

WAVEFORMS SPECTRA

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INTRODUCTION

The subscript "gd" is the acronym of "Global Data.xmcd"
The subscript "fs" is the acronym of "Fourier series.xmcd"
The subscript "s" is the acronym of "Signal List.xmcd"
The subscript "dp" is the acronym of "Dirac Pulse - formulae.xmcd"

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FILES REFERENCES

- Reference:E:\Pulses And Waveforms\Pulses and Waveforms formulae.xmcd(R)
- Reference:E:\Pulses And Waveforms\Pulse Train Data.xmcd(R)
- Reference:E:\Pulses And Waveforms\staircase pulse data.xmcd(R)
- Reference:E:\Pulses And Waveforms\staircase 2 pulse data.xmcd(R)
- Reference:E:\Pulses And Waveforms\staircase 3 pulse data.xmcd(R)
- Reference:E:\Pulses And Waveforms\staircase 4 pulse data.xmcd(R)
- Reference:E:\Pulses And Waveforms\sawtooth pulse data.xmcd(R)
- Reference:E:\Pulses And Waveforms\FM data.xmcd(R)
- Reference:E:\Pulses And Waveforms\PM data.xmcd(R)
- Reference:E:\Pulses And Waveforms\Signal's Bandwidth Calculation.xmcd(R)

PERIODIC WAVEFORMS' FREQUENCY SPECTRA

TEST Waveforms

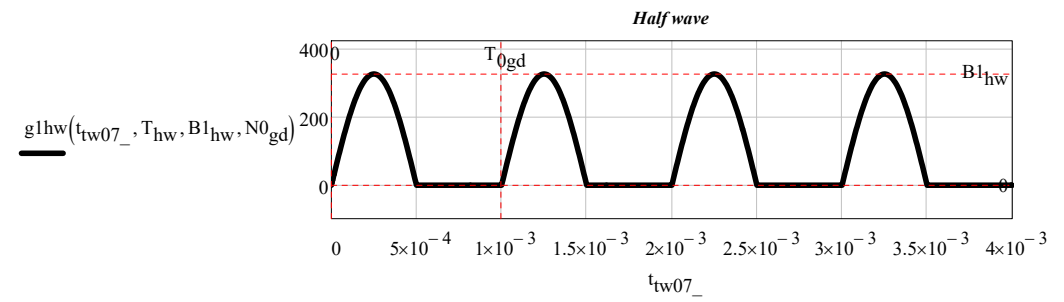
Periodic Waveforms Periodic Waveforms Periodic Waveforms

1) Half wave

Data file "general data.xmcd"

Amplitude: $B1_{hw} := 230 \cdot \sqrt{2} \cdot V$

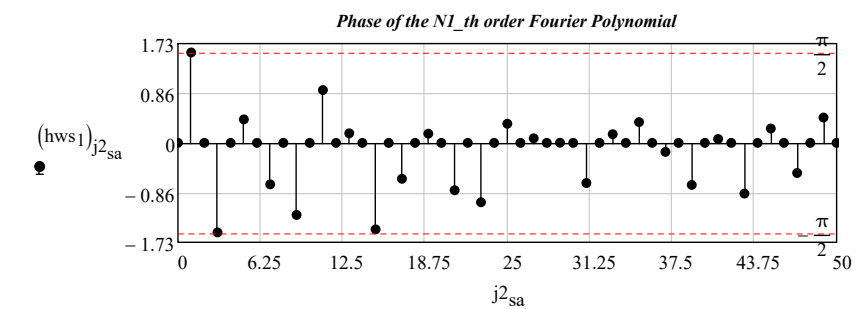
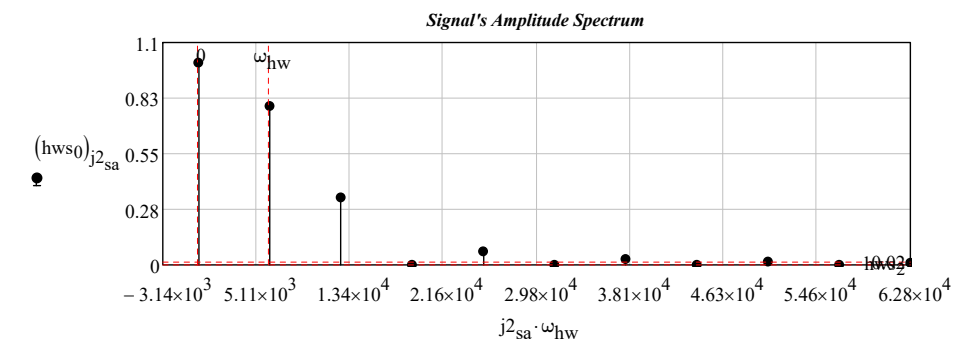
$$f_{hw} := \frac{1}{T_{0gd}} \quad T_{hw} := T_{0gd} \quad \text{Angular frequency:} \quad \omega_{hw} := \frac{2 \cdot \pi}{T_{0gd}} \quad T_{hw} = 1 \cdot \text{ms}$$



$$V_{hw}(t) := g1hw(t, T_{hw}, B1_{hw}, N0_{gd}) \quad B1_{hw} = 325.269 \text{ V}$$

$$hws := SPCT(V_{hw}, rt_{gd}, N1_, 0 \cdot \text{sec}, T_{hw}) \quad N1_ = 50$$

$$j2_{sa} := 0..rows(hws0) - 1$$



$$Bw_{sa} := hws3 \cdot \text{Hz} \quad Bw_{sa} = 0.019 \cdot \text{MHz}$$

$$\text{sampling frequency:} \quad fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.038 \cdot \text{MHz}$$

$$\text{relerr} := hws7 \quad \text{relerr} = 10 \cdot \%$$

$$k := 0..2^8 - 1$$

$$nptk := \frac{k}{fpt_{so}}$$

INTRODUCTION

This worksheet is a collection of some common (and not), signals used in electronics. It deals with the harmonic analysis of periodic signals satisfying the Dirichlet conditions. For each one first, it is plotted a graph, then is calculated its bandwidth in order to do a correct sampling of it. The Fourier harmonics and phase, are plotted in two graphs. Then, the sampled signal is rebuilt with the Shannon interpolation formula. Given the sampled signal, the fft function is applied and plotted to compare the result (first 18 functions only). The previous procedure is repeated for each signal (43). The numerical results are compared with the analytical one only for example 3 and 18. In some cases, calculating the signal bandwidth takes a long time.

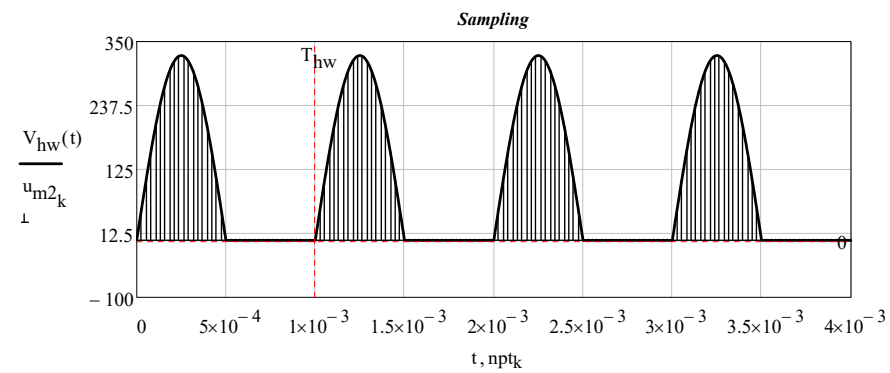
To simplify the realization of this file (this worksheet) and to make more agile and immediate viewing of signal's graphics, the data, for some signal, are defined in other worksheets as listed in the references just above this introduction. Hence, to change the data, one must open the relative file and modify them. Then one should save and close both files (relative file data and this one) to store the data variations done, then one can reopen "Waveforms & Spectra".

Units: Mrads and Grads stand for $10^6 \cdot \text{rad}$ and $10^9 \cdot \text{rad}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{hw}} = 6.737$

Signal sampling: $u_{m2_k} := V_{hw}(npt_k)$

$u_{m2}^T =$	0	1	2	3	4	...
	0	53.538	105.615	154.811		

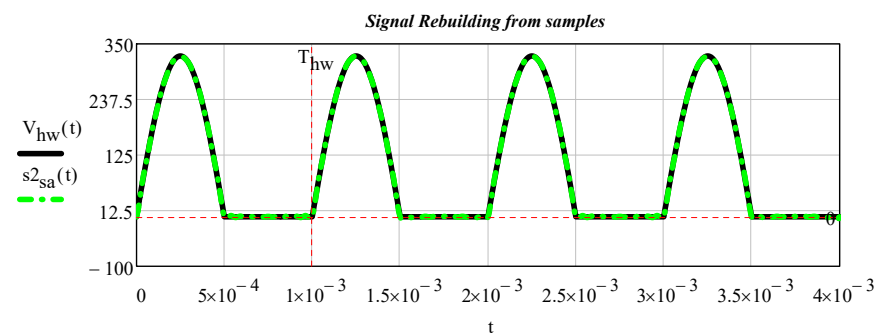


relerr = 10-% $\omega_{bw} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bw} = 0.119 \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bw}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s2_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m2_n} \cdot \text{sinc}(\omega_{bw} \cdot t - n \cdot \pi)) \right]$ $N0_{gd} - 1 = 255$ $u_{m2_{12}} = 297.873$
 rows(u_{m2}) = 256

relerr = 10-%

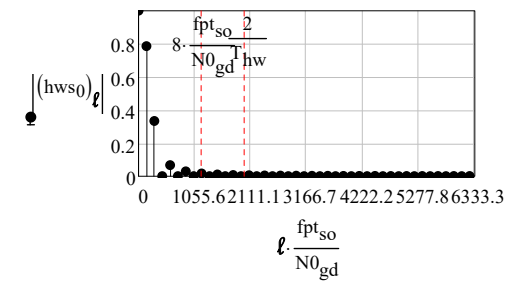
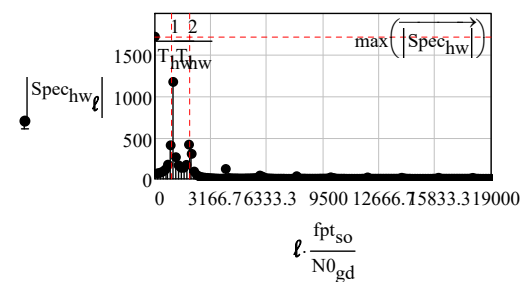


length(u_{m2}) = 256

$fpt_{so} = 38 \text{ kHz}$

$Spec_{hw} := \text{fft}(u_{m2})$ length($Spec_{hw}$) = 129

$l := 0 .. \frac{N0_{gd}}{2} - 1$ $\frac{N0_{gd}}{2} = 128$



TEST Waveforms

Periodic Waveforms

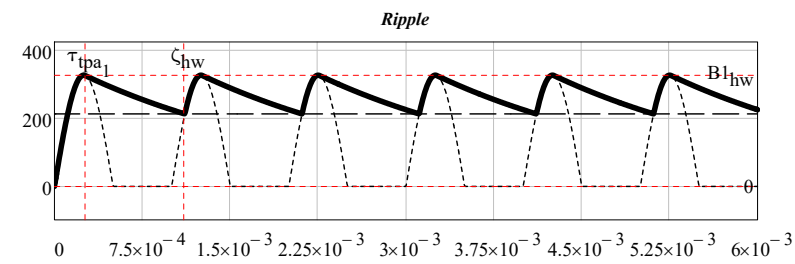
2 Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{hw} = 325.269 \cdot V$,
 Amplitude of the decreasing exponential for $t=0$: V_{pp} ,
 Exponential Time constant: $\tau_{hw1} := 2 \cdot T_{0gd}$
 Period: $T_{hw} = 1 \times 10^3 \cdot \mu s$,
 Pulsation: $\omega_{hw} := \frac{2 \cdot \pi}{T_{hw}} = 6.283 \cdot \frac{\text{krads}}{\text{sec}}$,
 Intersection abscissa between half wave and exponential: ζ (scalar),
 Tangent points abscissas between half wave and exponential: τ_{tpa} (vector)

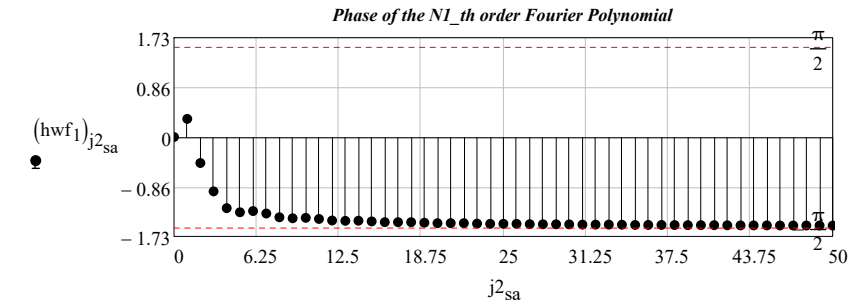
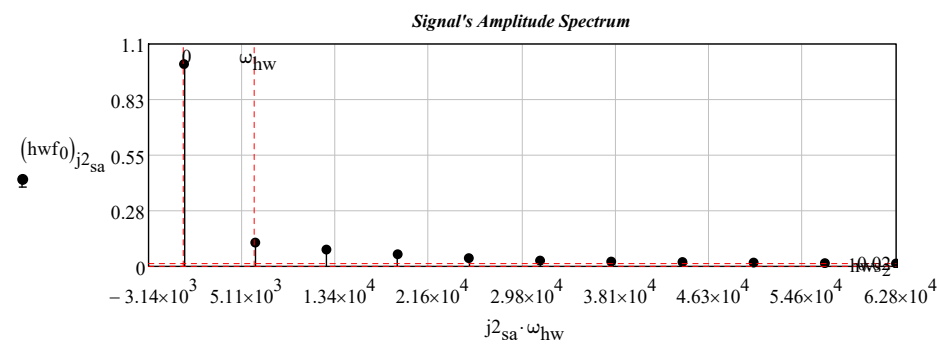
$$\tau_{tpa_{k_{sl}}} := \frac{\text{atan}(-\omega_{hw} \cdot \tau_{hw1}) + k_{sl} \cdot \pi}{\omega_{hw}}$$

$$V_{tpv} := B1_{hw} \cdot \sin(\omega_{hw} \cdot \tau_{tpa_1}) \cdot e^{-\frac{\tau_{tpa_1}}{\tau_{hw1}}} \quad V_{tpv} = 369.746 \text{ V}$$

$$\zeta_{hw} := Z01(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{tpv})$$



$B1_{hw} = 325.269 \text{ V}$ $V_{hwf}(t) := g2hw(t, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}, N0_{gd})$
 $hwf := \text{SPCT}(V_{hwf}, \tau_{gd}, N1_, 0 \cdot \text{sec}, T_{0gd})$ $N1_ = 50$
 $j2_{sa} := 0 \dots \text{rows}(hws0) - 1$

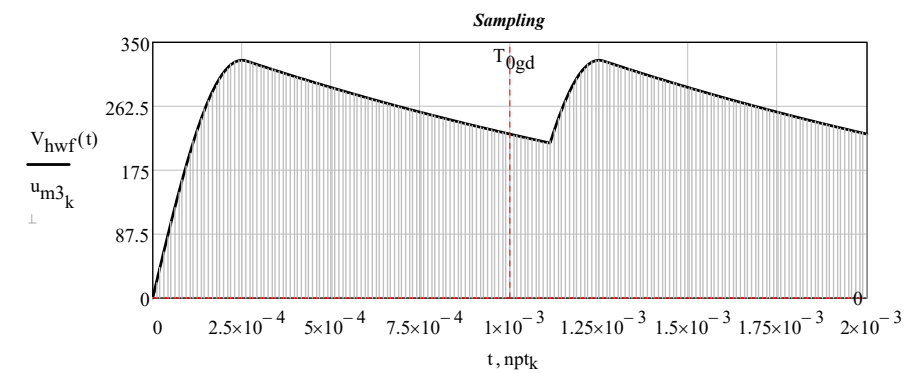


$Bw_{sa} := hwf_3 \cdot \text{Hz}$
 $Bw_{sa} = 0.048 \cdot \text{MHz}$
 sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.096 \cdot \text{MHz}$

$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

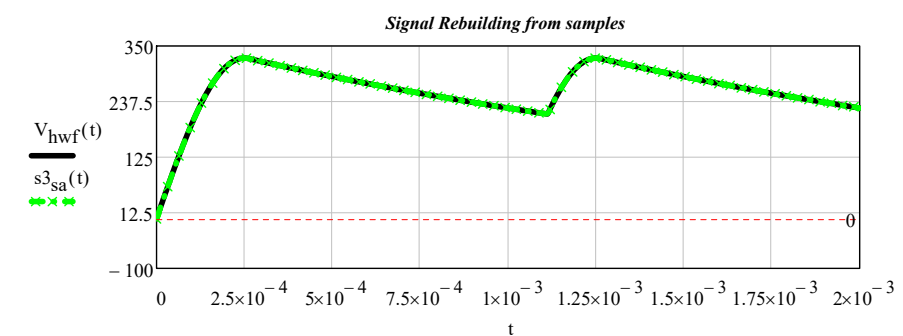
$$u_{m3_k} := V_{hwf}(npt_k)$$

$$u_{m3}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \begin{matrix} 0 \\ \vdots \end{matrix} & 0 & 21.274 & 42.456 & 63.457 & 84.186 & 104.554 & 124.475 & 143.863 & 162.635 & \dots \end{matrix}$$


relerr = 10% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.302 \cdot \frac{\text{Mrads}}{\text{sec}}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s3_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m3_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10%



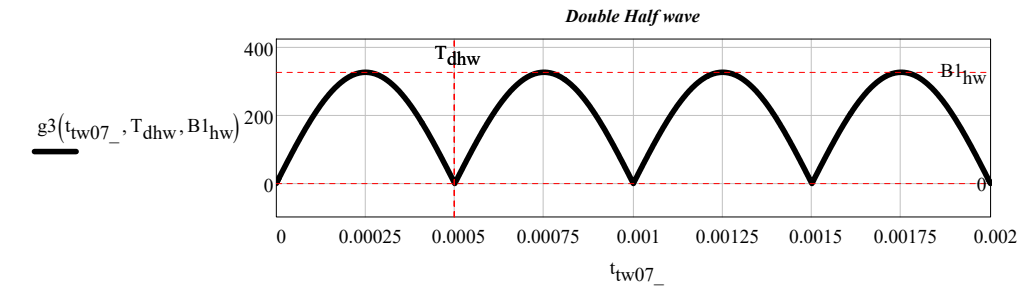
Symbol frequency:

TEST Waveforms

Periodic Waveforms

3 Double Half wave

$$T_{dhw} := \frac{T_{hw}}{2} \quad \omega_{dhw} := \frac{\pi}{T_{dhw}} \quad g_3(t_{sl}, T_{dhw}, B1_{hw}) := \frac{B1_{hw}}{V} \cdot \left| \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t_{sl}\right) \right|$$



Dirichlet conditions

A periodic function $s(t)=s(t+T)$, can be expressed by the Fourier series provided that (Dirichlet conditions):

- (1) it is discontinuous and presents a finite number of discontinuities in the period T ;
- (2) has a limited average value in the period T ;
- (3) it has a finite number of maximum positive or negative.

If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

- 1) signals of energy for which holds: $\int_{-\infty}^{\infty} (|s_{fs}(t)|)^2 dt < \infty$,
- 2) power signals for which holds: $\lim_{T \rightarrow \infty} \left[\frac{1}{T} \int_{-T}^T (|s_{fs}(t)|)^2 dt \right] < \infty$

Fourier series definition

$$s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(\omega \cdot k \cdot t) + b_k \cdot \sin(\omega \cdot k \cdot t))$$

The coefficients are defined as follows:

$$\frac{a_0}{2} = A_{fs} = \frac{1}{T} \int_{t_0}^{t_0+T} s_{fs}(t) dt$$

$$a_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t) dt$$

$$b_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t) dt$$

$$B1_{hw} = 325.269 V \quad s_{fs}(t) := \frac{B1_{hw}}{V} \cdot \left| \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) \right|$$

$$T_{dhw} := T_{dhw} \quad t := t \quad T_{hw} := T_{hw}$$

$$\frac{a_0}{2} = A_{fs} = \frac{2}{T_{hw}} \cdot \frac{B1_{hw}}{V} \cdot \int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) dt = \frac{2 \cdot B1_{hw}}{\pi \cdot V}$$

$$a_k = \frac{4}{T_{hw}} \cdot \frac{B1_{hw}}{V} \int_0^{T_{hw}} \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) \cdot \cos\left(\frac{2 \cdot \pi}{T_{hw}} \cdot k \cdot t\right) dt = \frac{2 \cdot B1_{hw} \cdot (\cos(\pi \cdot k) + 1)}{-\pi \cdot V \cdot (k^2 - 1)}$$

$$b_k = \frac{4}{T_{hw}} \cdot \frac{B1_{hw}}{V} \int_0^{T_{hw}} \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) \cdot \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot k \cdot t\right) dt = \frac{2 \cdot B1_{hw} \cdot \sin(\pi \cdot k)}{\pi \cdot V \cdot (k^2 - 1)}$$

$$s_{fs}(t) = \frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \sum_{k=1}^{\infty} \left[\frac{(\cos(\pi \cdot k) + 1)}{-(k^2 - 1)} \cos(\omega \cdot k \cdot t) + \frac{\sin(\pi \cdot k)}{(k^2 - 1)} \cdot \sin(\omega \cdot k \cdot t) \right] \right] \cos[k \cdot (\pi + \omega \cdot t)] = (-1)^k \cdot \cos(k \cdot \omega \cdot t)$$

$$\frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \sum_{k=1}^{\infty} \frac{\cos(\omega \cdot k \cdot t) + \cos[k \cdot (\pi + \omega \cdot t)]}{1 - k^2} \right] = \frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \sum_{k=1}^{\infty} \frac{\cos(\omega \cdot k \cdot t) + (-1)^k \cdot \cos(k \cdot \omega \cdot t)}{1 - k^2} \right]$$

$$\frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \sum_{k=1}^{\infty} \frac{\cos(\omega \cdot k \cdot t) + (-1)^k \cdot \cos(k \cdot \omega \cdot t)}{1 - k^2} \right] = \frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \frac{\pi \cdot \cos(\omega \cdot t) \cdot i}{2} + \sum_{k=2}^{\infty} \frac{[1 + (-1)^k] \cdot \cos(k \cdot \omega \cdot t)}{1 - k^2} \right]$$

$$\lim_{k \rightarrow 1^+} \frac{[1 + (-1)^k] \cdot \cos(k \cdot \omega \cdot t)}{1 - k^2} \rightarrow \frac{\pi \cdot \cos(\omega \cdot t) \cdot i}{2} \quad \lim_{k \rightarrow 1^-} \frac{[1 + (-1)^k] \cdot \cos(k \cdot \omega \cdot t)}{1 - k^2} \rightarrow \frac{\pi \cdot \cos(\omega \cdot t) \cdot i}{2}$$

$$s_{dhw}(t) = \frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \frac{\pi \cdot \cos\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) \cdot i}{2} + \sum_{k=2}^{\infty} \frac{[1 + (-1)^k] \cdot \cos\left(k \cdot \frac{2 \cdot \pi}{T_{hw}} \cdot t\right)}{1 - k^2} \right]$$

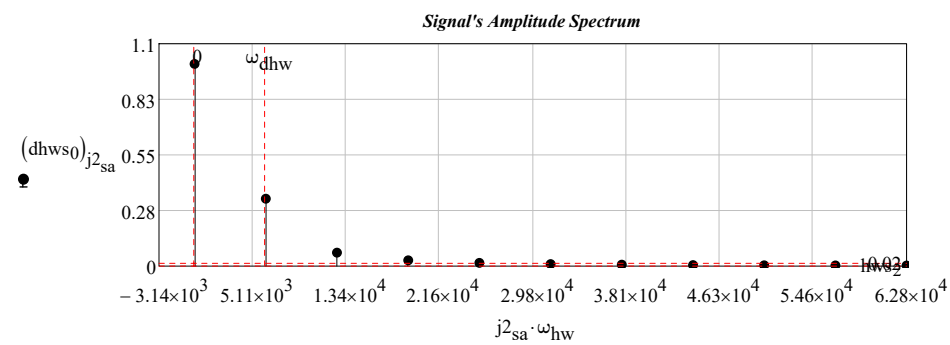
$$s_{dhw}(t) := \frac{2 \cdot B1_{hw}}{\pi \cdot V} \left[1 + \frac{\pi \cdot \cos\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right) \cdot i}{2} + \sum_{k=2}^{100} \frac{[1 + (-1)^k] \cdot \cos\left(k \cdot \frac{2 \cdot \pi}{T_{hw}} \cdot t\right)}{1 - k^2} \right]$$

$$s_{dhw}(0) = 2.05 + 325.269i \quad |s_{dhw}(0)| = 325.276 \quad s_{dhw}\left(\frac{T_{dhw}}{2}\right) = 325.249$$

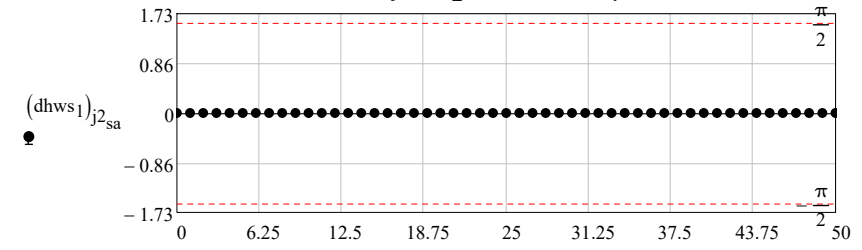
$$B1_{hw} = 325.269 V \quad V_{dhw}(t) := g3(t, T_{hw}, B1_{hw}) \quad \omega_{sa} := \omega_{dhw} \quad 2 \cdot \frac{B1_{hw}}{\pi \cdot V} = 207.073$$

$$dhw_s := SPCT(V_{dhw}, rt_{gd}, N1_, 0 \cdot sec, T_{dhw}) \quad N1_ = 50$$

$$j2_{sa} := 0 \dots rows(hws0) - 1$$



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := dhw_s \cdot Hz$$

$$Bw_{sa} = 0.03 \cdot MHz$$

$$\text{sampling frequency: } fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.06 \cdot MHz$$

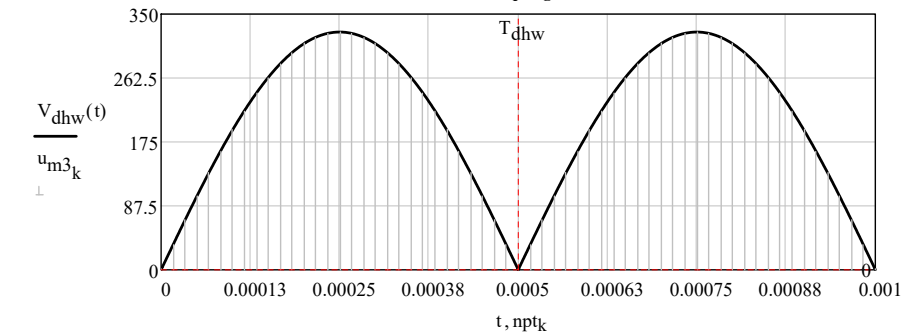
$$k := 0 \dots 2^8 - 1 \quad npt_k := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{dhw}} = 8.533$$

$$u_{m3}_k := V_{dhw}(npt_k)$$

$u_{m3}^T =$	0	1	2	3	4	
	0	34	67.627	100.514	...	

Sampling



$$relerr = 10 \cdot \%$$

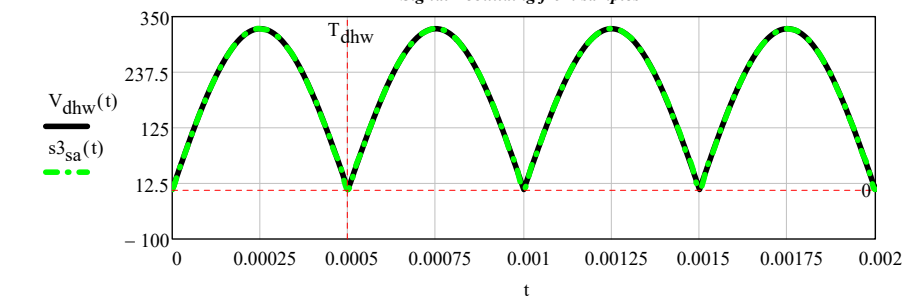
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.188 \frac{Mrads}{sec}$$

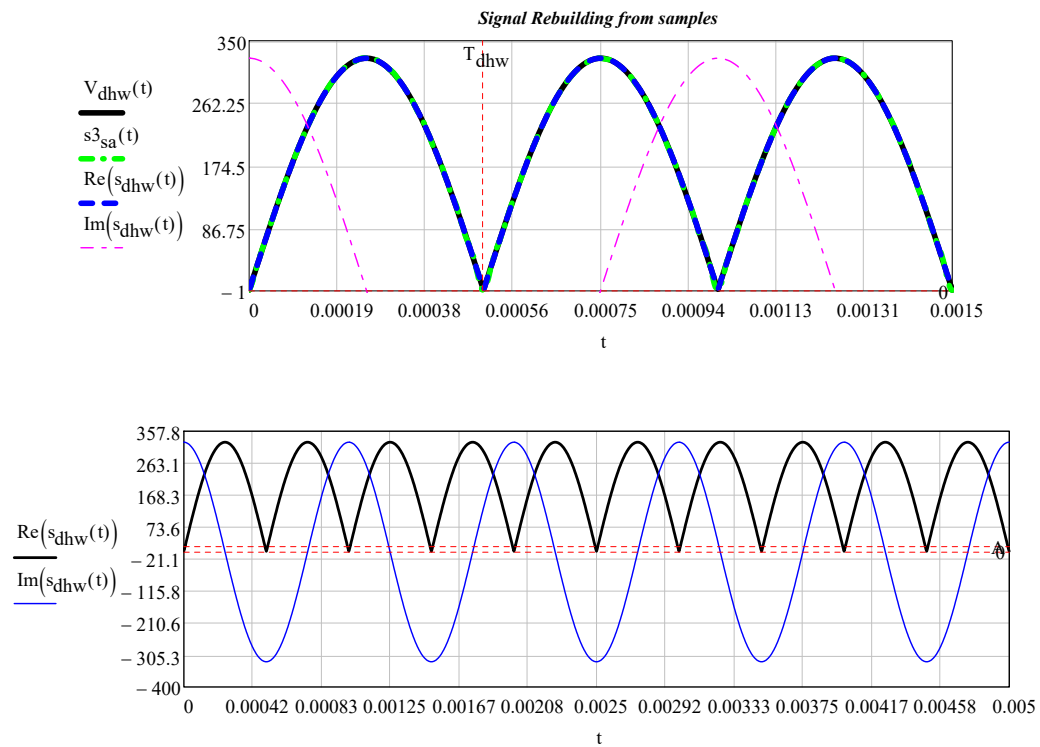
$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

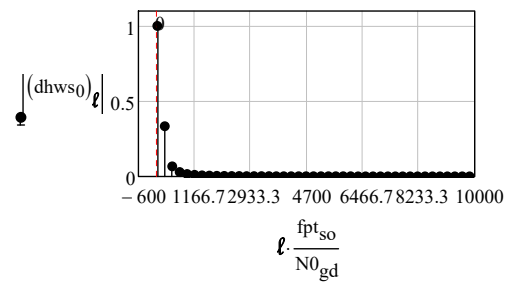
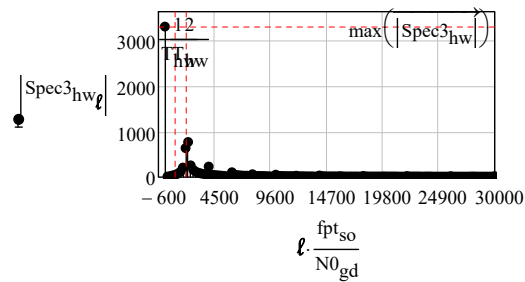
$$\text{interpolation formula: } s3_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m3}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \quad N0_{gd} - 1 = 255 \quad relerr = 10 \cdot \%$$

Signal Rebuilding from samples





$\text{length}(u_{m2}) = 256$
 $\text{fpt}_{so} = 60 \cdot \text{kHz}$
 $\text{Spec3}_{hw} := \text{fft}(u_{m3}) \quad \text{length}(\text{Spec3}_{hw}) = 129$
 $\ell := 0 \dots \frac{N0_{gd}}{2} - 1$

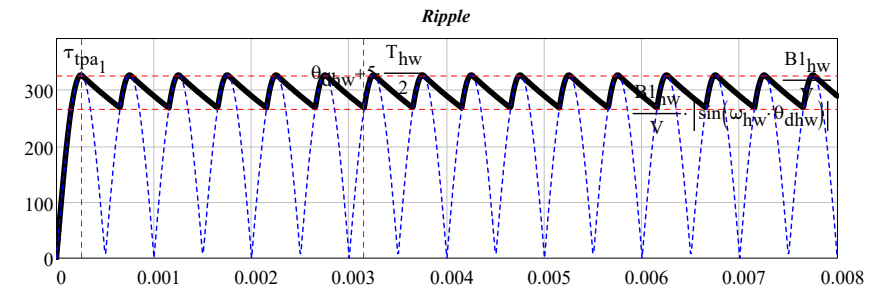


TEST Waveforms

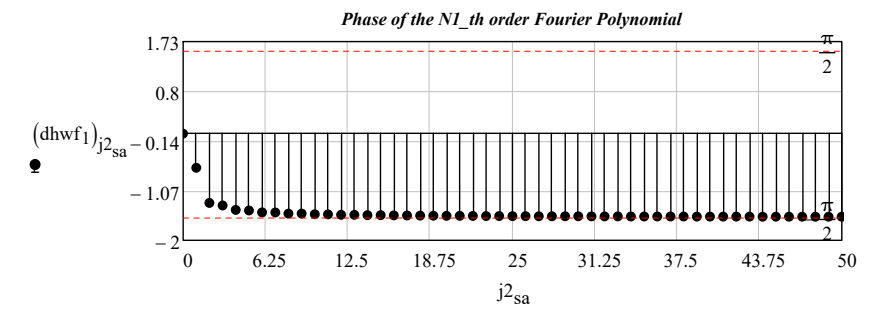
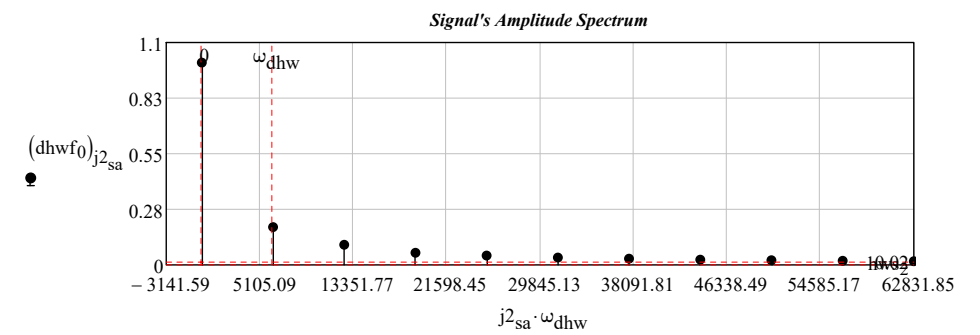
Periodic Waveforms
4 Double Half wave filtered

$$V_{ppt} := B1_{hw} \cdot \sin(\omega_{hw} \cdot \tau_{tpa1}) \cdot e^{\frac{\tau_{tpa1}}{\tau_{hw1}}} \quad \theta_{dhw} := Z1(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{ppt}) \quad \text{rip1} := \frac{\frac{B1_{hw}}{V} - \frac{B1_{hw}}{V} \cdot |\sin(\omega_{hw} \cdot \theta_{dhw})|}{\frac{B1_{hw}}{V}}$$

$V_{ppt} = 369.746 \text{ V}$



$B1_{hw} = 325.269 \text{ V}$ $V_{dhwf}(t) := g4(t, \tau_{hw1}, \tau_{tpa}, \theta_{dhw}, \omega_{hw}, B1_{hw}, V_{ppt}, N0_{gd})$
 $\text{dhwf} := \text{SPCT}(V_{dhwf}, \text{rt}_{gd}, N1_, 0 \cdot \text{sec}, T_{dhw}) \quad N1_ = 50$
 $j2_{sa} := 0 \dots \text{rows}(\text{dhws0}) - 1$

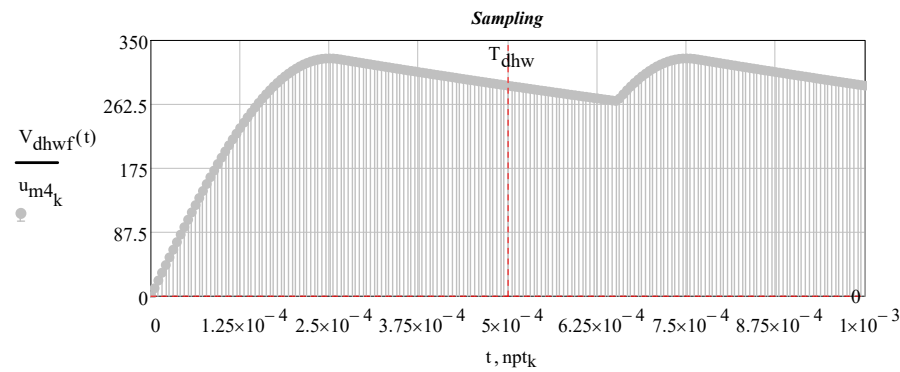


$Bw_{sa} := \text{dhwf}_3 \cdot \text{Hz}$
 $Bw_{sa} = 0.096 \cdot \text{MHz}$
 sampling frequency: $\text{fpt}_{so} := 2 \cdot Bw_{sa} \quad \text{fpt}_{so} = 0.192 \cdot \text{MHz}$

$npt_k := \frac{k}{\text{fpt}_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{dhw}} = 2.667$

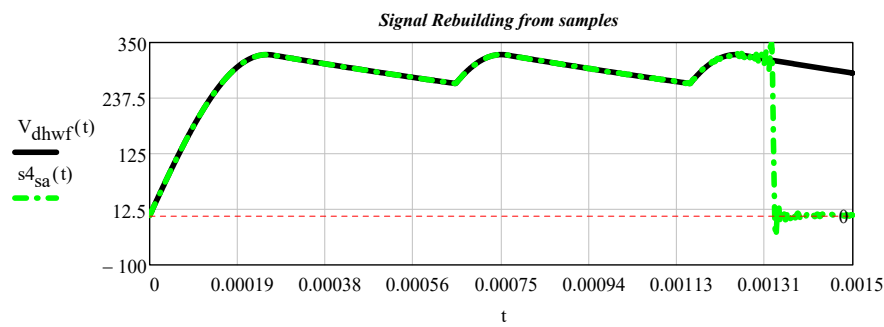
$u_{m4}_k := V_{dhwf}(npt_k)$

$$u_{m4}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{matrix} 0 \\ \dots \end{matrix} & 0 & 10.643 & 21.274 & 31.882 & 42.456 & 52.985 & \dots \end{matrix}$$


relerr = 10-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.603 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s^4_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m4}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10-%

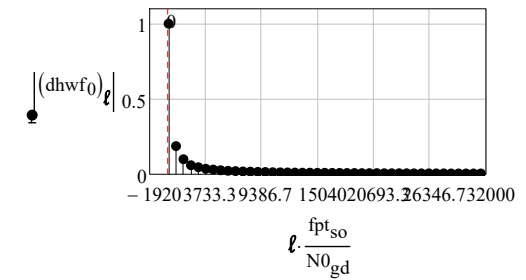
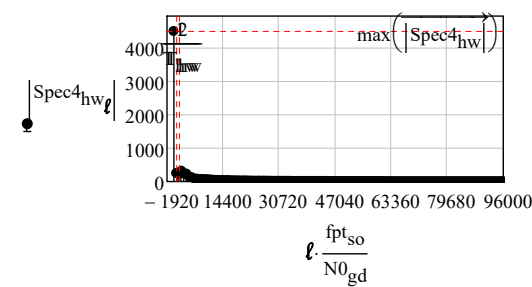


$\text{length}(u_{m4}) = 256$

$fpt_{so} = 192 \cdot \text{kHz}$

$\text{Spec4}_{hw} := \text{fft}(u_{m4})$ $\text{length}(\text{Spec4}_{hw}) = 129$

$\ell := 0 \dots \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$



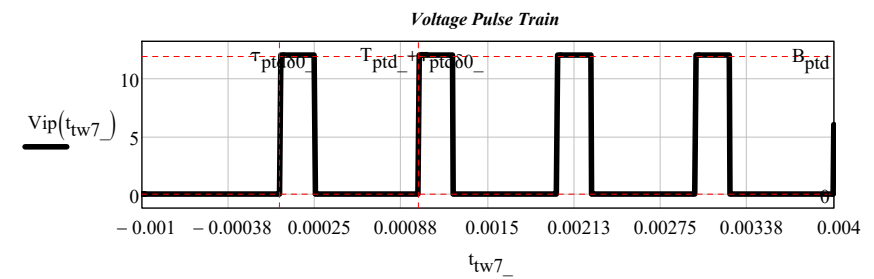
TEST Waveforms

Periodic Waveforms

5 Voltage Pulse Train

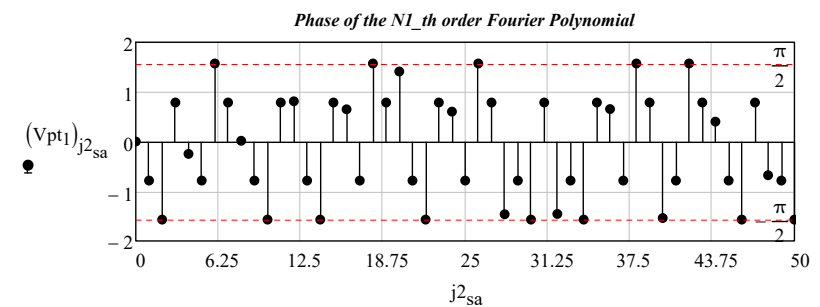
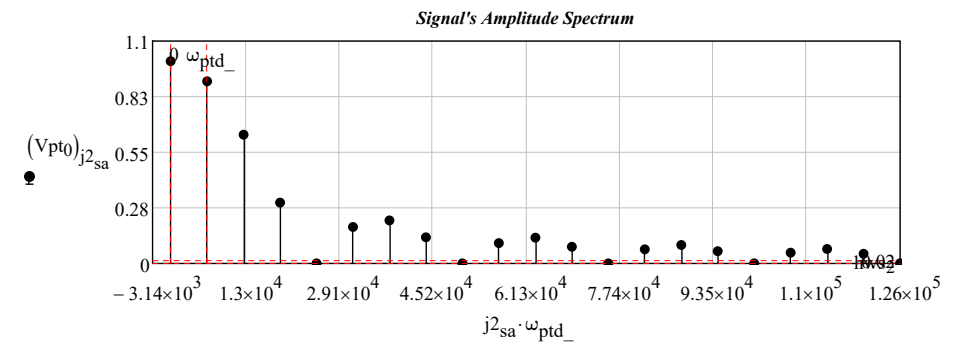
Data file " pulse_train_data.xmcd"

$Vip(t) := Vip1(t, T_{ptd_}, \tau_{ptd\delta 0_}, \delta_{ptd_}, B_{ptd}, N0_{gd})$



$Vpt := \text{SPCT}(Vip, rt_{gd}, N1_ , 0 \cdot \text{sec}, T_{ptd_})$ $N1_ = 50$

$j^2_{sa} := 0 \dots \text{rows}(dhw_{s0}) - 1$ $\omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$



$Bw_{sa} := Vpt3 \cdot \text{Hz}$

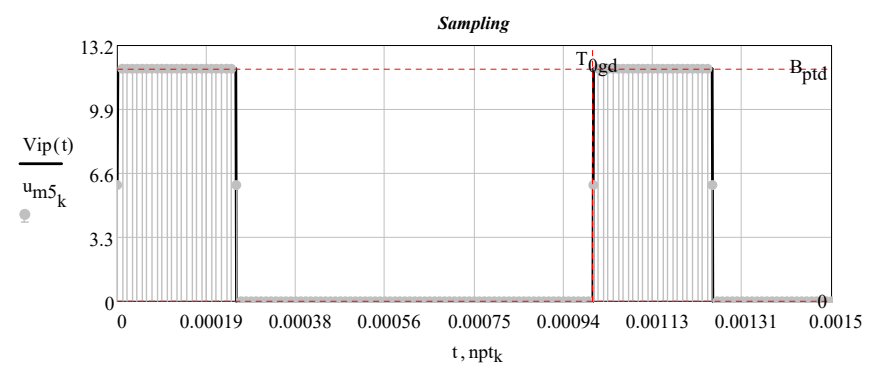
$Bw_{sa} = 0.048 \cdot \text{MHz}$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.096 \cdot \text{MHz}$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}} \cdot T0_{gd}} = 2.667$$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

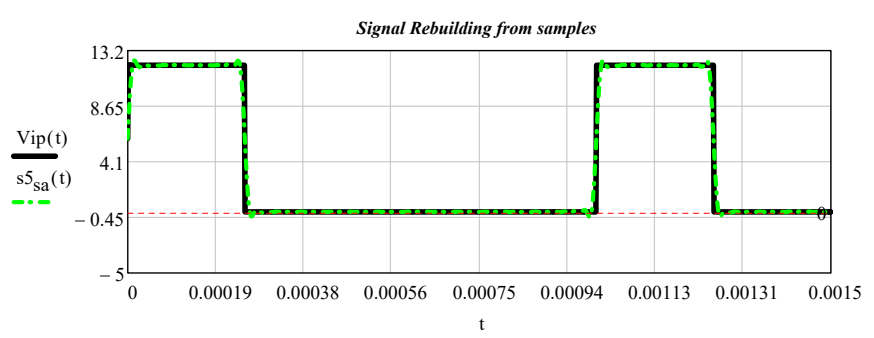
$$u_{m5_k} := \text{Vip}(n_{ptk})$$

$$u_{m5}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \begin{matrix} 0 \\ \dots \end{matrix} & 0 & 6 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & \dots \end{matrix}$$


$$\text{relerr} = 10\% \quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.302 \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s5_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m5_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$

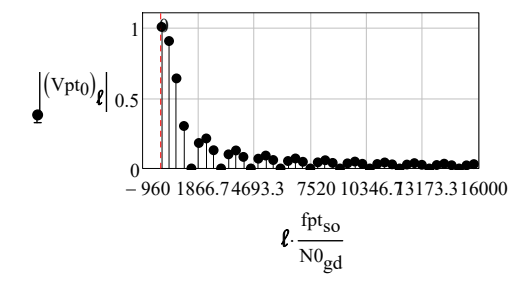
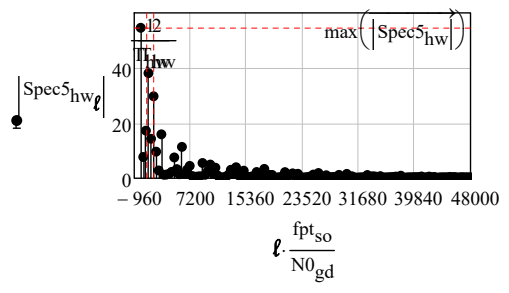


$$\text{length}(u_{m5}) = 256$$

$$f_{pt_{so}} = 96 \cdot \text{kHz}$$

$$\text{Spec5}_{hw} := \text{fft}(u_{m5}) \quad \text{length}(\text{Spec5}_{hw}) = 129$$

$$l := 0 \dots \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



TEST Waveforms

Periodic Waveforms

6 RF Pulse Train

Data file "rf pulse data.xmcd"

Step amplitude.....: $V_{rfpd} := B_{ptd}, V_{rfpd} = 12 \cdot V$

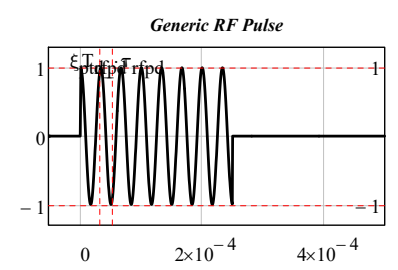
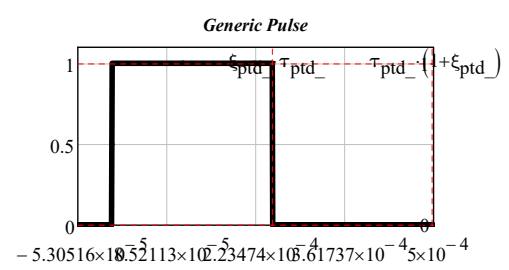
Signal frequency.....: $f_{rfpd} := 30 \cdot f_{ptd_}$

Signal period.....: $T_{rfpd} := \frac{1}{f_{rfpd}}$

Signal angular frequency.....: $\omega_{rfpd} := 2 \cdot \pi \cdot f_{rfpd} \quad \omega_{rfpd} = 0.188 \frac{\text{Mrads}}{\text{sec}}$

time constant.....: $\tau_{rfpd} := \frac{10}{\omega_{rfpd}}, \tau_{rfpd} = 53.052 \cdot \mu\text{s}$

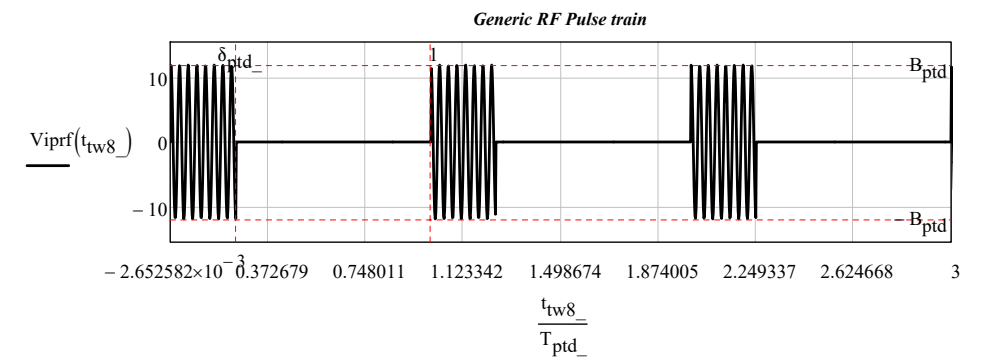
Rising edge delay.....: $\tau_{drfpd} := 0 \cdot \text{ns}$



Average value: $v_{ptmrfsl} := B_{ptd} \cdot \delta_{ptd_}$

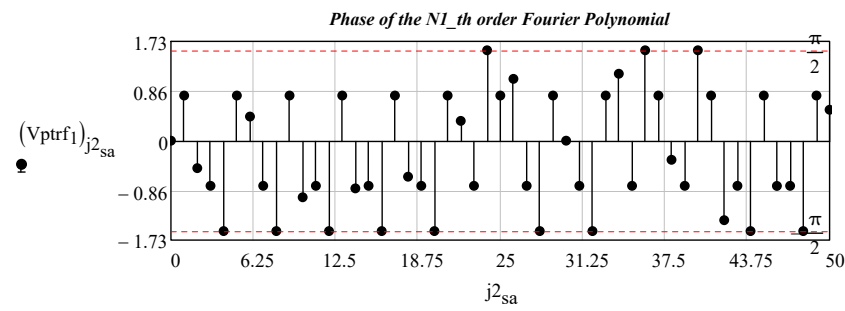
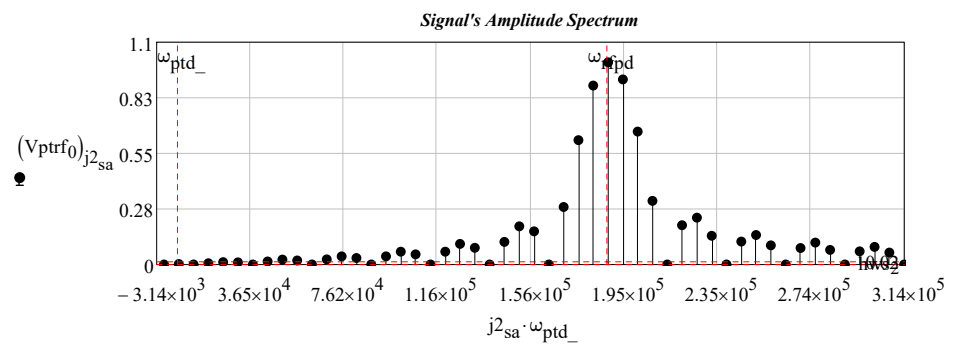
$$t_{tw8_} := -1 \cdot \tau_{ptd_} - 1 \cdot \tau_{ptd_} + \frac{4 \cdot T_{ptd_} + \tau_{ptd_}}{2000} \dots 4 \cdot T_{ptd_}$$

$$V_{iprf}(t) := v_{ptrf} \left(t, T_{ptd_}, \tau_{drfpd}, \delta_{ptd_}, \omega_{rfpd}, \frac{V_{rfpd}}{V}, N0_{gd} \right)$$

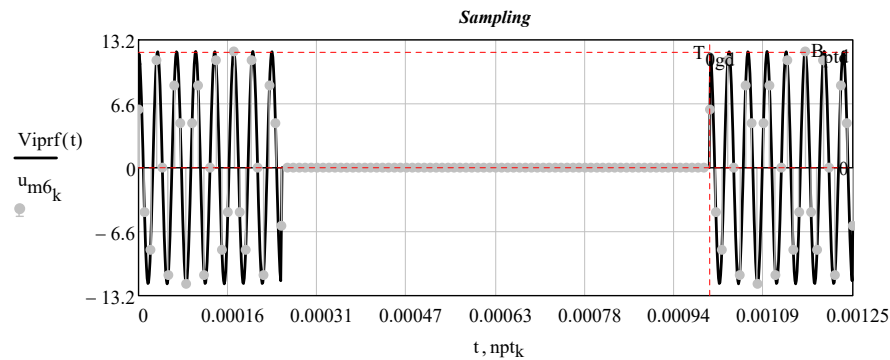


$$V_{ptrf} := \text{SPCT}(V_{iprf}, \tau_{gd}, N1_ , 0 \cdot \text{sec}, T_{ptd_}) \quad N1_ = 50$$

$$\omega_{\text{ptd}} = 6.283 \cdot \frac{\text{krads}}{\text{s}} \quad j2_{\text{sa}} := 0 \dots \text{rows}(\text{dhws0}) - 1 \quad \omega_{\text{rfpd}} = 188.496 \cdot \frac{\text{krads}}{\text{s}}$$



$Bw_{\text{sa}} := \text{Vptrf3} \cdot \text{Hz}$
 $Bw_{\text{sa}} = 0.048 \cdot \text{MHz}$
 sampling frequency: $f_{\text{pt}_{\text{so}}} := 2 \cdot Bw_{\text{sa}} \quad f_{\text{pt}_{\text{so}}} = 0.096 \cdot \text{MHz}$
 $n_{\text{pt}_k} := \frac{k}{f_{\text{pt}_{\text{so}}}}$
 Frequency resolution: $\frac{N0_{\text{gd}}}{f_{\text{pt}_{\text{so}}}} \cdot \frac{1}{T0_{\text{gd}}} = 2.667$
 $u_{\text{m6}_k} := \text{Viprf}(n_{\text{pt}_k})$

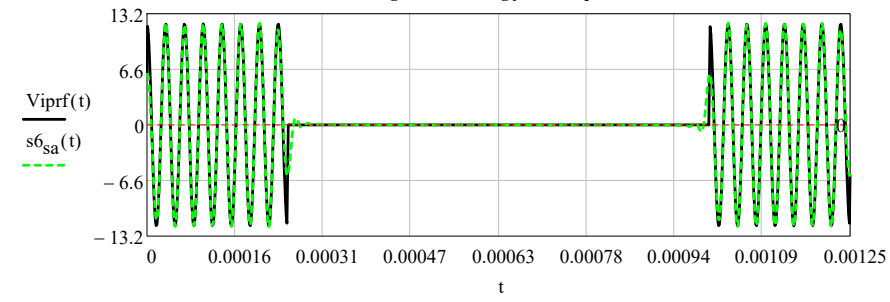
$$u_{\text{m6}}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 6 & -4.592 & -8.485 & 11.087 & \dots \end{bmatrix}$$


$\text{relerr} = 10\%$
 $\omega_{\text{bwr}} := 2 \cdot \pi \cdot Bw_{\text{sa}} \quad \omega_{\text{bwr}} = 0.302 \cdot \frac{\text{Mrads}}{\text{sec}}$
 $n \cdot \frac{\pi}{\omega_{\text{bwr}}} = n \cdot \frac{1}{2 \cdot Bw_{\text{sa}}}$

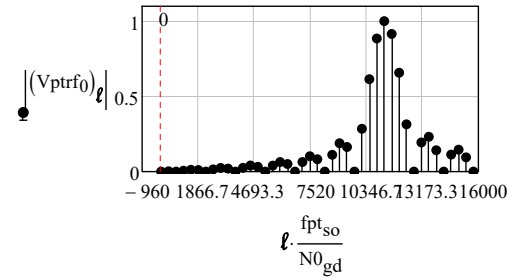
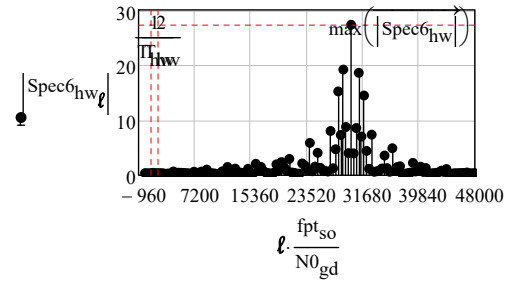
Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s6_{\text{sa}}(t) := \sum_{n=0}^{N0_{\text{gd}}-1} \left(u_{\text{m6}_n} \cdot \text{sinc}(\omega_{\text{bwr}} \cdot t - n \cdot \pi) \right)$ $N0_{\text{gd}} - 1 = 255$ $\text{relerr} = 10\%$

Signal Rebuilding from samples



$\text{length}(u_{\text{m6}}) = 256$
 $f_{\text{pt}_{\text{so}}} = 96 \cdot \text{kHz}$
 $\text{Spec6}_{\text{hw}} := \text{fft}(u_{\text{m6}}) \quad \text{length}(\text{Spec6}_{\text{hw}}) = 129$
 $\ell := 0 \dots \frac{N0_{\text{gd}}}{2} \quad \frac{N0_{\text{gd}}}{2} = 128$



TEST Waveforms

Periodic Waveforms

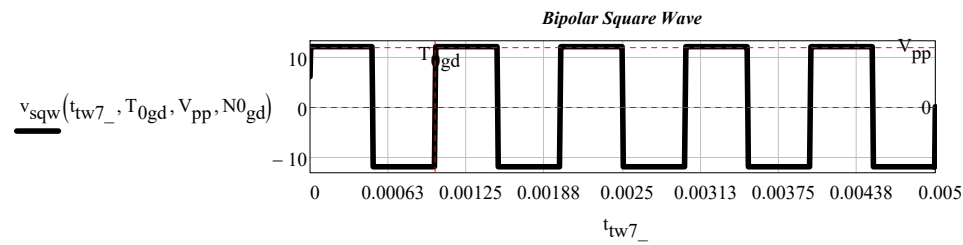
7 Bipolar Square Wave

Data file "pulse train data.xmcd"

Signal amplitude: $V_{pp} = 12\text{ V}$

Square wave period: $T_{0gd} = 1 \times 10^6\text{ ns}$

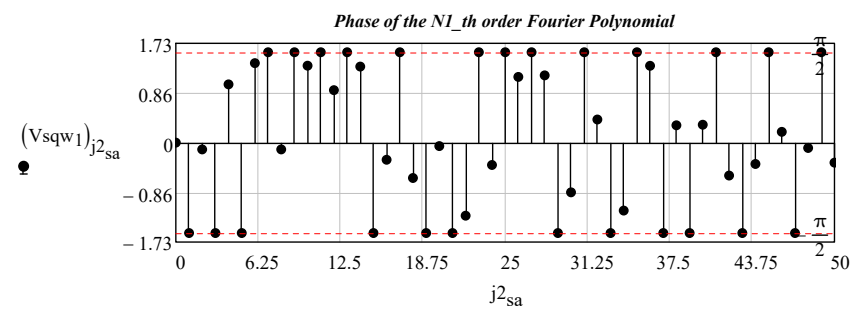
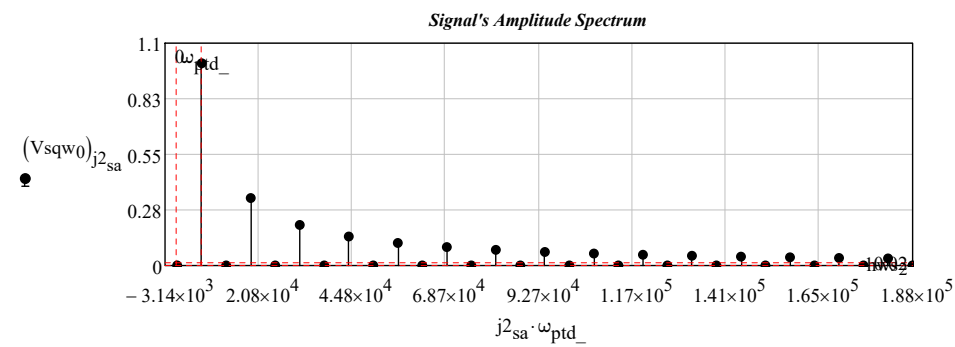
$\omega_{ptd} = 6.283 \times 10^{-6} \frac{\text{Grads}}{\text{sec}}$



$$V_{sqw}(t) := V^{-1} \cdot v_{sqw}(t, T_{0gd}, V_{pp}, N_{0gd})$$

$$V_{sqw} := \text{SPCT}(V_{sqw}, rt_{gd}, N1_, 0\text{ sec}, T_{0gd}) \quad N1_ = 50$$

$$j2_{sa} := 0.. \text{rows}(\text{dhws}_0) - 1 \quad \omega_{ptd} = 6.283 \times 10^{-3} \frac{\text{Mrads}}{\text{s}}$$



$$Bw_{sa} := V_{sqw3} \cdot \text{Hz}$$

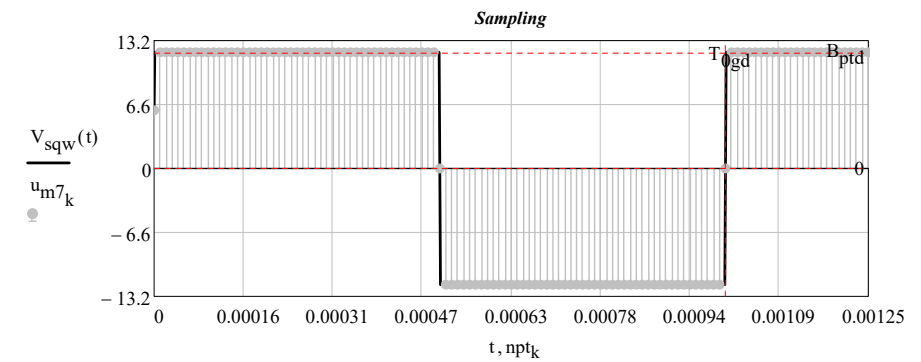
$$Bw_{sa} = 0.048\text{ MHz}$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.096\text{ MHz}$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N_{0gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{0gd}} = 2.667$

$$u_{m7}_k := V_{sqw}(npt_k)$$

$$u_{m6}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ 6 \\ -4.592 \\ -8.485 \\ 11.087 \\ \dots \end{matrix} & & & & & \end{matrix}$$


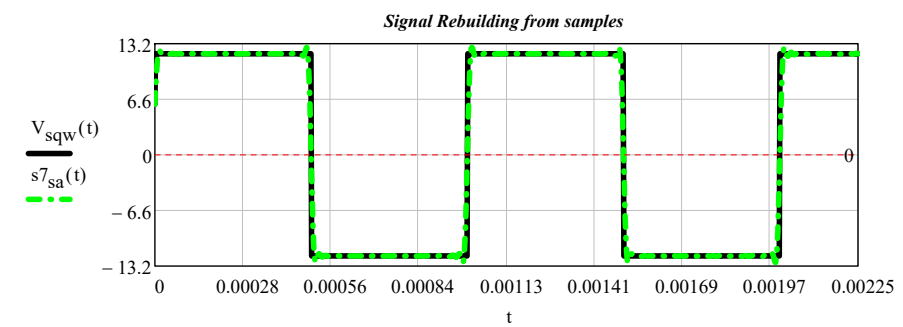
relerr = 10-%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.302 \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s7_{sa}(t) := \sum_{n=0}^{N_{0gd}-1} (u_{m7}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N_{0gd} - 1 = 255$ relerr = 10-%

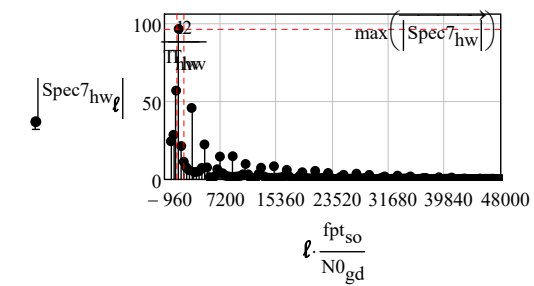


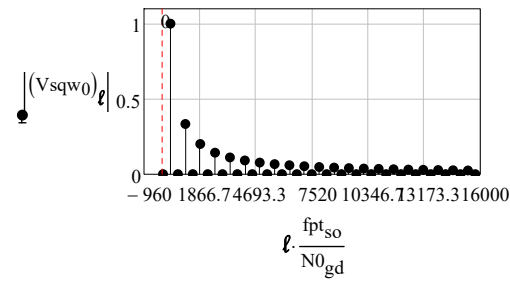
$$\text{length}(u_{m7}) = 256$$

$$f_{pt_{so}} = 96\text{ kHz}$$

$$\text{Spec7}_{hw} := \text{fft}(u_{m7}) \quad \text{length}(\text{Spec7}_{hw}) = 129$$

$$l := 0.. \frac{N_{0gd}}{2} \quad \frac{N_{0gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

8 Bipolar Square Wave 1

Data file "pulse train data.xmcd"

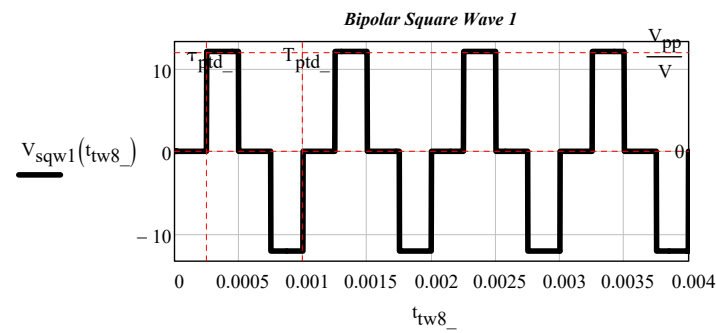
Signal amplitude: $V_{pp} = 12 \cdot V$

Square wave period: $T_{0gd} = 1 \times 10^6 \cdot ns$ $\xi_{twsl} := \xi_{ptd_}$

$\omega_{ptd_} = 6.283 \times 10^{-6} \cdot \frac{Grads}{sec}$ $\tau_{\delta sl} := -\tau_{ptd_} \cdot (1 - \xi_{twsl})$

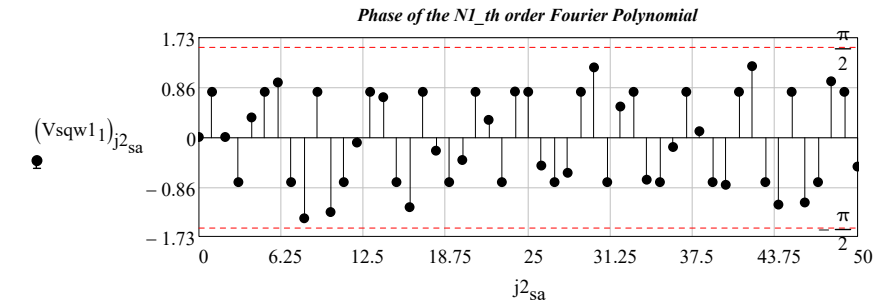
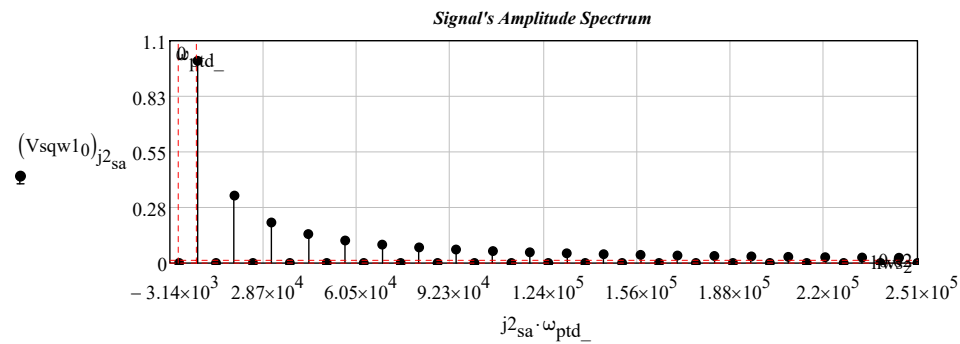
$\tau_{ptd_} = 2.5 \times 10^{-4} s$

$V_{sqw1}(t) := V_6(t, \tau_{\delta sl}, \tau_{ptd_}, T_{ptd_}, V_{pp}, N0_{gd})$



$V_{sqw1} := SPCT(V_{sqw1}, \tau_{gd}, N1_, \tau_{ptd_}, T_{ptd_})$ $N1_ = 50$

$j2_{sa} := 0..rows(V_{sqw1}) - 1$ $\omega_{ptd_} = 6.283 \cdot \frac{krads}{s}$



$Bw_{sa} := V_{sqw13} \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

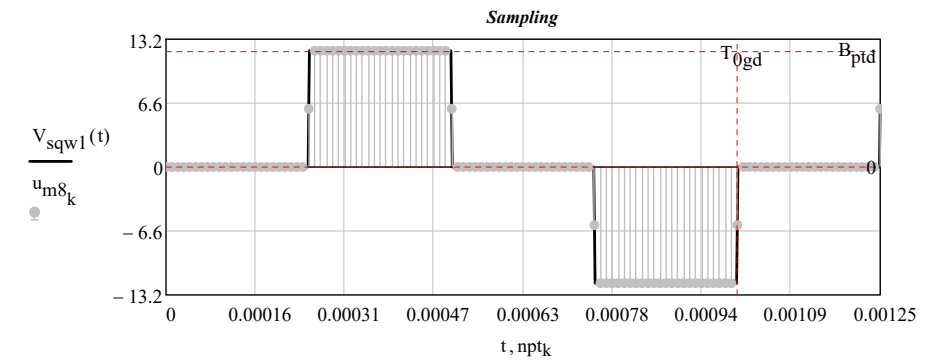
sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa}$ $f_{pt_{so}} = 0.096 \cdot MHz$

$npt_k := \frac{k}{f_{pt_{so}}}$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{0gd}} = 2.667$

$u_{m8}_k := V_{sqw1}(npt_k)$

$u_{m8}^T =$	0	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0	...



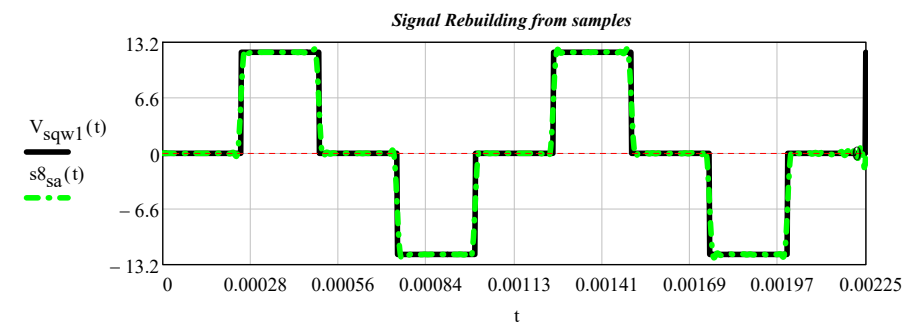
relerr = 10-%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.302 \cdot \frac{Mrads}{sec}$

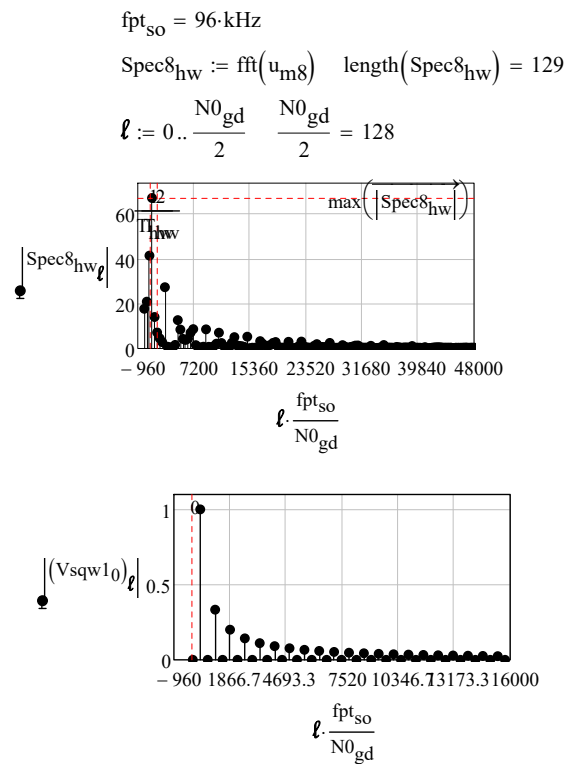
$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s8_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m8}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10-%



$length(u_{m8}) = 256$



TEST Waveforms

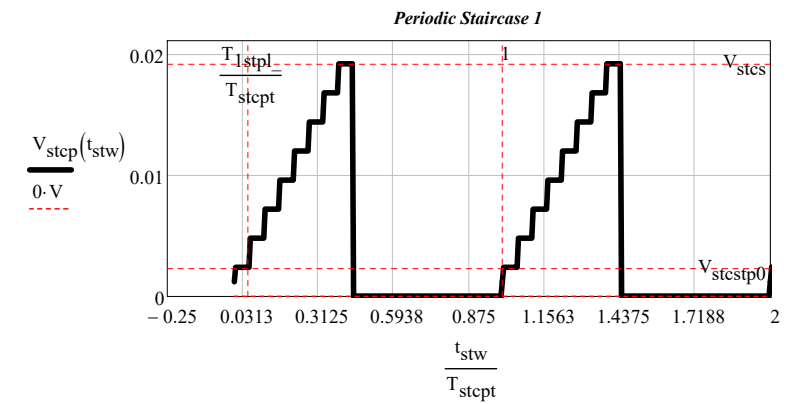
Periodic Waveforms

9 Staircase 1 Voltage Pulse Train

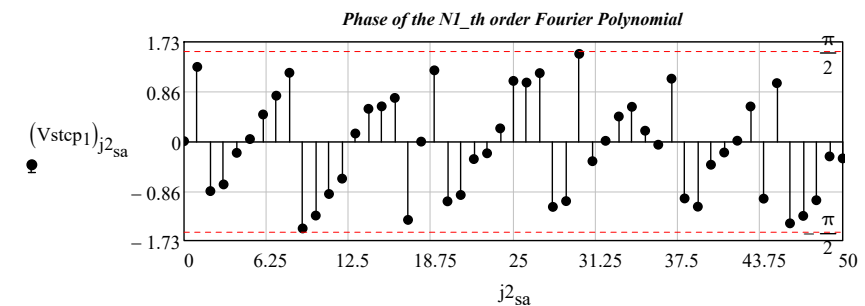
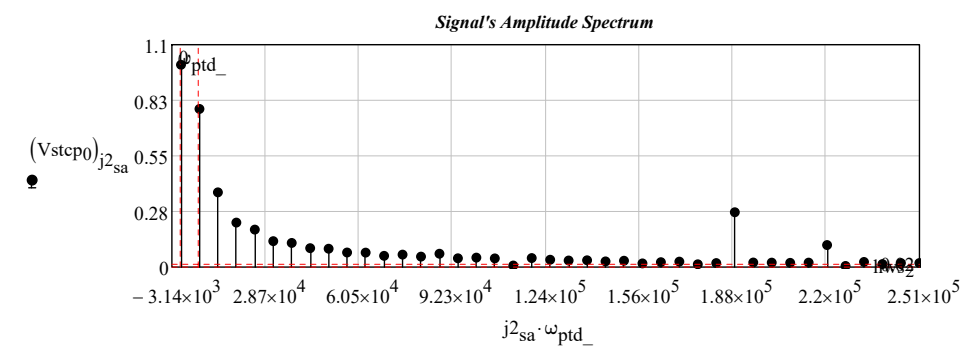
Description of the Function's parameters: $v_{stcp}(t_{s1}, \text{period}, \text{signal_amplitude}, \text{number_of_steps}, \text{max_number_of_periods})$
 $: v_{stc}(t_{s1}, \text{step_length}, \text{signal_amplitude}, \text{number_of_steps}, \text{max_number_of_periods})$

For data, see the worksheet "staircase pulse data.xmcd"

Period: $T_{stcpt} := (m1_{steps} + 1) \cdot T_{1stpl_}$
 Duty Cycle: $\delta_{stcpt} := \frac{m1_{steps} \cdot T_{1stpl_}}{T_{stcpt}}$
 Staircase frequency: $f_{stcpt} := \frac{1}{T_{stcpt}}$
 $\omega_{stcpt} := 2 \cdot \pi \cdot f_{stcpt} \quad \omega_{1stpl_} := \frac{2 \cdot \pi}{T_{1stpl_}}$
 Number of periods shown: $n_p := 20$
 $v_{stcptasl} := \frac{V_{stcs}}{2 \cdot m1_{steps} \cdot (m1_{steps} + 1)} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1) = 4.8 \text{ mV}$
 $t_{stw} := 0 \cdot T_{stcpt}, 0 \cdot T_{stcpt} + \frac{10 \cdot T_{stcpt}}{2000} .. 10 \cdot T_{stcpt}$
 $V_{stcp}(t) := \frac{v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd})}{V}$



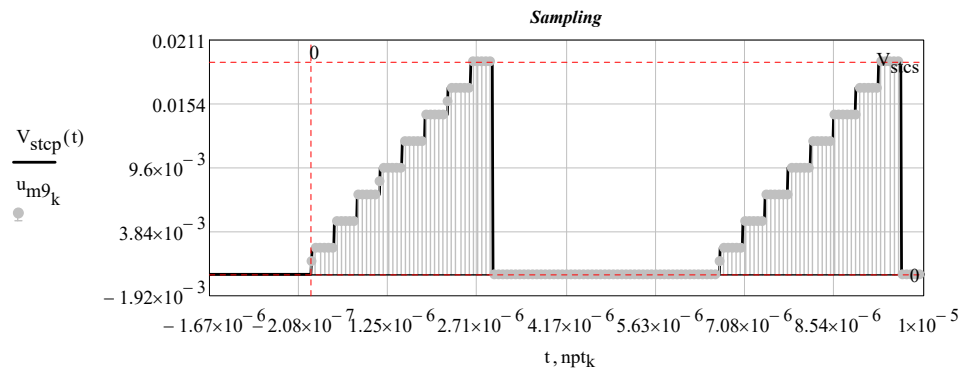
$V_{stcp} := \text{SPCT}(V_{stcp}, rt_{gd}, N1_, 0 \cdot s, T_{stcpt}) \quad N1_ = 50$
 $j2_{sa} := 0 .. \text{rows}(V_{stcp0}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \frac{\text{Mrads}}{s}$



$Bw_{sa} := V_{stcp3} \cdot \text{Hz}$
 $Bw_{sa} = 7.2 \text{ MHz}$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 14.4 \text{ MHz}$
 $npt_k := \frac{k}{f_{pt_{so}}}$
 Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{stcpt}} = 2.667$
 $u_{m9}_k := V_{stcp}(npt_k)$

$$u_{m9}^T =$$

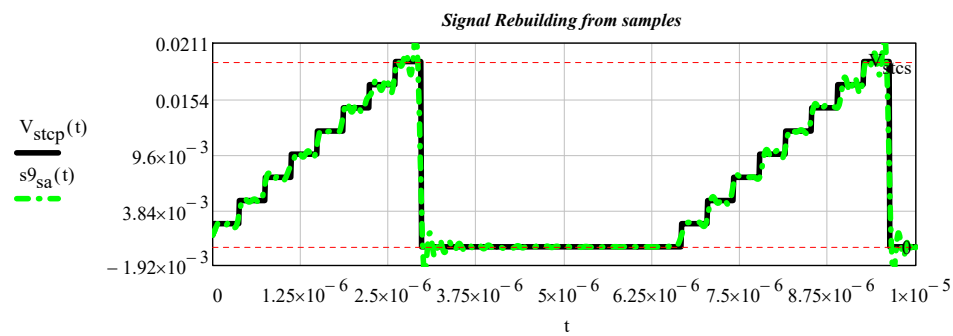
	0	1	2	3	4	5	6
0	$1.2 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$



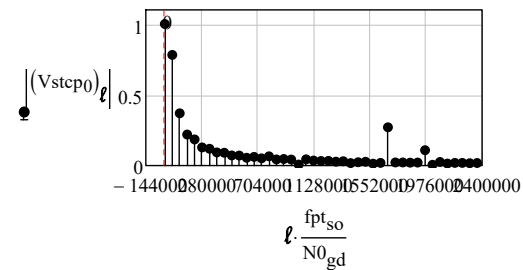
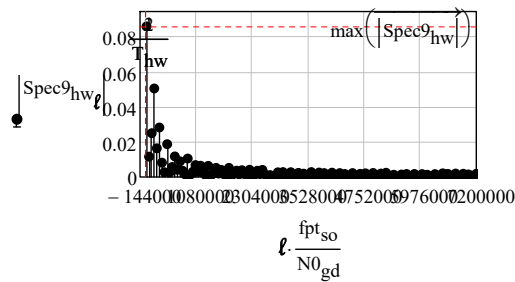
relerr = 10-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 45.239 \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{sa}^9(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m9}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10-%



$\text{length}(u_{m9}) = 256$
 $f_{pt_{so}} = 1.44 \times 10^4 \cdot \text{kHz}$
 $\text{Spec}_{hw}^9 := \text{fft}(u_{m9})$ $\text{length}(\text{Spec}_{hw}^9) = 129$
 $\ell := 0.. \frac{N0_{gd}}{2} - 1$ $\frac{N0_{gd}}{2} = 128$



TEST Waveforms

Periodic Waveforms

10 Staircase 2 Voltage Pulse Train

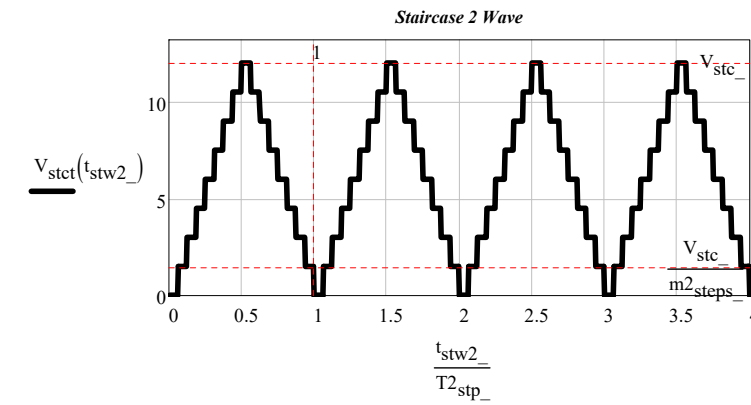
Description of the Function's parameters: $v_{stct}(time, period, max_amplitude, number_of_steps, max_number_of_periods)$

$v_{stcc}(t_{sl}, step_length, signal_amplitude, number_of_steps, number_max_of_periods)$

For data, see the worksheet "staircase 2 pulse data.xmcd"

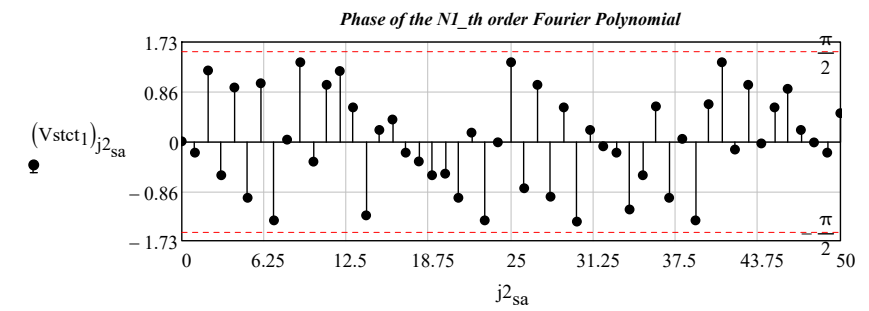
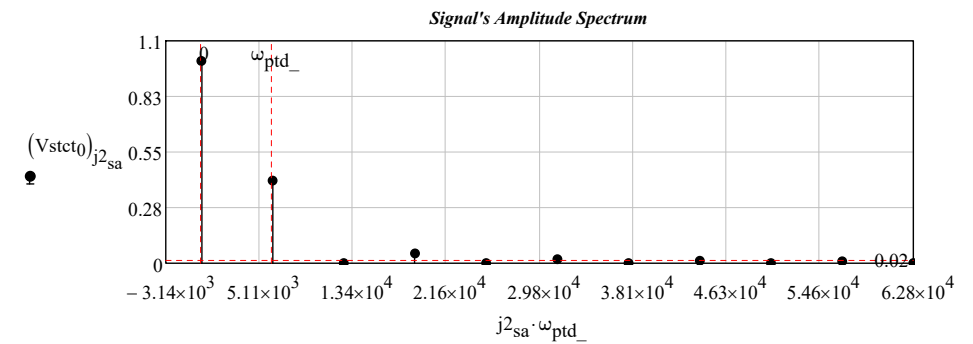
$t_{stw2_} := 0 \cdot T2_{stp_} + 0 \cdot T2_{stp_} + \frac{10 \cdot T2_{stp_}}{2000} .. 10 \cdot T2_{stp_}$

$V_{stct}(t) := \frac{v_{stct}(t, T2_{stp_}, V_{stc_}, m2_{steps_}, N0_{gd})}{V}$



$V_{stct} := \text{SPCT}(V_{stct}, rt_{gd}, N1_, 0 \cdot s, T2_{stp_})$ $N1_ = 50$

$j2_{sa} := 0.. \text{rows}(V_{stct0}) - 1$ $\omega_{ptd_} = 6.283 \times 10^{-3} \frac{Mrads}{s}$

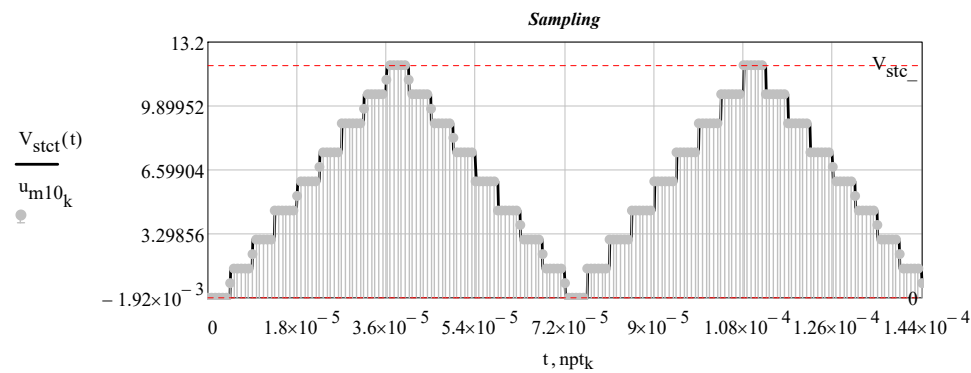


$Bw_{sa} := V_{stct3} \cdot \text{Hz}$
 $Bw_{sa} = 0.667 \cdot \text{MHz}$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa}$ $f_{pt_{so}} = 1.333 \cdot \text{MHz}$

$$npt_k := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp_}} = 2.667$$

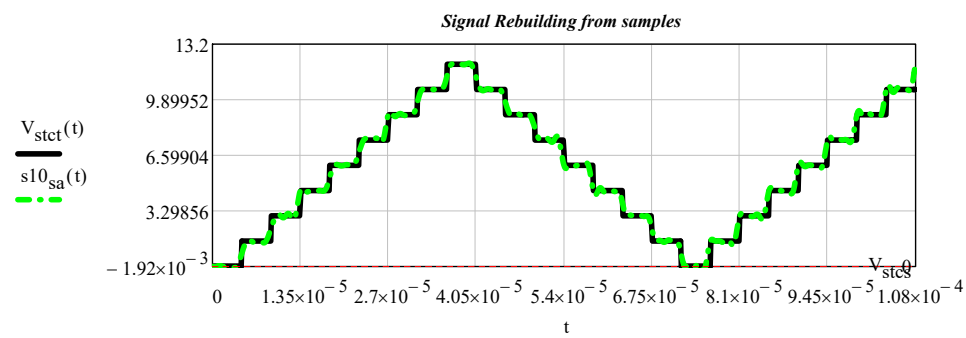
$$u_{m10}_k := V_{stct}(npt_k)$$

$$u_{m10}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 1.5 & \dots \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \dots \end{bmatrix}$$


$$relerr = 10\% \quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 4.189 \frac{Mrads}{sec} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s10_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m10}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad relerr = 10\%$$

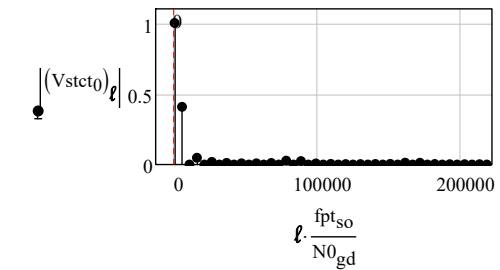
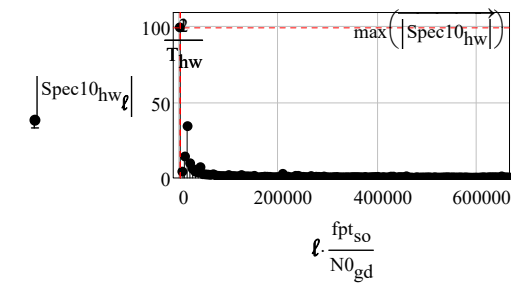


$$\text{length}(u_{m10}) = 256$$

$$fpt_{so} = 1.333 \times 10^3 \cdot \text{kHz}$$

$$\text{Spec10}_{hw} := \text{fft}(u_{m10}) \quad \text{length}(\text{Spec10}_{hw}) = 129$$

$$l := 0.. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



TEST Waveforms

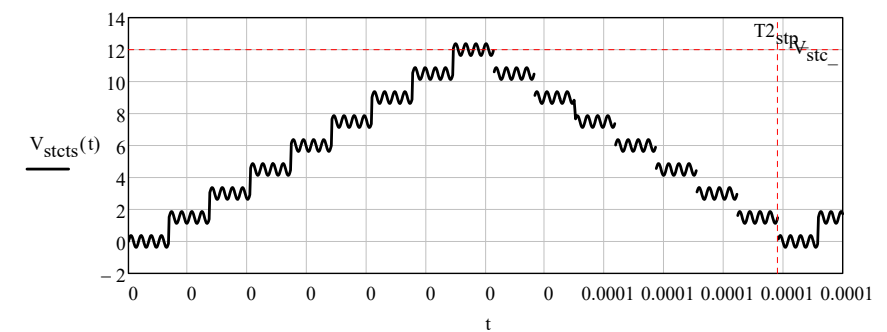
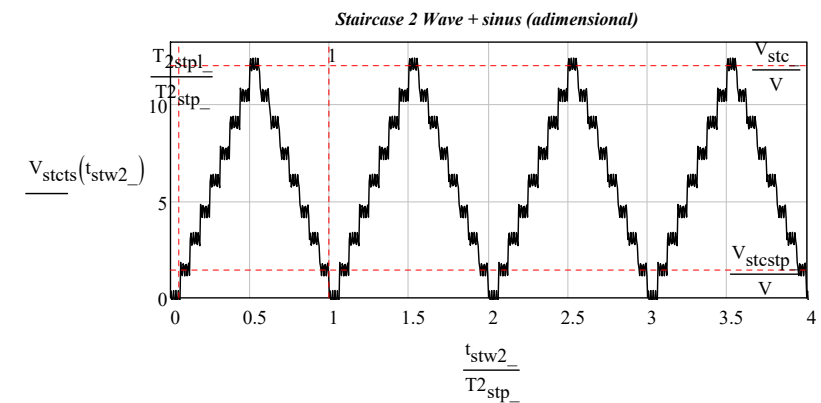
Periodic Waveforms

11 Staircase 2 Voltage Pulse Train + sinus

Description of the Function's parameters: Vstcsin(t_s1, period, max_amplitude, number_of_steps, max_number_of_periods)

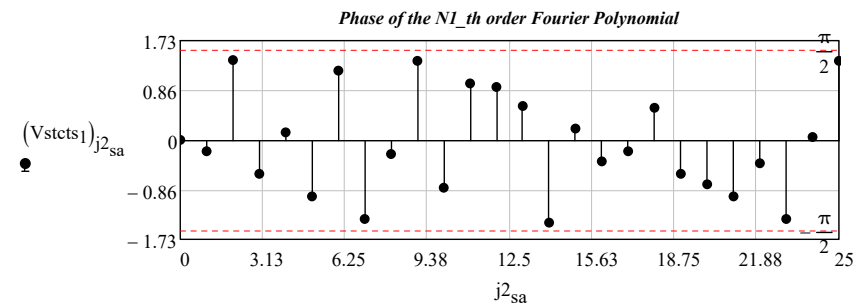
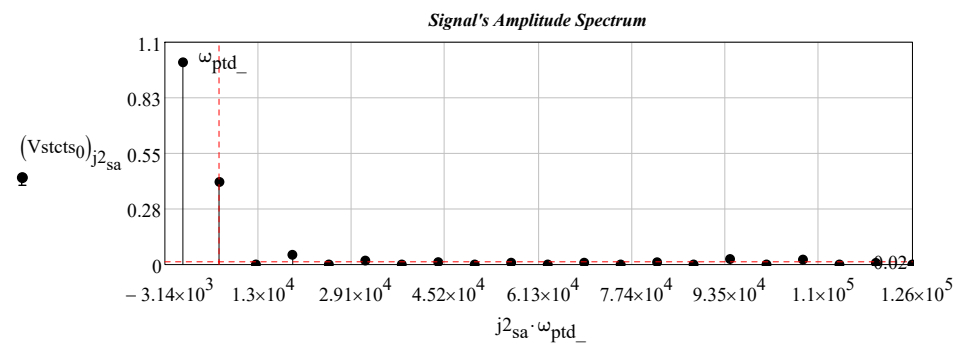
For data, see the worksheet "staircase 2 pulse data.xmcd"

$$V_{stcts}(t) := Vstcsin(t, T2_{stp_}, V_{stc_}, m2_{steps_}, N0_{gd})$$



$$N1_{stc} := 25 \quad V_{stcts} := \text{SPCT}(V_{stcts}, rt_{gd}, N1_{stc}, 0 \cdot s, T2_{stp_})$$

$$j^2_{sa} := 0..rows(Vstcts0) - 1 \quad \omega_{ptd_} = 6.283 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := Vstcts3 \cdot \text{Hz}$$

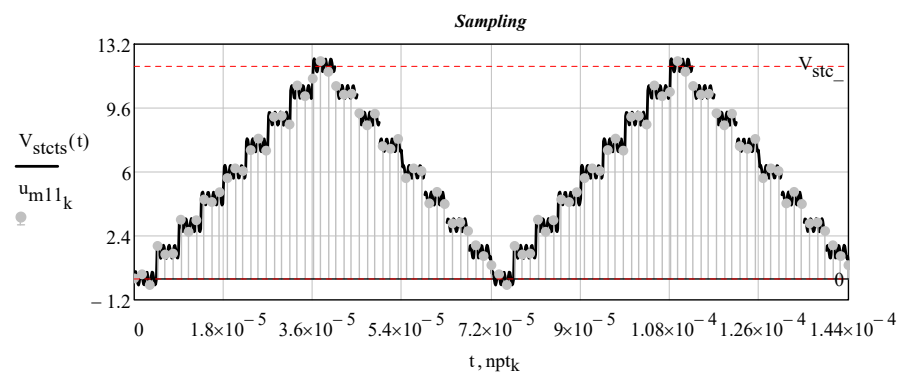
$$Bw_{sa} = 0.319 \cdot \text{MHz}$$

$$\text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.639 \cdot \text{MHz}$$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}} \cdot T2_{stp_}} = 5.565$$

$$u_{m11_k} := Vstcts(npt_k)$$

$$u_{m11}^T = \begin{array}{|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0.237 & -0.367 & 1.833 & \dots \\ \hline \end{array}$$


$$\text{relerr} = 10\%$$

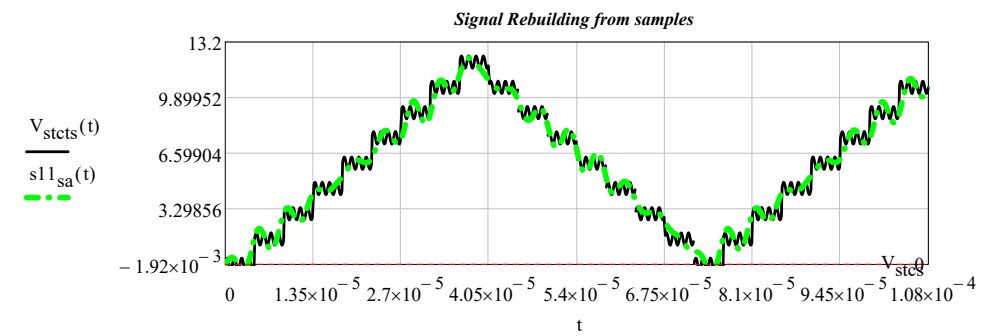
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 2.007 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s11_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m11_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right] \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$

$N1 = 25$

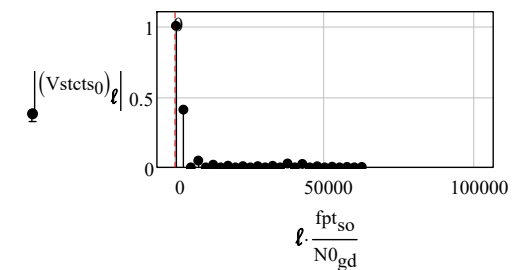
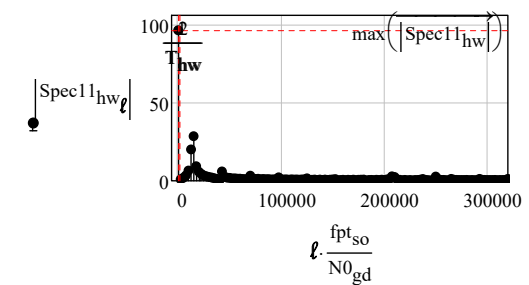


$$\text{length}(u_{m11}) = 256$$

$$f_{pt_{so}} = 638.889 \cdot \text{kHz}$$

$$\text{Spec11}_{hw} := \text{fft}(u_{m11}) \quad \text{length}(\text{Spec11}_{hw}) = 129$$

$$\ell := 0.. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



TEST Waveforms

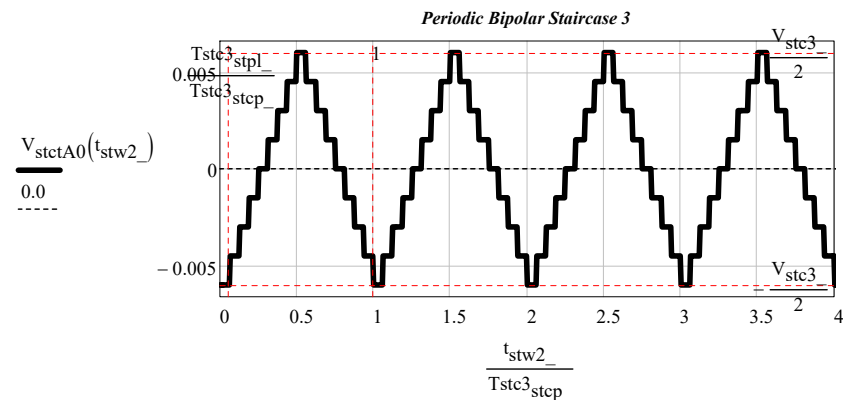
Periodic Waveforms

12 Staircase 3 Voltage Pulse Train

Description of the Function's parameters: $v_{stct}(t_{sl}, \text{period}, \text{step_amplitude}, \text{number_of_steps}, \text{max_number_of_periods})$
 $: v_{stctA0}[t_{sl}, (\text{period}, \text{step_amplitude}, \text{number_of_steps}, \text{max_number_of_periods})]$

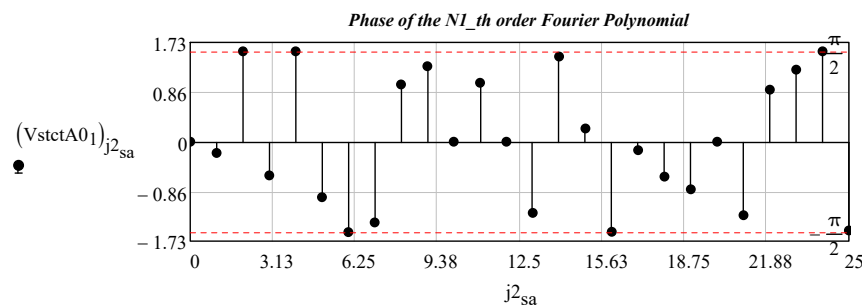
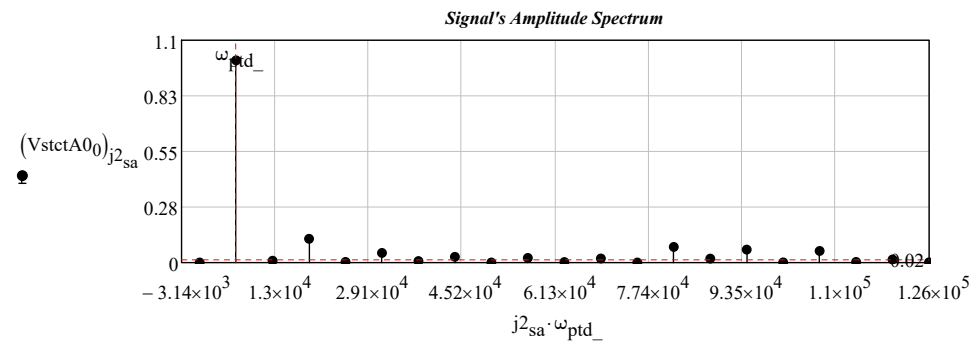
You can find the data in "staircase 3 pulse data.xmcd"

$$V_{stctA0}(t) := \frac{v_{stctA0}(t, Tstc3_{step_}, V_{stc3_}, mstc3_{steps_}, N0_{gd})}{V} \quad N1_ := 25$$



$$VstctA0 := SPCT(V_{stctA0}, rt_{gd}, N1_, 0, s, Tstc3_{step_}) \quad N1_ = 25$$

$$j2_{sa} := 0..rows(VstctA0) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$$



$$Bw_{sa} := VstctA0_3 \cdot \text{Hz}$$

$$Bw_{sa} = 0.328 \cdot \text{MHz}$$

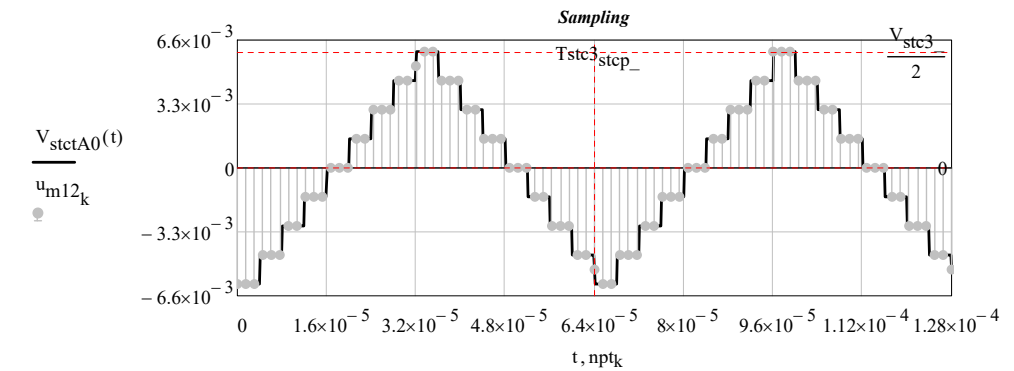
$$\text{sampling frequency: } fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.656 \cdot \text{MHz}$$

$$npt_k := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp_}} = 5.418$$

$$u_{m12}_k := V_{stctA0}(npt_k)$$

$u_{m12}^T =$	0	1	2	3	4	5
	0	$-6 \cdot 10^{-3}$	$-6 \cdot 10^{-3}$	$-6 \cdot 10^{-3}$	$-4.5 \cdot 10^{-3}$	$-4.5 \cdot 10^{-3}$
						...



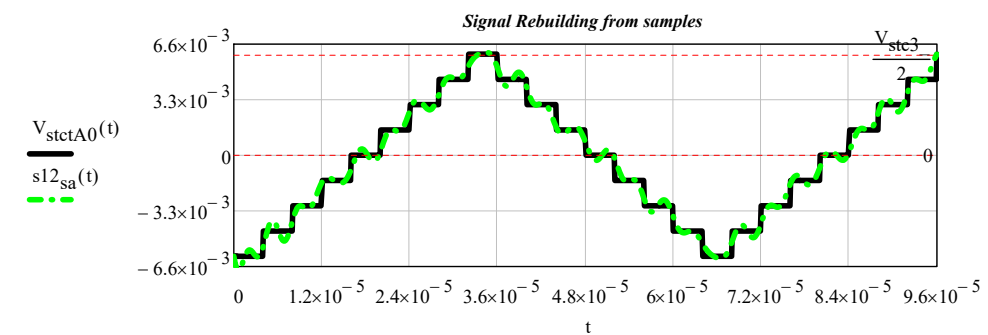
$$\text{reerr} = 10\%$$

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 2.062 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s12_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m12}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{reerr} = 10\%$$

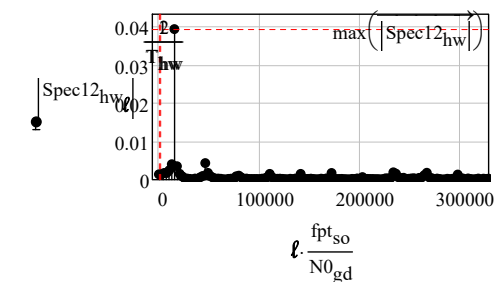


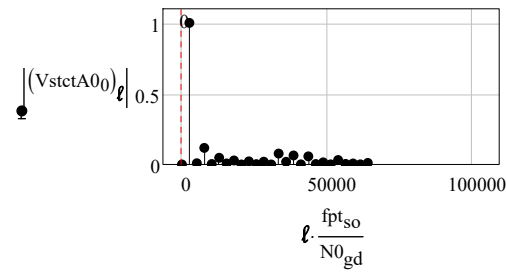
$$\text{length}(u_{m12}) = 256$$

$$fpt_{so} = 656.25 \cdot \text{kHz}$$

$$\text{Spec12}_{hw} := \text{fft}(u_{m12}) \quad \text{length}(\text{Spec12}_{hw}) = 129$$

$$\ell := 0.. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



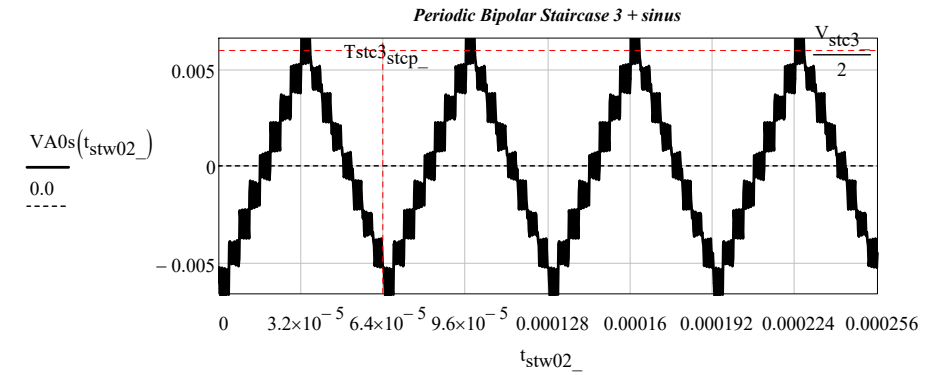


TEST Waveforms

Periodic Waveforms

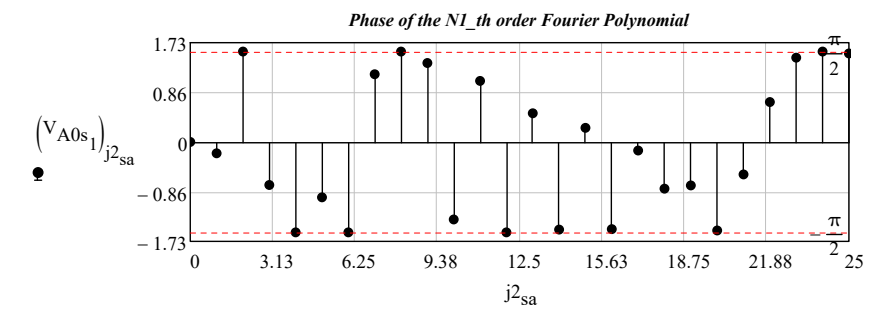
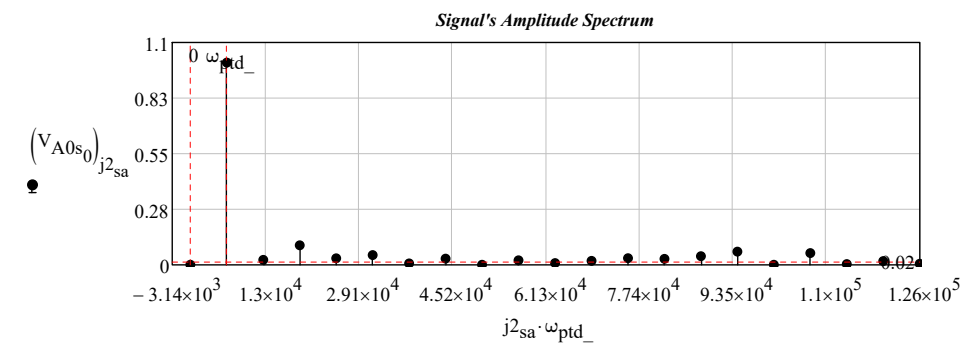
13 Staircase 3 Voltage Pulse Train + sinus

$$VA0s(t) := VistctA0sin(t, Tstc3_{stcp_}, V_{stc3_}, mstc3_{steps_}, N0_{gd})$$



$$VA0s := SPCT(VA0s, rt_{gd}, N1_, 0, s, Tstc3_{stcp_}) \quad N1_ = 25$$

$$j2_{sa} := 0..rows(VA0s_0) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \frac{Mrads}{s}$$



$$Bw_{sa} := VA0s_3 \cdot Hz$$

$$Bw_{sa} = 0.344 \text{ MHz}$$

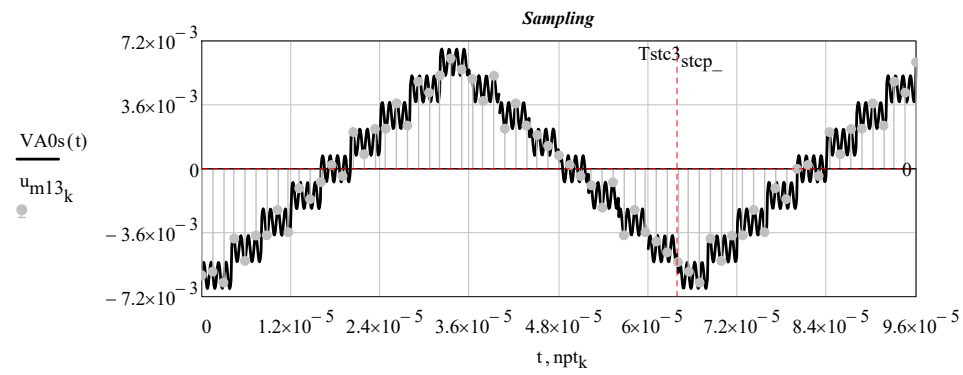
$$\text{sampling frequency: } fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.688 \text{ MHz}$$

$$nptk := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp_}} = 5.172$$

$$u_{m13}_k := VA0s(nptk)$$

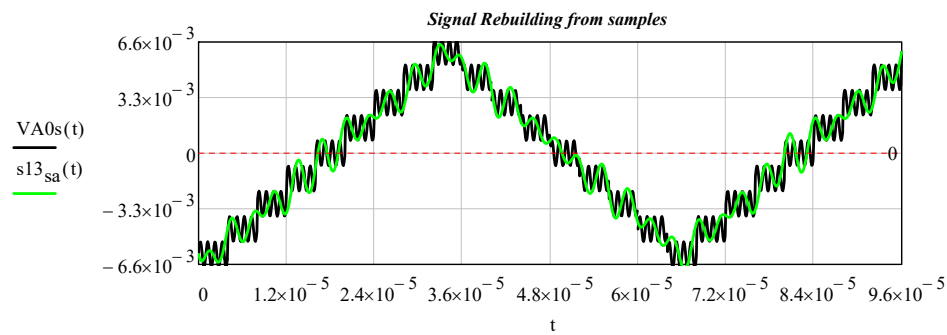
$$u_{m13}^T = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & \dots \end{matrix} \\ \begin{matrix} 0 \end{matrix} & \begin{matrix} -6 \cdot 10^{-3} & -5.789 \cdot 10^{-3} & -6.405 \cdot 10^{-3} & -3.933 \cdot 10^{-3} & \dots \end{matrix} \end{matrix}$$



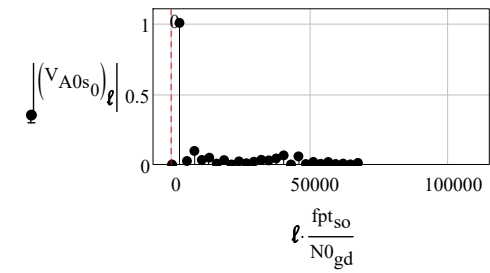
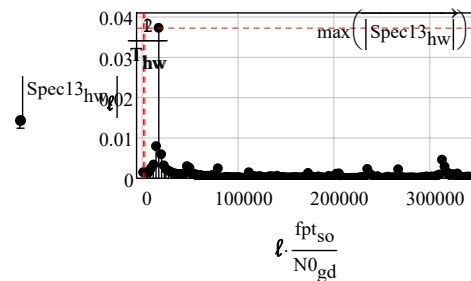
$$\text{relerr} = 10\% \quad \omega_{\text{bwr}} := 2 \cdot \pi \cdot \text{Bw}_{\text{sa}} \quad \omega_{\text{bwr}} = 2.16 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{\text{bwr}}} = n \cdot \frac{1}{2 \cdot \text{Bw}_{\text{sa}}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula} \quad s_{13_{\text{sa}}}(t) := \sum_{n=0}^{N0_{\text{gd}}-1} (u_{m13_n} \cdot \text{sinc}(\omega_{\text{bwr}} \cdot t - n \cdot \pi)) \quad N0_{\text{gd}} - 1 = 255 \quad \text{relerr} = 10\%$$



$$\begin{aligned} \text{length}(u_{m13}) &= 256 \\ \text{fpt}_{\text{so}} &= 687.5 \cdot \text{kHz} \\ \text{Spec13}_{\text{hw}} &:= \text{fft}(u_{m13}) \quad \text{length}(\text{Spec13}_{\text{hw}}) = 129 \\ \ell &:= 0.. \frac{N0_{\text{gd}}}{2} \quad \frac{N0_{\text{gd}}}{2} = 128 \end{aligned}$$



TEST Waveforms

Periodic Waveforms

14 Staircase 4 Voltage Pulse Train

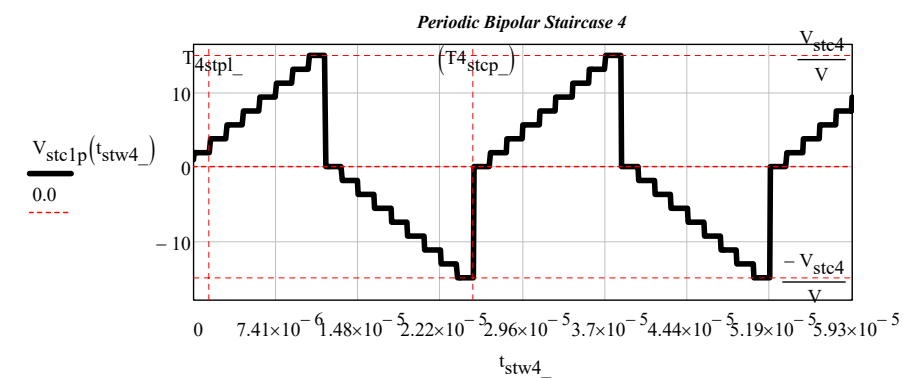
Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps, max number of periods)

To modify data, see the worksheet "staircase 4 pulse data.xmcd"

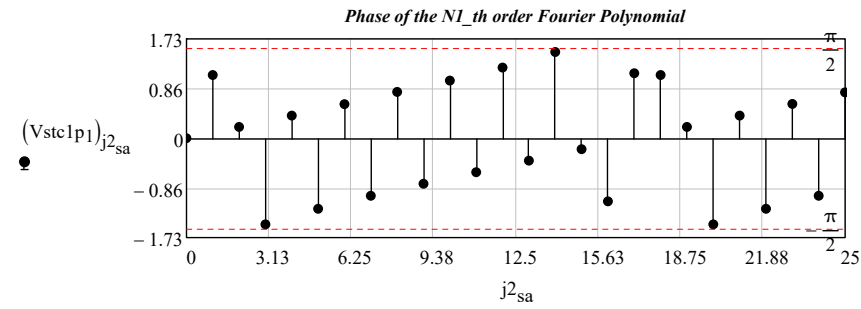
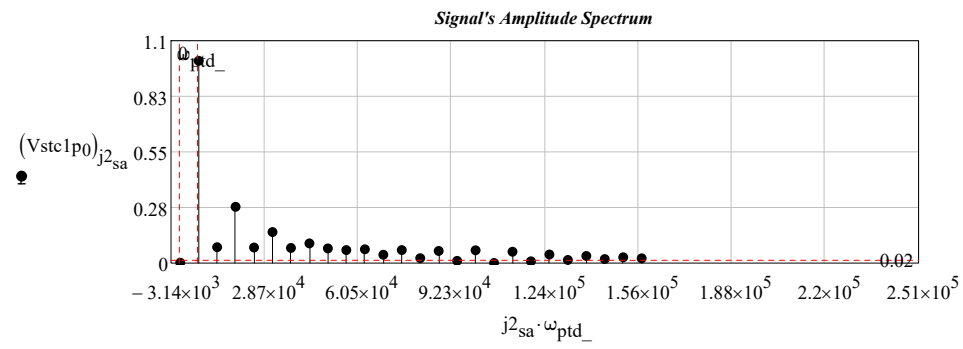
Step Amplitude:	$V_{\text{stc4}} = 15 \text{ V}$
Step length:	$T_{4\text{stp1}} = 1.481 \cdot \mu\text{s}$
Number of steps:	$m_{4\text{steps}} = 8$
Time constant:	$\tau_{4-} = 74.074 \cdot \text{ns}$
Period:	$T_{4\text{stcp}} = 0.025 \cdot \text{ms}$
Frequency:	$f_{44\text{stcp}} = 3.971 \times 10^4 \cdot \text{Hz}$
	$\omega_{44\text{stcp}} = 2.495 \times 10^5 \cdot \frac{\text{rad}}{\text{sec}}$

Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps)

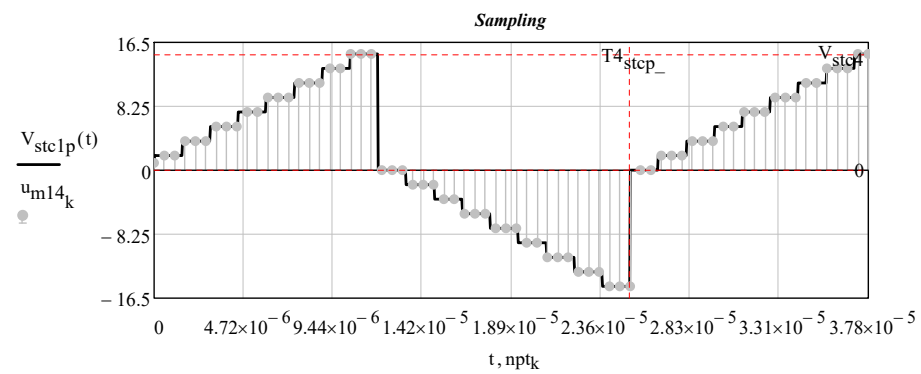
$$V_{\text{stc1p}}(t) := \frac{\text{vstc1p}(t, T_{4\text{stp1}}, V_{\text{stc4}}, m_{4\text{steps}}, N0_{\text{gd}})}{V}$$



$$\begin{aligned} V_{\text{stc1p}} &:= \text{SPCT}(V_{\text{stc1p}}, \text{rt}_{\text{gd}}, N1_, 0 \cdot \text{s}, T_{4\text{stcp}}) & N1_ &= 25 \\ j_{2_{\text{sa}}} &:= 0.. \text{rows}(V_{A0s_0}) - 1 & \omega_{\text{ptd}} &= 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{\text{s}} \end{aligned}$$



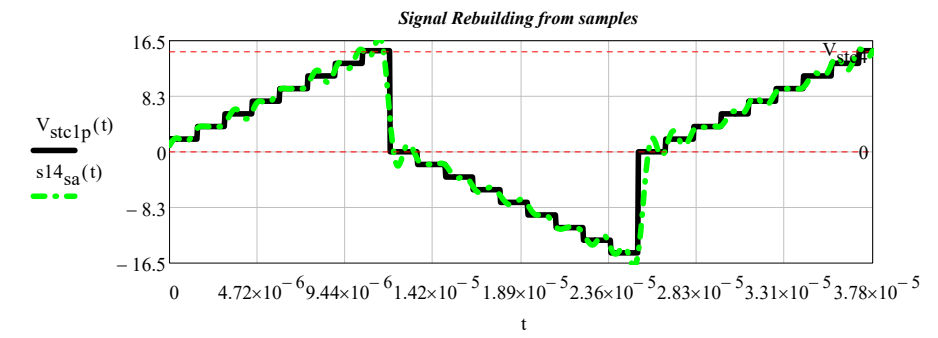
$Bw_{sa} := Vstc1p3 \cdot Hz$
 $Bw_{sa} = 0.913 \cdot MHz$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa}$ $f_{pt_{so}} = 1.826 \cdot MHz$
 $n_{ptk} := \frac{k}{f_{pt_{so}}}$
 Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T4_{stcp_}} = 5.565$
 $u_{m14_k} := Vstc1p(n_{ptk})$

$$u_{m14}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{matrix} 0 & 0.938 & 1.875 & 1.875 & 3.75 & 3.75 & 3.75 & \dots \end{matrix} \end{matrix}$$


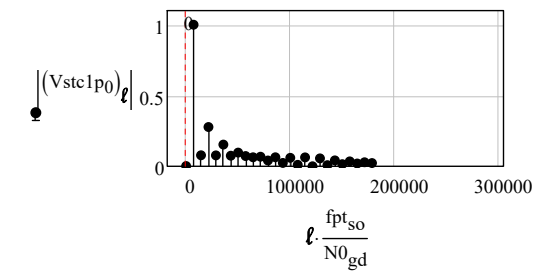
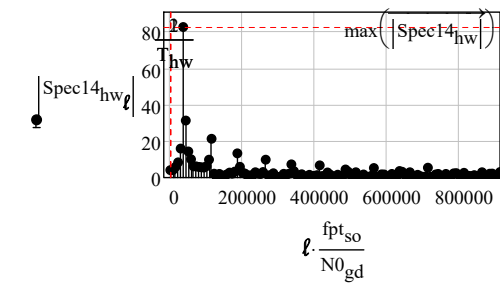
$relerr = 10\%$ $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 5.738 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{14_{sa}}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m14_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right)$ $N0_{gd} - 1 = 255$ $relerr = 10\%$



$length(u_{m14}) = 256$
 $f_{pt_{so}} = 1.826 \times 10^3 \cdot kHz$
 $Spec14_{hw} := \text{fft}(u_{m14})$ $length(Spec14_{hw}) = 129$
 $\ell := 0.. \frac{N0_{gd}}{2} - \frac{N0_{gd}}{2} = 128$



TEST Waveforms

Periodic Waveforms

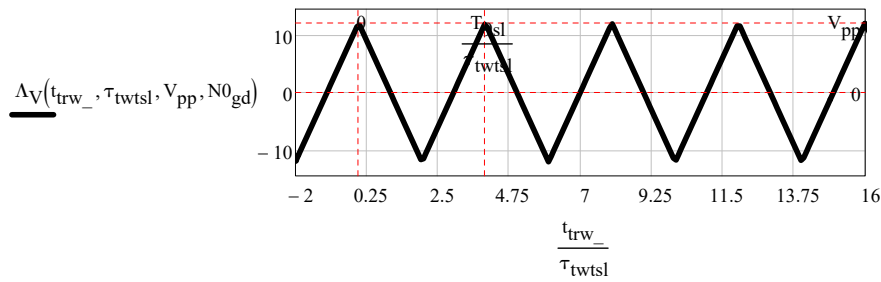
15 Bipolar Triangular Voltage Wave

Description of the Function's parameters : ΔV (time, triangle half base, triangle amplitude, max number of periods)

Time constant: $\tau_{twtsl} := 1 \cdot \mu s$

Period: $T_{9sl} := 4 \cdot \tau_{twtsl}$ $f_{9sl} := \frac{1}{T_{9sl}}$

$$t_{trw_} := -1 \cdot T_{9sl}, -1 \cdot T_{9sl} + \frac{20 \cdot T_{9sl} + 1 \cdot T_{9sl}}{1000} .. 20 \cdot T_{9sl}$$



Bipolar Triangular Voltage Wave Built using the Step Function

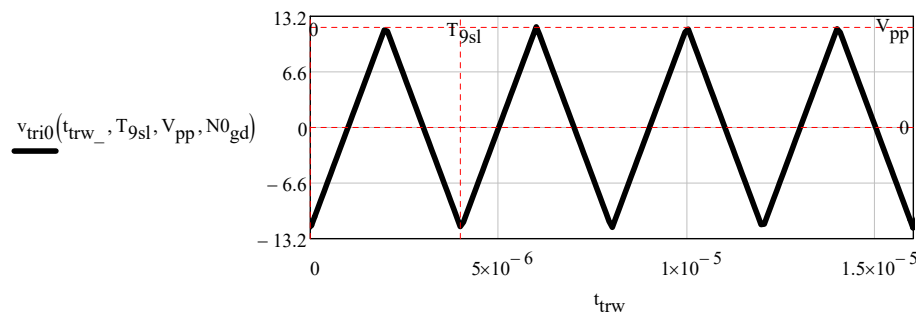
Signal amplitude: $V_{pp} = 12 \cdot V$

Time constant: $\tau_{twtsl} = 1 \cdot \mu s$

Period: $T_{9sl} = 4 \cdot \mu s$

$$\omega_{9sl} := 2 \cdot \pi \cdot f_{9sl} \quad \omega_{9sl} = 1.571 \times 10^6 \frac{\text{rad}}{\text{sec}}$$

$$N0_{gd} = 256$$

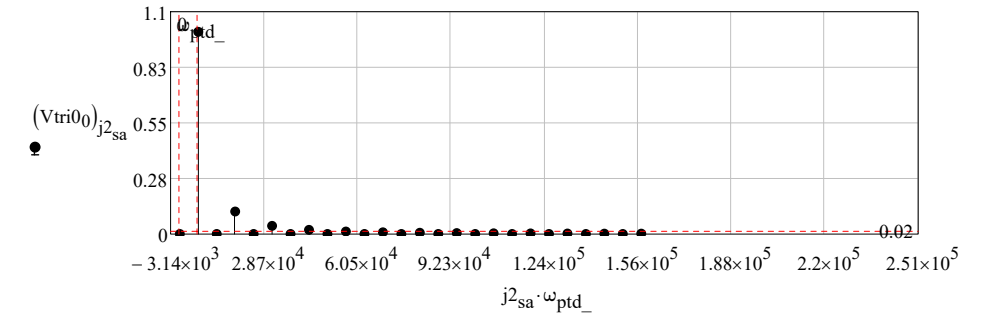


$$V_{tri0}(t) := \frac{v_{tri0}(t, T_{9sl}, V_{pp}, N0_{gd})}{V}$$

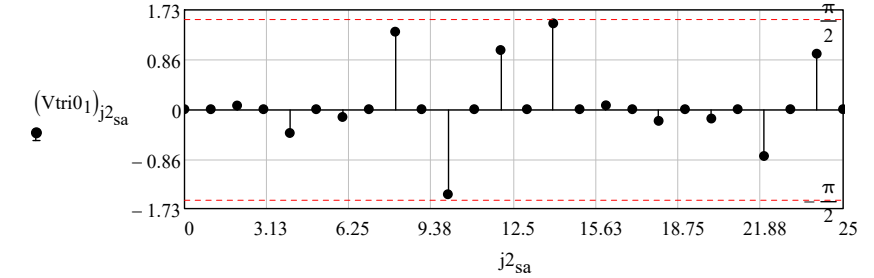
$$V_{tri0} := \text{SPCT}(V_{tri0}, \tau_{gd}, N1_, 0 \cdot s, T_{9sl}) \quad N1_ = 25$$

$$j2_{sa} := 0 .. \text{rows}(V_{tri0}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \frac{\text{Mrads}}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := V_{tri0} \cdot \text{Hz}$$

$$Bw_{sa} = 3.5 \cdot \text{MHz}$$

sampling frequency: $f_{pt_so} := 2 \cdot Bw_{sa}$ $f_{pt_so} = 7 \cdot \text{MHz}$

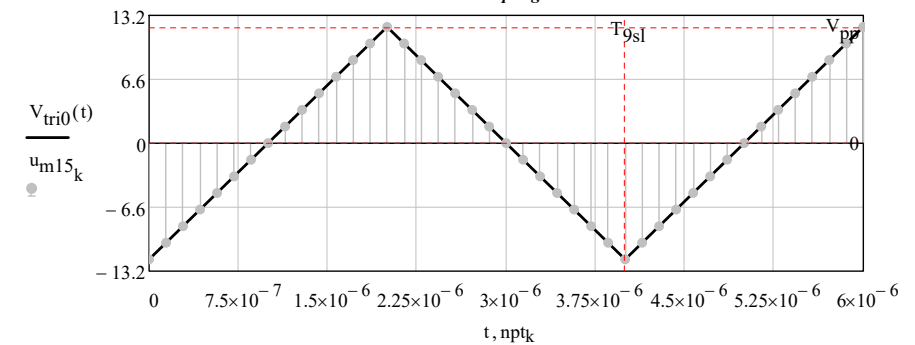
$$npt_k := \frac{k}{f_{pt_so}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_so}} \cdot \frac{1}{T4_{stcp_}} = 1.452$

$$u_{m15_k} := V_{tri0}(npt_k)$$

$$u_{m15}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ \dots \end{matrix} & -12 & -10.286 & -8.571 & -6.857 & \dots \end{matrix}$$

Sampling



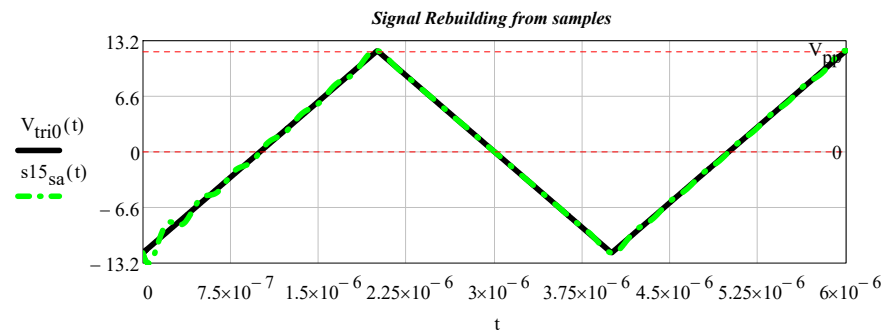
relerr = 10%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 21.991 \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s15_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m15_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \right]$ $N0_{gd} - 1 = 255$ $\text{relerr} = 10\%$

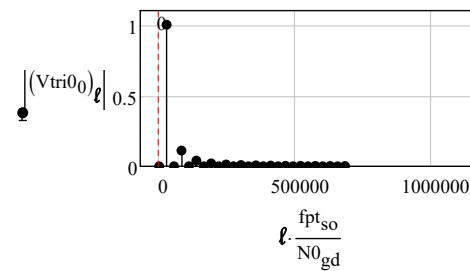
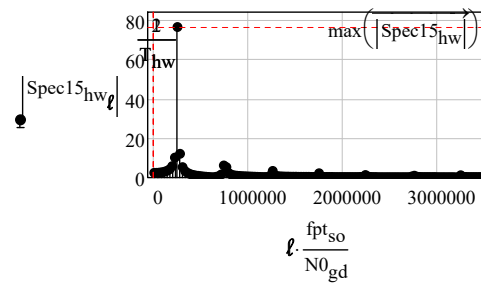


$$\text{length}(u_{m15}) = 256$$

$$f_{pt_{so}} = 7 \times 10^3 \cdot \text{kHz}$$

$$\text{Spec15}_{hw} := \text{fft}(u_{m15}) \text{ length}(\text{Spec15}_{hw}) = 129$$

$$l := 0 \dots \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



TEST Waveforms

Periodic Waveforms

16 Triangular Cusps Voltage Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot \text{V}$

Pulse width: $P_{wsl} := \tau_{ptd} \quad P_{wsl} = 250 \cdot \mu\text{s}$

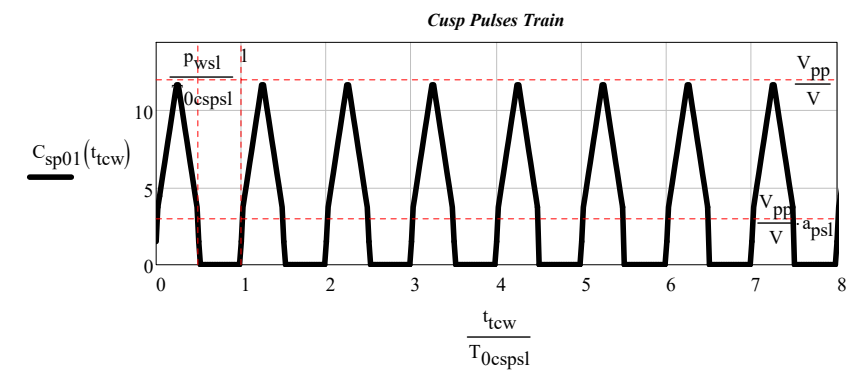
Max pulse amplitude and cusp ratio: $a_{psl} := \frac{1}{4} \quad a_{psl} < 1$

Cusp slope: $c_{ssl} := V_{pp} \cdot \frac{2 \cdot (1 - a_{psl})}{P_{wsl}} \quad c_{ssl} = 0.072 \cdot \frac{\text{V}}{\mu\text{s}}$

Period: $T_{0cspsl} := 2 \cdot P_{wsl}$

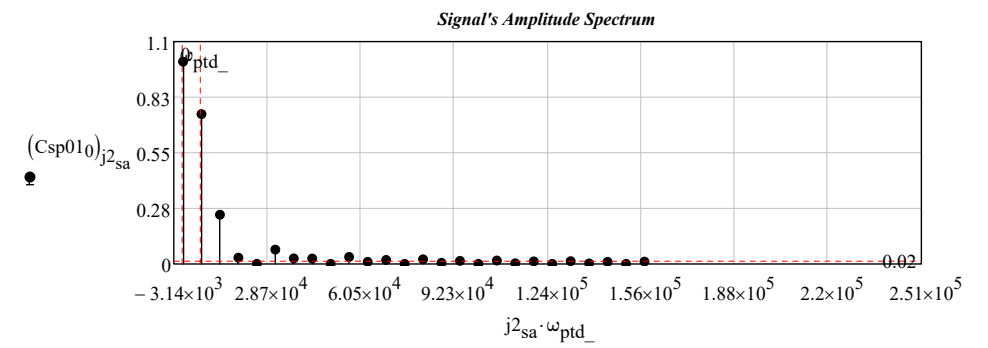
$$t_{tcw} := 0 \cdot T_{0cspsl}, 0 \cdot T_{0cspsl} + \frac{10 \cdot T_{0cspsl} - 0 \cdot T_{0cspsl}}{500} \dots 10 \cdot T_{0cspsl}$$

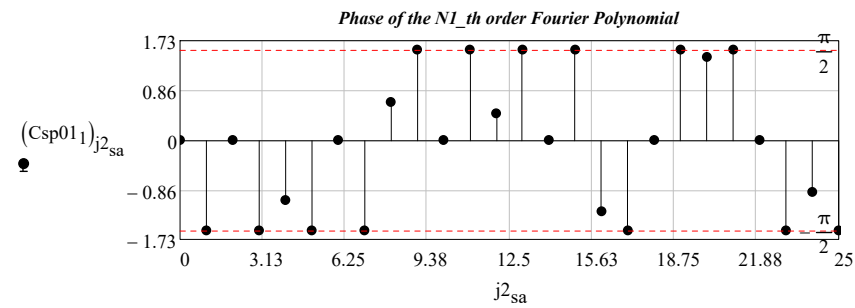
$$C_{sp01}(t) := \frac{\text{csp01}(t, P_{wsl}, a_{psl}, T_{0cspsl}, V_{pp}, N0_{gd})}{V}$$



$$C_{sp01} := \text{SPCT}(C_{sp01}, \tau_{gd}, N1_, 0 \cdot \text{s}, T_{0cspsl}) \quad N1_ = 25$$

$$j2_{sa} := 0 \dots \text{rows}(C_{sp01_0}) - 1 \quad \omega_{ptd} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{\text{s}}$$





$$Bw_{sa} := Csp013 \cdot \text{Hz}$$

$$Bw_{sa} = 0.046 \cdot \text{MHz}$$

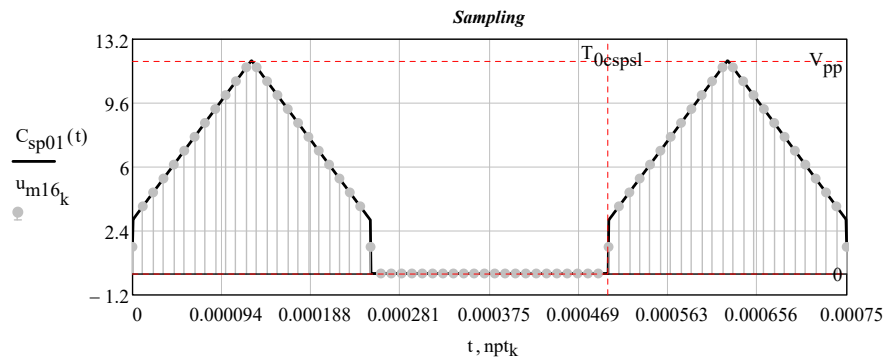
$$\text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.092 \cdot \text{MHz}$$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{step_}} = 110.486$$

$$u_{m16}_k := Csp01(npt_k)$$

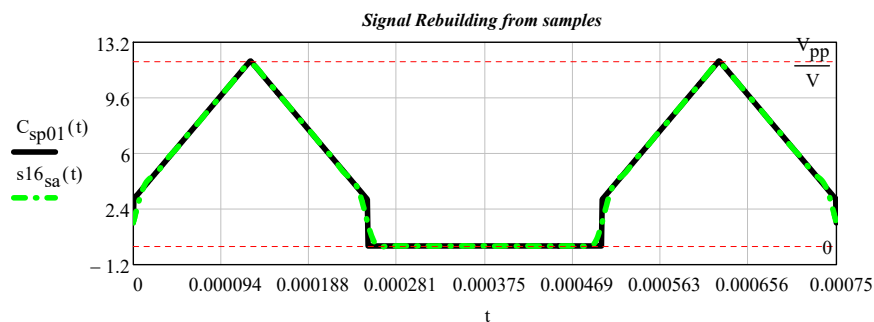
u_{m16}^T	0	1	2	3	4	5	6	7	
	0	1.5	3.783	4.565	5.348	6.13	6.913	7.696	...



$$relerr = 10\% \quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.289 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s16_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m16}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad relerr = 10\%$$

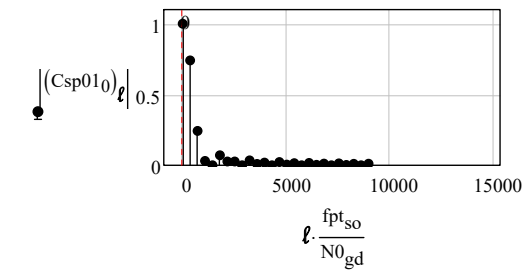
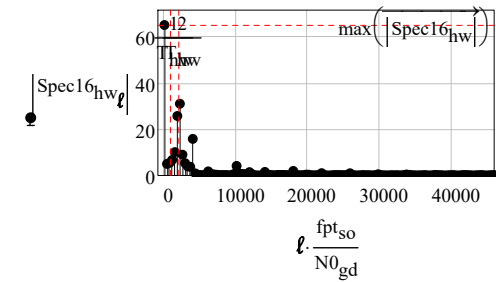


$$\text{length}(u_{m16}) = 256$$

$$f_{pt_{so}} = 92 \cdot \text{kHz}$$

$$\text{Spec16}_{hw} := \text{fft}(u_{m16}) \quad \text{length}(\text{Spec16}_{hw}) = 129$$

$$l := 0 \dots \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$



TEST Waveforms

Periodic Waveforms

17 Bipolar Sawtooth with positive slope Pulse Train

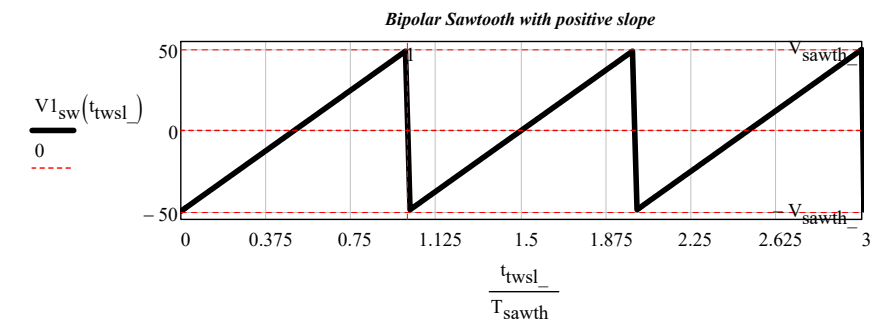
$$\text{Period: } T_{sawth_} := 1 \cdot \delta_{sawth_}$$

$$\text{Frequency: } f_{sawth_} := \frac{1}{T_{sawth_}} \quad f_{sawth_} = 1 \cdot \text{MHz}$$

$$T_{sawth_} = 1 \cdot \mu\text{s} \quad \omega_{sawth_} := 2 \cdot \pi \cdot f_{sawth_} \quad \omega_{sawth_} = 6.283 \cdot \frac{\text{Mrads}}{\text{sec}}$$

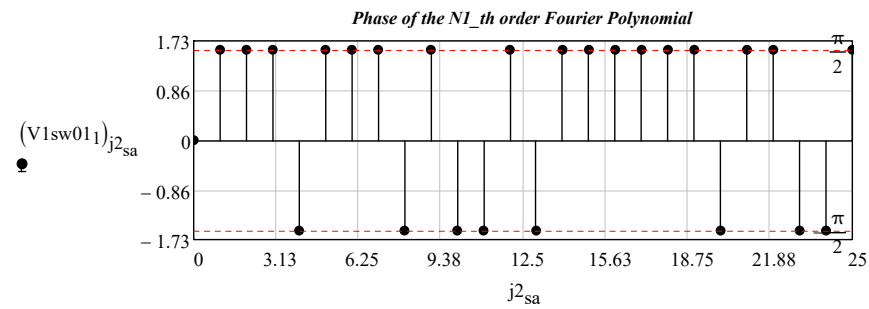
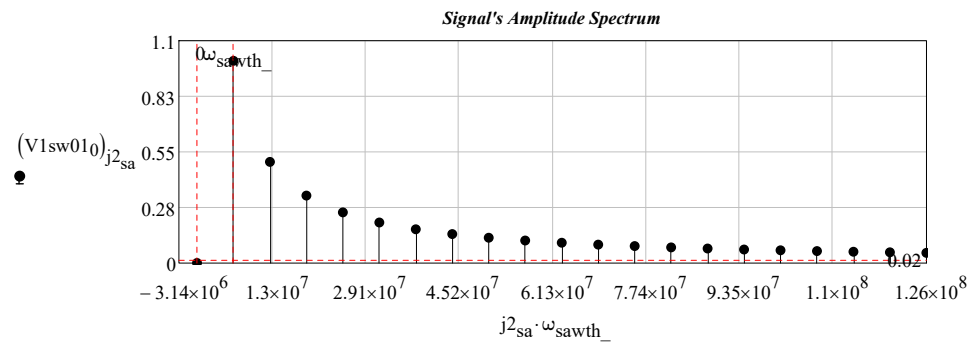
$$t_{twsl_} := 0, \frac{5 \cdot T_{sawth_}}{500} \dots 5 \cdot T_{sawth_}$$

$$V1_{sw}(t) := \frac{v1_{sw}(t, T_{sawth_}, V_{sawth_}, N0_{gd})}{V}$$



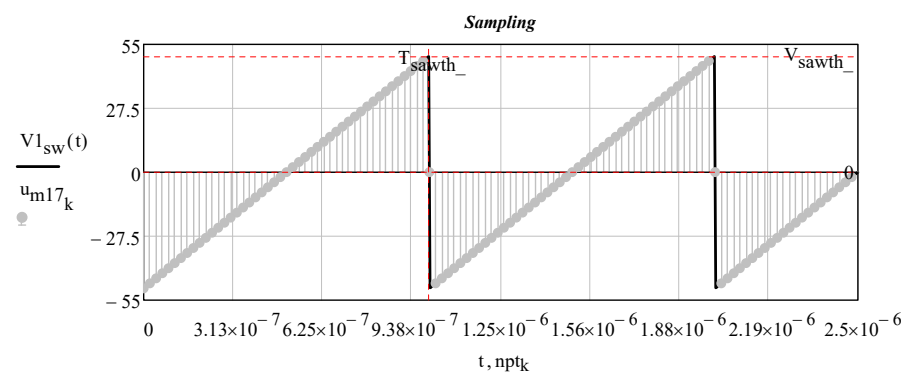
$$V1sw01 := \text{SPCT}(V1_{sw}, rt_{gd}, N1_, 0 \cdot s, T_{sawth_}) \quad N1_ = 25$$

$$j2_{sa} := 0 \dots \text{rows}(V1sw01) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{\text{s}}$$



$Bw_{sa} := V1sw013 \cdot Hz$
 $Bw_{sa} = 23 \cdot MHz$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa}$ $f_{pt_{so}} = 46 \cdot MHz$
 $n_{ptk} := \frac{k}{f_{pt_{so}}}$
 Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{sawth_}} = 5.565$
 $u_{m17_k} := V1_{sw}(n_{ptk})$

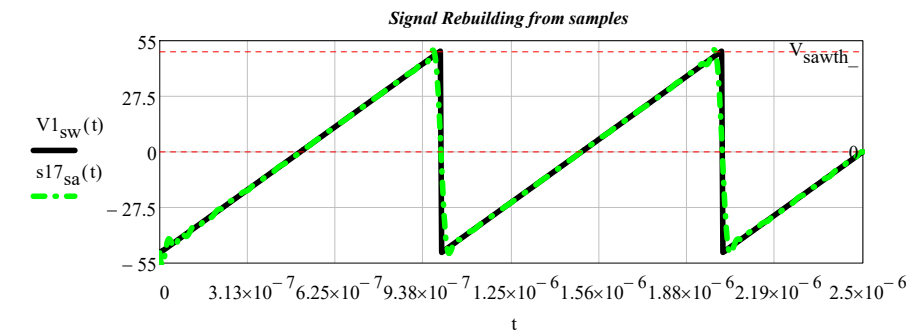
$u_{m17}^T =$	0	1	2	3	4	...
	-50	-47.826	-45.652	-43.478		



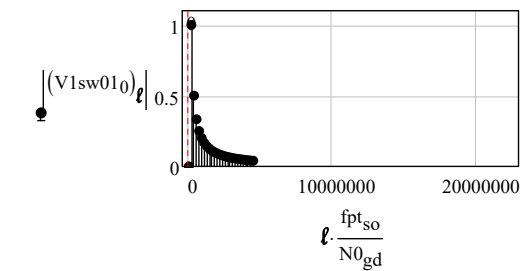
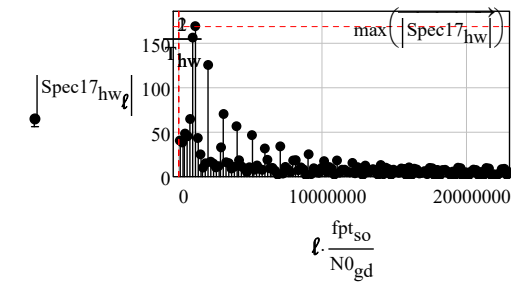
$relerr = 10\%$ $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 144.513 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s17_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m17_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right)$ $N0_{gd} - 1 = 255$ $relerr = 10\%$



$length(u_{m17}) = 256$
 $f_{pt_{so}} = 46 \cdot MHz$
 $Spec17_{hw} := \text{fft}(u_{m17})$ $length(Spec17_{hw}) = 129$
 $\ell := 0.. \frac{N0_{gd}}{2} - \frac{N0_{gd}}{2} = 128$



Periodic Waveforms

18 Bipolar Sawtooth with negative slope Pulse Train

Amplitude: $V_{\text{sawth}_-} = 50 \cdot V$

Sawtooth length: $\delta_{\text{sawth}_-} = 1 \cdot \mu\text{s}$

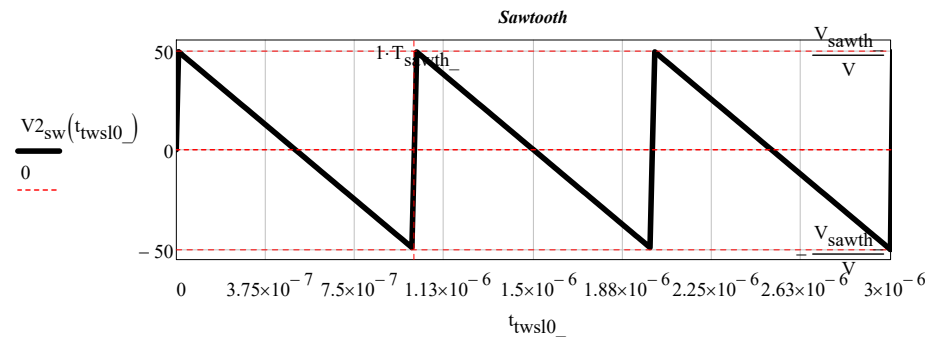
Slope: $sp_{\text{sawth}_-} = 50 \cdot \frac{V}{\mu\text{s}}$

Period: $T_{\text{sawth}_-} = 1 \cdot \mu\text{s}$

Frequency: $\frac{1}{T_{\text{sawth}_-}} = 1 \cdot \text{MHz}$

$$t_{\text{twsl0}_-} := -T_{\text{sawth}_-} \cdot 0, T_{\text{sawth}_-} \cdot 0 + \frac{5 \cdot T_{\text{sawth}_-} + T_{\text{sawth}_-} \cdot 0}{500} \dots 5 \cdot T_{\text{sawth}_-}$$

$$V2_{\text{sw}}(t) := \frac{v2_{\text{sw}}(t, T_{\text{sawth}_-}, V_{\text{sawth}_-}, N0_{\text{gd}})}{V}$$



Dirichlet conditions

A periodic function $s(t)=s(t+T)$, can be expressed by the Fourier series provided that (Dirichlet conditions):

- (1) it is discontinuous and presents a finite number of discontinuities in the period T;
- (2) has a limited average value in the period T;
- (3) it has a finite number of maximum positive or negative.

If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

1) signals of energy for which holds: $\int_{-\infty}^{\infty} (|s_{fs}(t)|)^2 dt < \infty$,

2) power signals for which holds: $\lim_{T \rightarrow \infty} \left[\frac{1}{T} \int_{-T}^T (|s_{fs}(t)|)^2 dt \right] < \infty$

Fourier series definition

$$s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(\omega \cdot k \cdot t) + b_k \cdot \sin(\omega \cdot k \cdot t))$$

The coefficients are defined as follows:

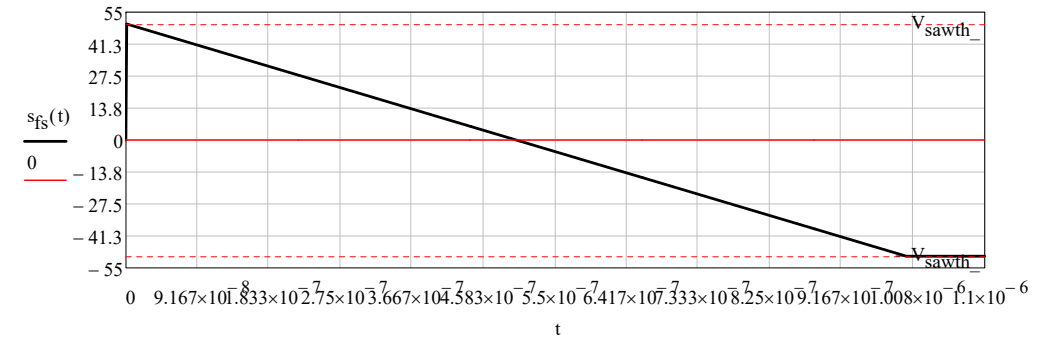
$$\frac{a_0}{2} = A_{fs} = \frac{1}{T} \int_{t_0}^{t_0+T} s_{fs}(t) dt$$

$$a_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t) dt$$

$$b_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t) dt$$

$V_{\text{sawth}_-} = 50V$

$$s_{fs}(t) := 2 \cdot V_{\text{sawth}_-} \cdot \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot (\Phi(t) - \Phi(t - T_{\text{sawth}_-})) - \frac{1}{2}$$



$$\frac{a_0}{2} = A_{fs} = \frac{2 \cdot V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_{t_0}^{t_0+T_{\text{sawth}_-}} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot (\Phi(t) - \Phi(t - T_{\text{sawth}_-})) - \frac{1}{2} dt = \frac{2 \cdot V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{T_{\text{sawth}_-}} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) dt$$

$$\frac{2 \cdot V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{T_{\text{sawth}_-}} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) dt = 0$$

$$a_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t) dt = 2 \cdot \frac{V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{T_{\text{sawth}_-}} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot \cos(\omega \cdot k \cdot t) dt$$

$$2 \cdot \frac{V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{T_{\text{sawth}_-}} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot \cos(\omega \cdot k \cdot t) dt = \frac{2 \cdot V_{\text{sawth}_-} \cdot \left(4 \cdot \sin\left(\frac{T_{\text{sawth}_- \cdot \omega \cdot k}{2}\right)^2 - T_{\text{sawth}_- \cdot \omega \cdot k \cdot \sin(T_{\text{sawth}_- \cdot \omega \cdot k)} \right)}{T_{\text{sawth}_-^2 \cdot \omega^2 \cdot k^2}$$

$$a_k = \frac{2 \cdot V_{\text{sawth}_-} \cdot \left(4 \cdot \sin\left(\frac{T_{\text{sawth}_- \cdot \omega \cdot k}{2}\right)^2 - T_{\text{sawth}_- \cdot \omega \cdot k \cdot \sin(T_{\text{sawth}_- \cdot \omega \cdot k)} \right)}{T_{\text{sawth}_-^2 \cdot \omega^2 \cdot k^2}$$

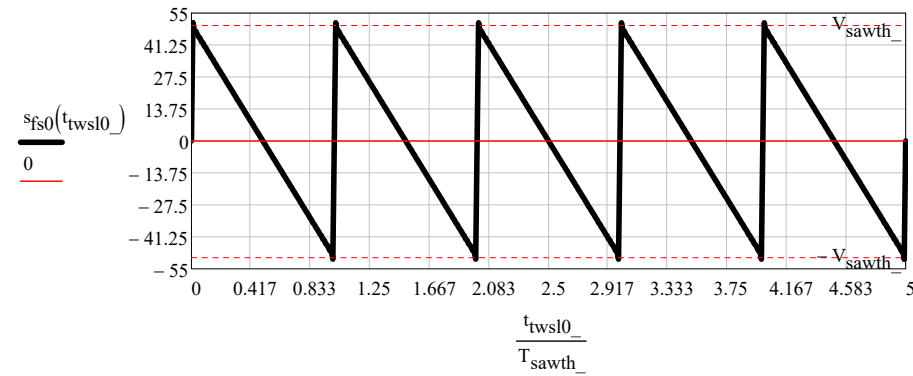
$$b_k = \frac{2}{T} \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t) dt = 2 \cdot \frac{V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{t_0+T} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot \sin(\omega \cdot k \cdot t) dt$$

$$2 \cdot \frac{V_{\text{sawth}_-}}{T_{\text{sawth}_-}} \int_0^{t_0+T} \left(\frac{-t}{T_{\text{sawth}_-}} + 1 \right) \cdot \sin(\omega \cdot k \cdot t) dt = \left(\cos\left(\frac{T_{\text{sawth}_- \cdot \omega \cdot k}{2}\right)^2 - \frac{\sin(T_{\text{sawth}_- \cdot \omega \cdot k)}{T_{\text{sawth}_- \cdot \omega \cdot k}} \right) \cdot \frac{4 \cdot V_{\text{sawth}_-}}{(T_{\text{sawth}_- \cdot \omega \cdot k)}$$

$$b_k = \left(\cos\left(\frac{T_{\text{sawth}_- \cdot \omega \cdot k}{2}\right)^2 - \frac{\sin(T_{\text{sawth}_- \cdot \omega \cdot k)}{T_{\text{sawth}_- \cdot \omega \cdot k}} \right) \cdot \frac{4 \cdot V_{\text{sawth}_-}}{(T_{\text{sawth}_- \cdot \omega \cdot k)}$$

$$\omega_{sf} := \omega_{sawth_}$$

$$s_{fs0}(t) := \frac{2 \cdot V_{sawth_}}{(T_{sawth_} \cdot \omega_{sf})} \sum_{k=1}^{N0_{gd}} \left[\frac{\left(4 \cdot \sin\left(\frac{T_{sawth_} \cdot \omega_{sf} \cdot k}{2}\right)^2 - T_{sawth_} \cdot \omega_{sf} \cdot k \cdot \sin(T_{sawth_} \cdot \omega_{sf} \cdot k) \right)}{T_{sawth_} \cdot \omega_{sf} \cdot k} \cos(\omega_{sf} \cdot k \cdot t) \dots \right. \\ \left. + \left(\cos\left(\frac{T_{sawth_} \cdot \omega_{sf} \cdot k}{2}\right) - \frac{\sin(T_{sawth_} \cdot \omega_{sf} \cdot k)}{T_{sawth_} \cdot \omega_{sf} \cdot k} \right) \frac{2}{k} \cdot \sin(\omega_{sf} \cdot k \cdot t) \right]$$

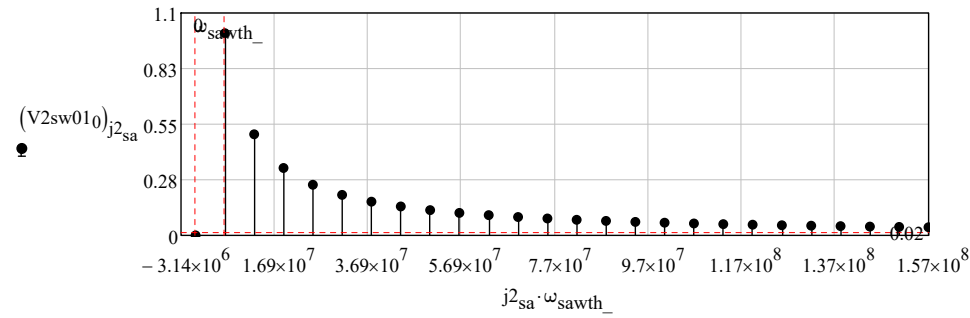


$$V_{sawth_} = 50 \text{ V}$$

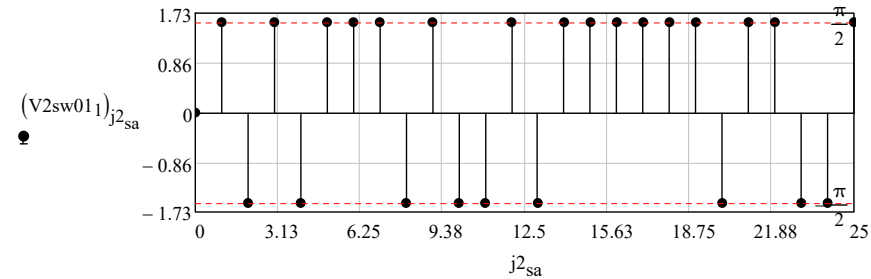
$$V2sw01 := \text{SPCT}(V2_{sw}, rt_{gd}, N1_, 0 \cdot s, T_{sawth_}) \quad N1_ = 25$$

$$j2_{sa} := 0 .. \text{rows}(V2sw01_0) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \frac{\text{Mrads}}{\text{s}}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := V2sw01_3 \cdot \text{Hz}$$

$$Bw_{sa} = 23 \cdot \text{MHz}$$

$$\text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 46 \cdot \text{MHz}$$

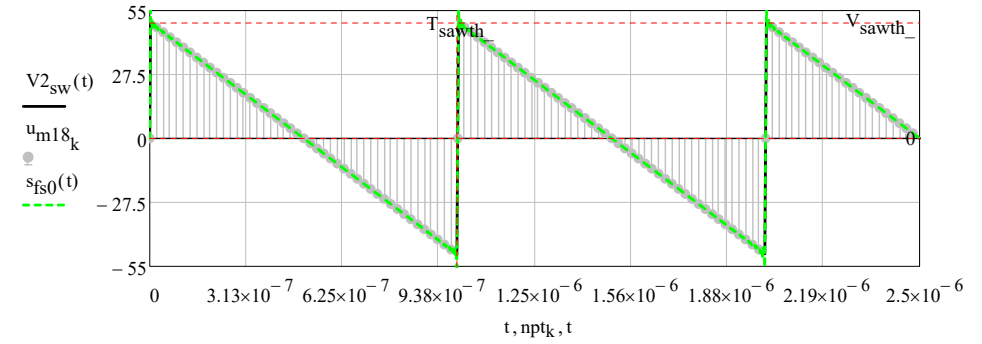
$$npt_k := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}} \cdot T_{sawth_}} = 5.565$$

$$u_{m18}_k := V2_{sw}(npt_k)$$

0	0	1	2	3	4	...
0	0	47.826	45.652	43.478

Comparison among the given signal, the sampled and the calculated series



$$\text{relerr} = 10\%$$

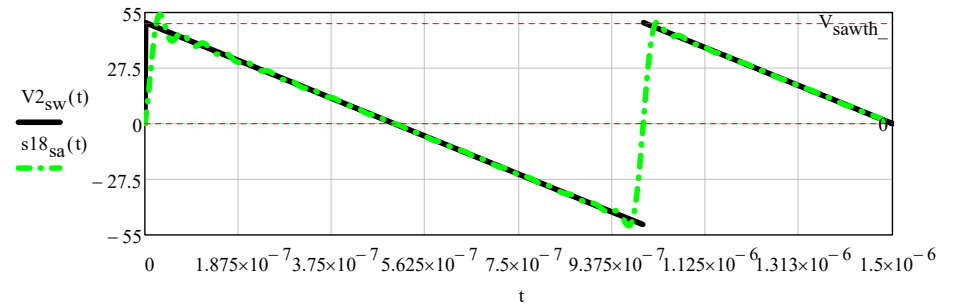
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 144.513 \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s18_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m18}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$

Signal Rebuilding from samples

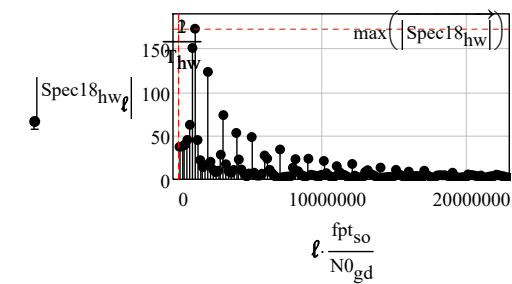


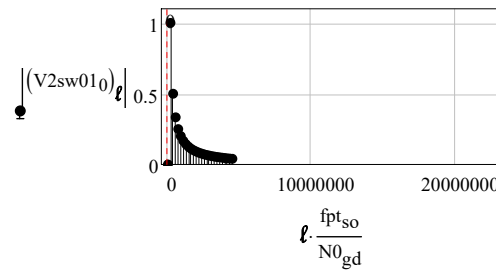
$$\text{length}(u_{m18}) = 256$$

$$f_{pt_{so}} = 46 \cdot \text{MHz}$$

$$\text{Spec18}_{hw} := \text{fft}(u_{m18}) \quad \text{length}(\text{Spec18}_{hw}) = 129$$

$$l := 0 .. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

19 Bipolar Sawtooth with adjustable rising and falling edges Pulse Train

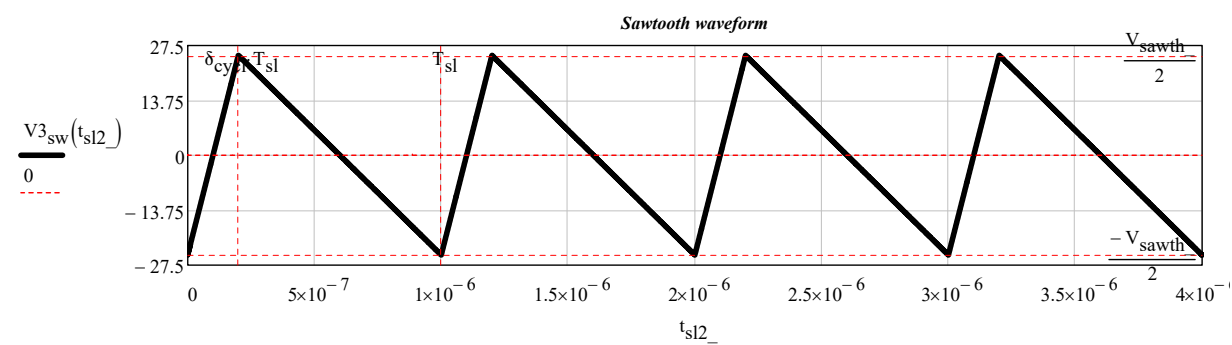
$$\delta_{cycl} \cdot T_{sl} = 200 \cdot \text{ns}$$

$$T_{sl} = 1 \cdot \mu\text{s} \quad f_{3sw} := \frac{1}{T_{sl}} \quad \omega_{3sw} := 2 \cdot \pi \cdot f_{3sw} \quad \omega_{3sw} = 6.283 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\delta_{cycl} = 20\%$$

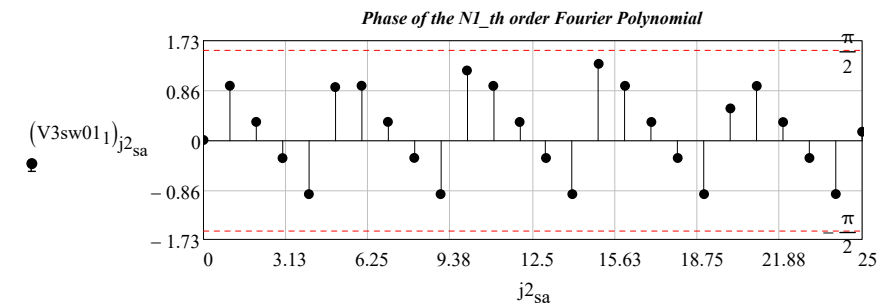
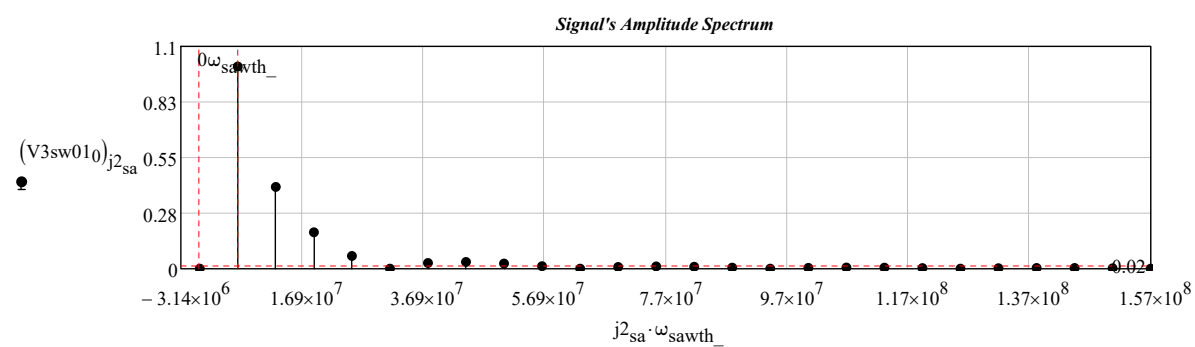
$$t_{sl2_} := 0 \cdot T_{sl}, 0 \cdot T_{sl} + \frac{4 \cdot T_{sl}}{10000} \dots 4 \cdot T_{sl}$$

$$V_{3sw}(t) := \frac{V_s(t \cdot \text{sec}^{-1}, T_{sl} \cdot \text{sec}^{-1}, \delta_{cycl}, V_{sawth_}, N_{gd})}{V}$$



$$V_{3sw01} := \text{SPCT}(V_{3sw}, rt_{gd}, N1_, 0 \cdot s, T_{sawth_}) \quad N1_ = 25$$

$$j2_{sa} := 0 \dots \text{rows}(V_{3sw01}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$$



$$Bw_{sa} := V_{3sw013} \cdot \text{Hz}$$

$$Bw_{sa} = 13 \cdot \text{MHz}$$

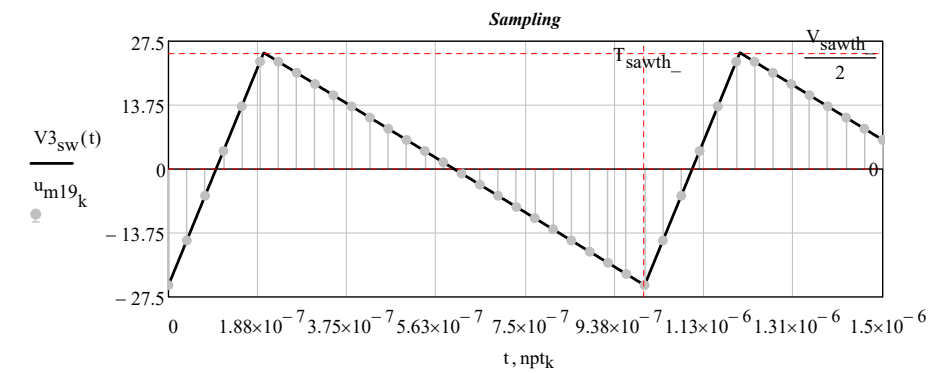
$$\text{sampling frequency: } f_{pt_so} := 2 \cdot Bw_{sa} \quad f_{pt_so} = 26 \cdot \text{MHz}$$

$$n_{ptk} := \frac{k}{f_{pt_so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_so}} \cdot \frac{1}{T_{sawth_}} = 9.846$$

$$u_{m19}_k := V_{3sw}(n_{ptk})$$

$u_{m19}^T =$	0	1	2	3	4	5	6
	-25	-15.385	-5.769	3.846	13.462	23.077	...



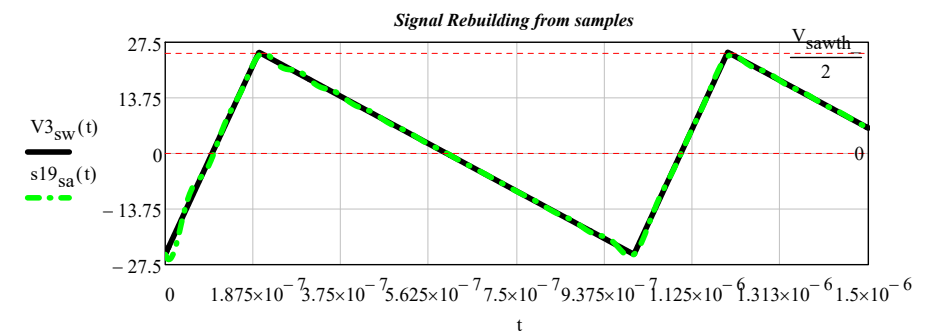
$$\text{relerr} = 10\%$$

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 81.681 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s_{19}_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m19}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$



TEST Waveforms

Periodic Waveforms

2D Dimensionless (RC) Pulse Train

$$T_{rcsl} := T_{0gd} \quad \alpha_{rcsl} := 0.3 \quad Bw_{rcsl} := \frac{1 + \alpha_{rcsl}}{2 \cdot T_{0gd}}$$

$$T_{0gd} = 1 \times 10^3 \cdot \mu s \quad Bw_{rcsl} = 650 \cdot Hz$$

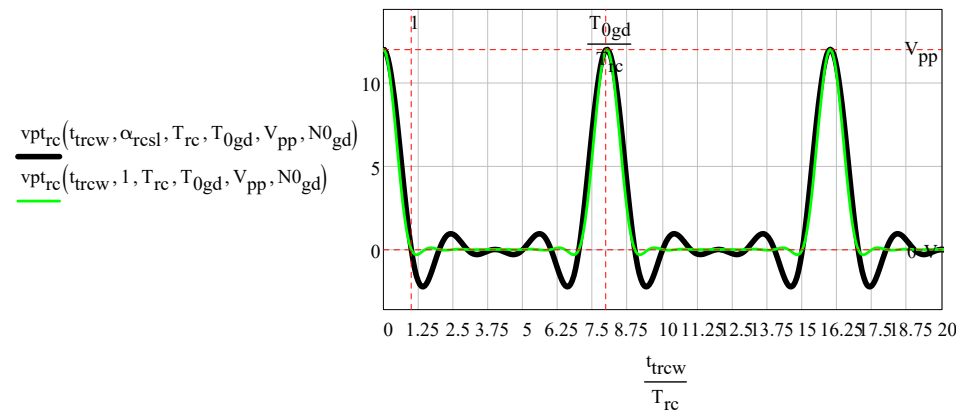
$$T_{rc} := \frac{T_{0gd}}{8}$$

$$V_{pt0rc}(t) := \frac{v_{ptrc}(t, \alpha_{rcsl}, T_{rc}, T_{0gd}, V_{pp}, N_{0gd})}{V}$$

$$\alpha_{rcsl} = 0.3$$

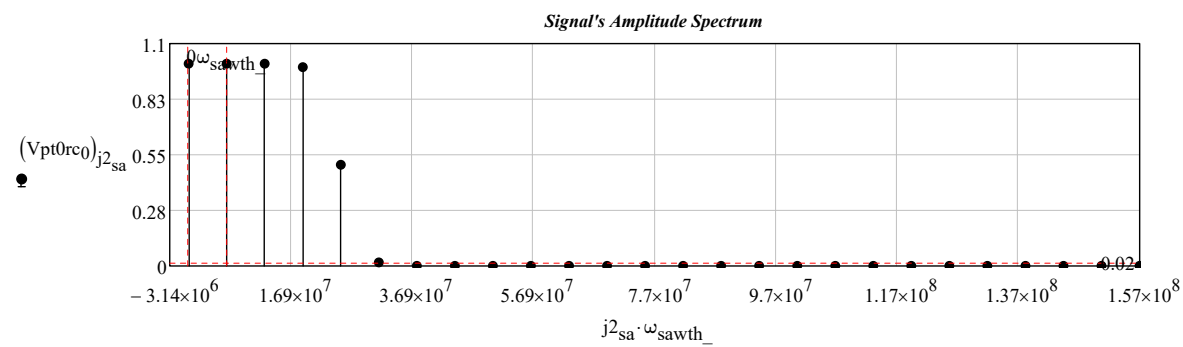
$$t_{trcw} := -20 \cdot T_{rc}, -20 \cdot T_{rc} + \frac{40 \cdot T_{rc} + 20 \cdot T_{rc}}{1000} \dots 40 \cdot T_{rc}$$

Raised-Cosine (RC) Pulse Train for $\alpha=0.3$ and $\alpha=1$

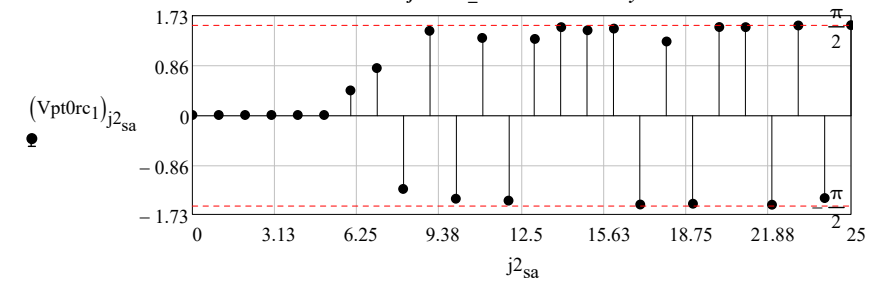


$$\alpha_{rcsl} = 0.3 \quad V_{pt0rc} := SPCT(V_{pt0rc}, t_{trcw}, N_{1_}, 0 \cdot s, T_{0gd}) \quad N_{1_} = 25$$

$$j_{2sa} := 0 \dots \text{rows}(V_{pt0rc}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$$



Phase of the $N_{1_}$ th order Fourier Polynomial



$$Bw_{sa} := V_{pt0rc} \cdot Hz$$

$$Bw_{sa} = 5 \times 10^{-3} \cdot MHz$$

$$\text{sampling frequency: } f_{pt_so} := 2 \cdot Bw_{sa} \quad f_{pt_so} = 0.01 \cdot MHz$$

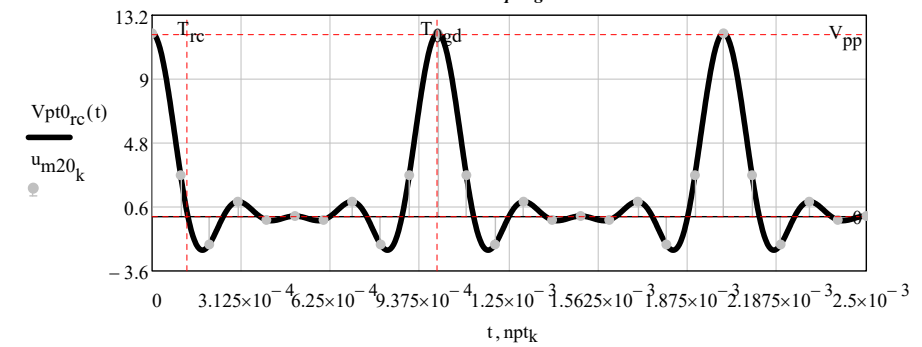
$$n_{ptk} := \frac{k}{f_{pt_so}}$$

$$\text{Frequency resolution: } \frac{N_{0gd}}{f_{pt_so}} \cdot \frac{1}{T_{rc}} = 204.8$$

$$u_{m20k} := V_{pt0rc}(n_{ptk})$$

$u_{m20}^T =$	0	1	2	3	4	...
	12	2.678	-1.871	0.944		

Sampling



$$\text{relerr} = 10\%$$

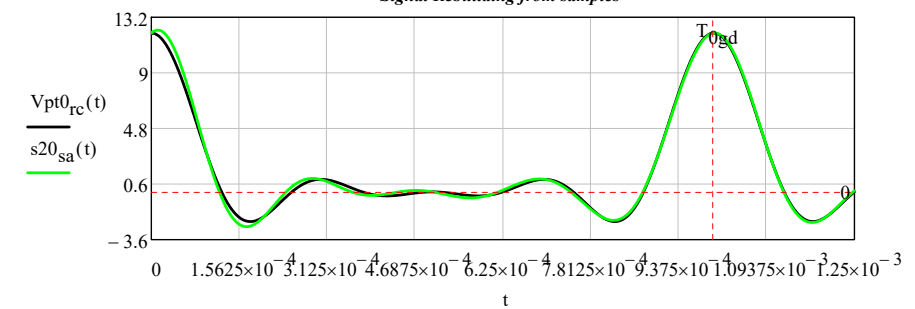
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.031 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s_{20sa}(t) := \sum_{n=0}^{N_{0gd}-1} (u_{m20n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N_{0gd} - 1 = 255 \quad \text{relerr} = 10\%$$

Signal Rebuilding from samples



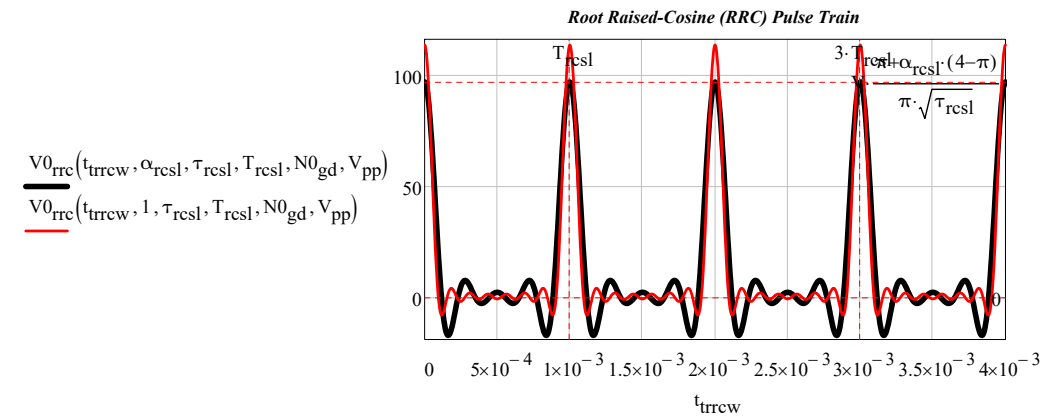
TEST Waveforms

Periodic Waveforms

21 Root Raised-Cosine (RC) Pulse Train

$$\tau_{rctl} := \frac{T_{0gd}}{8} \quad T_{0gd} = 1 \times 10^3 \cdot \mu s \quad Bw_{rctl} = 650 \cdot Hz$$

$$V_{0rrc}(t_{trcw}, \alpha_{rctl}, \tau_{rctl}, T_{rctl}, N_{0gd}, V_{pp}) := V_{rrc}\left(t_{trcw}, \alpha_{rctl}, \tau_{rctl}, T_{rctl}, N_{0gd}, \frac{V_{pp}}{V}\right)$$



$$\frac{V_{0rrc}(t_{trcw}, \alpha_{rctl}, \tau_{rctl}, T_{rctl}, N_{0gd}, V_{pp})}{V_{0rrc}(t_{trcw}, 1, \tau_{rctl}, T_{rctl}, N_{0gd}, V_{pp})}$$

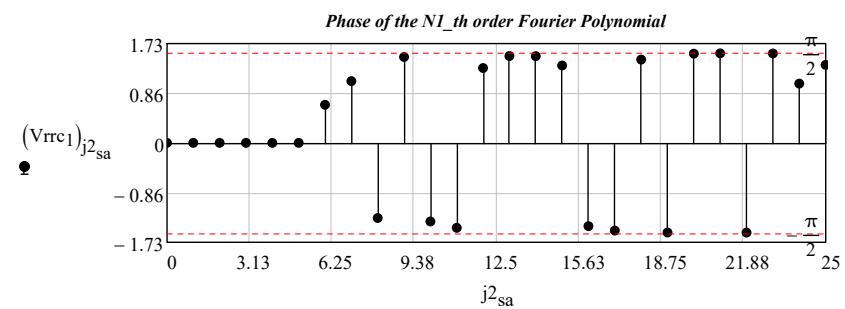
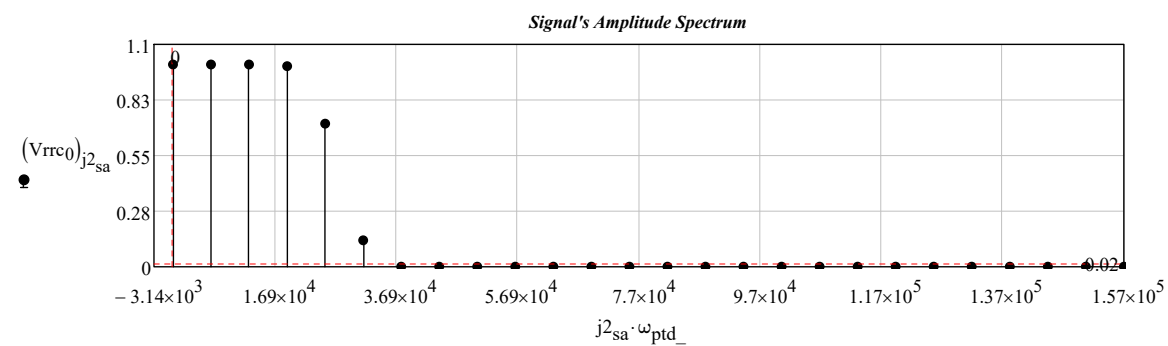
$$vrrc(t) := V_{rrc}\left(t, \alpha_{rctl}, \tau_{rctl}, T_{rctl}, N_{0gd}, \frac{V_{pp}}{V}\right)$$

$$t_{trcw} := -20 \cdot T_{rctl}, -20 \cdot T_{rctl} + \frac{20 \cdot T_{rctl} + 20 \cdot T_{rctl}}{1000} .. 20 \cdot T_{rctl}$$

$$V_{rrc} := SPCT(vrrc, rt_{gd}, N1_, 0 \cdot s, T_{rctl})$$

$$N1_ = 25$$

$$j2_{sa} := 0 .. \text{rows}(V_{pt0rc0}) - 1 \quad \omega_{ptd} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$$



$$Bw_{sa} := Vrc3 \cdot Hz$$

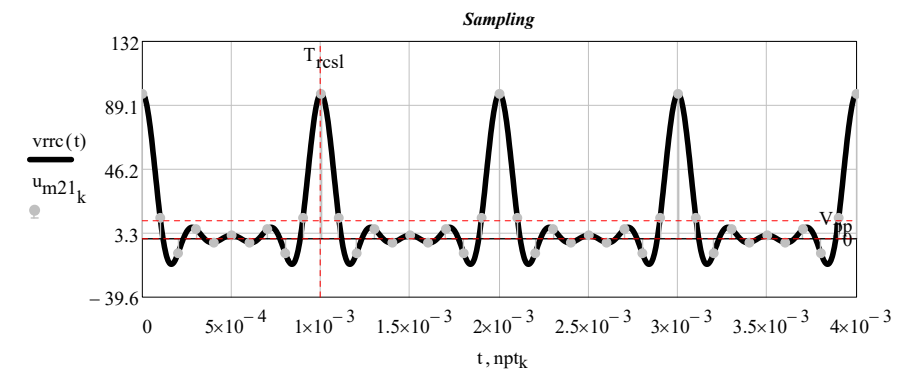
$$Bw_{sa} = 5 \times 10^{-3} \cdot MHz$$

$$\text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.01 \cdot MHz$$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N_{0gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{rctl}} = 25.6$$

$$u_{m21_k} := vrrc(n_{ptk})$$

$$u_{m21}^T = \begin{bmatrix} 0 & 96.801 & 13.619 & -10.131 & 6.083 & -3.022 & 1.904 & \dots \\ 0 & 96.801 & 13.619 & -10.131 & 6.083 & -3.022 & 1.904 & \dots \end{bmatrix}$$


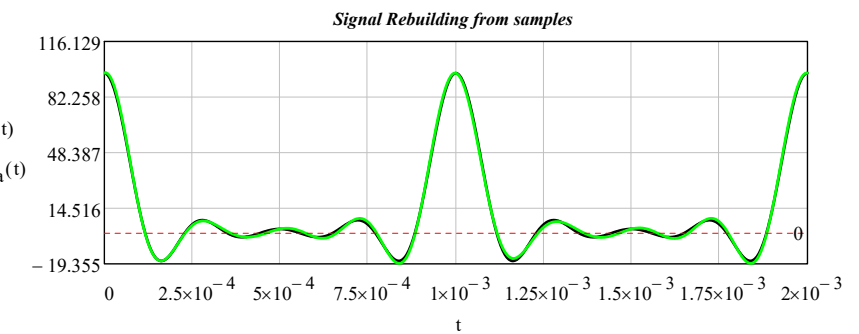
$$relerr = 10\%$$

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.031 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s21_{sa}(t) := \sum_{n=0}^{N_{0gd}-1} (u_{m21_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N_{0gd} - 1 = 255 \quad relerr = 10\%$$



TEST Waveforms

Periodic Waveforms

22AM test signal (single tone)

Carrier Amplitude: $A1_{sl} := 20 \cdot \text{volt}$

Modulating signal's amplitude: $B1_{sl} := 12 \cdot \text{volt}$ $\omega_{0gd} = 6.283 \cdot \frac{\text{krads}}{\text{s}}$

$$\omega_{1csl} := 15 \cdot \omega_{0gd} \quad T1_{csl} := \frac{2 \cdot \pi}{\omega_{1csl}} \quad \omega_{1msl} := \frac{\omega_{1csl}}{10} \quad T1_{msl} := \frac{2 \cdot \pi}{\omega_{1msl}} \quad f1_{msl} := \frac{\omega_{1msl}}{2 \cdot \pi} \quad f15sl := \frac{\omega_{1csl}}{2 \cdot \pi}$$

$$\omega_{1csl} = 94.248 \cdot \frac{\text{krads}}{\text{sec}} \quad \frac{\omega_{1csl}}{\omega_{1msl}} = 10$$

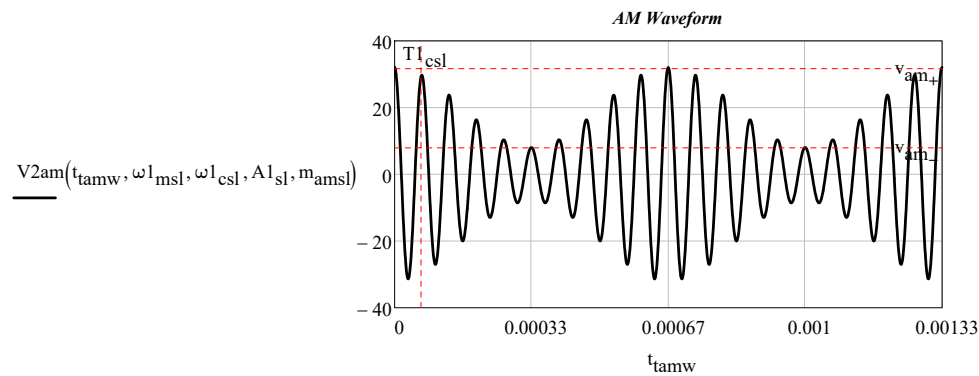
$$v_{am+} := A1_{sl} + B1_{sl} \quad v_{am-} := A1_{sl} - B1_{sl} \quad A1_{sl} = v_{am+} + v_{am-} \quad B1_{sl} = v_{am+} - v_{am-}$$

$$v_{am+} = 32 \cdot \text{volt}$$

$$v_{am-} = 8 \cdot \text{volt}$$

$$m_{amsl} := \frac{v_{am+} - v_{am-}}{v_{am+} + v_{am-}} \quad m_{amsl} = 60\% \quad \frac{B1_{sl}}{A1_{sl}} = 60\%$$

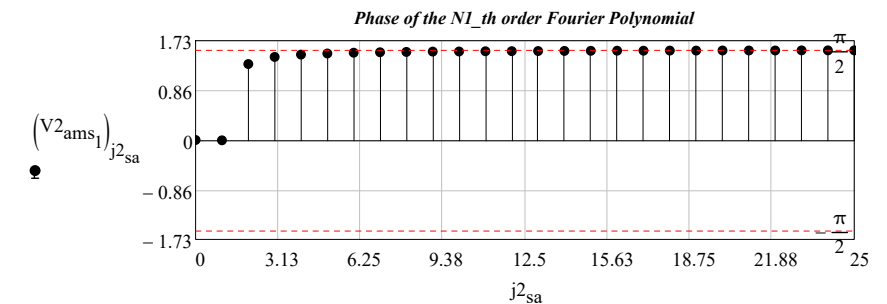
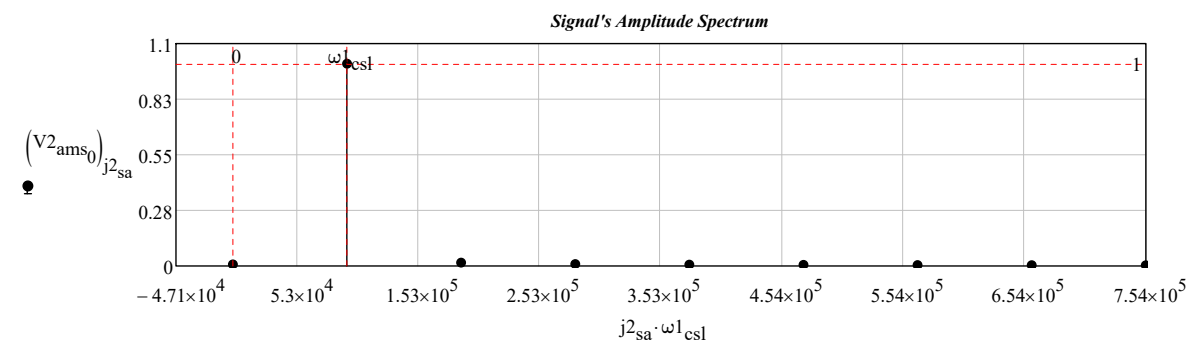
$$t_{tamw} := -T0gd \cdot 3, -T0gd \cdot 3 + \frac{20 \cdot T1_{csl} + T0gd \cdot 3}{5000} \dots 20 \cdot T1_{csl}$$



$$N1_ := 50 \quad V2_{am}(t) := V2am(t, \omega_{1msl}, \omega_{1csl}, A1_{sl}, m_{amsl})$$

$$V2_{ams} := SPCT(V2_{am}, rt_{gd}, N1_, 0 \cdot s, T1_{csl}) \quad N1_ = 25$$

$$j2_{sa} := 0 \dots \text{rows}(Vpt0rc0) - 1 \quad \omega_{ptd} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{\text{s}}$$



$$Bw_{sa} := V2_{ams3} \cdot \text{Hz}$$

$$Bw_{sa} = 0.075 \cdot \text{MHz}$$

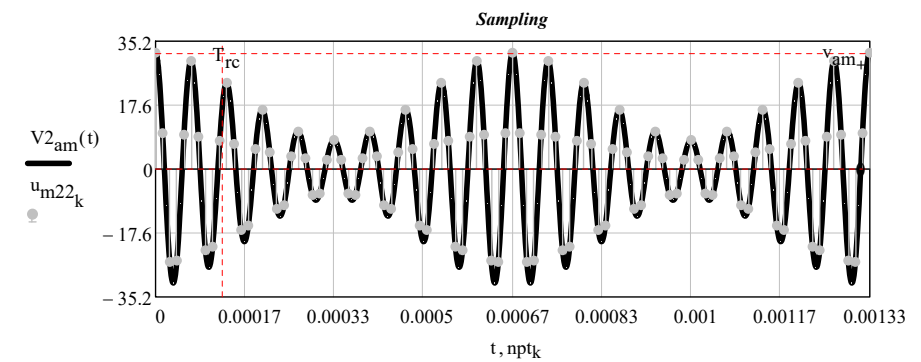
$$\text{sampling frequency: } fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.15 \cdot \text{MHz}$$

$$npt_k := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{rc}} = 13.653$$

$$u_{m22_k} := V2_{am}(npt_k)$$

$u_{m22}^T =$	0	1	2	3	4	5	6
	32	25.869	9.859	-9.823	-25.584	-31.413	...



trace 1	1	2	4	line
trace 2	2	1	1	ste

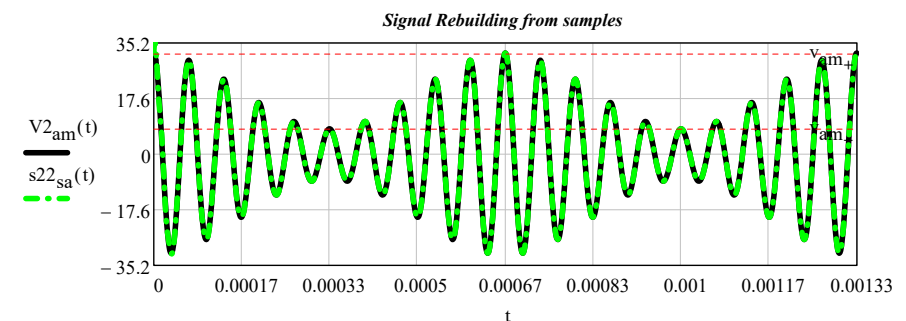
$$\text{relerr} = 10\%$$

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.471 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula} \quad s22_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m22_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$



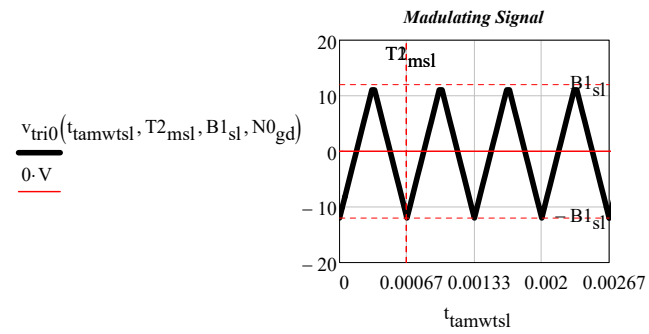
TEST Waveforms

Periodic Waveforms

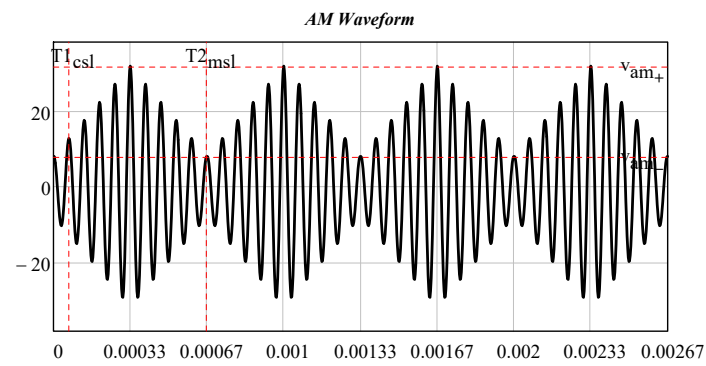
23AM test signal (triangular wave)

$$\omega_{2msl} := \frac{\omega_{1csl}}{10} \quad T_{2msl} := \frac{2 \cdot \pi}{\omega_{2msl}}$$

$$t_{tamwtsl} := 0 \cdot \text{sec}, 40 \cdot \frac{T_{2msl}}{1000} \dots 40 \cdot T_{2msl}$$



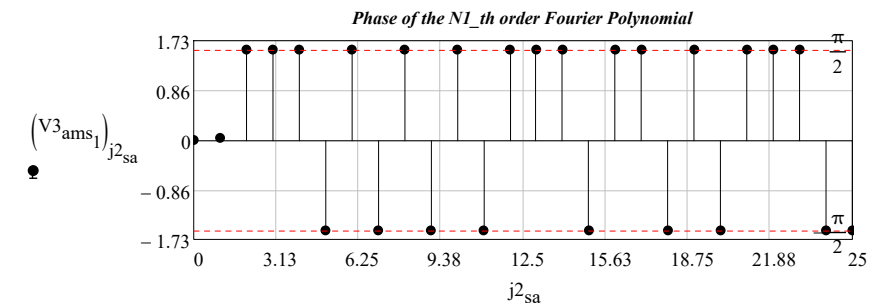
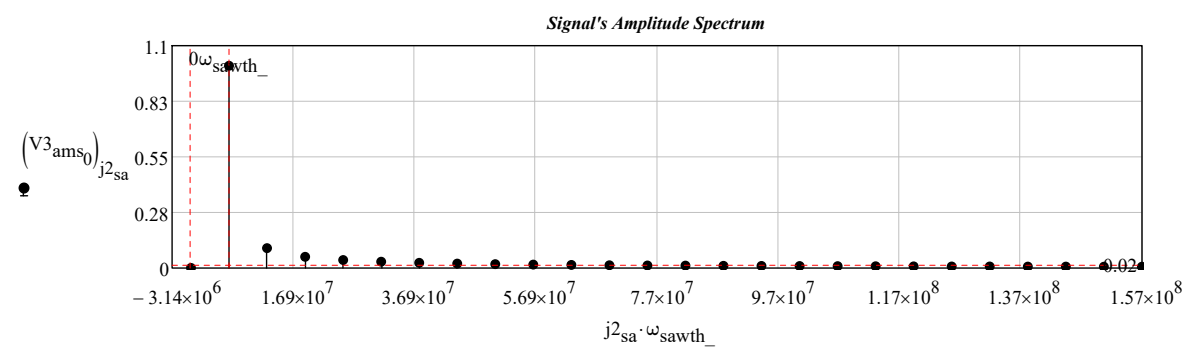
$$t_{twsl} := -T_{0gd} \cdot 3, -T_{0gd} \cdot 3 + \frac{8 \cdot T_{2msl} + T_{0gd} \cdot 3}{500} \dots 8 \cdot T_{2msl}$$



$$V_{3am}(t) := V_{3am}(t, \omega_{1msl}, \omega_{1csl}, m_{amsl}, A_{1sl}, B_{1sl}, N_{0gd})$$

$$V_{3ams} := \text{SPCT}(V_{3am}, rt_{gd}, N1, 0 \cdot s, T1_{csl}) \quad N1 = 25$$

$$j_{2sa} := 0 \dots \text{rows}(V_{pt0rc0}) - 1 \quad \omega_{ptd} = 6.283 \times 10^{-3} \frac{\text{Mrads}}{s}$$



$$Bw_{sa} := V_{3ams3} \cdot \text{Hz}$$

$$Bw_{sa} = 0.345 \cdot \text{MHz}$$

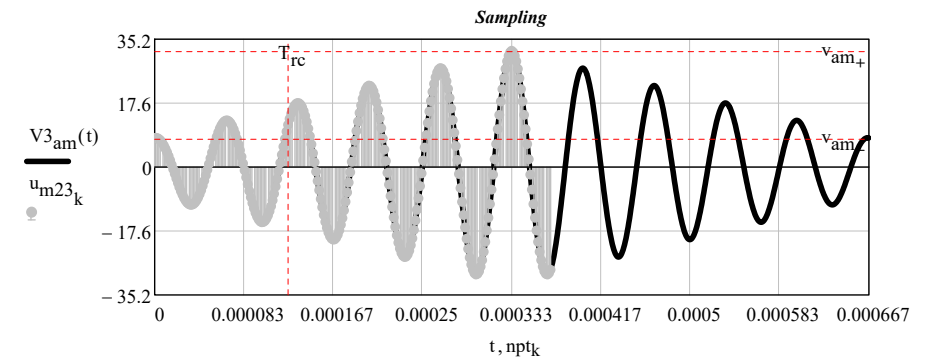
$$\text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.69 \cdot \text{MHz}$$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N_{0gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{rc}} = 2.968$$

$$u_{m23_k} := V_{3am}(n_{ptk})$$

u_{m23}^T	0	1	2	3	4	5	6
	8	8.029	7.904	7.625	7.192	6.61	...



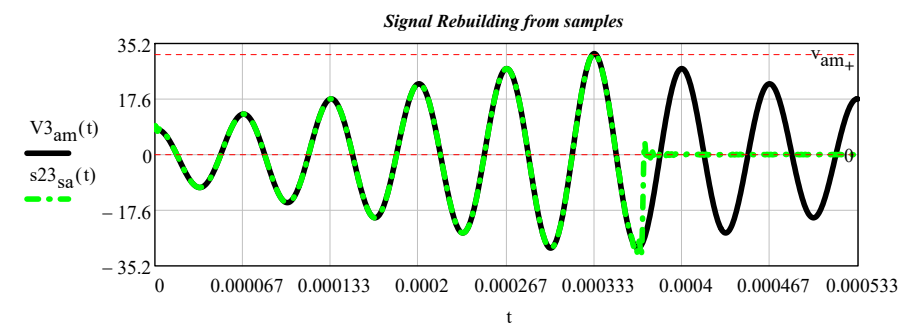
$$\text{relerr} = 10\%$$

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 2.168 \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s_{23sa}(t) := \sum_{n=0}^{N_{0gd}-1} (u_{m23_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N_{0gd} - 1 = 255 \quad \text{relerr} = 10\%$$



TEST Waveforms

Periodic Waveforms

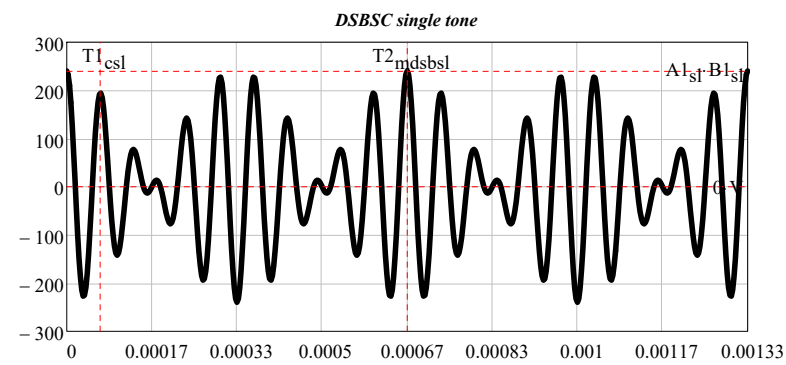
24 AM DSBC test signal (single tone)

$$\omega_{2msl} := \frac{\omega_{1csl}}{10} \quad T_{2m dsbsl} := \frac{2 \cdot \pi}{\omega_{2msl}} \quad \omega_{2msl} = \frac{2 \cdot \pi}{T_{2m dsbsl}} \quad \frac{A_{1sl} \cdot B_{1sl}}{2} = 120 \cdot \text{volt}^2$$

$$\omega_{1csl} = 94.248 \cdot \frac{\text{krads}}{\text{sec}} \quad \omega_{2msl} = 9.425 \cdot \frac{\text{krads}}{\text{sec}} \quad f_{2msl} := \frac{1}{T_{2msl}} \quad f_{1csl} := \frac{\omega_{1csl}}{2 \cdot \pi}$$

$$T_{1csl} := \frac{1}{f_{1csl}} \quad \nu_{sl} := 20$$

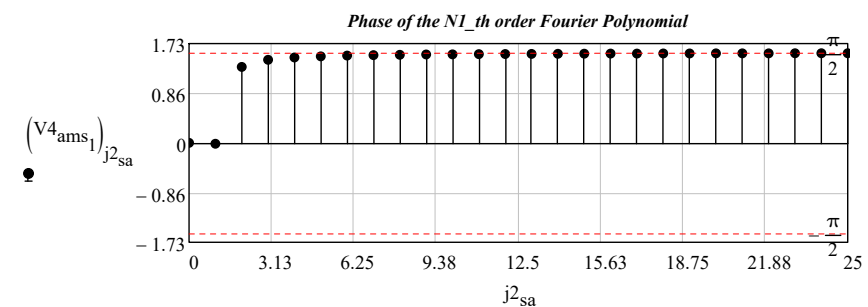
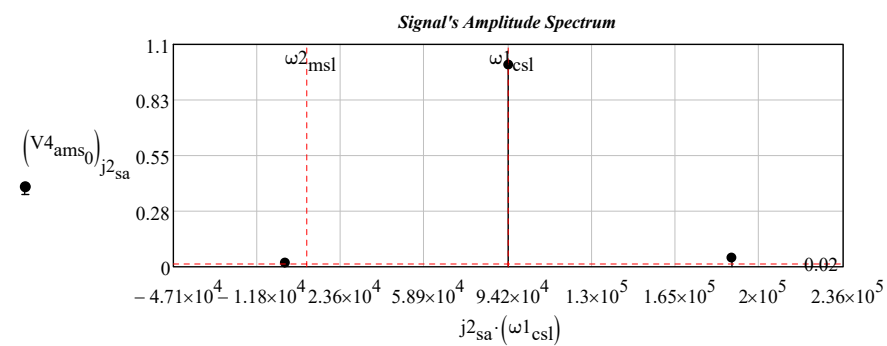
$$t_{dsbw} := 0 \cdot \text{sec}, \nu_{sl} \cdot \frac{T_{1csl}}{500} \dots \nu_{sl} \cdot T_{1csl}$$



$$V_{4am}(t) := V_{4dsbsc}(t, f_{1csl}, f_{2msl}, A_{1sl}, B_{1sl})$$

$$V_{4ams} := \text{SPCT}(V_{4am}, t_{gd}, N1_, 0 \cdot s, T_{1csl}) \quad N1_ = 25$$

$$j_{2sa} := 0 \dots \text{rows}(V_{4ams_0}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$$



$$Bw_{sa} := V_{4ams_3} \cdot \text{Hz}$$

$$Bw_{sa} = 0.195 \cdot \text{MHz}$$

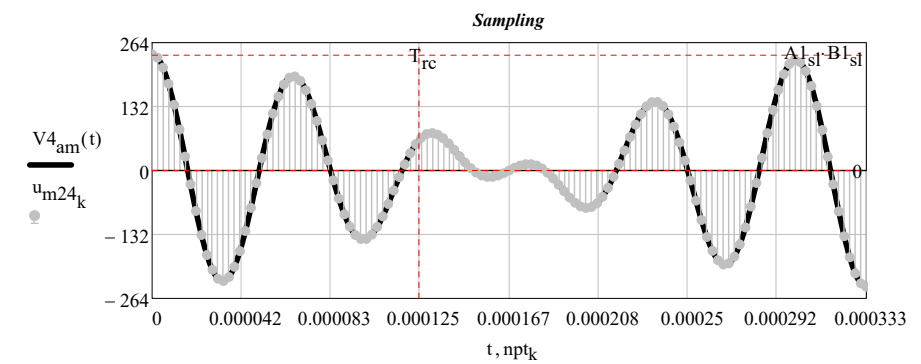
sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.39 \cdot \text{MHz}$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{rc}} = 5.251$

$$u_{m24_k} := V_{4am}(n_{ptk})$$

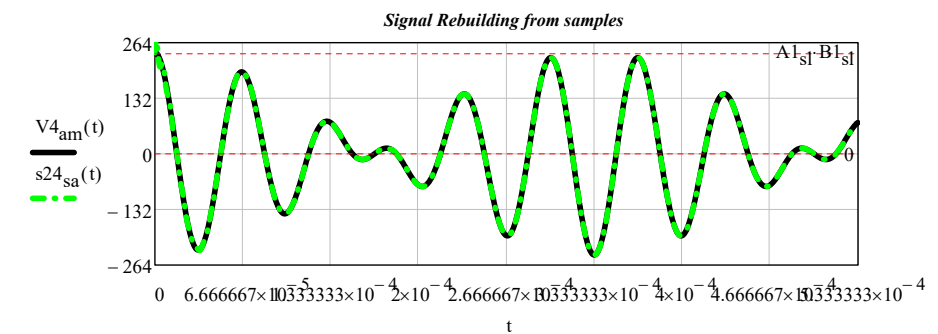
$u_{m24}^T =$	0	1	2	3	4
	240	232.958	212.261	179.171	...



relerr = 10.0% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 1.225 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{24_{sa}}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m24_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10.0\%$



TEST Waveforms

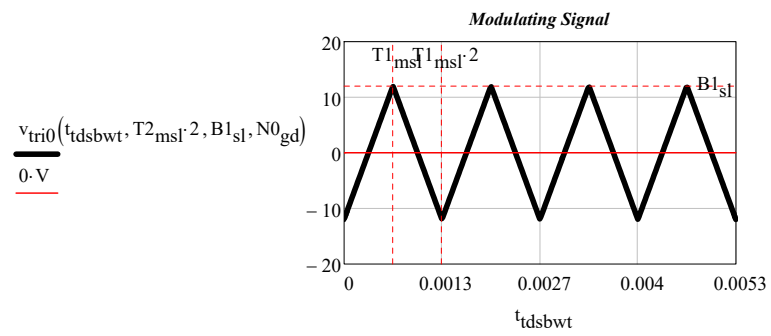
Periodic Waveforms

25AM DSBC test signal (triangular wave)

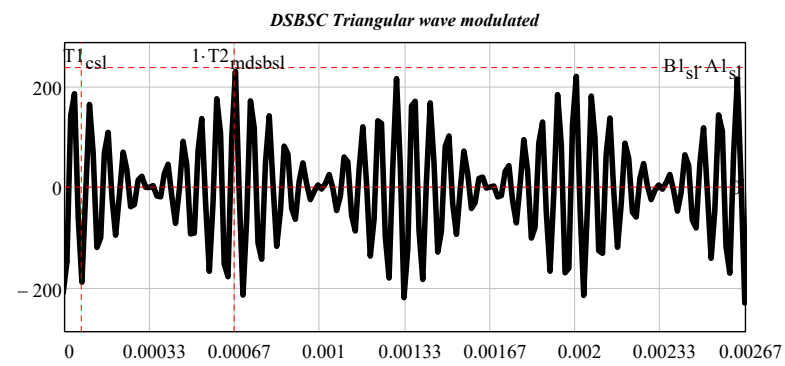
$$T_{18} := T_{2\text{mdsbsl}}$$

$$f_{18} := \frac{1}{T_{18}}$$

$$t_{\text{dsbwt}} := -T_{18} \cdot 3, -T_{18} \cdot 3 + \frac{8 \cdot T_{18} + T_{18} \cdot 3}{500} \dots 8 \cdot T_{18}$$



$$v_{\text{tri0}}(t_{\text{dsbwt}}, T_{2\text{msl}}^2, B_{1\text{sl}}, N_{0\text{gd}})$$



$$v_{5\text{dsbsc}}(t) := V_{5\text{dsbsc}}(t, T_{2\text{mdsbsl}}, f_{1\text{csl}}, f_{2\text{msl}}, A_{1\text{sl}}, B_{1\text{sl}}, N_{0\text{gd}})$$

$$v_{5\text{dsbsc}}(T_{2\text{md}}$$

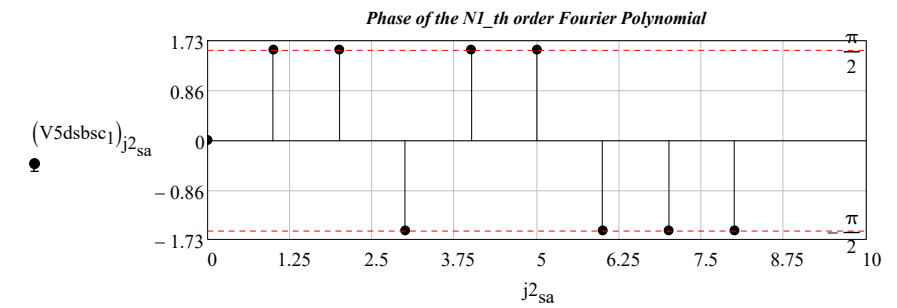
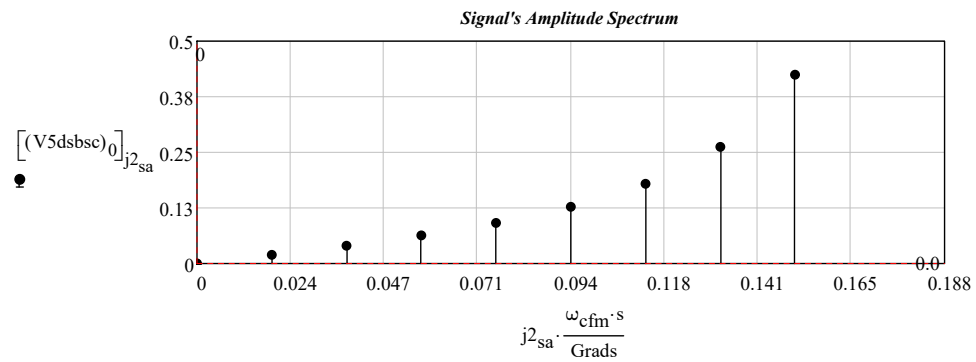
$$N_{1_} := 25$$

$$f_{\text{cfm}} = 3 \cdot \text{MHz}$$

$$\omega_{\text{cfm}} = 0.019 \cdot \frac{\text{Grads}}{\text{s}}$$

$$V_{5\text{dsbsc}} := \text{SPCT}(v_{5\text{dsbsc}}, r_{\text{gd}}, N_{1_}, 0 \cdot \text{s}, T_{2\text{mdsbsl}}) \quad N_{1_} = 25$$

$$j_{2\text{sa}} := 0 \dots \text{rows}(V_{5\text{dsbsc}}) - 1 \quad \omega_{\text{fmm}} = 0.942 \cdot \frac{\text{Mrads}}{\text{s}}$$



$$B_{w_{\text{sa}}} := V_{5\text{dsbsc}} \cdot \text{Hz}$$

$$B_{w_{\text{sa}}} = 0.035 \cdot \text{MHz}$$

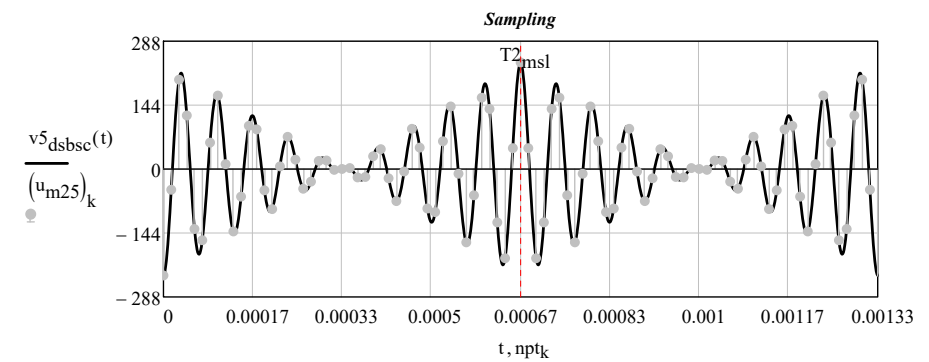
$$\text{sampling frequency: } f_{\text{pt}_{\text{so}}} := 2 \cdot B_{w_{\text{sa}}} \quad f_{\text{pt}_{\text{so}}} = 0.069 \cdot \text{MHz}$$

$$n_{\text{pt}_k} := \frac{k}{f_{\text{pt}_{\text{so}}}}$$

$$\text{Frequency resolution: } \frac{N_{0\text{gd}}}{f_{\text{pt}_{\text{so}}}} \cdot \frac{1}{T_{2\text{msl}}} = 5.565$$

$$(u_{m25})_k := v_{5\text{dsbsc}}(n_{\text{pt}_k})$$

u_{m25}^T	0	1	2	3	4
	-240	-46.706	200.989	120.351	...



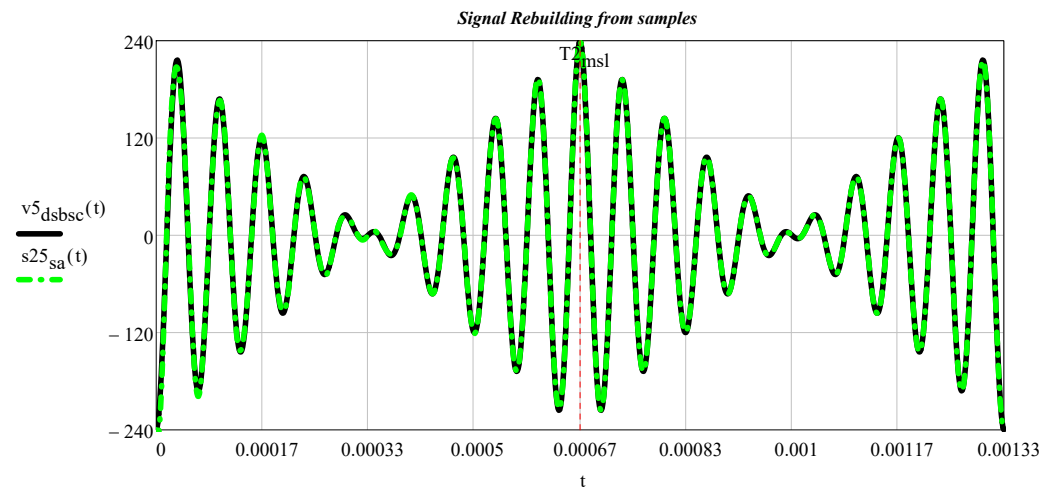
$$\text{relerr} = 10\%$$

$$\omega_{\text{bwr}} := 2 \cdot \pi \cdot B_{w_{\text{sa}}} \quad \omega_{\text{bwr}} = 0.217 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{\text{bwr}}} = n \cdot \frac{1}{2 \cdot B_{w_{\text{sa}}}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula} \quad s_{25\text{sa}}(t) := \sum_{n=0}^{N_{0\text{gd}}-1} (u_{m25}_n \cdot \text{sinc}(\omega_{\text{bwr}} \cdot t - n \cdot \pi)) \quad N_{0\text{gd}} - 1 = 255 \quad \text{relerr} = 10\%$$



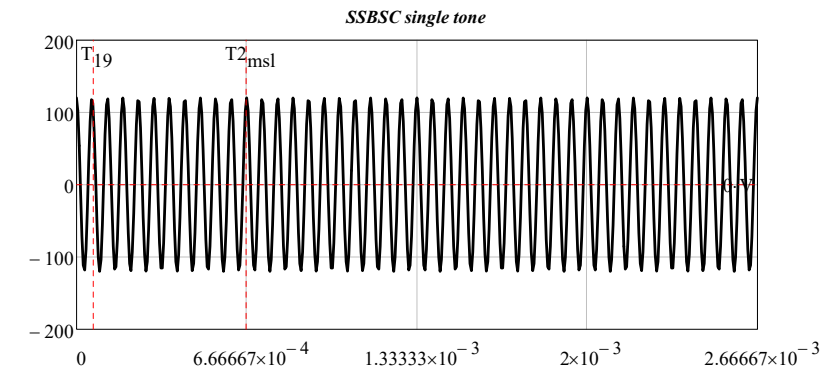
TEST Waveforms

Periodic Waveforms

26AM SSBSC test signal (single tone)

$$f_{19} := \frac{\omega_{1cs1}}{2 \cdot \pi} \quad T_{19} := \frac{1}{f_{19}}$$

$$t_{ssbw} := 0 \cdot \text{sec}, \frac{4 \cdot T_{2msl}}{500} .. 4 \cdot T_{2msl}$$



$$v_{ssbsc}(t) := \frac{V_{ssbsc}(t, f_{1cs1}, f_{2msl}, A_{1s1}, B_{1s1})}{V^2} \quad v_{ssbsc}(T_{19}) = 97.082$$

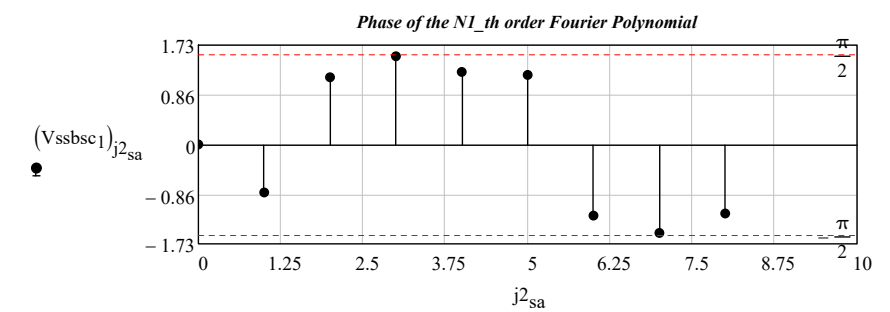
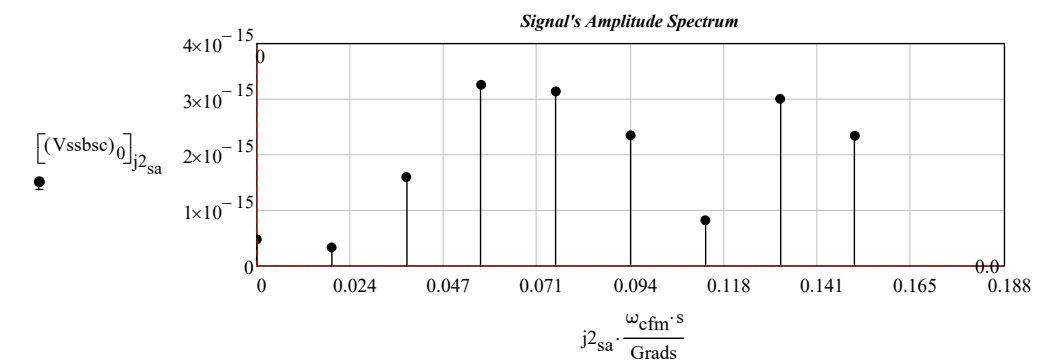
$$N1_{sa} := 25$$

$$f_{cfm} = 3 \cdot \text{MHz}$$

$$\omega_{cfm} = 0.019 \cdot \frac{\text{Grads}}{\text{s}}$$

$$V_{ssbsc} := \text{SPCT}(v_{ssbsc}, rt_{gd}, N1_{sa}, 0 \cdot \text{s}, T_{2msl}) \quad N1_{sa} = 25$$

$$j_{2sa} := 0 .. \text{rows}(V_{ssbsc}) - 1 \quad \omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{s}}$$



$$Bw_{sa} := V_{ssbsc3} \cdot \text{Hz}$$

$$Bw_{sa} = 0.023 \cdot \text{MHz}$$

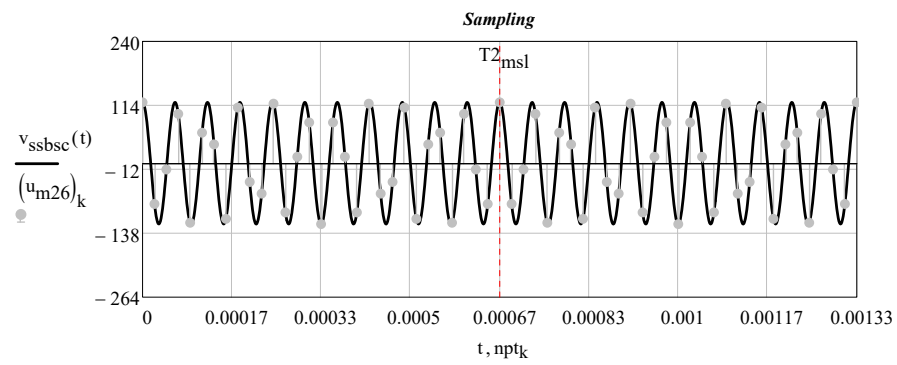
sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.045 \cdot \text{MHz}$

$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 8.533$

$$(u_{m26})_k := v_{ssbsc}(npt_k)$$

$u_{m26}^T =$	0	1	2	3	4	5	
	120	-80.296	-12.543	97.082	-117.378	...	



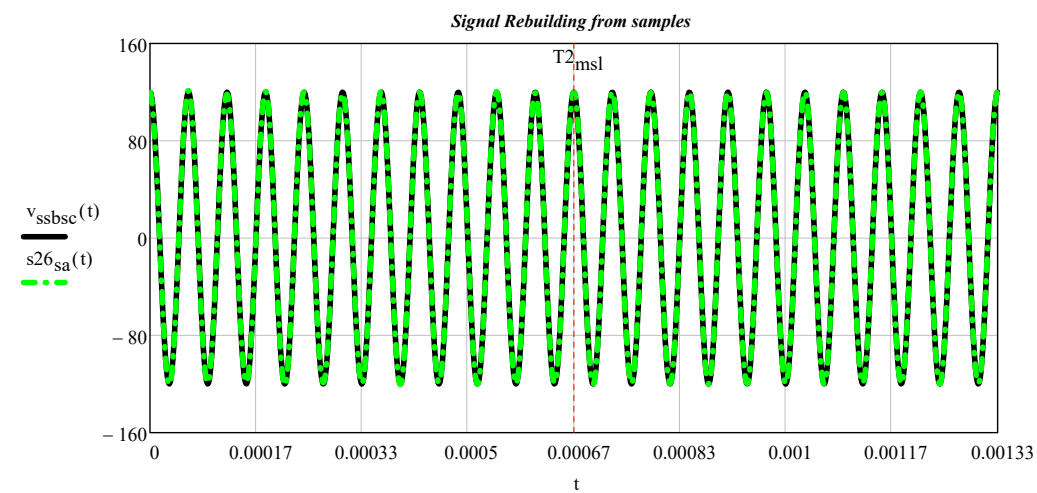
relerr = 10-%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.141 \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s26_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m26}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10-%

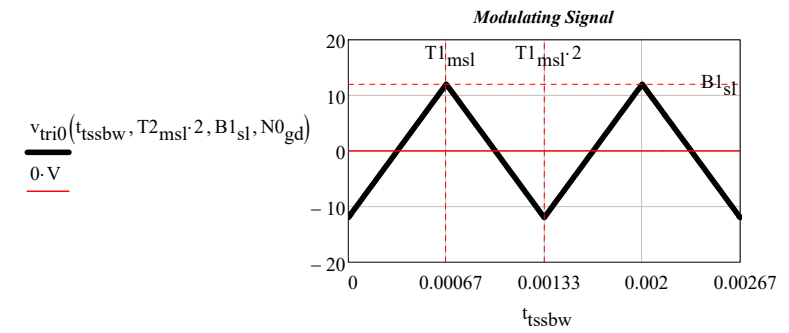


TEST Waveforms

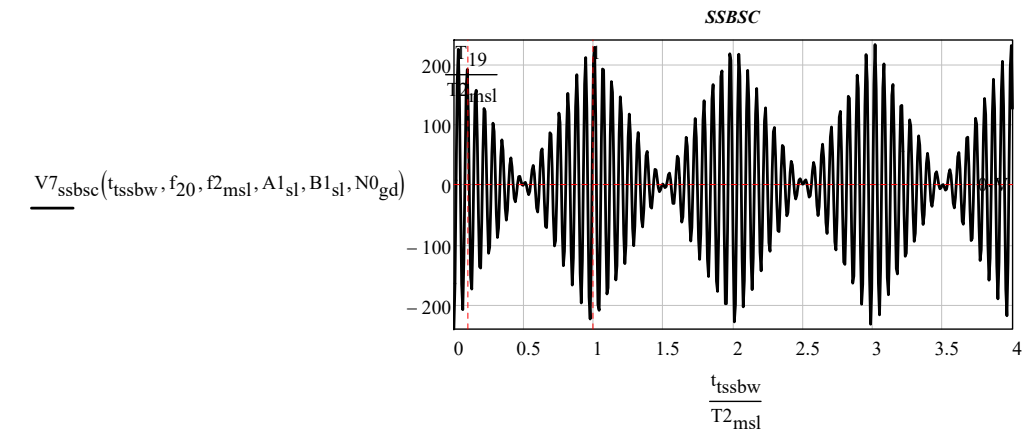
Periodic Waveforms

27AM SSBSC test signal (triangular wave)

$$f_{20} := \frac{10}{T1_{csl}}$$



$$\frac{A1_{sl} \cdot A1_{sl}}{2} = 200 \text{ V}^2$$



$N1_{sa} := 25$

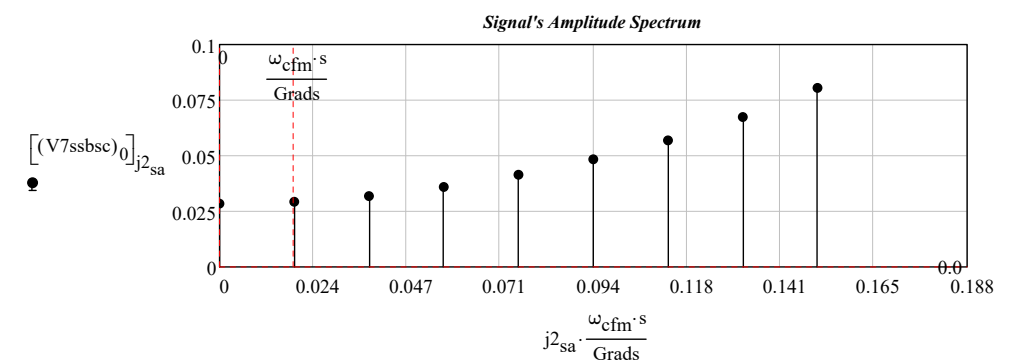
$$f_{cfm} = 3 \cdot \text{MHz}$$

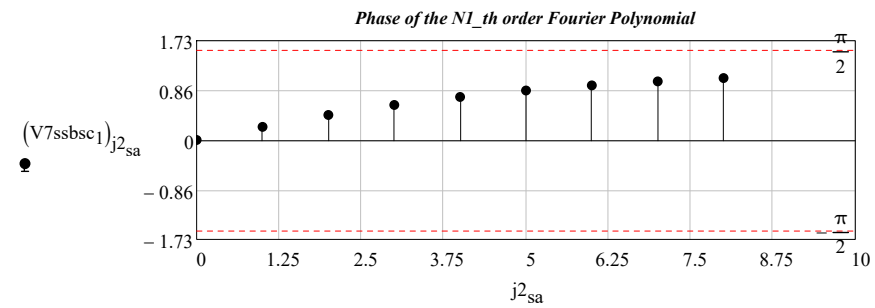
$$\omega_{cfm} = 0.019 \frac{\text{Grads}}{\text{s}}$$

$$v7_{ssbsc}(t) := V7_{ssbsc}(t, f_{20}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd})$$

$$V7_{ssbsc} := \text{SPCT}(v7_{ssbsc}, rt_{gd}, N1_{sa}, 0 \cdot \text{s}, T2_{msl}) \quad N1_{sa} = 25$$

$$j2_{sa} := 0 \dots \text{rows}(V7_{ssbsc}) - 1 \quad \omega_{fmm} = 0.942 \frac{\text{Mrads}}{\text{s}}$$





$$Bw_{sa} := V7ssbsc3 \cdot Hz$$

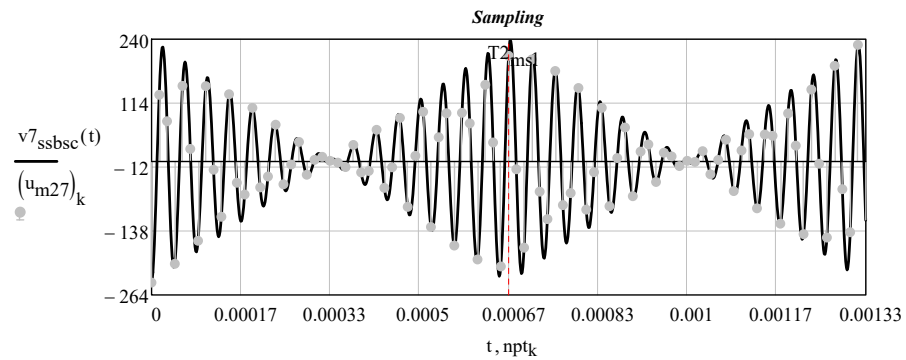
$$Bw_{sa} = 0.035 \cdot MHz$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.069 \cdot MHz$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T2_{msl}} = 5.565$

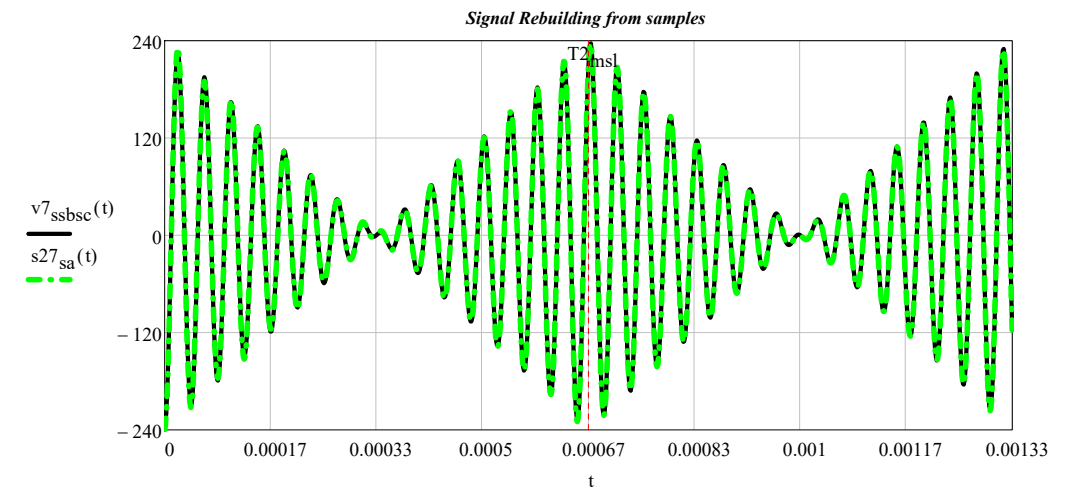
$$(u_{m27})_k := v7_{ssbsc}(npt_k)$$

$$u_{m27}^T = \begin{array}{|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & -240 & 130.212 & 78.129 & -202.786 & \dots \\ \hline \end{array}$$


relerr = 10.-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.217 \cdot \frac{Mrads}{sec} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s27_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m27}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10.-%$



TEST Waveforms

Periodic Waveforms

28 FM test signal (single tone) (change data in FM data.xmcd)

Carrier Amplitude:.....: $A_{fm} = 200 \cdot \text{mV}$

Carrier Frequency.....: $f_{cfm} = 3 \cdot \text{MHz}$

Carrier period.....: $T_{cfm} = 0.333 \cdot \mu\text{s}$

Angular frequency of the carrier.....: $\omega_{cfm} = 18.85 \cdot \frac{\text{Mrads}}{\text{sec}}$

Amplitude of the single tone modulating signal.....: $B_{fmm} = 15 \cdot \text{V}$

Period of the modulating signal.....: $T_{fmm} = 6.667 \cdot \mu\text{s}$

Frequency of the single tone modulating signal.....: $f_{fmm} := \frac{1}{T_{fmm}}$

Angular frequency of the single tone modulating signal:

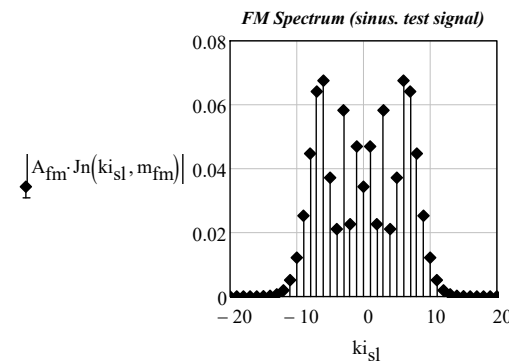
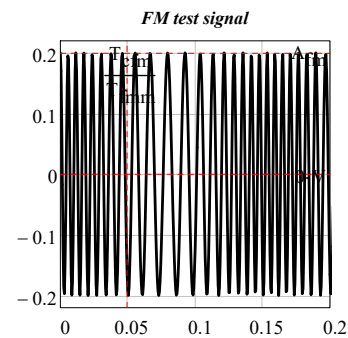
$\omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}}$

Frequency modulation index: $m_{fm} = 8$

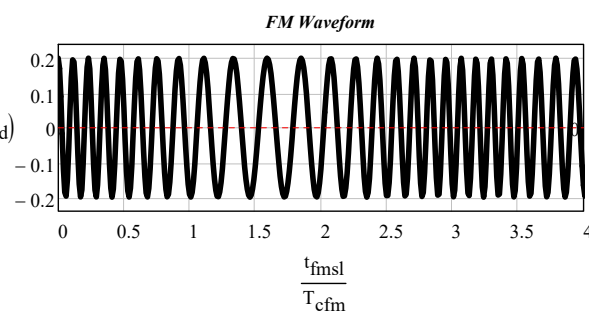
$\frac{T_{fmm}}{T_{cfm}} = 20$ $\frac{\omega_{cfm}}{\omega_{fmm}} = 20$

$m_{fm} = 8$ $k_{i_{sl}} := -30 \dots 30$

$$t_{fmsl} := T_{fmm} \cdot 0, T_{fmm} \cdot 0 + \frac{10 \cdot T_{fmm} - T_{fmm} \cdot 0}{20000} \dots 10 \cdot T_{fmm}$$



$$V7_{fm}(t_{fmsl}, \omega_{cfm}, \omega_{fmm}, A_{fm}, m_{fm}, N_{gd})$$



$m_{fm} = 8$
 $A_{fm} = 0.2 \text{ V}$
 $B_{fmm} = 15 \text{ V}$

$N1_{=} := 25$

$$f_{cfm} = 3 \cdot \text{MHz}$$

$$\omega_{cfm} = 0.019 \cdot \frac{\text{Grads}}{\text{s}}$$

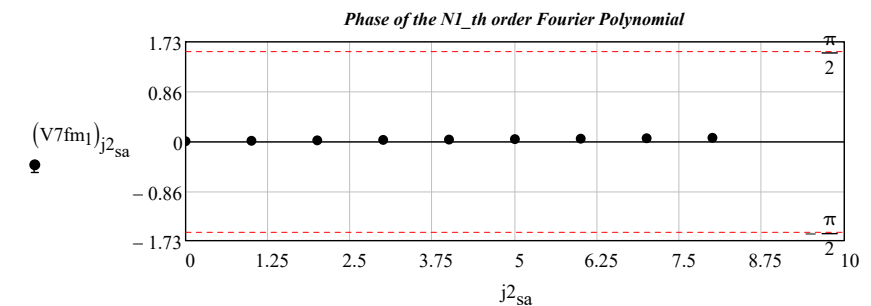
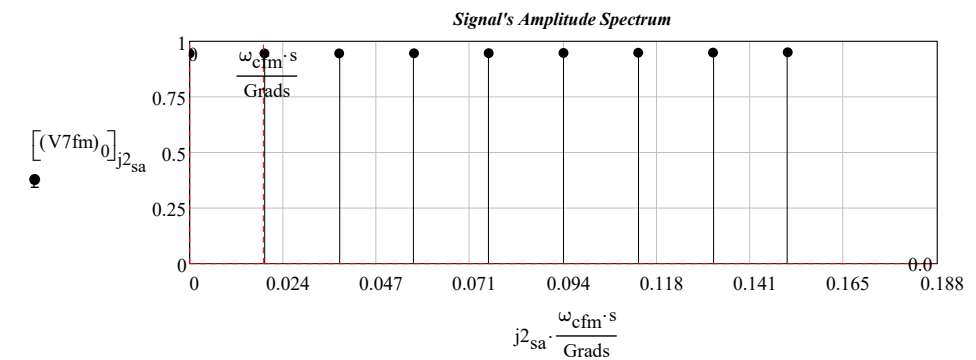
$$v7_{fm}(t) := V7_{fm}(t, \omega_{cfm}, \omega_{fmm}, A_{fm}, m_{fm}, N_{gd})$$

$$V7_{fm} := \text{SPCT}(v7_{fm}, rt_{gd}, N1_{}, 0 \cdot \text{s}, T_{fmm})$$

$$N1_{=} = 25$$

$$j2_{sa} := 0 \dots \text{rows}(V7_{fm}) - 1$$

$$\omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{s}}$$



$$Bw_{sa} := V7_{fm} \cdot \text{Hz}$$

$$Bw_{sa} = 3.45 \cdot \text{MHz}$$

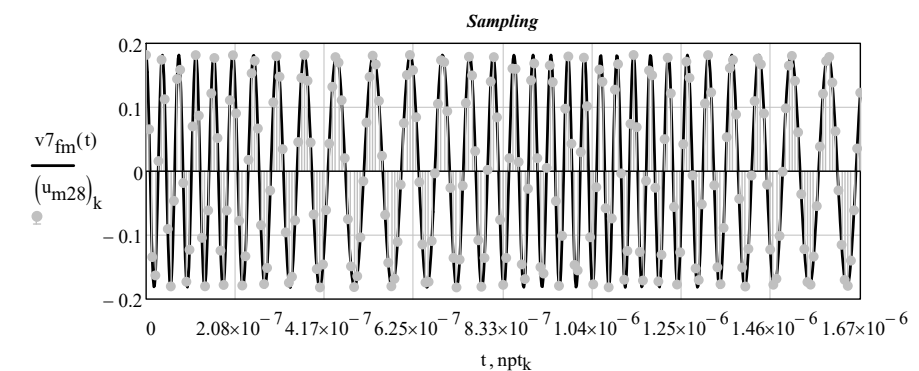
sampling frequency: $f_{pt_{so}} := 40 \cdot Bw_{sa}$ $f_{pt_{so}} = 138 \cdot \text{MHz}$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{fmm}} = 0.278$

$$(u_{m28})_k := v7_{fm}(n_{ptk})$$

$u_{m28}^T =$	0	1	2	3	4
	0.2	0.072	-0.148	-0.179	...



releer = 10-%

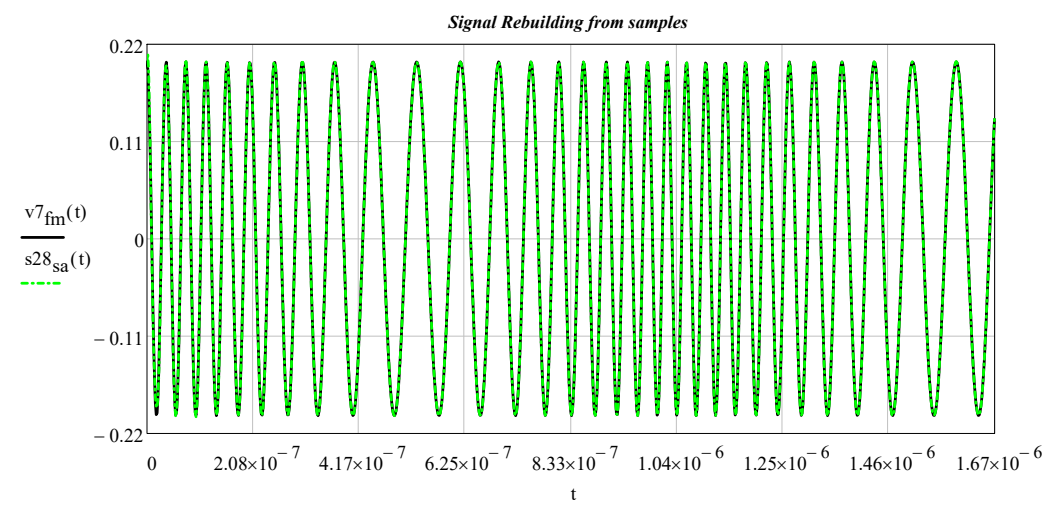
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \cdot 20$$

$$\omega_{bwr} = 433.54 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{28sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m28n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ $\text{relerr} = 10\%$

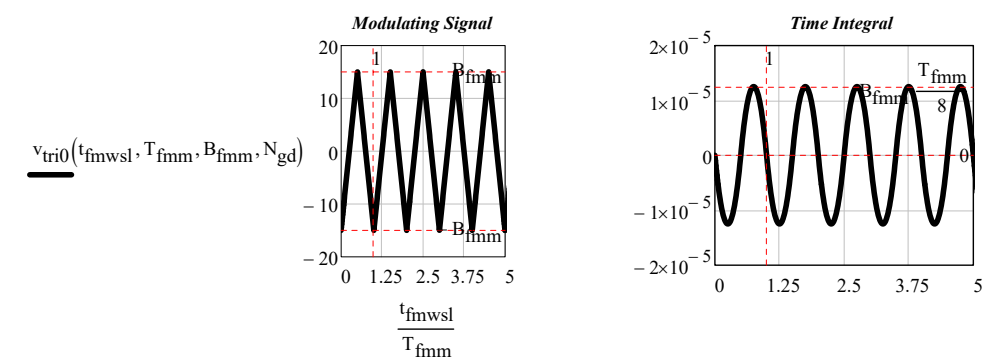


TEST Waveforms

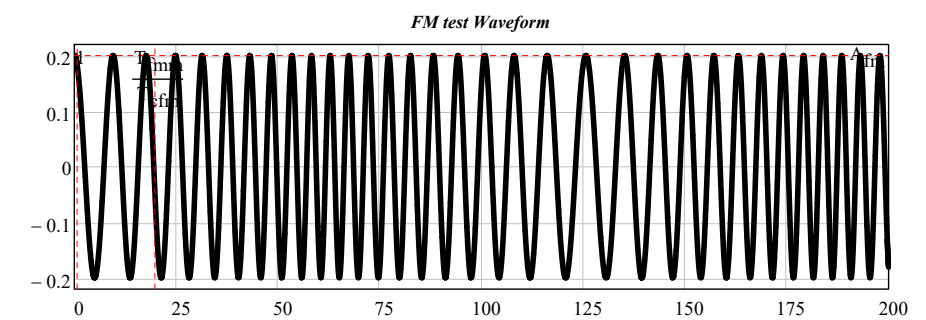
Periodic Waveforms

29 FM test signal (triangular wave)

$$t_{fmwsl} := 0 \cdot T_{fmm}, \frac{10 \cdot T_{fmm} - 0 \cdot T_{fmm}}{1000} .. 10 \cdot T_{fmm}$$



$$K_{stfmsl} := \frac{m_{fm} \cdot \omega_{fmm} \cdot f_{fmm}}{2 \cdot \pi \cdot B_{fmm}} := \frac{1}{T_{fmm}}$$



$N1_{w} := 25$

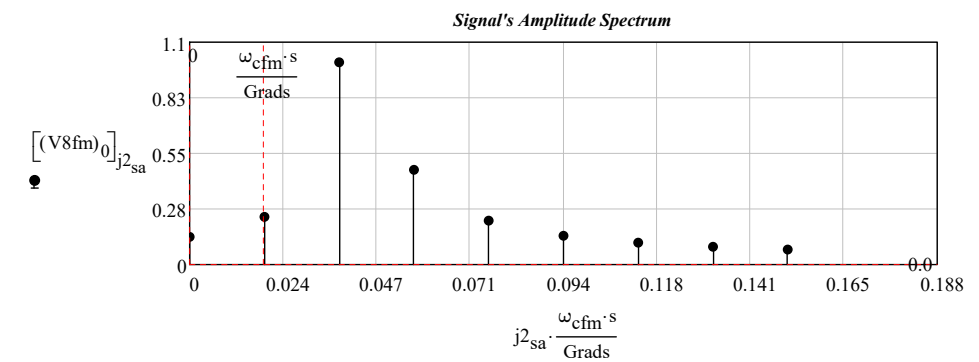
$$v_{8fm}(t) := V_{8fm}(t, T_{fmm}, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, K_{stfmsl}, N0_{gd})$$

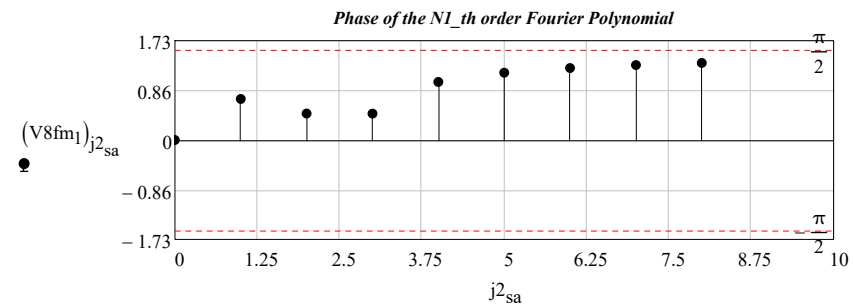
$$f_{cpm} = 0.12 \cdot \text{MHz}$$

$$V_{8fm} := \text{SPCT}(v_{8fm}, rt_{gd}, N1_{w}, 0 \cdot s, T_{fmm}) \quad N1_{w} = 25$$

$$\omega_{cpm} = 7.54 \times 10^{-4} \cdot \frac{\text{Grads}}{s}$$

$$j_{2sa} := 0 .. \text{rows}(V_{8fm}) - 1 \quad \omega_{pmm} = 0.075 \cdot \frac{\text{Mrads}}{s}$$





$$Bw_{sa} := V8fm3 \cdot Hz$$

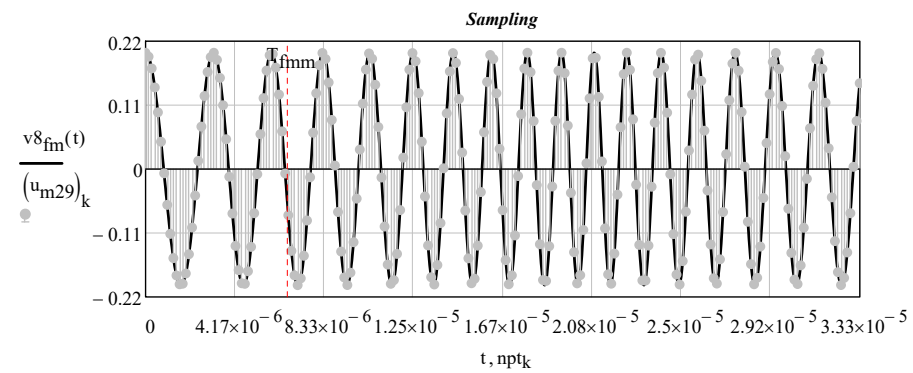
$$Bw_{sa} = 3.45 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 6.9 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{fmm}} = 5.565$

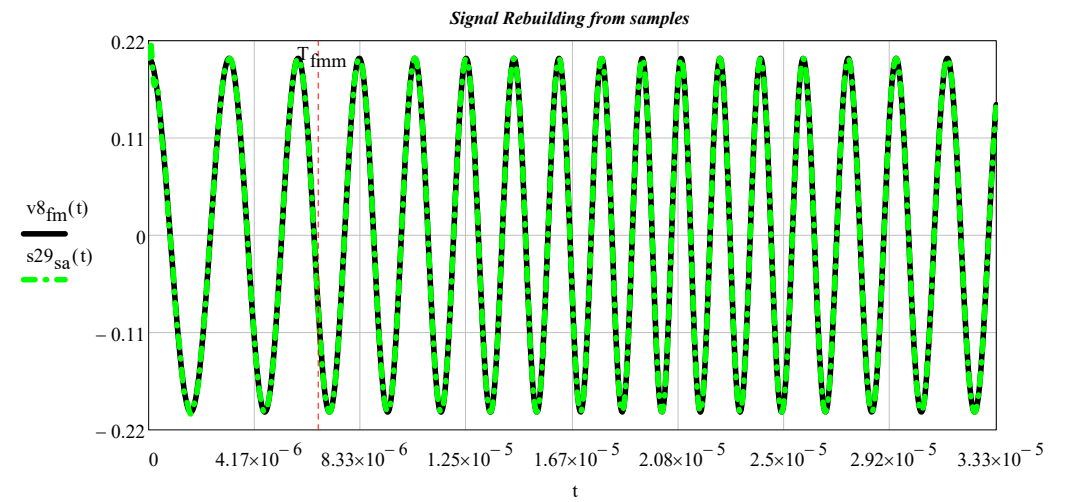
$$(u_{m29})_k := v8_{fm}(npt_k)$$

$$u_{m29}^T = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & \\ \hline 0 & 0.2 & 0.193 & 0.173 & 0.14 & \dots & \end{array}$$


relerr = 10-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 21.677 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s29_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} \left((u_{m29})_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right)$ $N0_{gd} - 1 = 255$ relerr = 10-%



TEST Waveforms

Periodic Waveforms

30 PM test signal (single tone)

Carrier Amplitude: $A_{pm} := 20 \cdot V$, $A_{pm} = 20 \cdot V$

Carrier Frequency: $f_{cpm} = 0.12 \cdot \text{MHz}$

Carrier period: $T_{cpm} = 8.333 \times 10^{-3} \cdot \text{ns}$

Angular frequency of the carrier: $\omega_{cpm} = 7.54 \times 10^{-4} \cdot \frac{\text{Grads}}{\text{sec}}$

Amplitude of the modulating signal: $B_{pm} = 5 \cdot V$

Modulating signal period: $T_{pmm} = 83.333 \cdot \mu\text{s}$

Frequency of the harmonic modulating signal: $f_{pmm} = 0.012 \cdot \text{MHz}$, $\frac{T_{pmm}}{T_{cpm}} = 10$

Angular frequency of the modulating signal: $\omega_{pmm} = 0.075 \cdot \frac{\text{Mrads}}{\text{sec}}$

Phase modulation index: $m_{pm} = 5 \cdot \text{rad}$

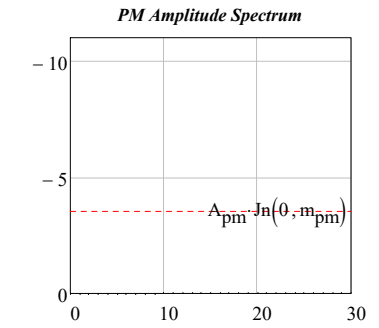
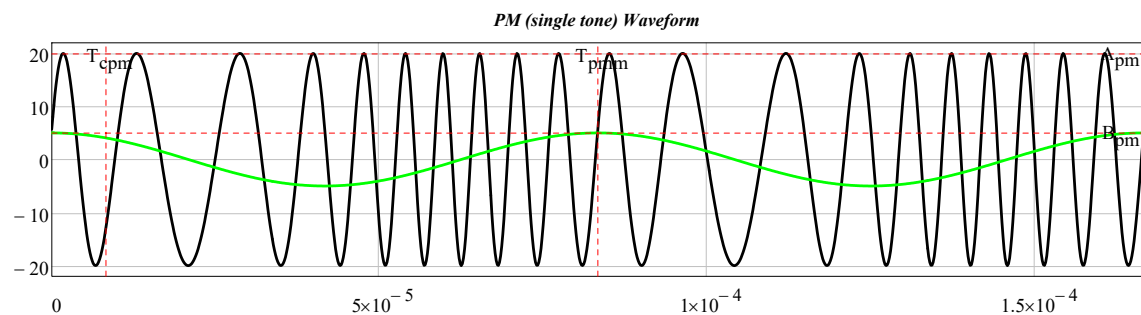
Phase-sensitivity factor: $k_{pm} = 1 \cdot \frac{\text{rad}}{V}$

$$k_{pm} = \frac{m_{pm}}{B_{pm}}$$

$$v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \text{Re} \left[A_{pm} \cdot e^{j \cdot 2 \cdot \pi \cdot f_{cpm} \cdot t} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left(e^{j \cdot \frac{k \cdot \pi}{2}} \cdot J_n(k, m_{pm}) \cdot \cos(k \cdot 2 \cdot \pi \cdot f_{pmm} \cdot t) \right) \right]$$

Dimensionless function: $v_{9pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \frac{v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})}{V}$

$f_{cpm} = 0.12 \cdot \text{MHz}$ $f_{pmm} = 0.012 \cdot \text{MHz}$
 $t_{pm} := T_{cpm} \cdot 0, T_{cpm} \cdot 0 + \frac{80 \cdot T_{cpm} - 0 \cdot T_{cpm}}{4000} .. 80 \cdot T_{cpm}$ $m_{pm} = 5$ $A_{pm} = 20 \cdot V$



$N1 := 25$

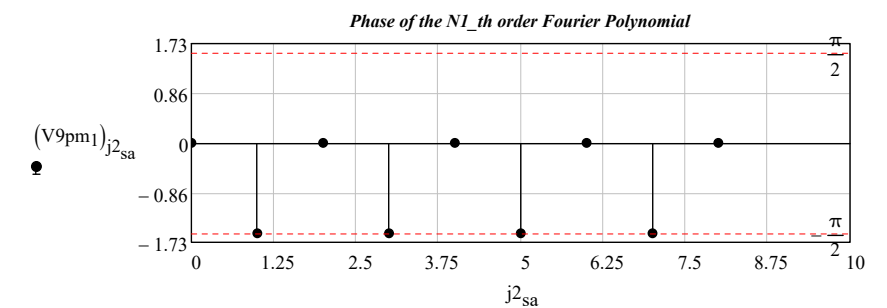
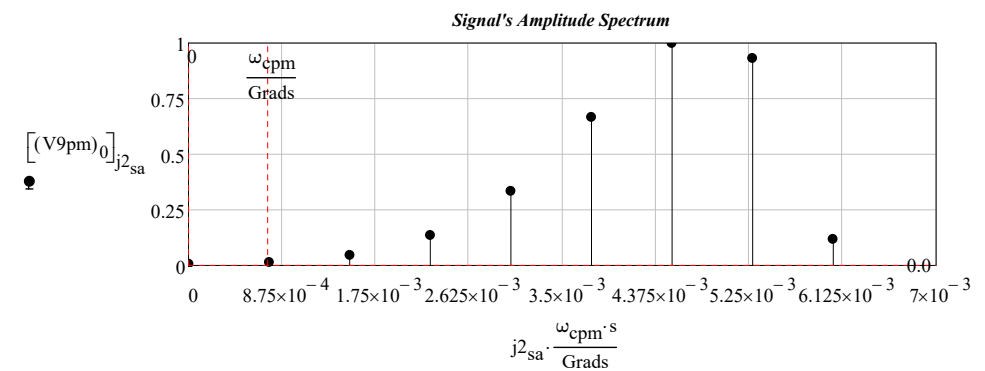
$f_{cpm} = 0.12 \cdot \text{MHz}$

$\omega_{cpm} = 7.54 \times 10^{-4} \cdot \frac{\text{Grads}}{\text{s}}$

$v_{9pm}(t) := V_{9pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})$

$V_{9pm} := \text{SPCT}(v_{9pm}, rt_{gd}, N1, 0, s, T_{pmm})$ $N1 = 25$

$j2_{sa} := 0 .. \text{rows}(V_{9pm}) - 1$ $\omega_{pmm} = 0.075 \cdot \frac{\text{Mrads}}{\text{s}}$



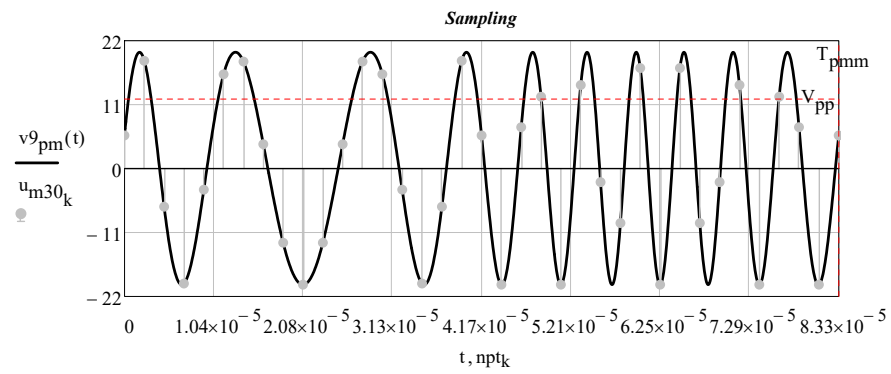
$Bw_{sa} := V_{9pm} \cdot 3 \cdot \text{Hz}$
 $Bw_{sa} = 0.216 \cdot \text{MHz}$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa}$ $f_{pt_{so}} = 0.432 \cdot \text{MHz}$

$npt_k := \frac{k}{f_{pt_{so}}}$
 Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{pmm}} = 7.111$

$(u_{m30})_k := v_{9pm}(npt_k)$

$u_{m30}^T =$

0	1	2	3	4	5	6
0	5.673	18.527	-6.578	-19.801	-3.66	16.21



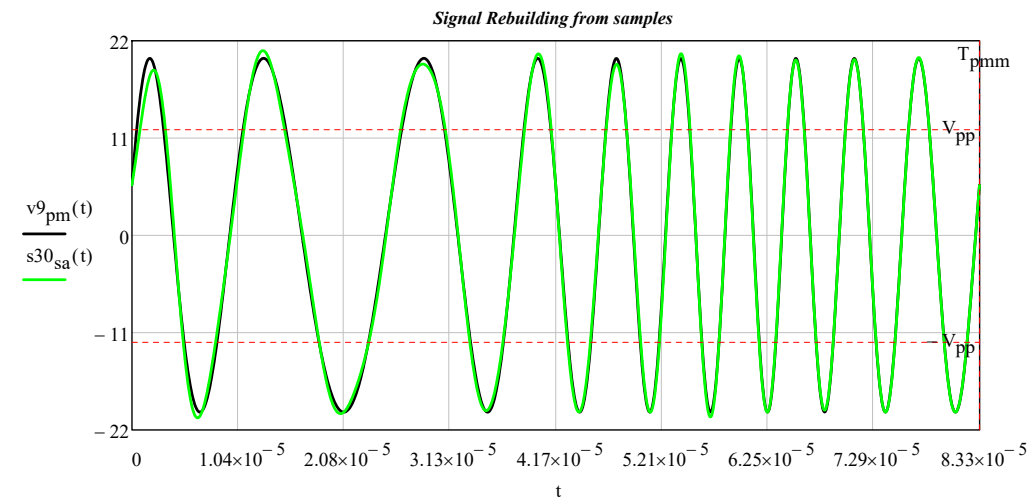
relerr = 10-%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 1.357 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula

$$s30_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m30}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$



TEST Waveforms

Periodic Waveforms

31 PM test signal (triangular wave)

$$T_{tri} := \frac{T_{pmm}}{2} \quad \omega_{cpm} := 0.377 \frac{\text{rad}}{\text{ns}} \quad v_{mtri}(t_{sl}) := v_{tri0}(t_{sl}, T_{tri}, A_{pm}, N0_{gd})$$

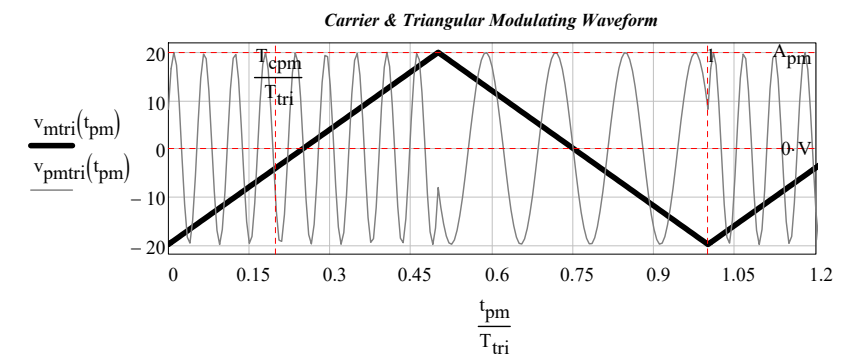
$$v_{pmtri}(t_{sl}) := A_{pm} \cdot \cos(\omega_{cpm} \cdot t_{sl} + k_{pm} \cdot v_{mtri}(t_{sl}))$$

$$k_{pm} = \frac{m_{pm}}{B_{pm}} \quad k_{pm} = 1 \frac{1}{V}$$

$$v_{pmtri}(t, T_{pmm}, f_{cpm}, k_{pm}, A_{pm}, B_{pm}, N0_{gd}) = A_{pm} \cdot \cos(2 \cdot \pi \cdot f_{cpm} \cdot t + k_{pm} \cdot v_{tri0}(t, T_{tri}, B_{pm}, N0_{gd}))$$

$$V10_{pm}(t, T_{pmm}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd}) = \frac{v_{pmtri}(t, T_{tri}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd})}{V}$$

$$t_{pm} := T_{tri} \cdot 0, T_{tri} \cdot 0 + \frac{5 \cdot T_{tri} - 0 \cdot T_{tri}}{1000} .. 5 \cdot T_{tri}$$



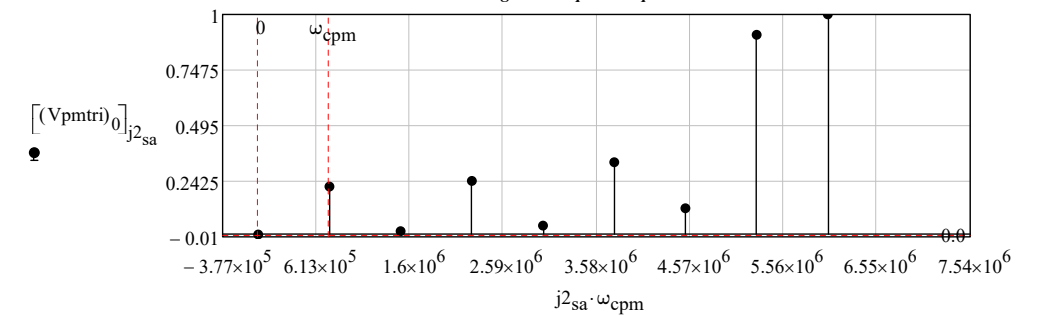
$$N1_{sa} := 25$$

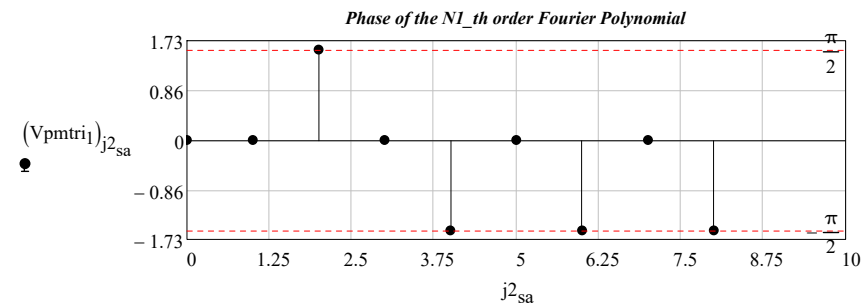
$$v_{pmtri}(t) := \frac{v_{pmtri}(t)}{V}$$

$$V_{pmtri} := \text{SPCT}(v_{pmtri}, rt_{gd}, N1_{sa}, 0, s, T_{tri}) \quad N1_{sa} = 25$$

$$\text{relerr} := V_{pmtri} \cdot j2_{sa} := 0 .. \text{rows}(V_{pmtri}) - 1 \quad \omega_{pmm} = 0.075 \cdot \frac{\text{Mrads}}{s}$$

Signal's Amplitude Spectrum





$$Bw_{sa} := Vpmtri3 \cdot Hz$$

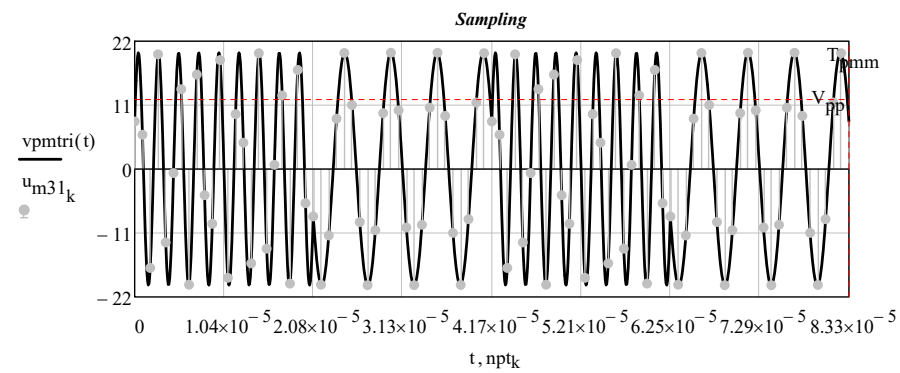
$$Bw_{sa} = 0.552 \cdot MHz$$

$$\text{sampling frequency: } fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 1.104 \cdot MHz$$

$$k := 0..2^8 - 1 \quad npt_k := \frac{k}{fpt_{so}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pmm}} = 2.783$$

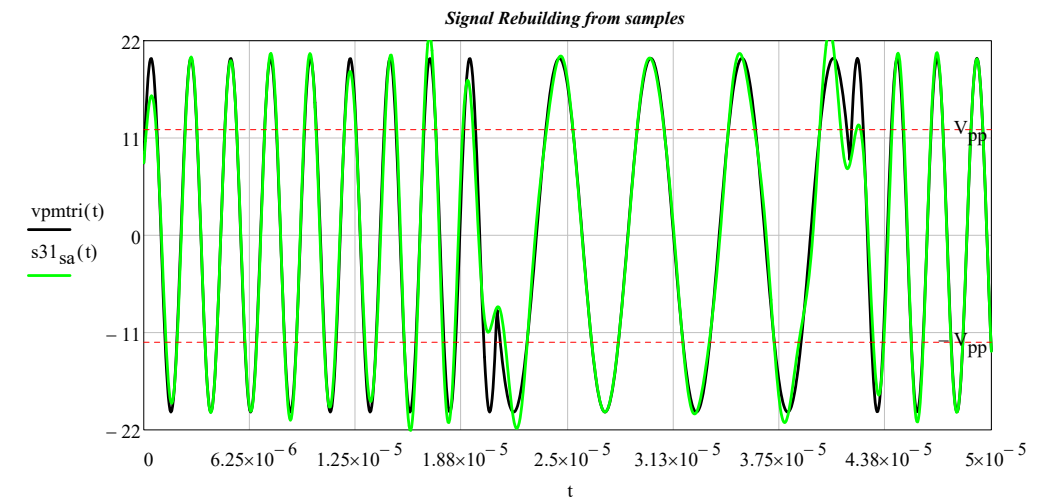
$$(u_{m31})_k := vpmtri(npt_k)$$

$$u_{m31}^T = \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 8.162 & 5.894 & -17.028 & 19.721 & -12.637 & -0.712 & \dots \\ \hline \end{array}$$


$$\text{relerr} = 10\% \quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 3.468 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula} \quad s31_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m31}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \right] \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$



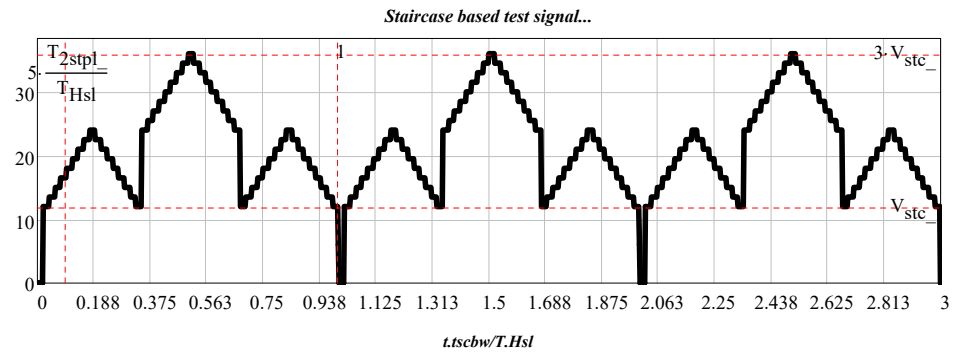
TEST Waveforms

Periodic Waveforms

32 Staircase based test signal

shift := 1

$$T_{Hsl} := (6 \cdot m2_{steps_} + shift + 3) \cdot T_{2stpl_} \quad t_{tscbw} := 0 \cdot T_{Hsl} \cdot 0 \cdot T_{Hsl} + \frac{5 \cdot T_{Hsl}}{2000} \dots 5 \cdot T_{Hsl}$$



N1_ := 25

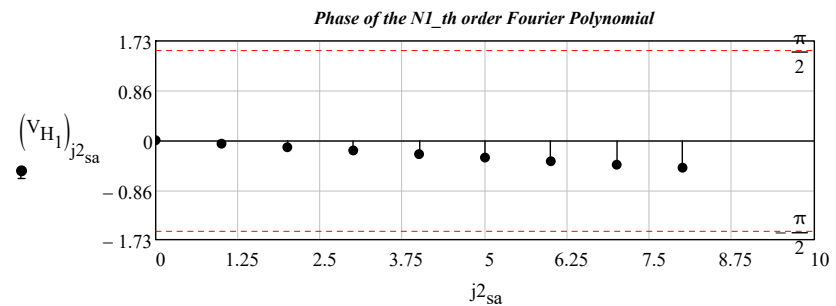
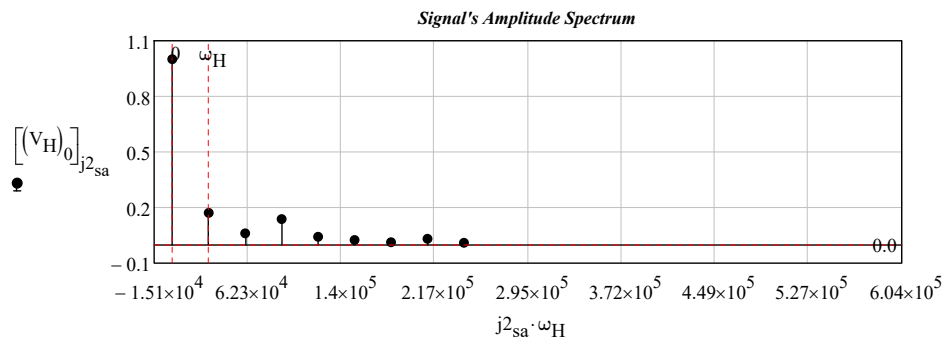
$$\omega_H := \frac{2 \cdot \pi}{T_{Hsl}}$$

$$VH(t) := V_H(t, T_{Hsl}, T_{2stpl_}, V_{stc_}, mstc3_{steps_}, shift, N_{gd})$$

$$5 \cdot T_{2stpl_} = 20 \cdot \mu s$$

$$V_{H1} := SPCT(VH, t_{gd}, N1_, 5 \cdot T_{2stpl_}, T_{Hsl}) \quad N1_ = 25$$

$$j2_{sa} := 0 \dots \text{rows}(V_H) - 1 \quad \omega_H = 30.208 \cdot \frac{\text{k rads}}{s}$$



$$Bw_{sa} := V_{H3} \cdot Hz$$

$$Bw_{sa} = 0.111 \cdot MHz$$

sampling frequency:

$$f_{pt_{sov}} := 2 \cdot Bw_{sa}$$

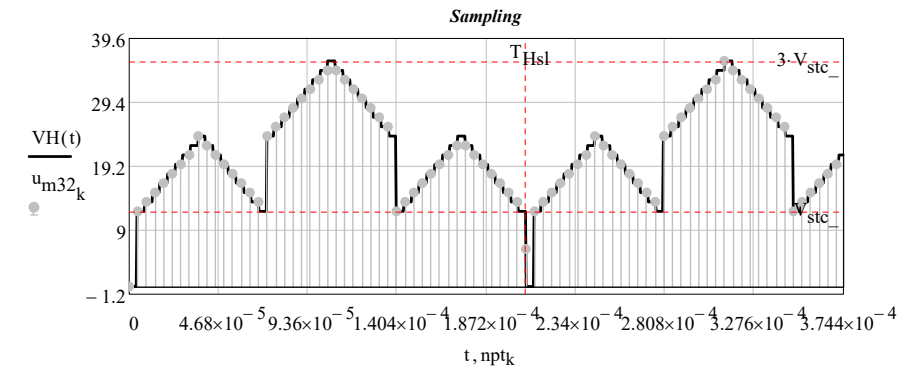
$$f_{pt_{so}} = 0.221 \cdot MHz$$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{Hsl}} = 5.565$$

$$(u_{m32})_k := VH(npt_k)$$

0	0	1	2	3	4	5	6	7	8	9
0	0	12	13.5	15	16.5	18	19.5	21	24	...



relerr = 10.0%

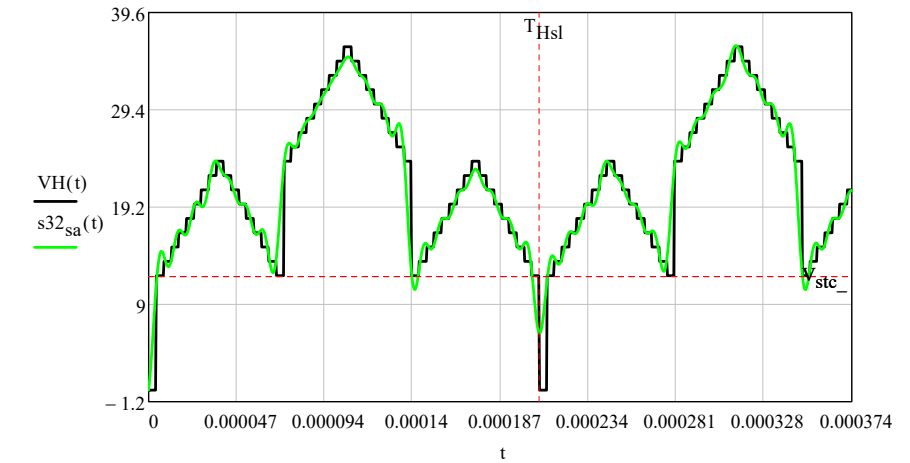
$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.695 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

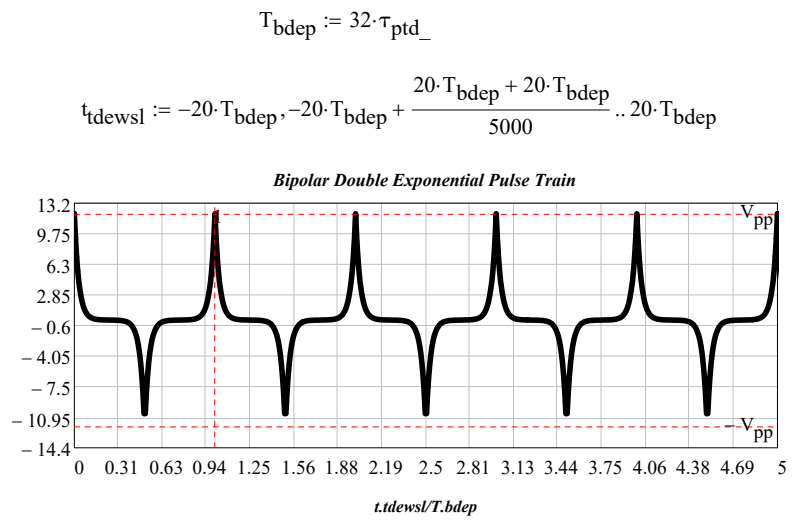
$$\text{interpolation formula } s32_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m32}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10.0\%$$

Signal Rebuilding from samples

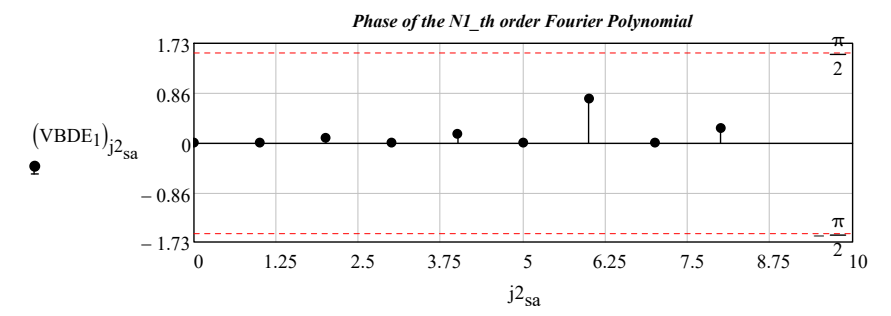
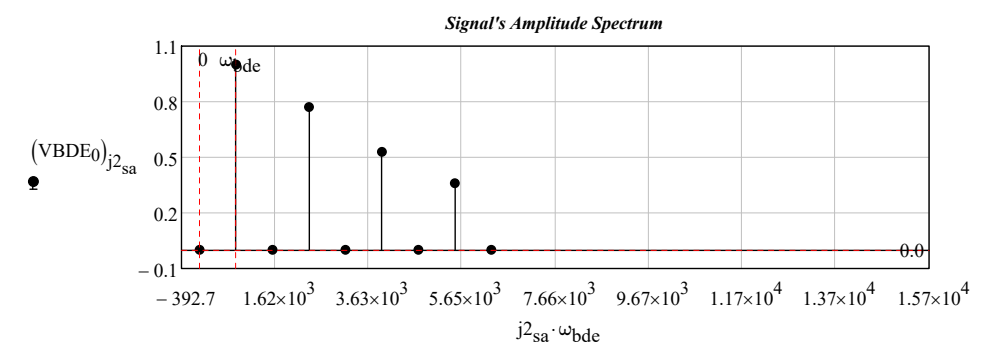


Periodic Waveforms

33 Bipolar Double Exponential Pulse Train



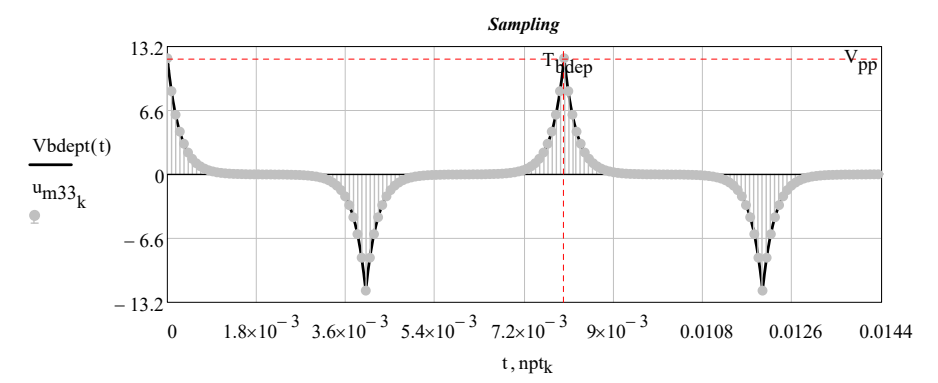
$N1_{sa} := 50$
 $\omega_{bde} := \frac{2 \cdot \pi}{T_{bdep}}$
 $Vbdept(t) := \frac{V_{bdept}(t, \tau_{ptd_}, T_{bdep}, V_{pp}, N0_{gd})}{V}$
 $VBDE := SPCT(Vbdept, \tau_{gd}, N1_{sa}, 0 \cdot s, T_{bdep}) \quad N1_{sa} = 50$
 $j2_{sa} := 0 .. \text{rows}(VBDE) - 1 \quad \omega_{bde} = 0.785 \frac{\text{krads}}{s}$



$Bw_{sa} := VBDE3 \cdot \text{Hz}$
 $Bw_{sa} = 6 \times 10^{-3} \cdot \text{MHz}$
 sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.012 \cdot \text{MHz}$
 $npt_k := \frac{k}{f_{pt_{so}}}$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{bdep}} = 2.667$
 $(u_{m33})_k := Vbdept(npt_k)$

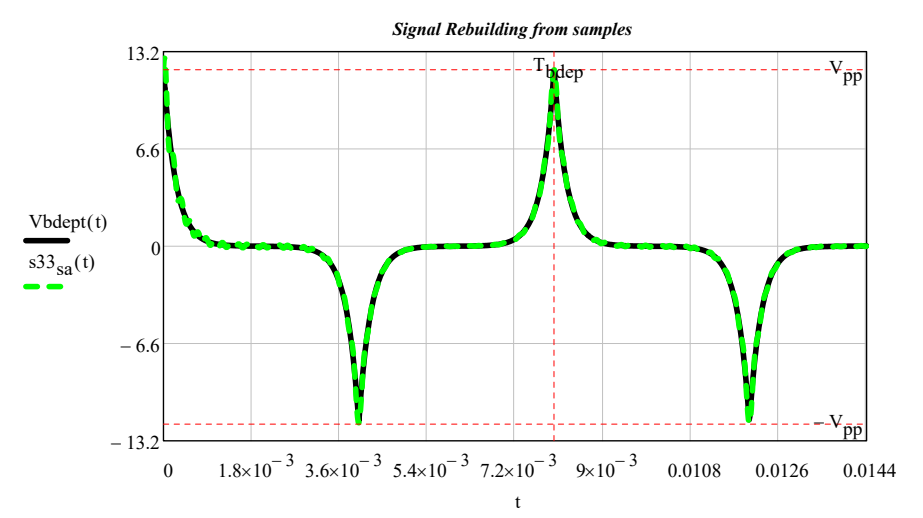
$u_{m33}^T =$	0	1	2	3	4
	12	8.598	6.161	4.415	...



$\text{reerr} = 10\%$
 $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.038 \frac{\text{Mrads}}{\text{sec}}$
 $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s33_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m33}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \right]$ $N0_{gd} - 1 = 255 \quad \text{reerr} = 10\%$

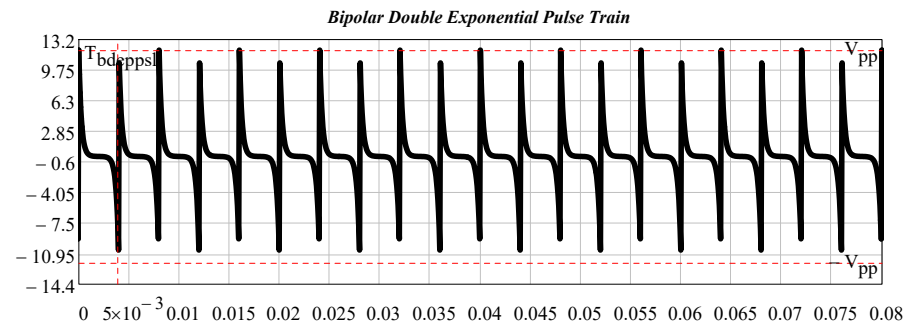


TEST Waveforms

Periodic Waveforms

34 Bipolar Double Exponential Odd symmetric Pulse Train

$$T_{bdeppsl} := 16 \cdot \tau_{ptd_}$$

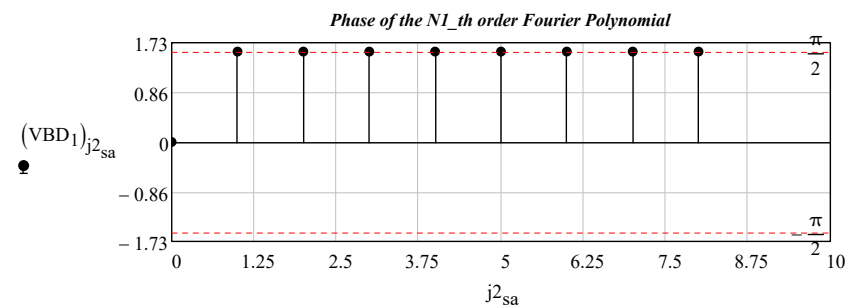
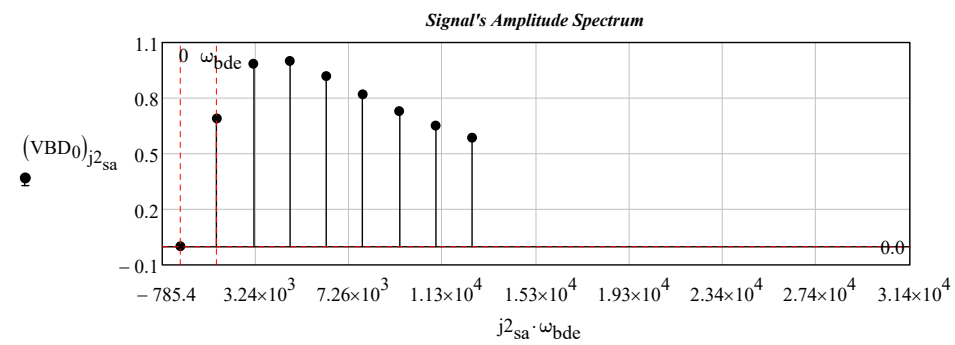


$$N1 := 50$$

$$\omega_{bde} := \frac{2 \cdot \pi}{T_{bdeppsl}} \quad V_{bdeospp}(t) := \frac{V_{bdeospp}(t, \tau_{ptd_}, T_{bdeppsl}, V_{pp}, N0_{gd})}{V}$$

$$VBD := SPCT(V_{bdeospp}, \tau_{gd}, N1, 0 \cdot s, T_{bdeppsl}) \quad N1 = 50$$

$$j2_{sa} := 0 \dots \text{rows}(VBD) - 1 \quad \omega_{bde} = 1.571 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := VBD3 \cdot \text{Hz}$$

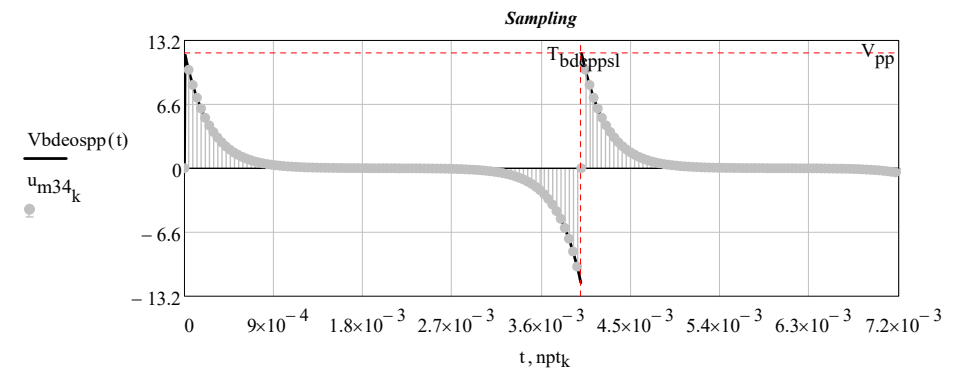
$$Bw_{sa} = 0.012 \cdot \text{MHz}$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.024 \cdot \text{MHz}$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{bdeppsl}} = 2.667$

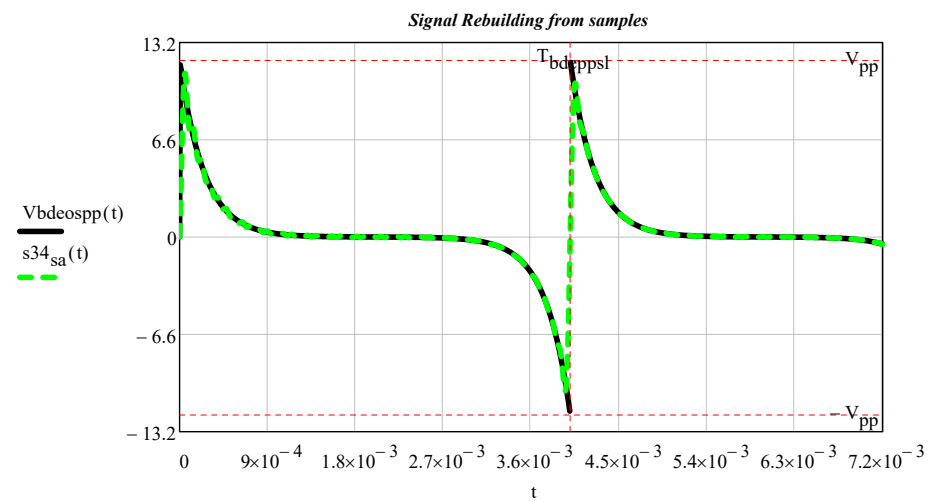
$$(u_{m34})_k := V_{bdeospp}(npt_k)$$

$$u_{m34}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{matrix} 0 \\ -1.35 \cdot 10^{-6} \\ 10.158 \\ 8.598 \\ 7.278 \end{matrix} & \dots \end{matrix}$$


relerr = 10.-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.075 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

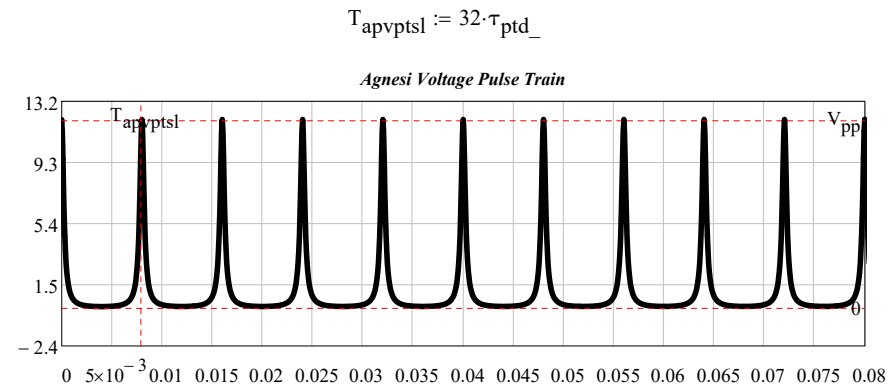
interpolation formula $s34_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m34}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10.-%$



TEST Waveforms

Periodic Waveforms

35 Agnesi Profile Voltage Pulse Train

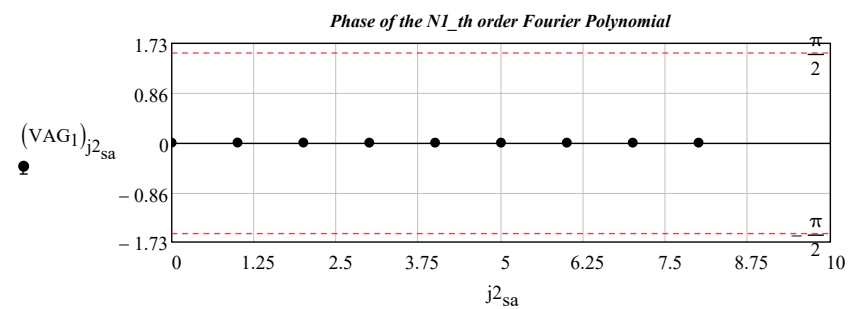
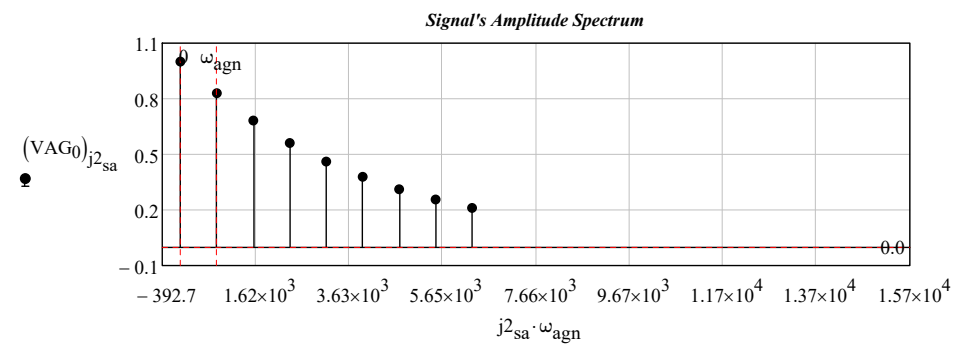


$N1 := 50$

$$\omega_{agn} := \frac{2 \cdot \pi}{T_{apvptsl}} \quad V_{agnp}(t) := \frac{V_{agnp}(t, \tau_{ptd_}, T_{apvptsl}, V_{pp}, N0_{gd})}{V}$$

$$VAG := SPCT(V_{agnp}, rt_{gd}, N1, 0, s, T_{apvptsl}) \quad N1 = 50$$

$$j2_{sa} := 0..rows(VAG) - 1 \quad \omega_{agn} = 0.785 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := VAG3 \cdot \text{Hz}$$

$$Bw_{sa} = 2.875 \times 10^{-3} \cdot \text{MHz}$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 5.75 \times 10^{-3} \cdot \text{MHz}$

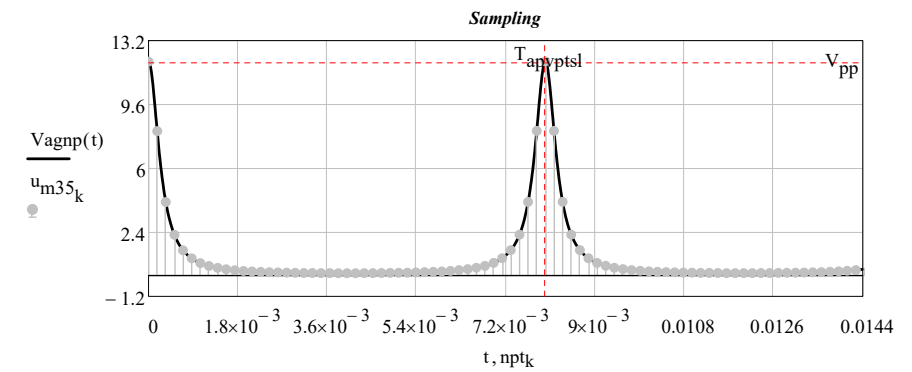
$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{bdeppsl}} = 11.13$

$$(u_{m35})_k := V_{agnp}(npt_k)$$

$$u_{m35}^T =$$

0	1	2	3	4	5	6	7	
0	12.019	8.106	4.108	2.262	1.395	0.939	0.675	...



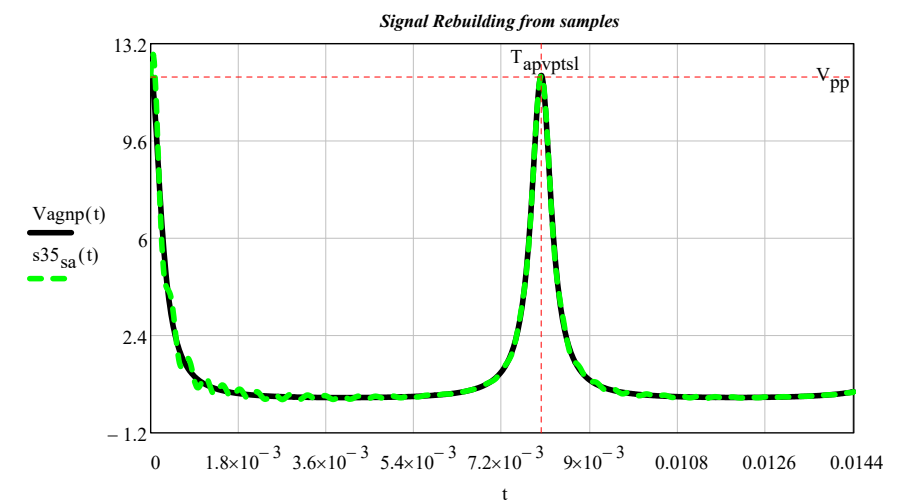
relerr = 10-%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.018 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

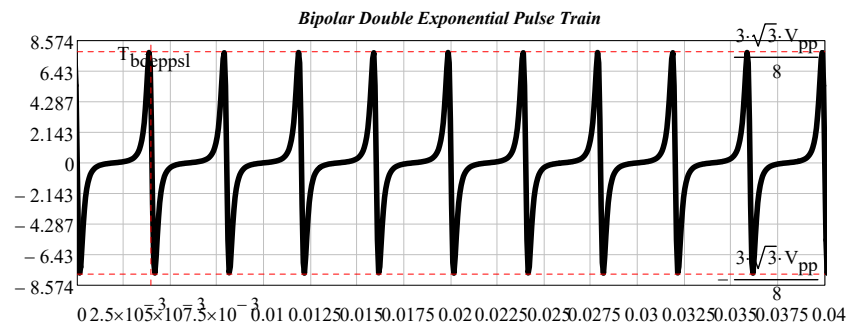
Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s35_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m35}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10-%



Periodic Waveforms

36.Agesi Derivative Profile Voltage Pulse Train

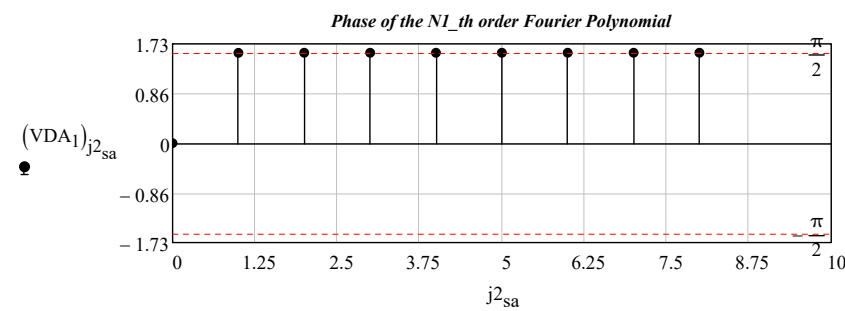
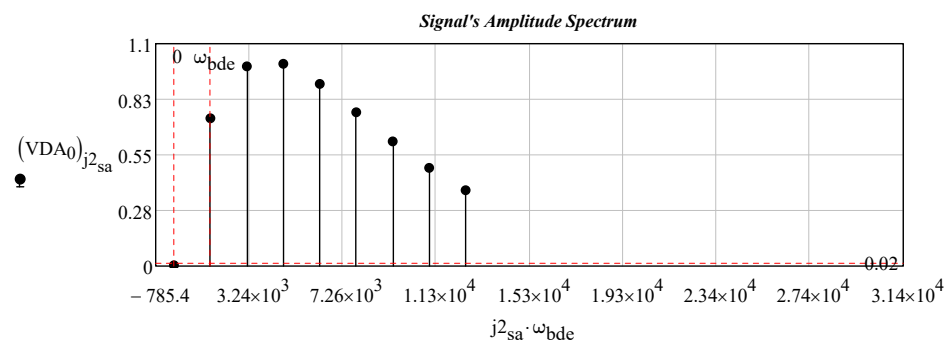


$N1_{gd} := 50$

$$\omega_{bde} := \frac{2 \cdot \pi}{T_{bdeppsl}} \quad VDagnp(t) := \frac{VDagnp(t, \tau_{ptd}, T_{bdeppsl}, V_{pp}, N0_{gd})}{V}$$

$$VDA := SPCT(VDagnp, \tau_{gd}, N1_{gd}, 0 \cdot s, T_{bdeppsl}) \quad N1_{gd} = 50$$

$$j2_{sa} := 0 \dots \text{rows}(VDA) - 1 \quad \omega_{bde} = 1.571 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := VDA_3 \cdot \text{Hz}$$

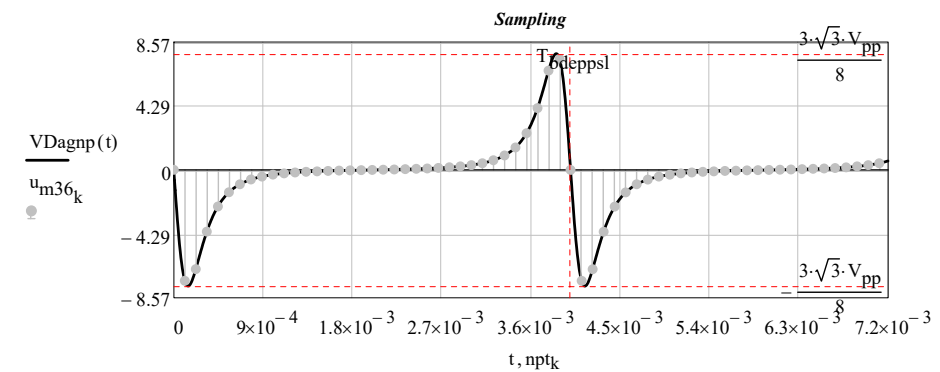
$$Bw_{sa} = 4.5 \times 10^{-3} \cdot \text{MHz}$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 9 \times 10^{-3} \cdot \text{MHz}$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{bdeppsl}} = 7.111$

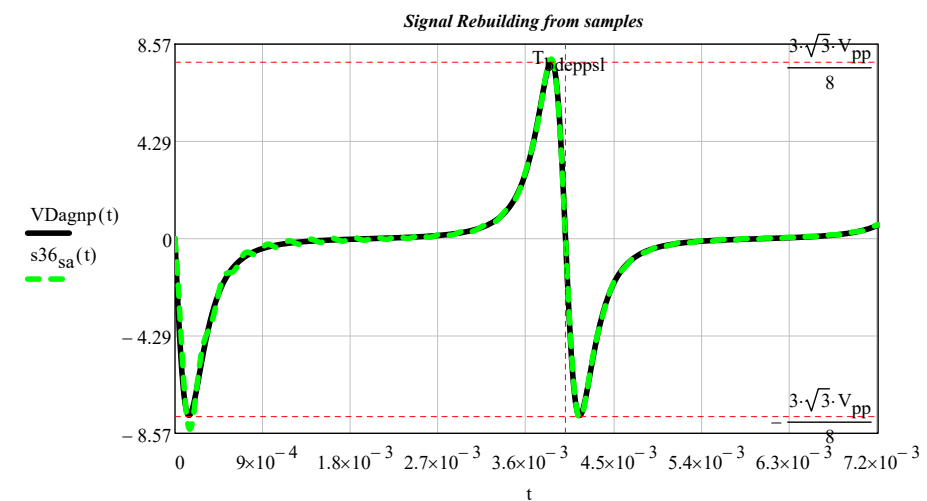
$$(u_{m36})_k := VDagnp(npt_k)$$

$$u_{m36}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ \dots \end{matrix} & 6.996 \cdot 10^{-3} & -7.43 & -6.649 & -4.138 & \dots \end{matrix}$$


relerr = 10% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.028 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s36_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m36}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$

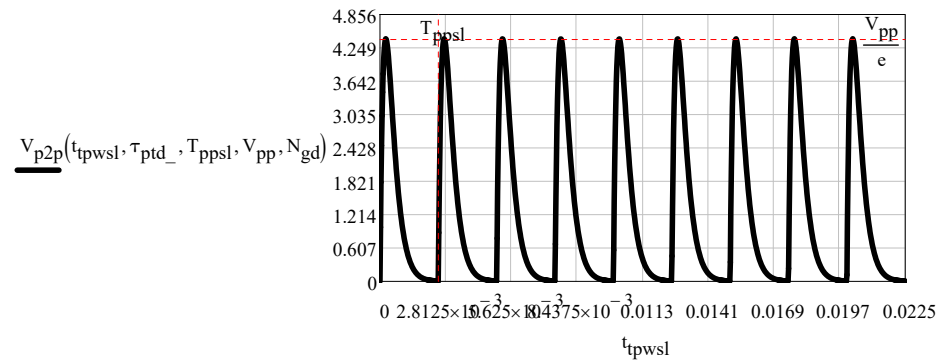


Periodic Waveforms

37 Poisson Profile Voltage Pulse Train

$$T_{ppsl} := 10 \cdot \tau_{ptd_}$$

$$t_{tpw} := 0 \cdot \tau_{ptd_}, 0 \cdot \tau_{ptd_} + \frac{200 \cdot \tau_{ptd_}}{500} .. 200 \cdot \tau_{ptd_}$$

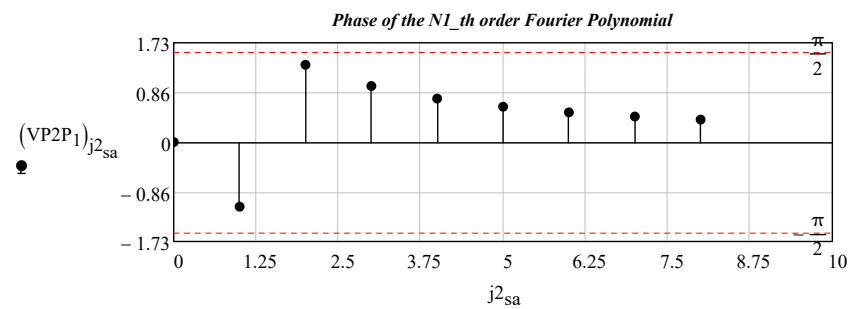
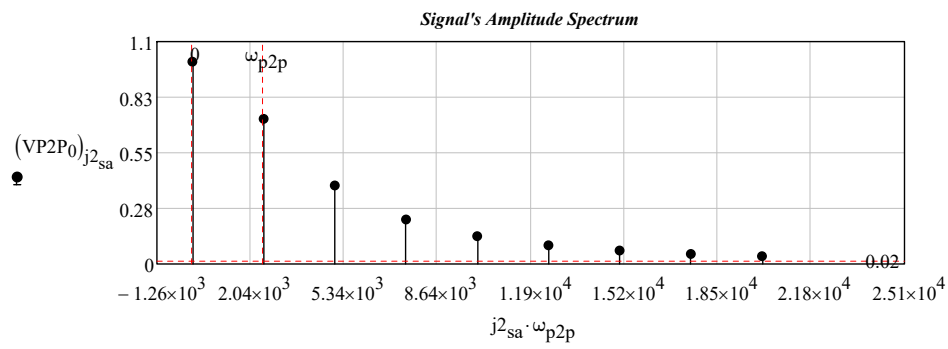


$$\omega_{p2p} := \frac{2 \cdot \pi}{T_{ppsl}}$$

$$V_{p2p}(t) := \frac{V_{p2p}(t, \tau_{ptd_}, T_{ppsl}, V_{pp}, N_{gd})}{V}$$

$$VP2P := SPCT(V_{p2p}, \tau_{gd}, N1_, 0 \cdot s, T_{ppsl}) \quad N1_ = 50$$

$$j2_{sa} := 0 .. \text{rows}(VP2P) - 1 \quad \omega_{p2p} = 2.513 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := VP2P_3 \cdot \text{Hz}$$

$$Bw_{sa} = 0.01 \cdot \text{MHz}$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 0.02 \cdot \text{MHz}$

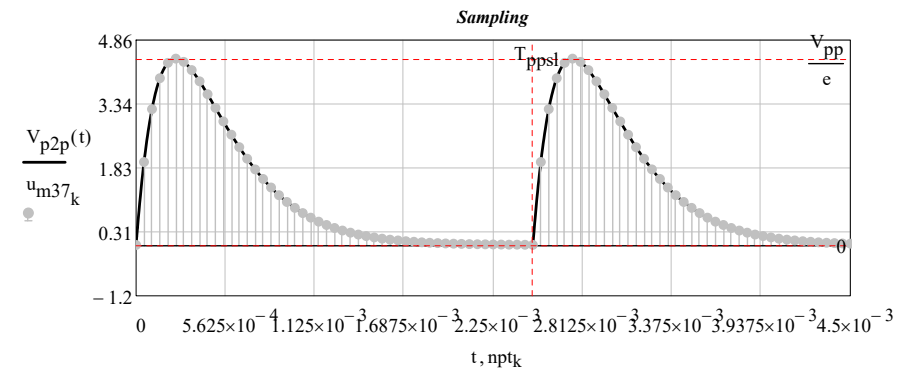
$$nptk := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{ppsl}} = 5.12$

$$(u_{m37})_k := V_{p2p}(nptk)$$

$$u_{m37}^T =$$

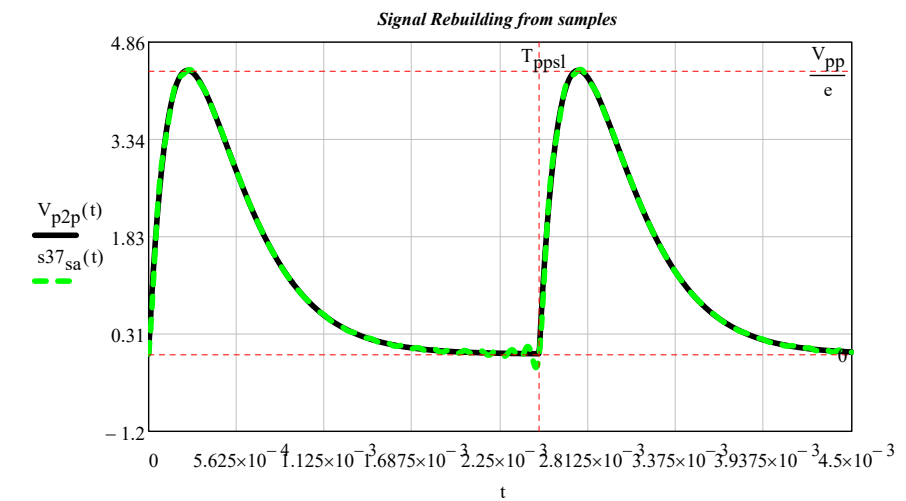
	0	1	2	3	4
0	0	1.965	3.218	3.951	...



relerr = 10-% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.063 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

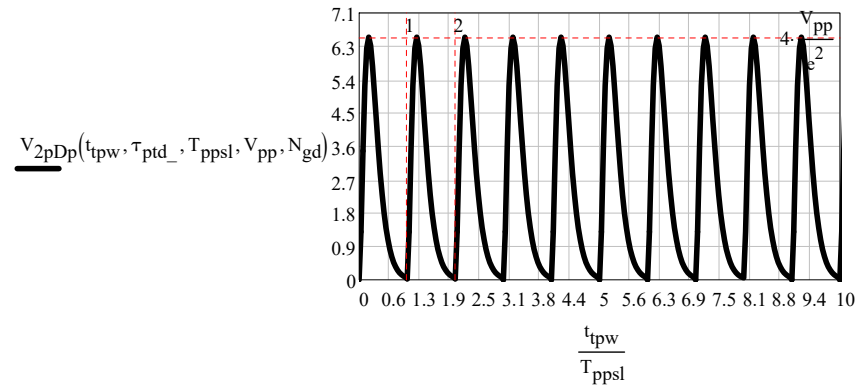
interpolation formula $s37_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m37}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$



TEST Waveforms

Periodic Waveforms

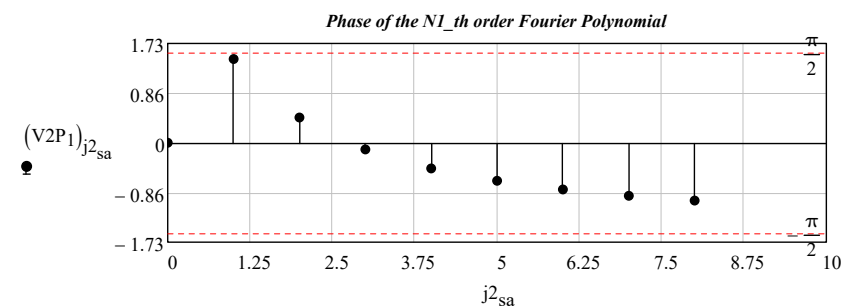
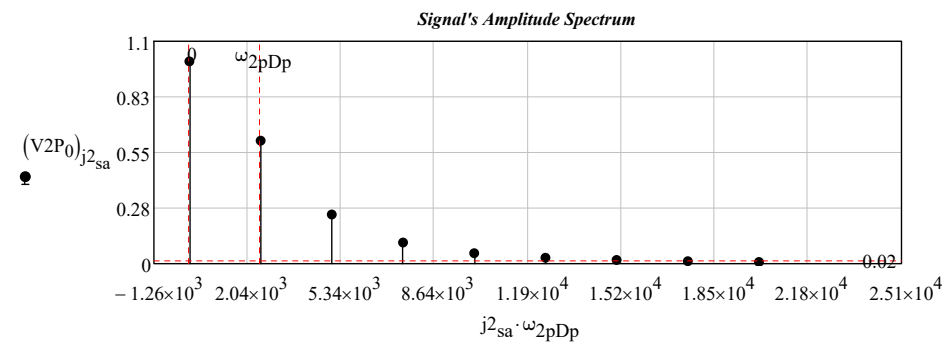
38 Poisson Derivative Profile Voltage Pulse Train



$$\omega_{2pDp} := \frac{2 \cdot \pi}{T_{ppsl}} \quad V_{2pDp}(t) := \frac{V_{2pDp}(t, \tau_{ptd}, T_{ppsl}, V_{pp}, N_{gd})}{V}$$

$$V_{2P} := SPCT(V_{2pDp}, \tau_{gd}, N1_, 0 \cdot s, T_{ppsl}) \quad N1_ = 50$$

$$j_{2sa} := 0 \dots \text{rows}(V_{2P}) - 1 \quad \omega_{2pDp} = 2.513 \cdot \frac{\text{krads}}{s}$$



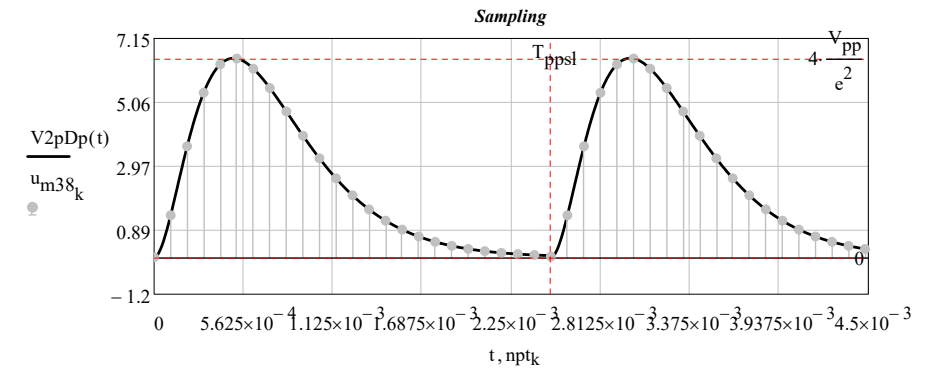
$$Bw_{sa} := V_{2P3} \cdot \text{Hz} \\ Bw_{sa} = 4.8 \times 10^{-3} \cdot \text{MHz} \\ \text{sampling frequency: } f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 9.6 \times 10^{-3} \cdot \text{MHz} \\ npt_k := \frac{k}{f_{pt_{so}}}$$

$$\text{Frequency resolution: } \frac{N0_{gd}}{f_{pt_{so}} \cdot T_{ppsl}} = 10.667$$

$$(u_{m38})_k := V_{2pDp}(npt_k)$$

$$u_{m38}^T =$$

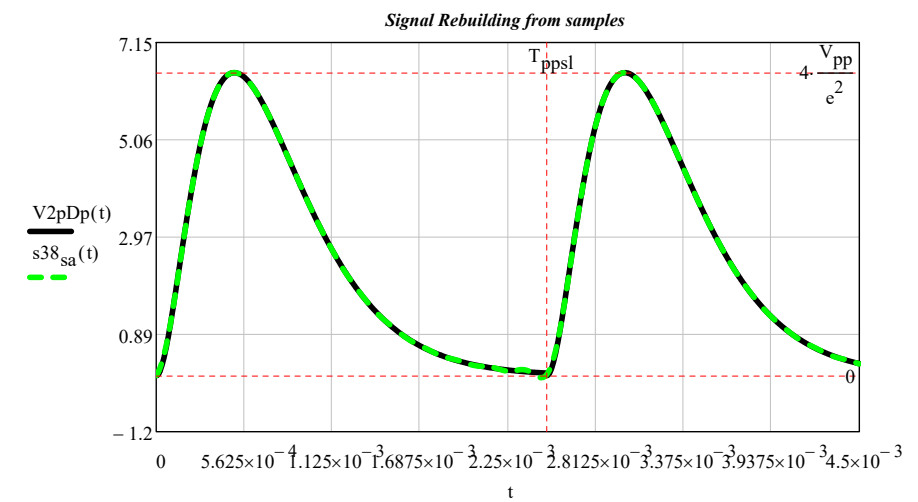
0	1	2	3	4	5	6	7	8	
0	0	1.373	3.622	5.372	6.296	6.485	6.156	5.524	...



$$\text{relerr} = 10\% \quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.03 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

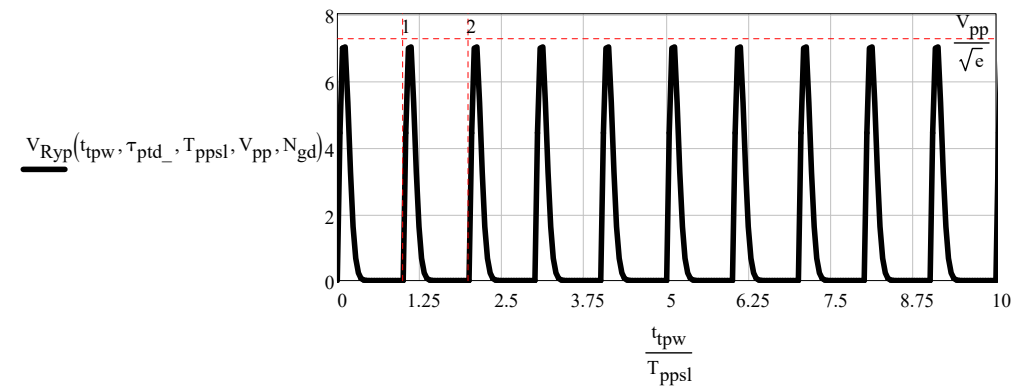
Signal reconstruction according to the Shannon sampling theorem:

$$\text{interpolation formula } s_{38_{sa}}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m38_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)) \right] \quad N0_{gd} - 1 = 255 \quad \text{relerr} = 10\%$$



Periodic Waveforms

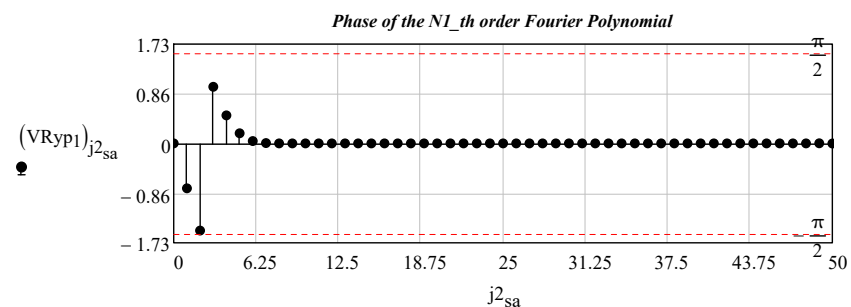
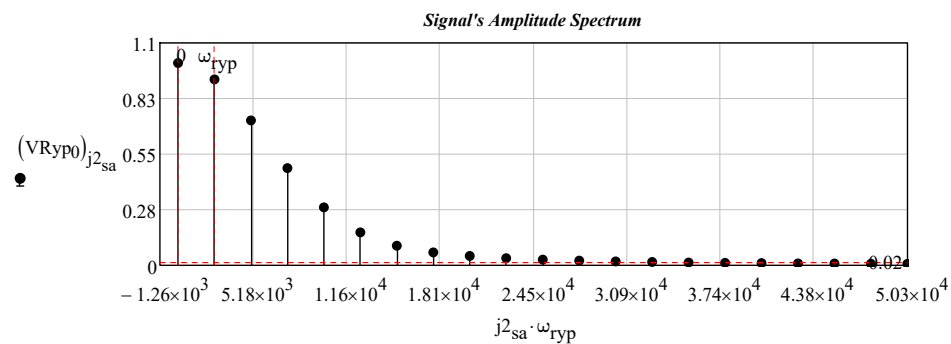
39 Rayleigh Profile Voltage Pulse Train



$$\omega_{ryp} := \frac{2 \cdot \pi}{T_{ppsl}} \quad V_{Ryp}(t) := \frac{V_{Ryp}(t, \tau_{ptd}, T_{ppsl}, V_{pp}, N_{gd})}{V}$$

$$VRyp := SPCT(V_{Ryp}, \tau_{gd}, N1, 0, s, T_{ppsl}) \quad N1 = 50$$

$$j2_{sa} := 0..rows(VRyp0) - 1 \quad \omega_{ryp} = 2.513 \cdot \frac{\text{krads}}{s}$$



$$Bw_{sa} := VRyp3 \cdot \text{Hz}$$

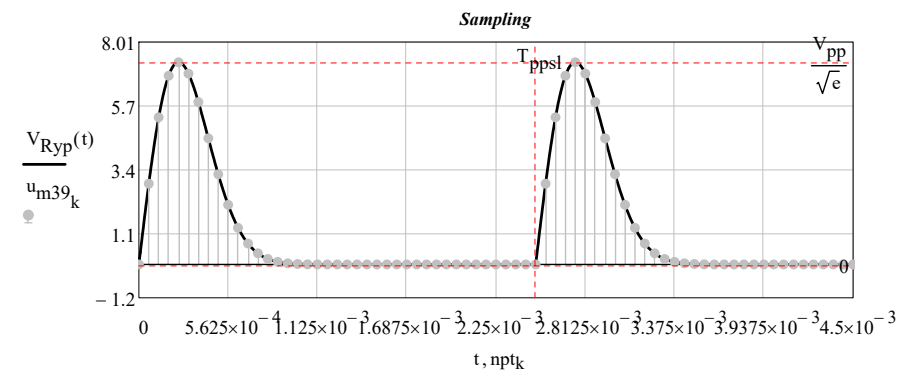
$$Bw_{sa} = 8 \times 10^{-3} \cdot \text{MHz}$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \quad fpt_{so} = 0.016 \cdot \text{MHz}$

$$nptk := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{ppsl}} = 6.4$

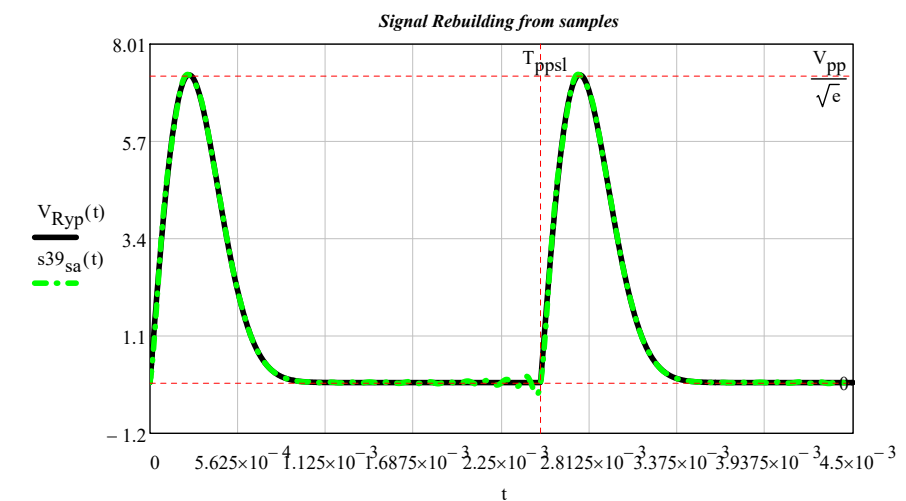
$$(u_{m39})_k := V_{Ryp}(nptk)$$

$$u_{m39}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ \dots \end{matrix} & 0 & 2.908 & 5.295 & 6.794 & \dots \end{matrix}$$


$$relerr = 10\% \quad \omega_{low} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.05 \cdot \frac{\text{Mrads}}{\text{sec}} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s39_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m39}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ $relerr = 10\%$



TEST Waveforms

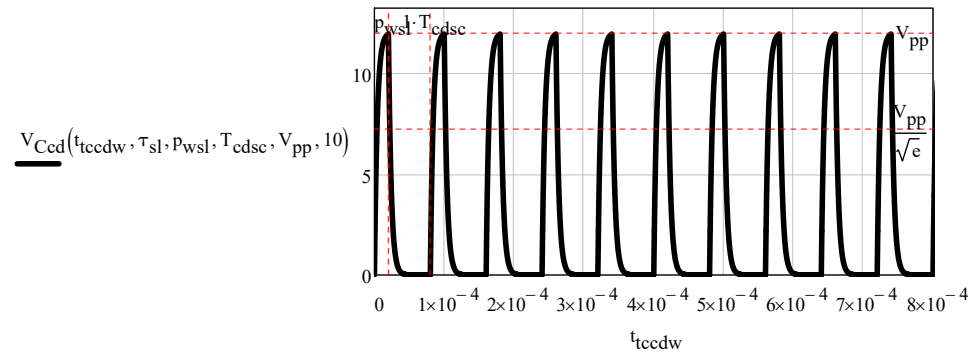
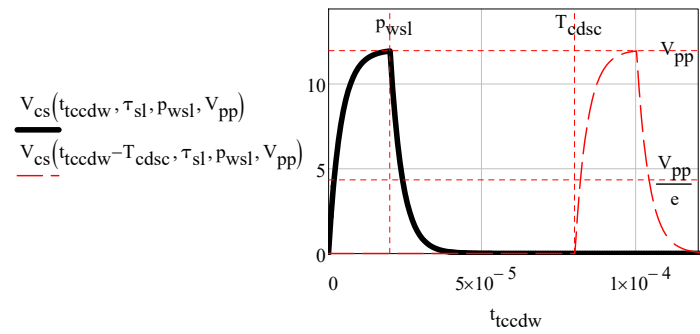
Periodic Waveforms

40 Cap. Charge and Discharge Pulse Train

pulse width: $p_{wsl} := 20 \cdot \mu s$
 time constant $\tau_{sl} := \frac{p_{wsl}}{5}$
 Period: $T_{cdsc} := 4 \cdot p_{wsl}$ $\omega_{cdsc} := \frac{2 \cdot \pi}{T_{cdsc}}$

$t_{ccdw} := 0 \cdot T_{cdsc}, 0 \cdot T_{cdsc} + \frac{100 \cdot T_{cdsc}}{10000} .. 100 \cdot T_{cdsc}$

Cap. Voltage Charge and Discharge

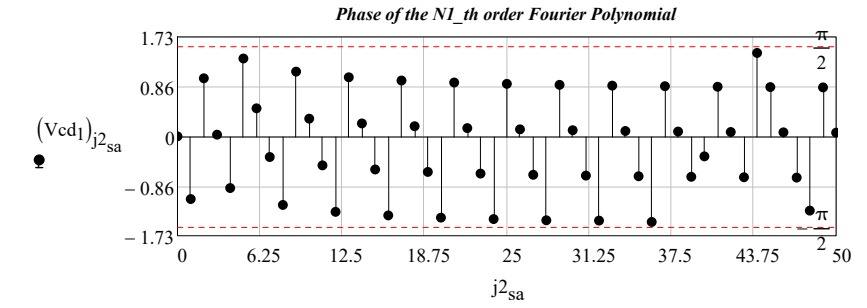
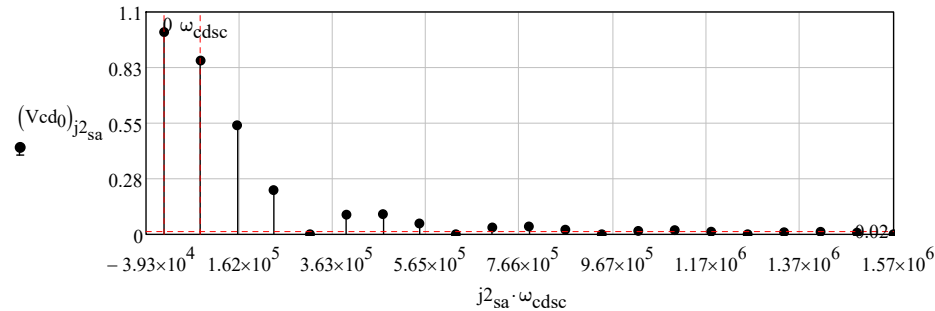


$V_{Ccd}(t) := \frac{V_{Ccd}(t, \tau_{sl}, p_{wsl}, T_{cdsc}, V_{pp}, 10)}{V}$

$Vcd := SPCT(V_{Ccd}, rt_{gd}, N1_, 0 \cdot s, T_{cdsc})$ $N1_ = 50$

$j2_{sa} := 0 .. rows(Vcd_0) - 1$ $\omega_{cdsc} = 78.54 \cdot \frac{k\text{rads}}{s}$

Signal's Amplitude Spectrum



$Bw_{sa} := Vcd_3 \cdot \text{Hz}$

$Bw_{sa} = 0.325 \cdot \text{MHz}$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.65 \cdot \text{MHz}$

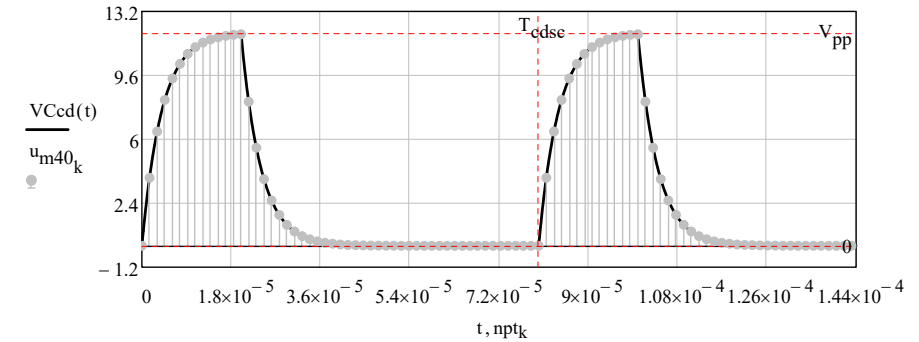
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{cdsc}} = 4.923$

$u_{m40}_k := VCcd(npt_k)$

$u_{m40}^T =$	0	1	2	3	4
	0	3.831	6.44	8.215	...

Sampling



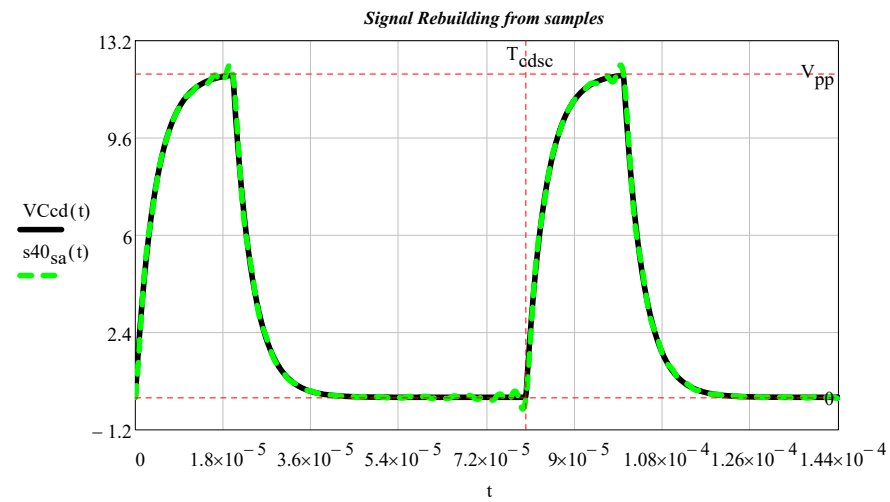
relerr = 10.0%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 2.042 \cdot \frac{\text{Mrads}}{\text{sec}}$

$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s40_{sa}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m40}_n \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10.0%



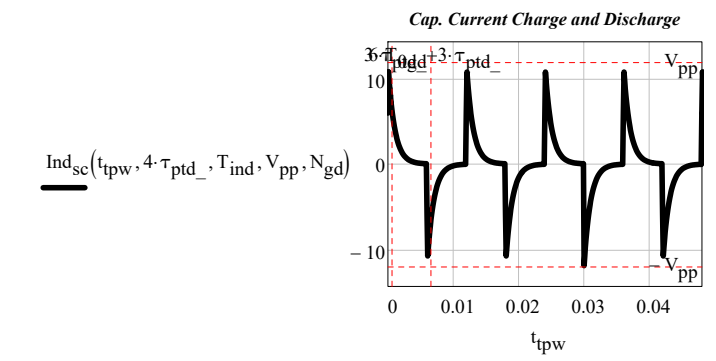
TEST Waveforms

Periodic Waveforms

41 Induct Charge and Discharge Pulse Train

$$T_{ind} := 6 \cdot T_{0gd}$$

$$Ind_{sc}(t_{pw}, \tau_{ptd}, T_{ind}, V_{pp}) := \sum_{k=0}^{N_{gd}} [(-1)^k \cdot V_{dis}(t_{pw} - k \cdot T_{ind}, \tau_{ptd}, V_{pp})]$$



$$Indsc(t) := \frac{Ind_{sc}(t, 4 \cdot \tau_{ptd}, T_{ind}, V_{pp}, N_{gd})}{V}$$

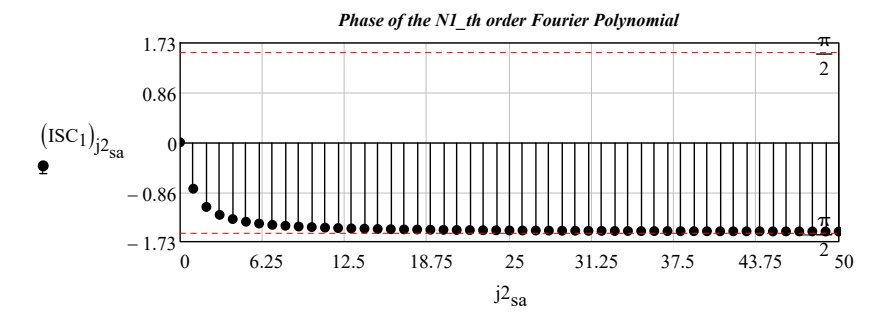
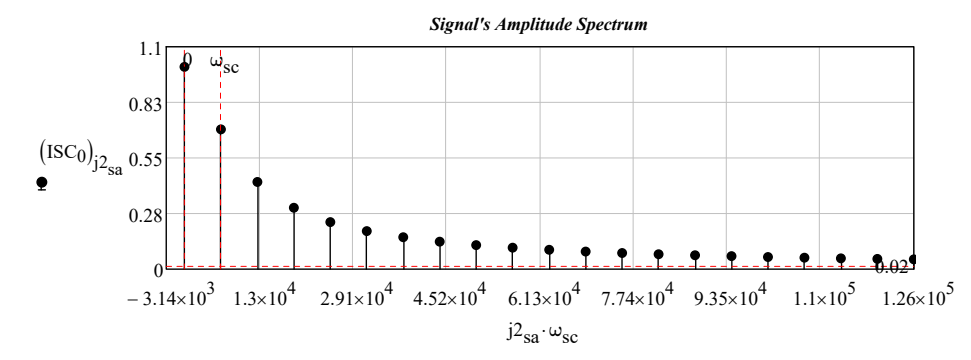
$$\omega_{sc} := \frac{2 \cdot \pi}{T_{0gd}}$$

$$ISC := SPCT(Ind_{sc}, rt_{gd}, N1_, 0 \cdot s, T_{ind})$$

$$N1_ = 50$$

$$j2_{sa} := 0..rows(ISC_0) - 1$$

$$\omega_{ptd} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$



$$Bw_{sa} := ISC_3 \cdot Hz$$

$$Bw_{sa} = 8 \times 10^{-3} \cdot MHz$$

sampling frequency:

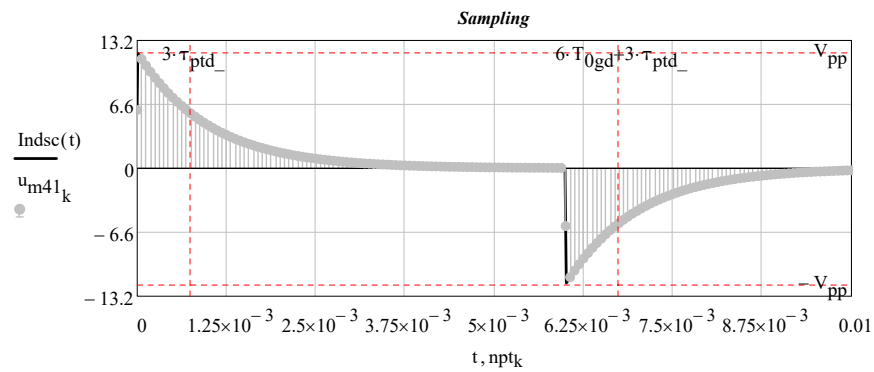
$$f_{pt_{so}} := 2 \cdot Bw_{sa}$$

$$f_{pt_{so}} = 0.016 \cdot MHz$$

$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}} \cdot T_{ind}} = 2.667$

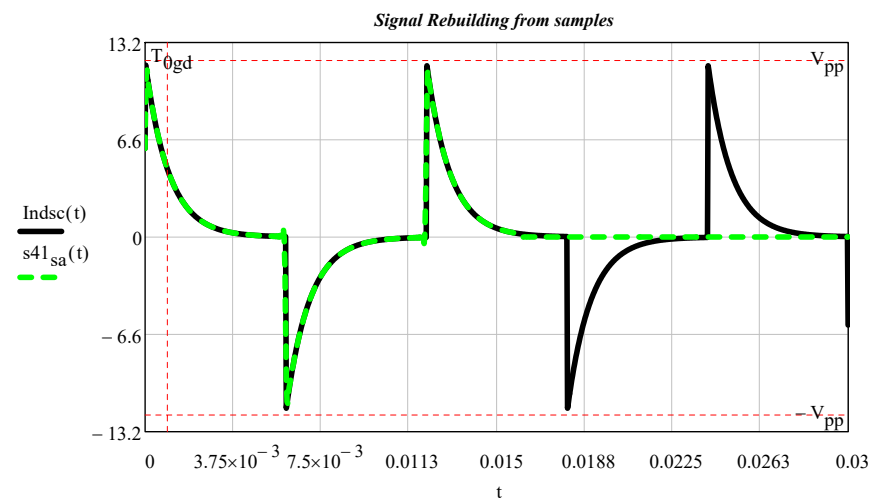
$u_{m41_k} := \text{Indsc}(npt_k)$

$$u_{m41}^T = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{matrix} 0 \\ 6 \\ 11.273 \\ 10.59 \\ 9.948 \\ 9.346 \\ 8.779 \\ \dots \end{matrix} \end{matrix}$$


relerr = 10% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.05 \cdot \frac{\text{Mrads}}{\text{sec}}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{41_{sa}}(t) := \sum_{n=0}^{N0_{gd}-1} (u_{m41_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi))$ $N0_{gd} - 1 = 255$ relerr = 10%



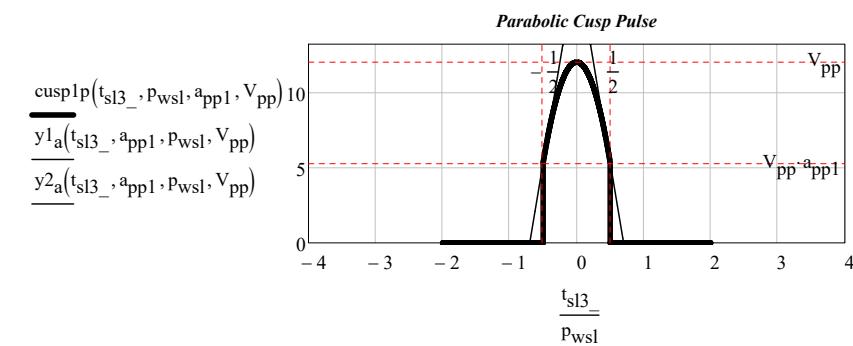
TEST Waveforms

Periodic Waveforms

42 Parabolic Cusps Pulse Train

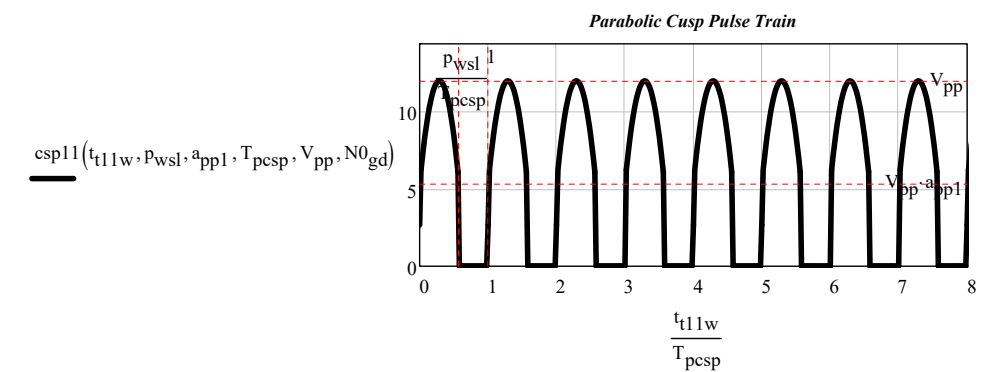
Signal amplitude: $V_{pp} = 12 \cdot V$
 Pulse width: $P_{wsl} = 20 \cdot \mu s$
 Duty cycle: $\delta_{cysl} := \gamma$
 Period: $T_{pcsp} := \frac{P_{wsl}}{\delta_{cysl}}$ $\omega_{pcsp} := \frac{2 \cdot \pi}{T_{pcsp}}$
 Max pulse amplitude and cusp ratio: $a_{pp1} := \frac{4}{9}$

$t_{sl3_} := -2 \cdot P_{wsl}, -2 \cdot P_{wsl} + \frac{(2 \cdot P_{wsl} + 2 \cdot P_{wsl})}{10000} .. 2 \cdot P_{wsl}$



$\text{cusp1p}(t_{sl3_}, P_{wsl}, a_{pp1}, V_{pp})$
 $y1_a(t_{sl3_}, a_{pp1}, P_{wsl}, V_{pp})$
 $y2_a(t_{sl3_}, a_{pp1}, P_{wsl}, V_{pp})$

$t_{t11w} := 0 \cdot T_{pcsp}, 0 \cdot T_{pcsp} + \frac{10 \cdot T_{pcsp} - 0 \cdot T_{pcsp}}{500} .. 10 \cdot T_{pcsp}$

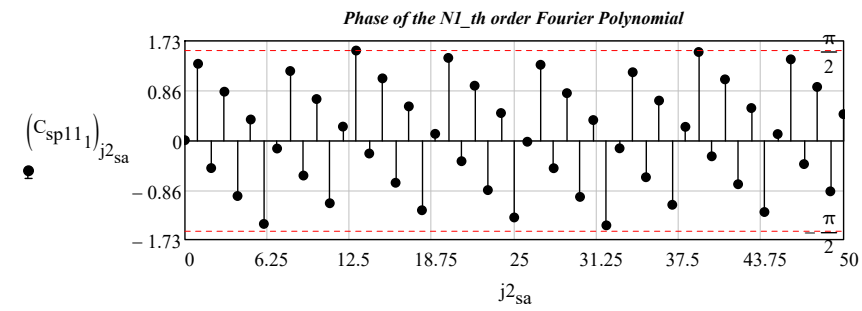
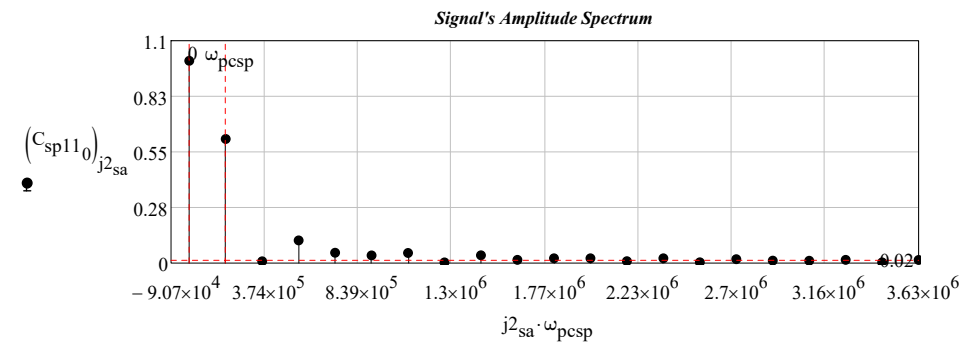


$\text{csp11}(t_{t11w}, P_{wsl}, a_{pp1}, T_{pcsp}, V_{pp}, N0_{gd})$

$C_{sp11}(t) := \frac{\text{csp11}(t, P_{wsl}, a_{pp1}, T_{pcsp}, V_{pp}, N0_{gd})}{V}$

$C_{sp11} := \text{SPCT}(C_{sp11}, rt_{gd}, N1_, 0 \cdot s, T_{pcsp})$ $N1_ = 50$

$j2_{sa} := 0 .. \text{rows}(C_{sp11_0}) - 1$ $\omega_{ptd_} = 6.283 \times 10^{-3} \cdot \frac{\text{Mrads}}{s}$



$$Bw_{sa} := C_{sp11_3} \cdot Hz$$

$$Bw_{sa} = 1.385 \cdot MHz$$

sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 2.771 \cdot MHz$

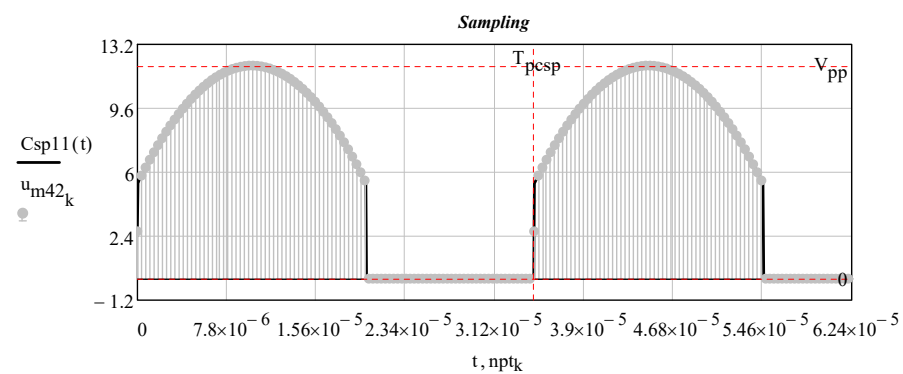
$$npt_k := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{pcsp}} = 2.667$

$$u_{m42_k} := C_{sp11}(npt_k)$$

$$u_{m42}^T =$$

0	1	2	3	4	5	6	7	...
2.667	5.806	6.261	6.699	7.119	7.522	7.908	...	



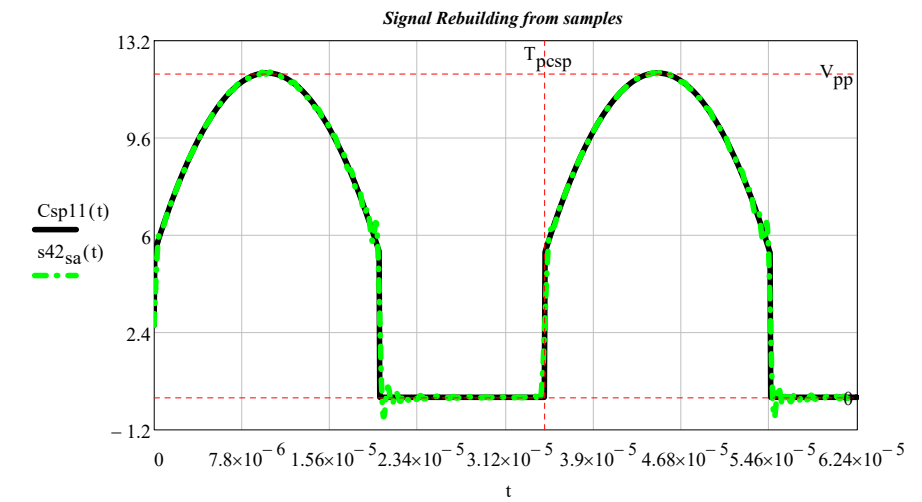
relerr = 10.%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 8.704 \frac{Mrads}{sec}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{42_{sa}}(t) := \sum_{n=0}^{N0_{gd}-1} \left(u_{m42_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right)$ $N0_{gd} - 1 = 255$ relerr = 10.%



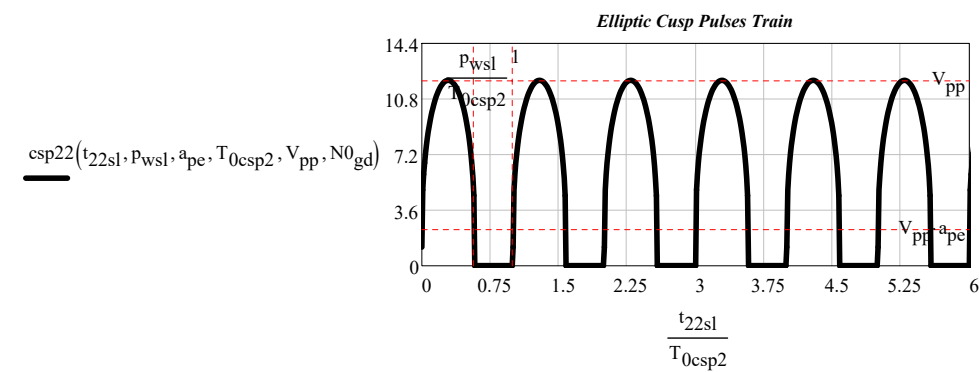
TEST Waveforms

Periodic Waveforms

43 Elliptic Cusps Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$
 Pulse width: $P_{wsl} = 20 \cdot \mu s$
 Duty cycle: $\delta_{cysl} := \gamma$
 Period: $T_{0csp2} := \frac{P_{wsl}}{\delta_{cysl}}$
 Max pulse amplitude and cusp ratio: $a_{pe} := \frac{2}{10}$

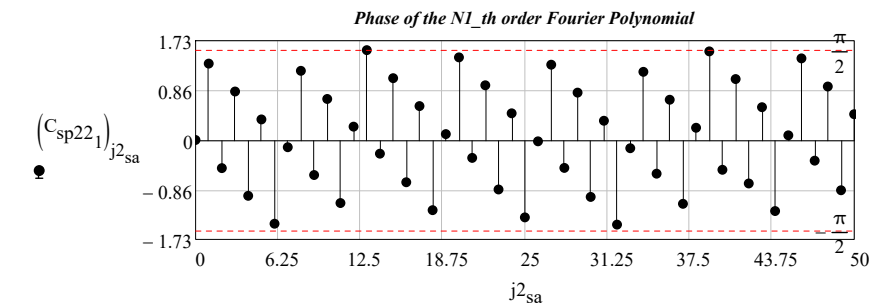
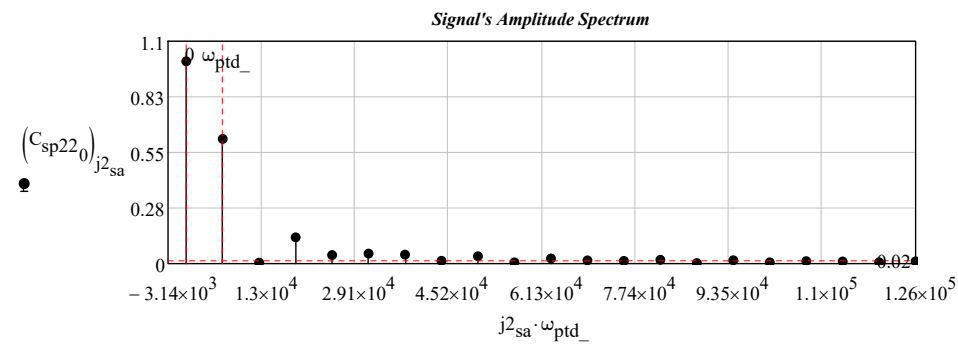
$$t_{22sl} := 0 \cdot T_{0csp2}, 0 \cdot T_{0csp2} + \frac{10 \cdot T_{0csp2} - 0 \cdot T_{0csp2}}{1000} \dots 10 \cdot T_{0csp2}$$



$$C_{sp22}(t) := \frac{csp22(t, P_{wsl}, a_{pe}, T_{0csp2}, V_{pp}, N0_{gd})}{V}$$

$$C_{sp22} := SPCT(C_{sp22}, rt_{gd}, N1_, 0 \cdot s, T_{0csp2}) \quad N1_ = 50$$

$$j^2_{sa} := 0 \dots rows(C_{sp22_0}) - 1 \quad \omega_{ptd_} = 6.283 \times 10^{-3} \frac{Mrads}{s}$$



$$Bw_{sa} := C_{sp22_3} \cdot Hz$$

$$Bw_{sa} = 1.385 \cdot MHz$$

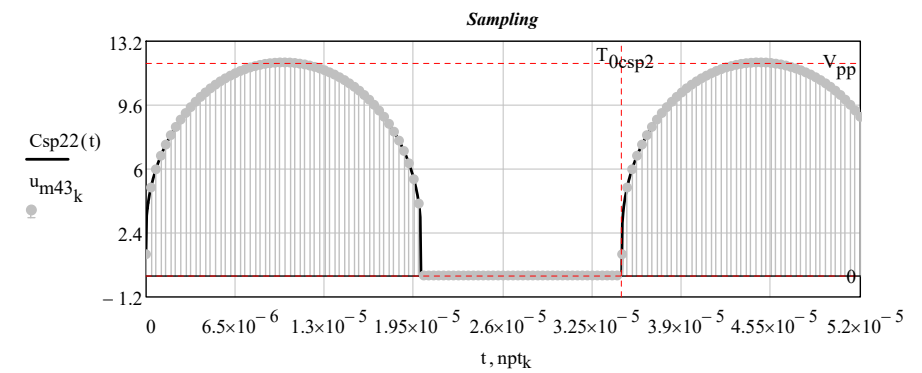
sampling frequency: $f_{pt_{so}} := 2 \cdot Bw_{sa} \quad f_{pt_{so}} = 2.771 \cdot MHz$

$$n_{ptk} := \frac{k}{f_{pt_{so}}}$$

Frequency resolution: $\frac{N0_{gd}}{f_{pt_{so}}} \cdot \frac{1}{T_{0csp2}} = 2.667$

$$u_{m43_k} := C_{sp22}(n_{ptk})$$

$u_{m43}^T =$	0	1	2	3	4	5	6	7	
	0	1.2	4.956	5.981	6.745	7.369	7.901	8.366	...



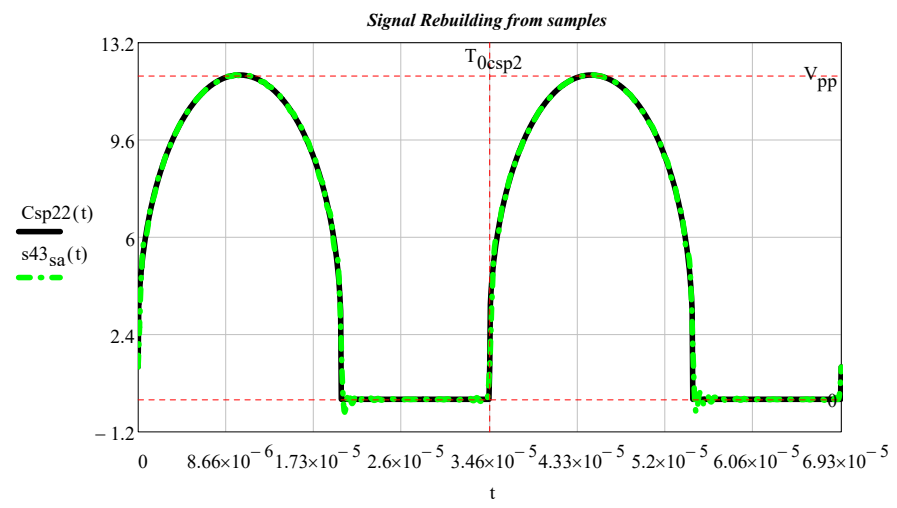
relerr = 10.-%

$$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 8.704 \frac{Mrads}{sec}$$

$$n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula $s_{43_{sa}}(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u_{m43_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)) \right] \quad N0_{gd} - 1 = 255 \quad relerr = 10.-%$



Fine