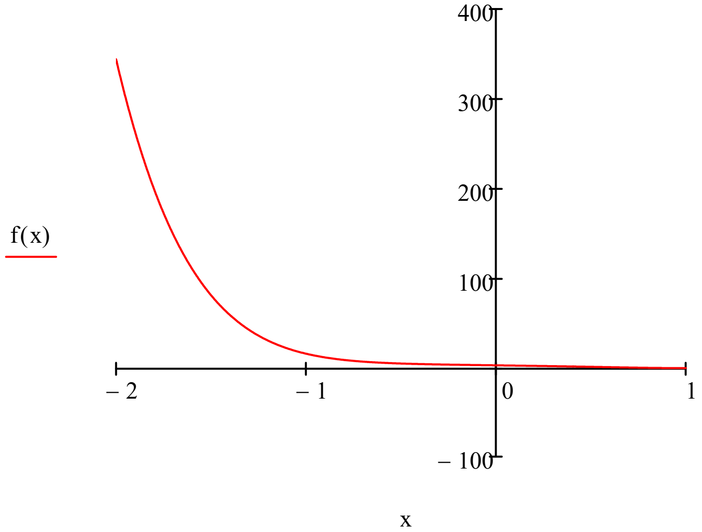
Newton Raphson example

$f1(x) := 3.x^6 - 3\cdot x^5 + 2\cdot x^4 - x^3\cdot 2 - 3\cdot x + 2$

$f2(x) := \sin(3\cdot x - 4)$

$f(x) := f1(x)$



$last := 10$

$x_0 := -0.3$

$fp(x) := \frac{d}{dx}f(x)$ $\qquad NR(x) := \frac{f(x)}{fp(x)}$

$i := 1 .. last$

$x_i := x_{i-1} - NR\left(x_{i-1}\right)$

$f\left(x_{last}\right) = 0$

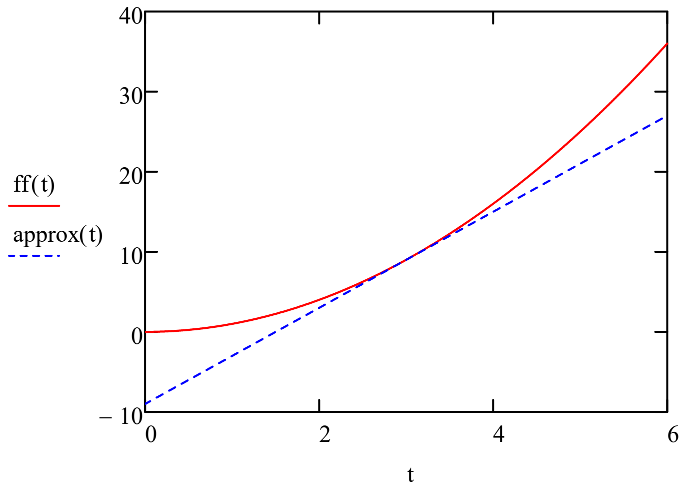
x =

	0
0	-0.3
1	0.46
2	0.587
3	0.583
4	0.583
5	0.583
6	0.583
7	0.583
8	0.583
9	0.583
10	0.583

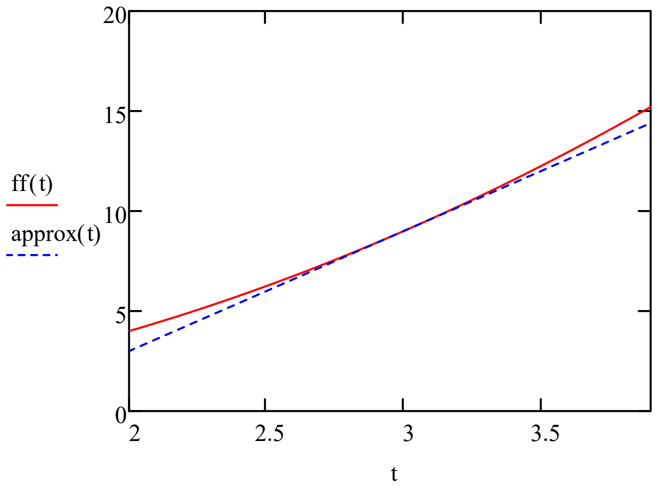
First order Taylor approximation of t-squared

$ff(t) := t^2$

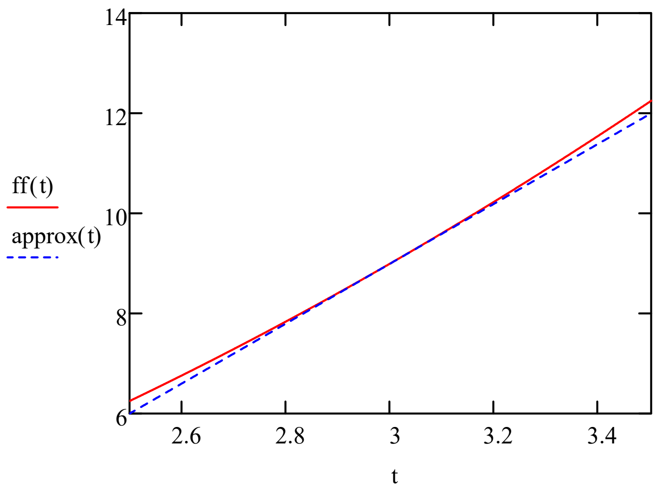
$approx(t) := 6\cdot t - 9$



40
30
20
10
0
− 10
0
2
4
6
t
ff(t)
approx(t)



20
15
10
5
0
2
2.5
3
3.5
t
ff(t)
approx(t)



14
12
10
8
6
2.6
2.8
3
3.2
3.4
t
ff(t)
approx(t)

Second order Taylor Polynomial

$f3(x) := \sin(3\cdot x + 2) - 2$ $\qquad$ $f3(0) = -1.091$

Around a = 0.35

$a := 0.35$

$fa := f3(a) = -1.909$

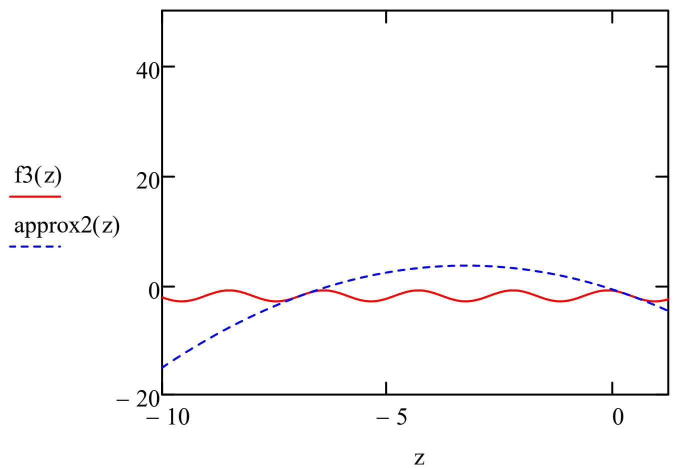
$fp(x) := \frac{d}{dx} f3(x)$

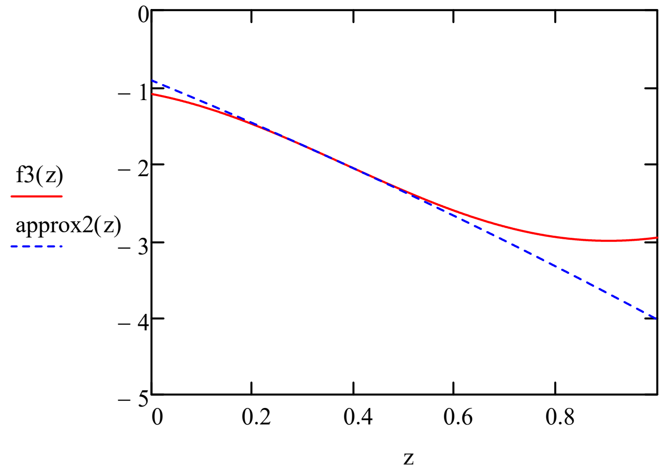
$fpa := fp(a) = -2.987$

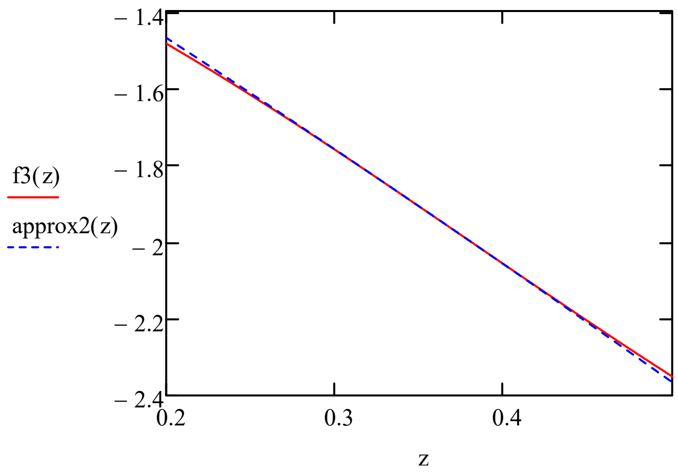
$fpp(x) := \frac{d^2}{dx^2} f3(x)$

$fppa := fpp(a) = -0.823$

$approx2(x) := fa + (x - a)\cdot fpa + (x - a)^2 \cdot \frac{fppa}{2}$ $\qquad$ $approx2(a) = -1.909$







Numerical Integration

$A := 3.6$

$f4(x) := \sin(3 \cdot x - A)$

$g(x) := \frac{-1}{3} \cdot \cos(3 \cdot x - A)$

$ExactSol := g(7) - g(2) = -0.286$ This is the exact solution

$\int_{2}^{7} f4(x)\,dx = -0.286$ This is the MathCad approximation using their numerical algorithm

Trapezium Rule

Nareas must be an even number

$Nareas := 20$ $\qquad$ $Npoints := Nareas + 1$

$Npoints = 21$

$xx_0 := 2$

$j := 1 .. Npoints - 1$

$$xx_j := xx_{j-1} + \frac{(7-2)}{Nareas}$$

$$h := \frac{7-2}{Nareas} = 0.25$$

xx =

	0
0	2
1	2.25
2	2.5
3	2.75
4	3
5	3.25
6	3.5
7	3.75
8	4
9	4.25
10	4.5
11	4.75
12	5
13	5.25
14	5.5
15	...

$k := 0 .. Npoints - 1$

$yy_k := f4(xx_k)$

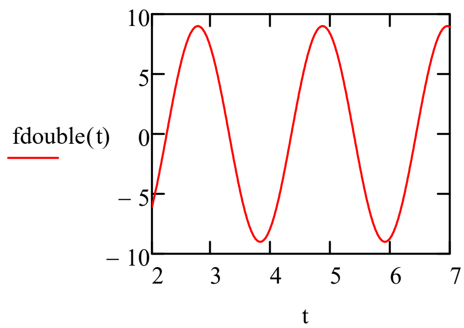
$$AreaTrap := \sum_{n=0}^{Npoints-2} \left[\left(\frac{yy_n + yy_{n+1}}{2}\right)\cdot h\right]$$

$AreaTrap = -0.273$

Error Analysis

Minimum M given by max value of f'' in the interval

$fdouble(x) := -9\cdot\sin(3\cdot x - A)$



$Mmin := 9$

$StepSize := \frac{5}{Nareas} = 0.25 \qquad StepSizeSq := StepSize^2 = 0.063$

$MaxErr := \frac{5}{12} \cdot StepSizeSq \cdot Mmin = 0.234$

$Error := |AreaTrap - ExactSol| = 0.014$

Simpson rule

$$AreaSimp := \frac{h}{3} \cdot \left[yy_0 + \left(4 \cdot \sum_{m=0}^{\frac{Npoints-3}{2}} yy_{2 \cdot m+1} \right) + \left(2 \cdot \sum_{m=1}^{\frac{Npoints-3}{2}} yy_{2 \cdot m} \right) + yy_{Npoints-1} \right]$$

$Npoints = 21$

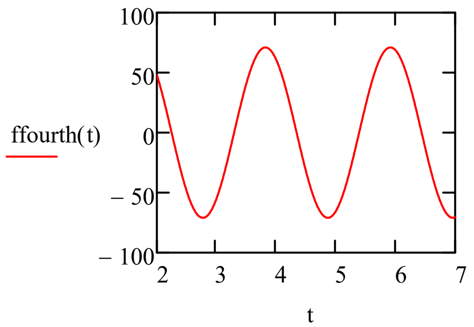
$AreaSimp = -0.287$

So you see that the Simpson rule is more accurate than the Trapezium rule

Error Analysis

Minimum M given by max value of f("") in the interval

$ffourth(x) := 71 \cdot \sin(3 \cdot x - A)$



$Mmin := 71$

$$StepSize := \frac{5}{Nareas} = 0.25 \qquad StepSize4th := StepSize^4 = 3.906 \times 10^{-3}$$

$$MaxErr := \frac{5}{180} \cdot StepSize4th \cdot Mmin = 7.704 \times 10^{-3}$$

$$Error := \left| AreaSimp - ExactSol \right| = 5.387 \times 10^{-4}$$