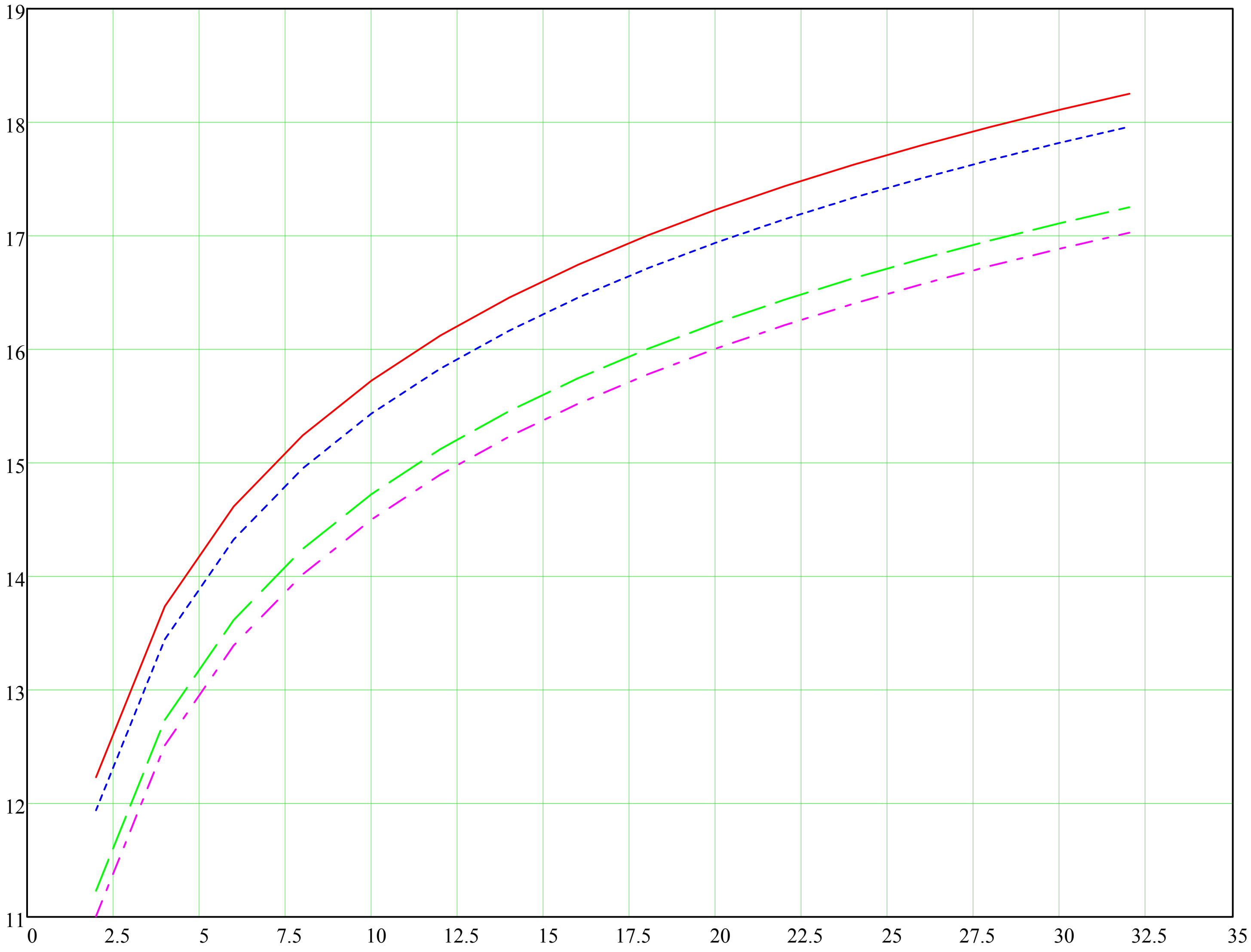
$$M_{L\_W2\_SCT\_fp3} := M_{L\_W2}\left[D_{ps}, \overrightarrow{\left(D_{ps}\cdot FR_{ps}\right)}, f_{p3}, T_{ps}, M_{NEp}\right]$$

$$M_{L\_W2\_Newt\_fp1} := M_{L\_W2}\left[D_{pn}, \overrightarrow{\left(D_{pn}\cdot FR_{pn}\right)}, f_{p1}, T_{pn}, M_{NEp}\right]$$

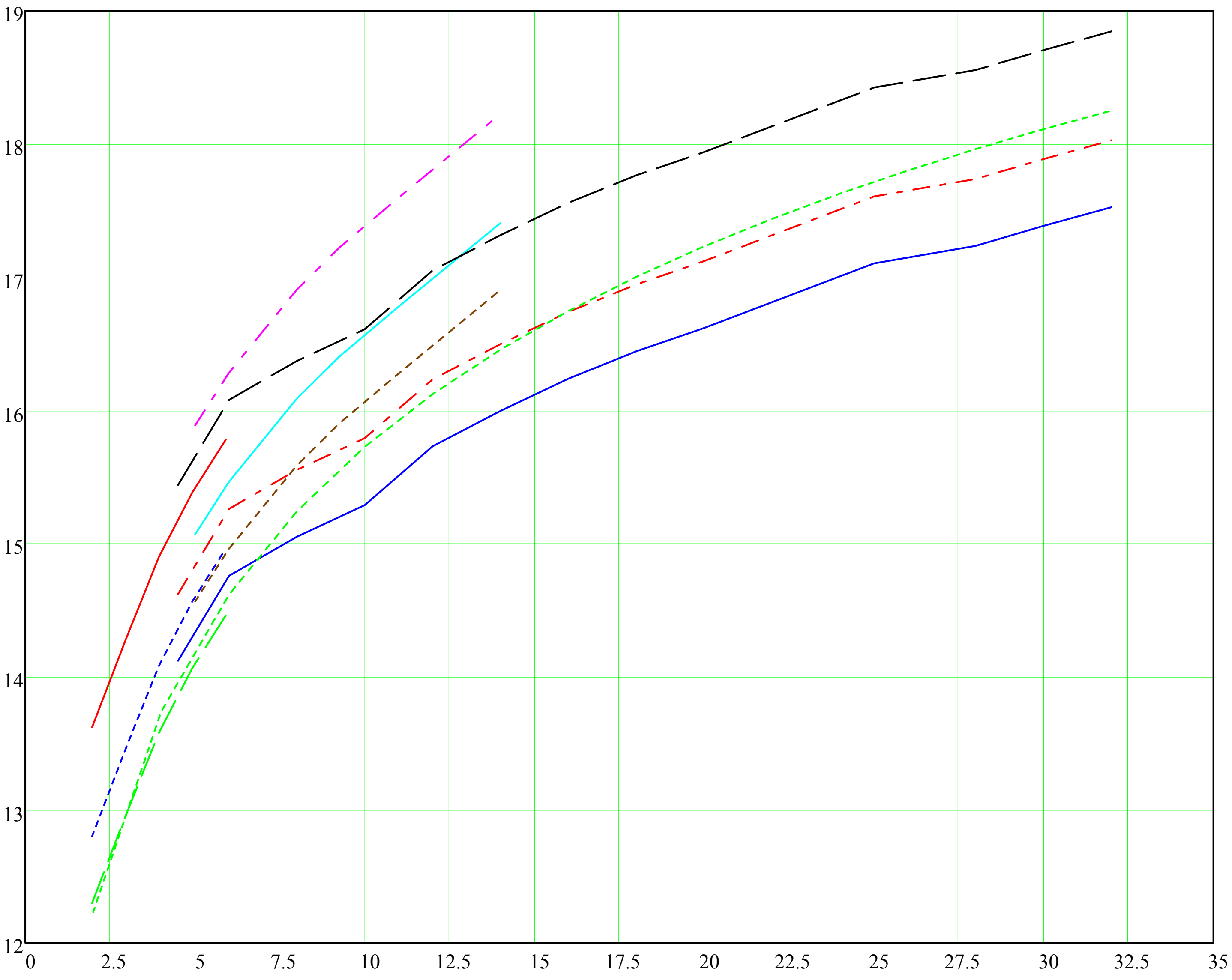
$M_{L\_W2\_Newt\_fp2} := M_{L\_W2}\left[D_{pn}, \overrightarrow{\left(D_{pn} \cdot FR_{pn}\right)}, f_{p2}, T_{pn}, M_{NEp}\right]$

$M_{L\_W2\_Newt\_fp3} := M_{L\_W2}\left[D_{pn}, \overrightarrow{\left(D_{pn} \cdot FR_{pn}\right)}, f_{p3}, T_{pn}, M_{NEp}\right]$

Plot the first four formulas



Plot Hodart (W2) Formula



NOTES

{1} [a] Non-astronomers usually ask how far we can see, not how faint we can see. [b] Because its light is not spread out, a point source is easier to see than a non-point source with the same visual magnitude. [c] All other things being equal, a skilled observer can see much deeper than a "newbie" observer. In my experience, this can amount to a difference of at least two magnitudes. Similarly, a younger observer will generally have better eyes than an older observer. [d] Telescopes with central obstructions (i.e. secondary mirror assemblies) lose a little bit of limiting magnitude due to slightly reduced collecting area and due to reduced contrast from diffraction effects. On the other hand, 8" Newtonians and 8" SCTs are probably the most common amateur telescopes in the world, while few amateurs have refractors larger than 4". In the game of visual limiting magnitude, aperture rules. [e] About ten years ago I replaced the 1.25" Celestron prism diagonal on my Celestron C5+ SCT with a 1.25" TeleVue Everbright mirror diagonal. This change gained 1.0 (±) magnitude for visual observing. [f] I also brought the 5" SCT that same weekend in 2009, but I never turned it toward Pluto for a comparison because it was busy as a camera platform for widefield astrophotography.

{2} The data tables compare my two primary telescopes with the five eyepieces I usually use with each one. These tables can be expanded or contracted as needed. Light transmission is estimated based on various sources. Both of my telescopes have Celestron's StarBright coatings and high-quality mirror diagonals. Even so, I doubt that total light transmission through the entire optical train much exceeds 85%. The eyepieces listed for each telescope are not the same because I have "assigned" certain eyepieces exclusively to one telescope or the other. For example, the 27-mm TeleVue Panoptic and 40-mm TMB that I use with the 8" SCT both have 2" barrels and cannot be used with the 1.25" diagonal on the 5" SCT, so I use 25-mm and 32-mm Plossls with the 5" SCT. The other eyepiece all have 1.25" barrels and can used with either telescope, although I usually keep a 2" adapter on the 13-mm Ethos for use with the 8" SCT. My Pluto observations were made with the excellent 10-mm Vixen

Lanthanum and 17-mm Celestron Plossl eyepieces, butfor this worksheetI didn't include the 10-mm eyepiece with the 8" SCT because it has been "retired" to the 5" SCT in favor of the 13-mm TeleVue Ethos and 8-mm TeleVue Delos eyepieces, both of which are even better. I still use the 17-mm Plossl with the 8" SCT because it is a surprisingly excellent eyepiece for being only about $50 (it's better than the other four Celestron Plossls that I own). Finally, if you don't use a focal reducer, you can eliminate it and the subsequent calculations that use it, or you can change it to a Barlow to see the effect of increased magnification. I have a 2x Barlow, but I don't use it very often. When I use the Barlow, it is almost exlusively with with the 5" SCT and a 6" Newtonian that I didn't include in this worksheet.

{3} The pupil diameter for dark adapted eyes is mostly age dependent. Teenagers can exceed 7 mm, while senior citizens may be limited to less than 5 mm. I picked a middle value of 6 mm.

{4} If you don't have a sky quality meter (**http://www.unihedron.com/projects/darksky/**) or app (**http://www.darkskymeter.com/**), the easiest way to estimate the maximum "naked-eye" limiting magnitude for a particular observing site is to use the Bortle Scale (**http://en.wikipedia.org/wiki/Bortle_scale** or the table on the next page, including suggested mean values) and a light pollution map. Unfortunately, sky conditions are rarely at their best, so a more conservative number based on experience should be used for observation planning. Light pollution maps can be found at various sites on the Internet, including **http://www.lightpollution.it/worldatlas/pages/fig1.htm** and **http://www.jshine.net/astronomy/dark_sky/**. I start with Attilla Danko's Clear Sky Charts page (**http://cleardarksky.com/csk/**), which predicts astronomical seeing conditions up to 48 hours in advance for nearly 4,700 North American observing sites. Danko's website formats the light pollution maps to put the observing site in question in the exact center of the map. Here is the map for Courtright Reservoir, from where I observed Pluto: **http://cleardarksky.com/lp/CrtghRsvCAlp.html?Mn=cameras**.

NOTES (cont.)

Bortle Scale for Night Sky Brightness

$$\mathrm{Class} := \begin{pmatrix}1\\2\\3\\4\\5\\6\\7\\8\\9\end{pmatrix} \qquad M_{NE\_low} := \begin{pmatrix}7.6\\7.1\\6.6\\6.1\\5.5\\5.1\\4.6\\4.1\\4\end{pmatrix} \qquad M_{NE\_high} := \begin{pmatrix}8\\7.5\\7\\6.5\\6\\5.5\\5\\4.5\\4\end{pmatrix}$$

$$i := \mathrm{Class}_1 .. \mathrm{last}(\mathrm{Class})$$

Suggested Mean Values for M_{NE}

$$\mathrm{mean}\left(M_{NE\_low_i}, M_{NE\_high_i}\right) =$$

7.8
7.3
6.8
6.3
5.75
5.3
4.8
4.3
4

Table variables:
Class = Bortle Class; M_{NE_low} & M_{NE_high} = Range of "naked-eye" limiting magnitude.

{5} The empirical equation provided by Celestron is the one I see used most often. The two Wikipedia formulas incorporate additional relevant variables. Hodart's Formula (W2) is probably the most accurate for an experienced observer under ideal conditions of steadiness and transparency. To simplify things overall, I modified Hodart's equation slightly by replacing the magnification or power (P) with the calculation for magnification ($\frac{F}{f}$). P is separately calculated for each telescope-eyepiece combination in {2} for informational purposes, but it is not used in subsequent calculations. The Wikipedia article has a good discussion about this whole topic as well as several useful links.

{6} Clark's and Hodart's Formulas come closest to my Pluto observations. The other three formulas are much more conservative, but are probably better for observation planning, since the skies are rarely as good as they could be.

{7} The range of telescope diameters in this worksheet is based on the following. I use 50-mm (2-in) finder scopes on both of my SCTs and the largest telescopes I have looked through are 24- and 25-inch Newtonians. I expanded the range a bit beyond these largest scopes. The telescope list can be modified as desired. I used Celestron's suite of SCTs (5"14") rather than Meade's (8"16") because I like the Celestron scopes better. As the W2 curves shows, longer telescope focal lengths and/or shorter eyepiece focal lengths (which together produce higher magnifications) result in higher visual limiting magnitudes. This is because higher magnification darkens the background sky and improves contrast.

GLOBAL DEFINITIONS

$\mathrm{ORIGIN} \equiv 1$ $\quad$ $\mathrm{times} \equiv 1$