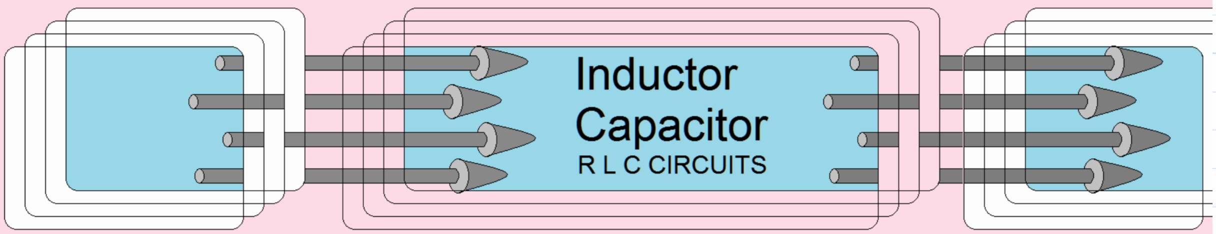
Engineering Circuits Analysis Notes And Example Problems - Schaums Outline 6th Edition.
My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: Electric Circuits 6th Ed., Nahvi & Edminister. Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill.
Karl S. Bogha.

Part 2 - B.
Chapter 7 Schaums Outlines: Electric Circuits 6th Edition.
Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Part 2 - B
(Intermediate Level)

May 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

First Order Circuits - Part B.

This is the chapter in Schaum's Outline (Series) where the initial conditions of the L and C come to play in the circuit.

There is some explanation in chapter 7 in 6th edition of Schaums, and not as in depth as an electrical engineering circuit textbook. Chapter 7 has at least 1 example after each section. Purpose here is to attempt the examples in the study material, and some additional solved problems. You need your textbook in hand.

Chapter 7 First Order Circuits - Schaums Outline Electric Circuits 6th Edition:

Continued

7.10 Response of first order circuits to a pulse.
7.11 Impulse response of RL and RC circuits.
7.12 Summary of step and impulse responses in RL and RC circuits.
7.13 Response of RL and RC circuits to sudden exponential excitations
7.14 Response of RL and RC circuits to sudden sinusoidal excitations
7.15 Summary of forced response in first order circuits
7.16 First order active circuits.<--- NOT doing this because the component is Op-amp. Op-amps are great, but here we focus on the components which Schaum focus on in Laplace Transforms in Chapther 16. Does NOT impact the studies for Laplace Transforms. Would require a refresher topic on Op-amps, that may be excessive. Not difficult to solve the Op-amp circuits problems in comparison.

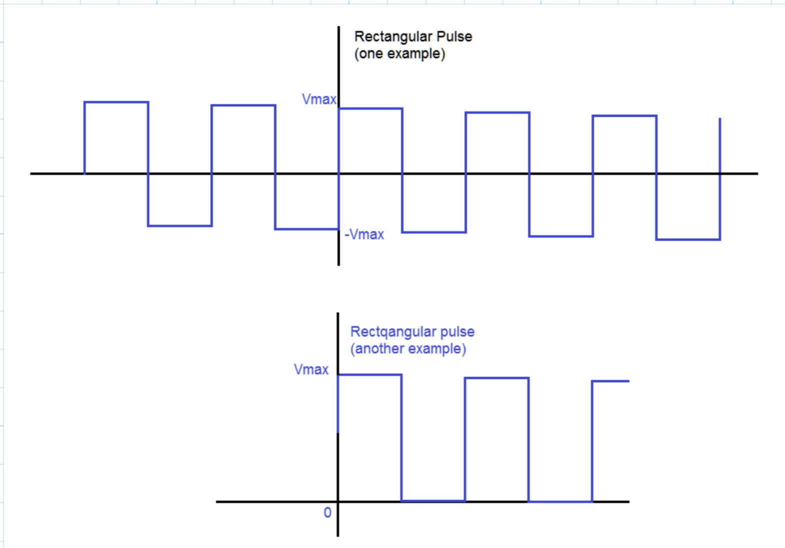
Sections 7.1 - 7.9 in Part A of First Order Circuits file.

Placed here again the comments from Part A.
Too much subject material. Just the examples after each section should do it here. Primarily to get more solving techniques. From my observation on the exercise problems with no solutions, most are similar to the solved ones with need of some changes or tricks in the solution. Schaums variety of solved problems simplify solving exercise problems in other electrical textbooks similarly.

This book is the 6th edition. Some appreciation should be given for as many years it has been in publication, since the mid-60's.

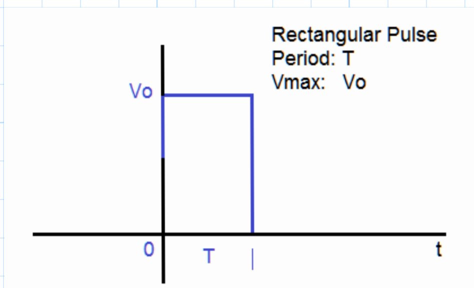
7:10 Response of First Order Circuits to a Pulse.

Here we derive the response of a 1st order circuit to a 'rectangular pulse'.
This applies to RC and RL circuits. The input to this circuit can be a current or voltage.
Usually we are given voltage find the current response. Can be both either.
In our next derivation we use a series RC circuit. *Hopefully the other circuits series RL or parallel RC and RL....RLC...are similarly achieved with some modifications/adjustments/ re-arranging/....*



Some forms of rectangular pulse. You may say it looks like some u(t) function been upgraded maybe.... Not wrong. Maybe one way to look at it is a upgraded unit step function, u(t). It comes on periodically.

Circuits used here, a series RC circuit with the voltage source delivering a pulse of duration T and height Vo. Height would be the amplitude.

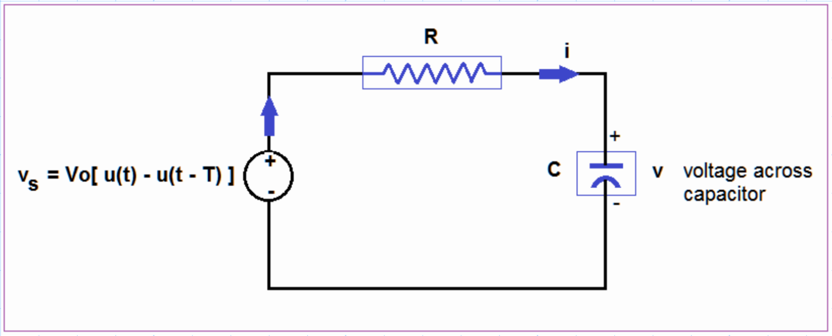


<---Now it may be this rectangular pulse. Schaums does not provide a figure for it but does provide its description in the circuit diagram. (<---This is the one).

For t<0, v(t) and i(t) are zero.
For the duration of the pulse, we use from section 7.3:

$$v(t) = V_o \cdot \left(1 - e^{\frac{-t}{RC}}\right) \qquad \text{For } 0 < t < T$$

$$i(t) = \frac{V_o}{R} \cdot e^{\frac{-t}{RC}} \qquad \text{For } 0 < t < T$$



<---This is the circuit as shown in Schaums. The voltage waveform does remind us of an example in a previous chapter.

Side tracking a little, just so we get a better understanding of the voltage source v_s waveform. *And why not.* We do similar as before. *Because this waveform figure is not provided in textbook.* See below, this may be it, STOP at t=1 and goes 0.

clear(t) $\quad t := -5, -4.9 .. 5 \quad T := 1$

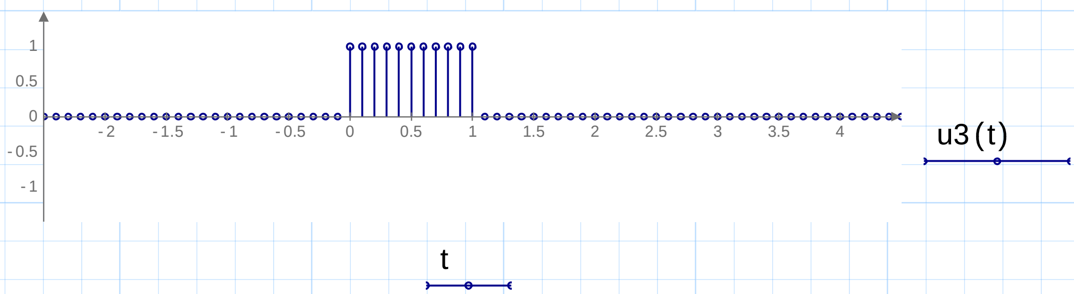
u2(t) is set up at t=T as a u(t).
..... so that in the u3(t) expression the 'desired magnitude' is achieved.

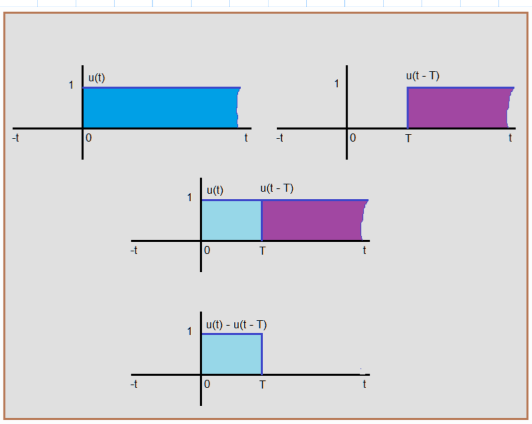
$$u1(t) := \begin{array}{l} \text{if } t \geq 0 \\ \quad \| \; u1 \leftarrow 1 \\ \text{else if } t < 0 \\ \quad \| \; u1 \leftarrow 0 \end{array}$$

$$u2(t) := \begin{array}{l} \text{if } (t - T) > 0.1 \\ \quad \| \; u2 \leftarrow 1 \\ \text{else if } (t - T) < 0.1 \\ \quad \| \; u2 \leftarrow 0 \end{array}$$

$$u3(t) := (u1(t) - u2(t))$$

Magnitude of voltage Vo = 1 here.
Used the stem plot, using vertical lines at point.





<----Should be the form of the rectangular pulse, is only for the period T shown at bottom of figure. Our 'if then else' loop did just that. You may have better ways of doing it in your software, you may skectch it too like here, not required it be done in software.

We are told when the pulse ceases, the circuit is source free.
Should be agreeable the waveform above so when its t>T (t=1=T) the voltage source v_s drops to zero.

The circuit becomes source free, with the capacitor at an initial voltage of V_T OR capacitor terminal voltage v_T. This will be lower than Vo, where v_s equal 1.

$$V_T = V_o \cdot \left(1 - e^{\frac{-T}{R \cdot C}}\right)$$

Here t has been replaced by T in the exponent term, which is the time at point T. T is NOT t= 0, rather t equal the time T second, which in the plot later is set to 2s. We know for the pulse duration period T the v_s voltage is Vo we set to 1 for the plot.

For the time t<T, 0 to T, the voltage source v_s charges up the capacitor. Time period T is where the voltage source v_s is providing a voltage. The capacitor is also charging up, the capacitor voltage increasing from 0 to some value. Capacitor maximum voltage was found to be 0.865 at end of t=T, in our example calculation for plot. *Calculations provided later*. After the period T, the voltage source is turned off. Now t>T, capacitor is discharging into the circuit and providing the voltage to the components in the circuit, this capacitor voltage will be decreasing to zero. Similalry for current but with a negative sign for t>T.

We have gone from t=0 to t=T and to t>T. *Although u(t) here starts at t=0.*
For time 0<t<T we identify to t in the function u(t) ---> v_s is the voltage source.
For time t>T we identify to the time shift function u(t-T) ---> capacitor voltage.
This time shift (t-T)>0 is the time when the capacitor is discharging.

Next we update the v and i expressions from section 7.3 presented here for the time shift (t-T).

$$v_T(t) = V_T \cdot \left(1 - e^{\frac{-(t-T)}{RC}}\right) \quad \text{For } t>T$$

$$i(t) = -\left(\frac{V_T}{R}\right) \cdot e^{\frac{-(t-T)}{RC}} \quad \text{For } t>T.$$

Negative current sign because current is flowing out of capacitor in the opposite direction to the current supplied by the voltage source v_s.

We may be able to plot v and i with some values for the variables.

Follow the expressions closely, if in an expression there are two terms, work out where each of the term applies to the plot. The expressions have been followed closely for a continous plot for time t=0 to time t>T.

New here!

Voltage plot:

clear $(t, t1, t2)$ $V_0 := 1$ $V_T := 1$ $R := 1000$ $C := 1 \cdot 10^{-3}$

$\tau_{RC} := R \cdot C = 1$ $T := 2$

$t1 := 0, 0.25 .. 2.0$ $t2 := 2.0, 2.25 .. 6$

$V_0(t1) := 1$ <---Rectangular pulse voltage for 0<t<T

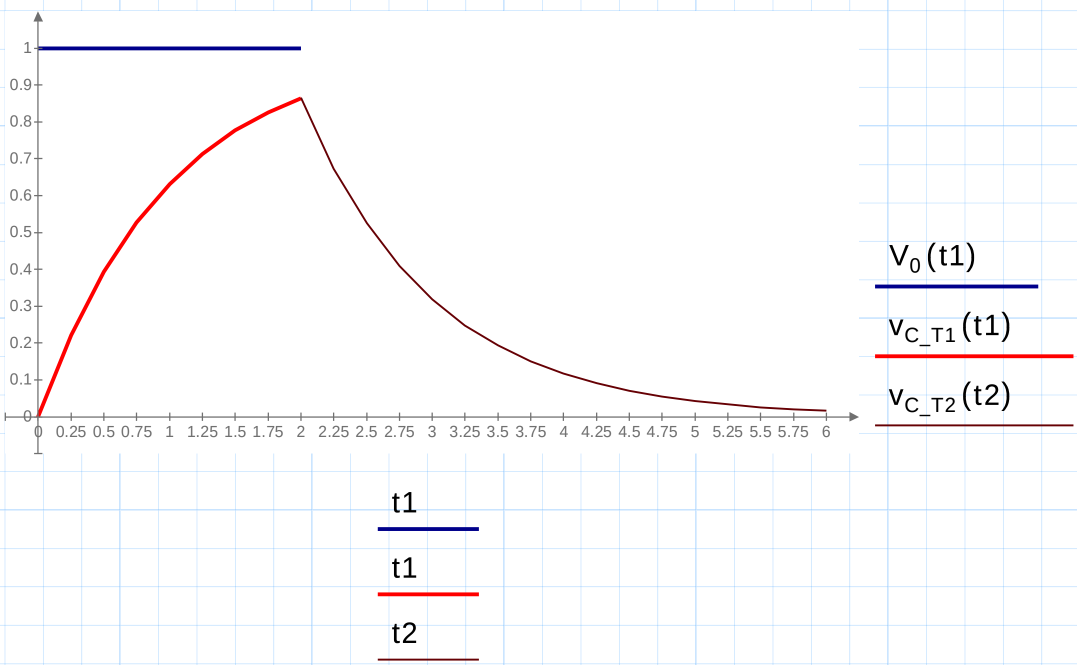
$V_{0\_1} := 1$ *<---Vo_1 = Vo(t1) so the Vo(t1)=1 does NOT conflict in calculations below.*

$v_{C\_T1}(t1) := V_{0\_1} \cdot \left(1 - e^{\frac{-(t1)}{\tau_{RC}}}\right)$ Capacitor voltage 0<t<T, which was assisted by the voltage v_s.

$V_{C\_T1\_2s} := V_{0\_1} \cdot \left(1 - e^{\frac{-(2)}{\tau_{RC}}}\right) = 0.865$ $V_{C\_T1\_2s} := 0.865$ *<---Capacitor voltage at 2 s. This applied in the next expression for t>T*

$v_{C\_T2}(t2) := V_{C\_T1\_2s} \cdot \left(e^{\frac{-(t2 - T)}{\tau_{RC}}}\right)$ Capacitor voltage from t>T

Plot curve below is correct - same as Schaums.



Capacitor voltage rises to a peak then drops to zero.

Next we try for the current plot. May be similar situation in terms of how to get it as was for v(t) above.

Current plot:

clear $(t, t1, t2)$ $\quad V_0 := 1 \quad V_T := 1 \quad R := 1000 \quad C := 1 \cdot 10^{-3}$

$\tau\_RC := R \cdot C = 1 \quad T := 2 \quad t1 := 0, 0.25 .. 2.0 \quad t2 := 2.0, 2.25 .. 6$

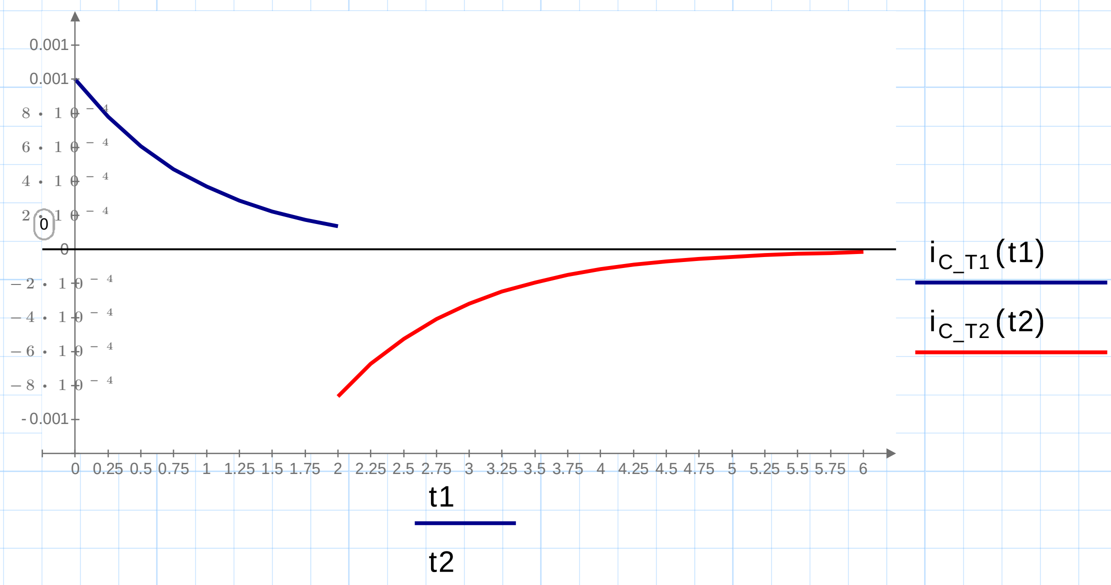
$I_0 := \dfrac{V_0}{R} = 0.001$ Current Io at time t=0, generated by v_s, and the resistance. The capacitor begins to charge consume current from t=0 to t>0.

$i_{C\_T1}(t1) := I_0 \cdot e^{\frac{-(t1)}{\tau\_RC}}$ <---Capacitor current for 0<t<T

$I_{0\_1} := I_0 = 0.001$ *<---Io_1 so the Io(t1)=0.001 does NOT conflict in calculations below.*

$i_{C\_T1\_2s} := \left(\dfrac{V_{C\_T1\_2s}}{R}\right) = 0.0009$ *Capacitor current at 2 s. As seen in plot below at 2s. This is V_T/R, which is capacitor voltage at 2s divided by resistance R. This become the initial current in the next expression.*

$i_{C\_T2}(t2) := -i_{C\_T1\_2s} \cdot \left(e^{\frac{-(t2-T)}{\tau\_RC}}\right)$ Remember negative sign on current, so it starts from -ve from t>T. Capacitor current from t>T
Plot curve below is correct - same as Schaums.



Capacitor current for t<T drops from Io (Vo/R). Drops because capacitor is storing charge, resulting with less current for the circuit. Next for t>T starts at (V at 2s/R) = 0.865A) but has a negative sign then drops to 0. Both curves leading to zero.

Go through slowly putting togther the terms for the voltage and current for their respective plots.

Before we move on to an example, what if we changed t from 2 to 3, what would the plot of v and i look like.

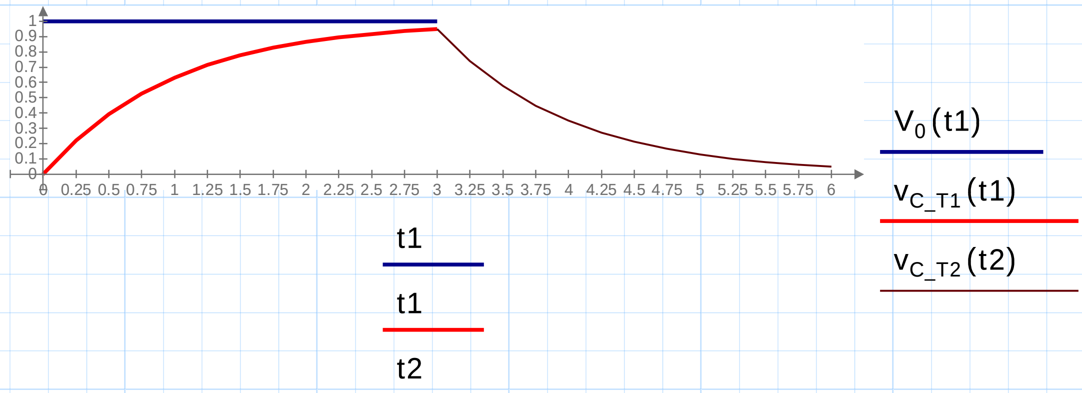
Voltage plot:

$V_{0\_1} := 1 \qquad \tau_{RC} := R \cdot C = 1 \qquad T := 3 \qquad t1 := 0, 0.25 .. 3.0 \qquad t2 := 3.0, 3.25 .. 6$

$V_0(t1) := 1 \qquad v_{C\_T1}(t1) := V_{0\_1} \cdot \left(1 - e^{\frac{-(t1)}{\tau_{RC}}}\right) \qquad V_{C\_T1\_2s} := V_{0\_1} \cdot \left(1 - e^{\frac{-(3)}{\tau_{RC}}}\right) = 0.95$

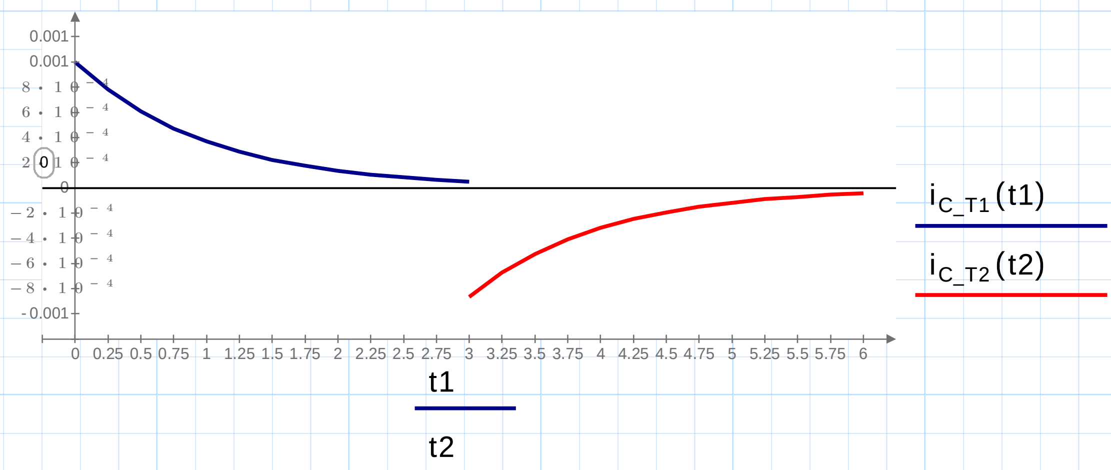
$v_{C\_T2}(t2) := V_{C\_T1\_2s} \cdot \left(e^{\frac{-(t2-T)}{\tau_{RC}}}\right)$

Closer to 1V, higher T gets closer to Vo. More time capacitor has to fully get charged to Vo.



$i_{C\_T1}(t1) := I_0 \cdot e^{\frac{-(t1)}{\tau_{RC}}} \qquad i_{C\_T2}(t2) := -i_{C\_T1\_2s} \cdot \left(e^{\frac{-(t2-T)}{\tau_{RC}}}\right)$

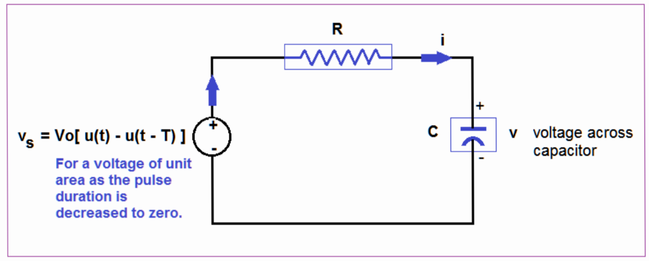
Current changes sign at T=3. Giving more charge time to capacitor, compared to T=2.



Example 7.11 Response of 1st Order Circuit to a Pulse.

In the circuit shown below.
Let R = 1k ohm, and C = 1 uF.
Let the voltage source be a pulse of height Vo for a duration T.

Find i and v for:
a). Vo = 1V and T=1 ms.
b). Vo = 10V and T=0.1ms
c). Vo = 100V and T=0.01 ms.



Hint:

Use the expressions provided below for 0<t<T, and t>T.
With the time constant RC = 1 ms.
For convenience express time in ms, voltage in V, and current in mA.
Also use the approximation e^(-t) aproximately equal (1 - t), when t<<1.

Shown here:

$t := 0.001 \qquad e^{-0.001} = 0.999$

So its aproximately good for 1 ms, ---> $1 - t = 0.999$
gets better for smaller than 1 ms.

$v(t) \;=\; V_0 \cdot \left(1 - e^{\frac{-t}{RC}}\right)$ For 0< t < T $\qquad i(t) \;=\; \frac{V_0}{R} \cdot e^{\frac{-t}{RC}}$ For 0< t < T

$v_T(t) \;=\; V_T \cdot \left(1 - e^{\frac{-(t-T)}{RC}}\right)$ For t>T $\qquad i(t) \;=\; -\left(\frac{V_T}{R}\right) \cdot e^{\frac{-(t-T)}{RC}}$ For t>T

Remember: V_T is the capacitor initial voltage it is NOT Vo which is the source voltage amplitude.

Solution:

$R := 1000$ Ohm $\qquad C := 1 \cdot 10^{-6}$ F.

$R \cdot C = 0.001 \qquad \tau_{RC} := R \cdot C = 0.001 \qquad \frac{1}{\tau_{RC}} = 1 \cdot 10^{3}$

a).

$V_0 := 1$ V $\qquad T := 1 \cdot 10^{-3}$ s or 1 ms.

For 0<t<T=1 ms:

$v(t) := \left(1 - e^{-t}\right)$ <--- So we see why we were given this expression for voltage.

$$v(t) = V_o \cdot \left(1 - e^{\frac{-t}{RC}}\right)$$ For 0< t < T

Lets say between 0 and T and maximum t is 1 ms, we try to use that in the expression above. Plug in for Vo, R and C, with t = 1ms = 0.001s.

$$v(t) = 1\left(1 - e^{\frac{-t}{\tau_{RC}}}\right) \qquad \frac{0.001}{\tau_{RC}} = 1 \qquad \text{rearrange:} \frac{1}{\tau_{RC}} = 1 \cdot 10^3 \qquad \frac{1}{0.001} = 1 \cdot 10^3$$

$$v(t) = 1\left(1 - e^{-t \cdot 1 \cdot 10^3}\right) = 1 - e^{-1000 \cdot t}$$ That familiar 1000 from another exercise!

Now lets say the unit for t is ms, which you know is trying to 'fix' things in here to make it work, but otherwise why bother with the (1 - e^-t)? How else could we get to that expression other than the appreciation of the unit of t in miliseconds, which is 10^-3. So if we state it like a coefficeint N(ms) than what ever that N was 1, 3, 5, 11,....its in ms. You may have a better angle/path/direction to that expression.

$$v(t) = 1\left(1 - e^{-t \cdot 1 \cdot 10^3}\right) = 1 - e^{-1000 \cdot t\,(s)}$$ place the, s, second first in the exponent

$$= 1 - e^{-1000 \cdot t\,(ms)}$$ place the, ms, milisecond for s, next multiply out the ms.

$$= 1 - e^{-1000 \cdot t\left(1 \cdot 10^{-3}\right)}$$

$$= 1 - e^{-t}$$ <---There.

Comment: *When I saw that equation the first time, I was a little taken aback, meaning something like intimidated/lowered/lacking......really lacking is closer, you may had felt the same. We seen the equations given prior and now we see something so simple (1-e^-t) it raises questions! So, hopefully thats how to get there. Check with your local? Engineer.*

Next the expression for current can be simplified:

$$i(t) = \frac{V_o}{R} \cdot e^{\frac{-t}{RC}}$$ We have sorted the exponential term and it could be placed in here.

$$= \frac{1}{1 \cdot 10^3} \cdot e^{-t}$$ Problem? But if we do what the author-engineer said use milliamp for current instead of Amp then it may sort things out.

$$= 0.001 \cdot e^{-t} \quad A$$

$$= e^{-t} \quad mA$$ *Happy!*

For 0<t<T=1 ms:

$v(t) := \left(1 - e^{-t}\right)$ and $i(t) := e^{-t}$

Now if we substitute t =T=1 ms = 0.001s, in the expressions above, we should get some numerical value for the answer for this time range. Plug in 1 because the expression is setup for milliseconds, and likewise for current i(t).

$V_T := \left(1 - e^{-1}\right) = 0.632$ V. Answer. <---voltage at end of 1 ms.

and

$i_{1ms} := e^{-1} = 0.368$ A. Answer? <---current at end of 1 ms.
The answer in Schaums was just leaving it as the expression without pluging in t=1ms.

$i := e^{-t}$ Answer. During the pulse rise time, slope on left side of rectangle, the capacitor got charged. Then it maintained a charge while the voltage was Vo (1V), when the voltage dropped to 0V, capacitor began to discharge this is in time t>1ms. Capacitor is 'open circuited' for a dc voltage of 0V. In our circuit's figure 'i' is the current in the series circuit coming from v_s. If the theory example was not clear on it, i.e. prior example, this should make it clearer.

For t>1 ms:

$v_T(t > T) = V_T \cdot \left(e^{\frac{-(t-T)}{RC}}\right)$

We need to adjust for T in the exponent power. Now its fixed, all in ms.

$= V_T \cdot \left(e^{-(t-T)}\right)$

Next, substitute in V_T calculated in the 0<t<T leave t in the exponent the same because we do not have any specific point in time past t=T to evaluate the expression its a general expression for t>T>1ms. But plug in for T = 1 because that shows its past t>T.

$= 0.632 \cdot \left(e^{-(t-1)}\right)$

$= 0.632 \cdot e^{1} \cdot e^{-t}$

$0.632 \cdot e^{1} = 1.72$

$= 1.72\ e^{-t}$ Answer.

For current at time t=1 ms:

$i := e^{-t}$ $\quad i := e^{-1} = 0.368$

Voltage at end of 1 ms: $\quad v_{1ms} := 0.632$

Current at end of 1 ms in the circuit i_1ms = V_T/R:

$$i_{1ms} := \frac{v_{1ms}}{R} = 6.32 \cdot 10^{-4}$$ A.

$$i(t > T) = -\left(\frac{V_T}{R}\right) \cdot e^{\frac{-(t-T)}{RC}}$$

REMEMBER: current will be in the opposite direction coming out of the capacitor into the circuit for t>T.

$$= -(i_{1ms}) \cdot e^{-(t-1)}$$

Bring in the progress we made in prior steps.

$$(i_{1ms}) \cdot e^{1} = 0.001718$$

This is mA next

$$\frac{(i_{1ms}) \cdot e^{1}}{1 \cdot 10^{-3}} = 1.717954$$

mA, next we pull in the remaining term like we did in the voltage expression.

$$= -1.72 \cdot e^{-t}$$ Answer.

Comment: Seemed simple but there were some simplifications required - deeper insight. They say 'Engineering' is not easy, here what seemed so simple had twists and turns. Maybe if I question more often, STOP, and not follow thru the procedure outlined, UNTIL things are clearer, then its a better learning experience for me. You may have your ideas on this. These techniques are applied in higher level courses. So, this may do it. Karl Bogha - "they say the high quality students pass the engineering degrees, quality may be subjective, depending on who holds the standard at any any one given time, however you know you must do the work to get thru". Jokes aside, in Asian nations due to limited seat capacity in institutions, admission is competitive. And I was not one of them admitted there, I graduated in US. Thanks.

Hopefully for part (b) things can be re-used to cut down on time.

Continued on next page.

b). $V_0 := 10 \ \ V$ $\quad T := 0.1 \cdot 10^{-3}$ s or 0.1 ms. We increased the voltage by 10 and reduced T by 10.

For 0<t<T=0.1 ms:

$v(t) := \left(1 - e^{-t}\right)$ We solved this with V=1, now V = 10V.

$v(t) := 10 \cdot \left(1 - e^{-t}\right)$ Answer.

Similarly for current expression: $\frac{V_0}{R} = 0.01$

$i(t) = (0.01) \cdot e^{-t}$ A

$= 10 \cdot e^{-t}$ mA. Answer.

For the next time range, we first need to calculate the initial voltage existing at time t=T=0.1 ms.

$v_{0.1ms} = 10 \cdot \left(1 - e^{-0.1}\right) = 10 - 10 \cdot e^{-0.1} = 10 - 9.04837 = 0.9516$

$v_{0.1ms} = 0.95 \ \ V$

For t>0.1 ms:

$v_T(t > T) = V_T \cdot \left(e^{\frac{-(t-T)}{RC}}\right)$

$= V_T \cdot \left(e^{-(t-T)}\right)$

$v(t > T) = 0.95 \left(e^{-(t-T)}\right)$

$= 0.95 \ e^{-(t-0.1)}$ $\qquad e^{0.1} = 1.105$

$= 0.95 \cdot (1.105) \cdot e^{-(t)}$ $\qquad 0.95 \cdot (1.105) = 1.05$

$= 1.05 \ e^{-(t)}$ Answer.

The coefficient will be the same in the current case because its the math expression.

$v_{0.1ms} := 0.95 \ \ V$

$$i(t) = -\left(\frac{V_T}{R}\right)\cdot e^{\frac{-(t-T)}{RC}}$$

$$\left(\frac{V_T}{R}\right) = i_{0.1ms} := \frac{v_{0.1ms}}{R} = 9.5\cdot 10^{-4} \quad \text{A}$$

$$\frac{i_{0.1ms}}{1\cdot 10^{-3}} = 0.95 \quad \text{mA}$$

$i(t>T) = -0.95\cdot e^{-(t-T)}$ mA. Bring in the progress we made in prior steps on the exponent term.

$= -0.95\cdot e^{-(t-0.1)}$ mA $\qquad e^{0.1} = 1.105$

$= -0.95\cdot 1.105\cdot e^{-t}$ $\qquad 0.95\cdot(1.105) = 1.05$

$= -1.05\cdot e^{-t}$ Answer.

Comment: Standarisation in solution process is appreciated, but you can see some checks need be in place to catch 'careless' mistakes.

c). $V_0 := 100V$ $\quad T := 0.01\cdot 10^{-3}$ s or 0.01 ms. We increased the voltage by 10 on part b, and reduced T by 10 on part b.

For 0<t<T=0.01 ms:

$v(t) := (1 - e^{-t})$ $\qquad$ Vo = 100V.

$v(t) := 100\cdot(1 - e^{-t})$ Answer.

Similarly for current expression: $\frac{V_0}{R} = 0.1$

$i(t) = (0.1)\cdot e^{-t}$ A

$= 100\cdot e^{-t}$ mA. Answer. Remember we had e^-t aprox = 1-t, we can do that here. So i(t) approximately = **100(1 - t).** Answer.

For the next time range, we first need to calculate the initial voltage existing at time t=T=0.1 ms.

$$v_{0.01ms} = 100\cdot(1 - e^{-0.01}) = 100 - 100\cdot e^{-0.01} = 100 - 99.005 = 0.995$$

$v_{0.01ms} = 0.995 \quad V$

For t>0.01 ms:

$$v_T(t>T) = V_T \cdot \left(e^{\frac{-(t-T)}{RC}} \right)$$

$$= V_T \cdot \left(e^{-(t-T)} \right)$$

$$v_T(t>T) = 0.995 \left(e^{-(t-T)} \right)$$

$$= 0.995\, e^{-(t-0.01)} \qquad e^{0.01} = 1.0101$$

$$= 0.995 \cdot (1.0101) \cdot e^{-t} \qquad 0.995 \cdot (1.0101) = 1.01 \text{ 2 decimal places.}$$

$$= 1.01\, e^{-t}$$ Answer.

The coefficient will be the same in the current case because its the same math expression.

$v_{0.01ms} := 0.995\,V$

$$i(t>T) = -\left(\frac{V_T}{R}\right) \cdot e^{\frac{-(t-T)}{RC}}$$

$$\left(\frac{V_T}{R}\right) = i_{0.01ms} := \frac{v_{0.01ms}}{R} = 9.95 \cdot 10^{-4} \ A$$

$$\frac{i_{0.01ms}}{1 \cdot 10^{-3}} = 0.995 \quad mA$$

$$i(t>T) = -0.995 \cdot e^{-(t-T)} \quad mA.$$ Bring in the progress we made in prior steps on the exponent term.

$$= -0.995 \cdot e^{-(t-0.01)} \quad mA \qquad e^{0.01} = 1.01005$$

$$= -0.995 \cdot 1.105 \cdot e^{-t} \qquad 0.995 \cdot (1.01005) = 1.005 = 1.01 \text{ 2 decimal places}$$

$$= -1.01 \cdot e^{-t}$$ Answer.

Schaums: As the input pulse approaches an impulse, *when Vo got bigger and T got smaller the rectangular pulse was becoming more like an impulse - from part a to part b then to* c, the capacitor voltage approach v = e^(-t) u(t), and the current approach i = *d* (t)-e^(-t) u(t).

Next we visit the Impulse Response, but **first** we plot the voltages and currents.

Discussion: Exponential plots can be surprising, takes experience, and regular use of them. The plots are straight lines for voltages and currents, all v(t) same and all i(t) same. You may verify using your software/ Excel. These plots maybe helpful visualisations.

The plots lead to us question why else a u(t) be implicated to a d(t) impulse/delta function, Schaums Engineers-Authors wrote '.....capacitor voltage v = e^(-t) u(t), and the current approach i = *d* (t)-e^(-t) u(t).'

$$v_C(t) = e^{-t} \cdot u(t)$$
$$i_C(t) = \delta(t) \cdot \left(-e^{-t} \cdot u(t)\right)$$

These plots raise alot of questions. Maybe by reviewing the plot straight lines cause us to relates to u(t) at given amplitudes and then ......followed by how the delta/impulse function may multiplied to it. You study it. Discuss it. Verify it. Raises Questions? All straight lines! Almost straight lines. Correct errors as required.

clear $(t, t1, t2, t3, t4, t5, t6)$

$$t1 := 0, 0.25 \cdot 10^{-3} .. 1 \cdot 10^{-3}$$

$$t2 := 1 \cdot 10^{-3}, 1.125 \cdot 10^{-3} .. 2 \cdot 10^{-3}$$

$$t3 := 0, 0.025 \cdot 10^{-3} .. 0.1 \cdot 10^{-3}$$

$$t4 := 0.1 \cdot 10^{-3}, 0.125 \cdot 10^{-3} .. 0.2 \cdot 10^{-3}$$

$$t5 := 0, 0.0025 \cdot 10^{-3} .. 0.01 \cdot 10^{-3}$$

$$t6 := 0.01 \cdot 10^{-3}, 0.0125 \cdot 10^{-3} .. 0.02 \cdot 10^{-3}$$

a). For 0<t<T (T=1 ms): — For t> 1ms

$v(t1) := \left(1 - e^{-t1}\right)$ — $v(t2) := 1.72\ e^{-t2}$

$i(t1) := e^{-t1}$ mA — $i(t2) := -1.72 \cdot e^{-t2}$

b). For 0<t<T (T=0.1 ms): — For t> 0.1ms

$v(t3) := 10 \cdot \left(1 - e^{-t3}\right)$ — $v(t4) := 1.05 \cdot e^{-(t4)}$

$i(t3) := 10 \cdot e^{-t3}$ — $i(t4) := -1.05 \cdot e^{-t4}$

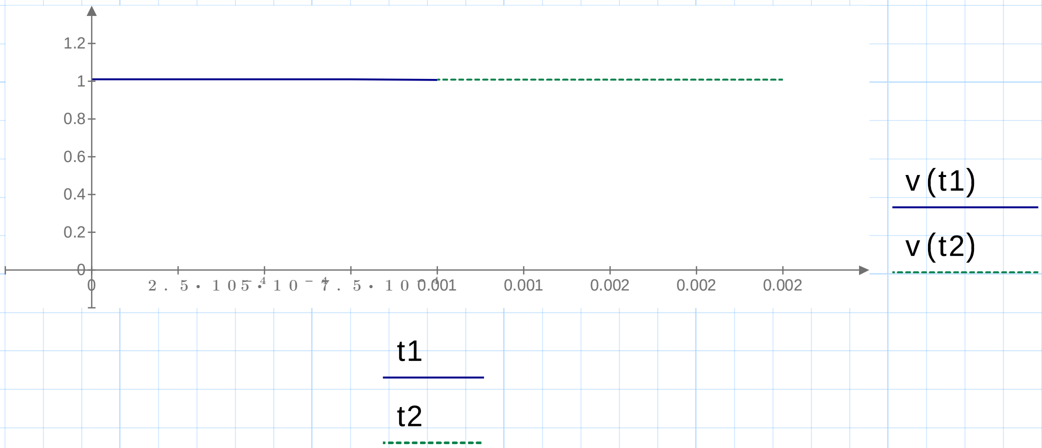
c). For 0<t<T (T=0.01 ms): — For t> 0.01ms

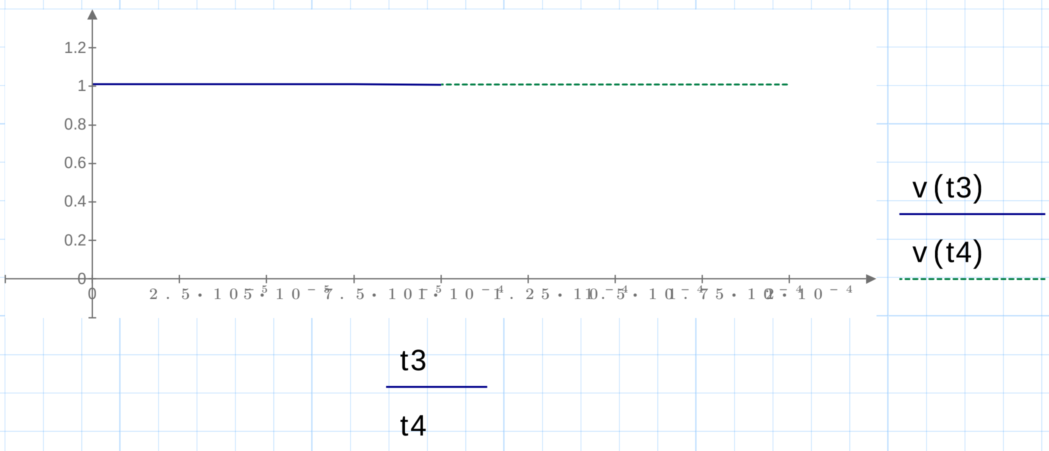
$v(t5) := 100 \cdot \left(1 - e^{-t5}\right)$ — $v(t6) := 1.01\ e^{-t6}$

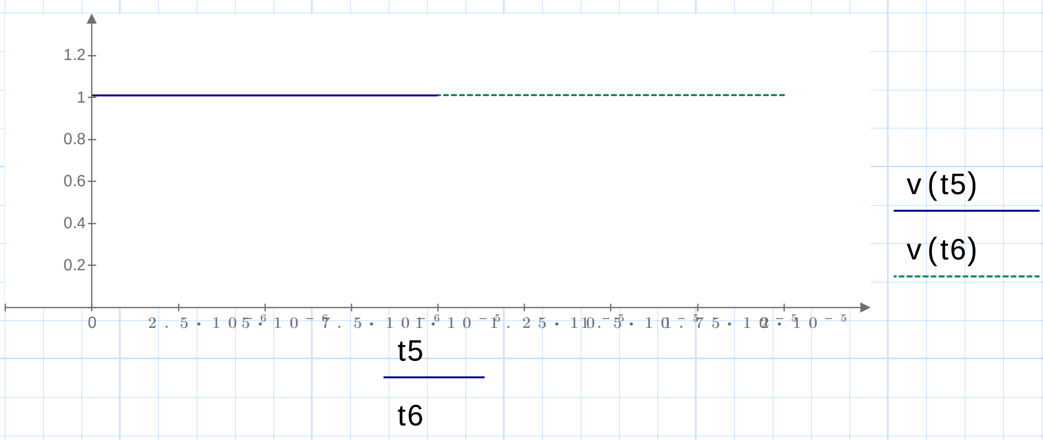
$i(t5) := 100 \cdot e^{-t5}$ — $i(t6) := -1.01 \cdot e^{-t6}$

Chapter 5 Part B. Engineering Circuits Analysis Notes And Example Problems - Schaums Outline 6th Edition.
My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis.
Source of study material: Electric Circuits 6th Ed., Nahvi & Edminister. Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill.
Karl S. Bogha.

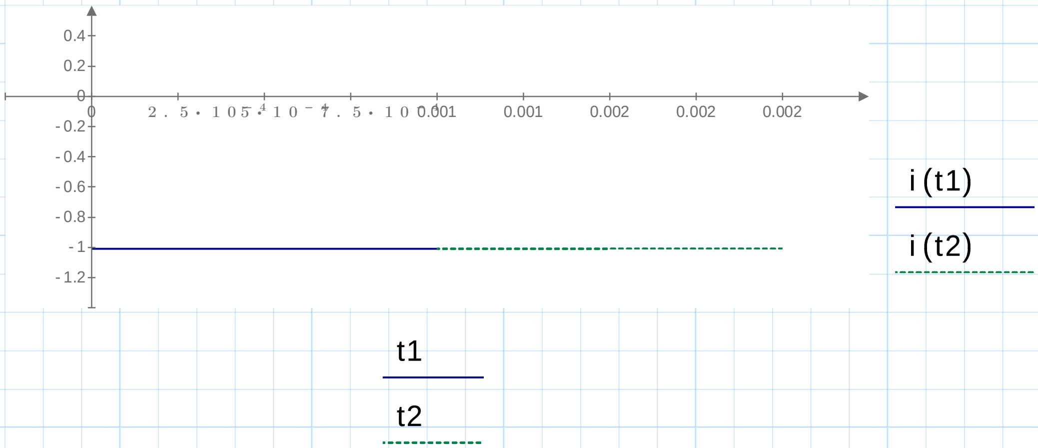
Voltage plots:

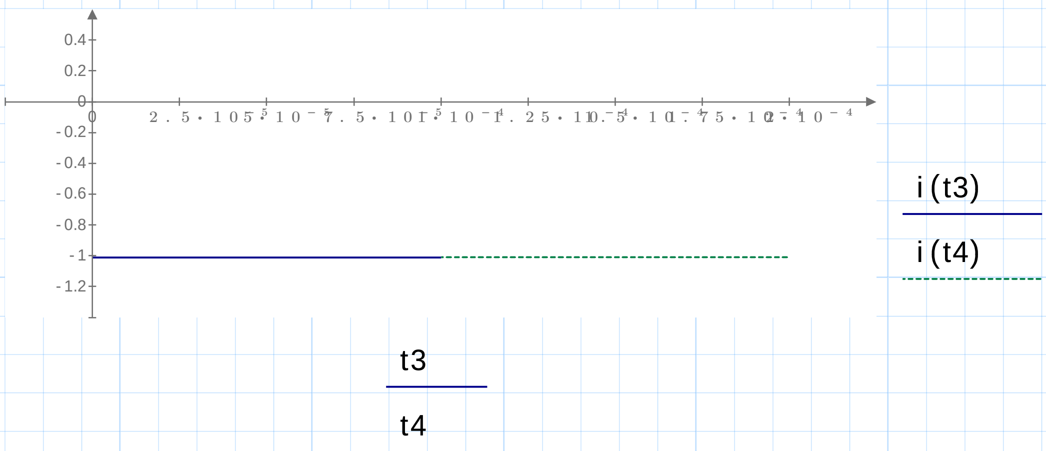


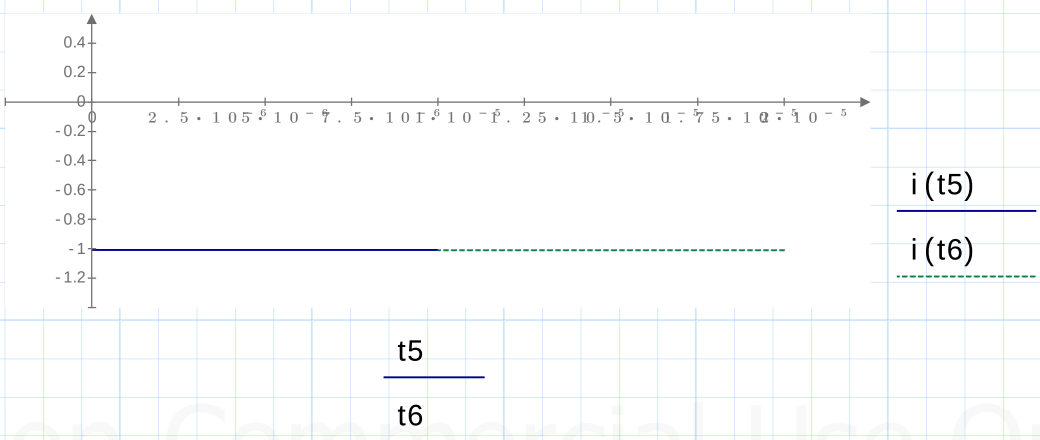




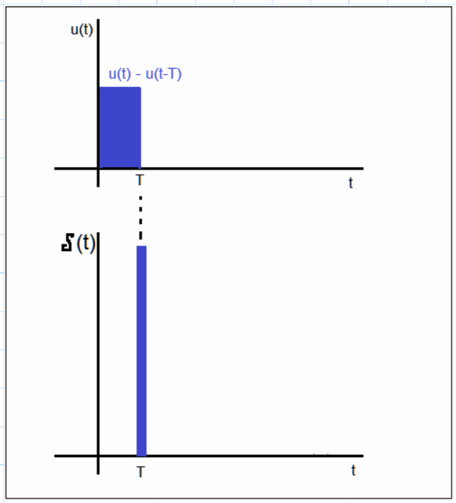
Current plots:







7.11 Impulse Response of RC and RL circuits.



From section 7.10 Repeated:
As the input voltage pulse approaches an impulse, the capacitor voltage and current approach v=e^(-t) u(t) V and i = d(t)-e^(-t)u(t). Schaums page 156.

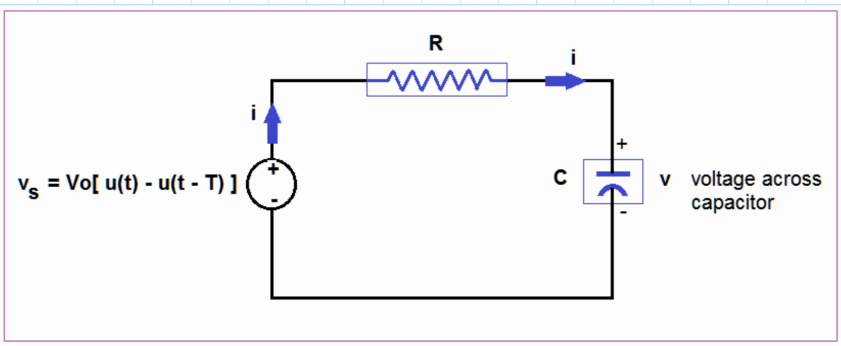
<----A narrow pulse can be modeled as an impulse with the area under the pulse indicating its strength. Impulse response is a useful tool in the analysis and synthesis of circuits.

It may be derived in several ways: take the limit of the response to a narrow pulse, to be called the limit approach, as illustrated in example 7-11 and 7-12; take the derivative of the step response ; or solve the differential equation directly. The impulse response is often designated by h(t) - page 156 Schaums.

Example 7.12 (Impulse response of RC & RL circuit). TROUBLING EXAMPLE.

You may go to 7.12 and 7.13 first, then return. All interconnected. 13 pages for 7.12.

Find the limits of i and v of the circuit in figure provided below (series RC circuit) for a voltage pulse unit area as the pulse duration is decreased to zero.



Same circuit of previous 2 examples.

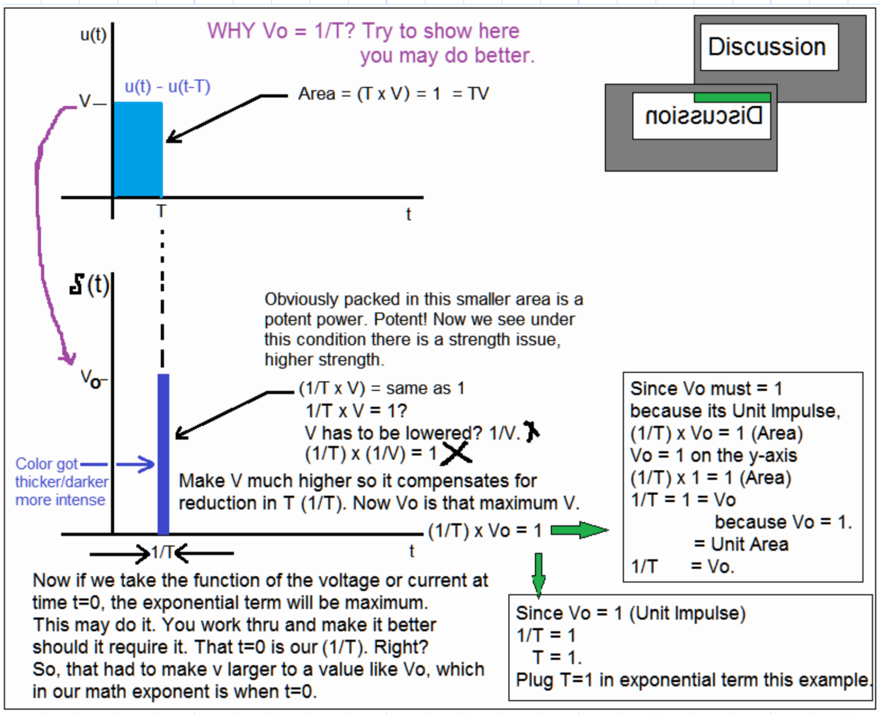
Solution:

We apply the pulse (rectangular) responses from section 7.10 (shown below) with Vo = 1/T and find their limits as T approaches zero.

$$V_T = V_0 \cdot \left(1 - e^{-\left(\frac{T}{RC}\right)}\right)$$

<---Equation from section 7.10. When the pulse ceases circuit is source free with the capacitor at an initial voltage V_T

Discussion in figure on next page to try to show how Vo = 1/T.
How could voltage equal inverse of T? Next page.



<--- Why Vo = 1/T
May be 2002 ways you can explain the impulse function relative to rectangular pulse function in waveform, and why the engineers choose to make Vo =1/T here.
(1/T) x V = 1
1/T = 1/V
Invert expression,
T= V? No go!
(1/T) x V = 1 (Unit Area)?
1/T << T correct.
So if (1/T) gets small its like saying limit T-->0.
Correct dependent on T value.
Obviously then for the area to remain 1, V has to go higher, that higher V is Vo.
(1/T) x Vo = 1; Vo =? = 1.
Vo is a unit impulse.
(1/T) x 1 = 1
(1/T) x 1 = Vo
(1/T) = Vo.
Vo = 1/T.
Also then T = 1/Vo
= 1/1 (Vo=1)
T = 1.
? Maybe good enough.
You work on it if not.

$$V_T = V_0 \cdot \left(1 - e^{-\left(\frac{T}{RC}\right)}\right)$$

$$V_T = \left(\frac{1}{T}\right) \cdot \left(1 - e^{-\left(\frac{T}{RC}\right)}\right)$$

Lim T-->0   Lim T-->0

Lets look at the T terms when T approaches 0

$e^{-0.001} = 0.999$ <---approaches 1, case here.

$e^{-1000} = 0$ <---when higher approaches 0.

$\frac{1}{T} = \frac{1}{1 \cdot 10^{-6}} = 1 \cdot 10^{6}$ <---T get smaller
inverse T gets larger.
No Go!

So we update Vo in the RHS term to 1/T.
T in the exponent power term? 1. From the discussion above.

$$= \left(\frac{1}{T}\right) \cdot \left(1 - e^{-\left(\frac{1}{RC}\right)}\right)$$

$$= \left(\frac{1}{1}\right) \cdot \left(1 - e^{-\left(\frac{1}{RC}\right)}\right)$$

$V_T = \left(1 - e^{-\left(\frac{1}{RC}\right)}\right)$ Made it so far...so good...*looks that way* but next what?

Lets work each term ie R and C, and RC. What is their impact:
(We want (1/RC) <<<1 explain later).

$R := 1000$ $C := 1 \cdot 10^{-6}$ $t\_constant1 := \frac{1}{(R \cdot C)} = 1 \cdot 10^{3}$

$e^{-(t\_constant1)} = 0$ Larger the t_constant1 closer to 0 the value of exponent.

$R := 1000 \cdot 10^{6}$ $C := 1 \cdot 10^{-6}$ $t\_constant2 := \frac{1}{(R \cdot C)} = 0.001$ Made R larger.

$e^{-(t\_constant2)} = 0.999$ Smaller t-constant2 exponent begins to move away from 0 to some positive value closer to 1. This by making R much larger.

Basically explained the exponential function to the negative power; closer to 0 and 1.
Again NO GO! What next?

What if we say (1/RC) = t. REMEMBER example 7.11 in the begining of the solution in hint we got to where the approximation e^-t approximatley equal (1 - t) when t<<1. When the circuit component resistor was 1000M ohm), lower the t_constant2 got (0.001), then its exponential term e^-(t_constant2) got closer to 1 (0.999). We kept the capacitor the same size for t_constant 1 and 2. Capacitor decreasing in size helps t_constant2 get even closer to 1. For now we got t = 0.001. So this is the condition (1/RC) << 1. And we got (1/RC) = 0.001. Ok for 0.001, lets proceed.

Presented here from example 7.11:

$v(t) = 1\left(1 - e^{-t \cdot 1 \cdot 10^{3}}\right) = 1 - e^{-1000 \cdot t\,(s)}$ place the, s, second first in the exponent

$= 1 - e^{-1000 \cdot t\,(ms)}$ place the, ms, milisecond for s, next multiply out the ms.

$= 1 - e^{-1000 \cdot t\,\left(1 \cdot 10^{-3}\right)}$

$v(t) = 1 - e^{-t}$ <---There.

Shown here:

$t := 0.001$ $e^{-0.001} = 0.999$

So its aproximately good for 1 ms, ---> $e^{-t} = 1 - t = 0.999$
gets better for smaller than 1 ms.

$v(t) = 1 - e^{-t} = 1 - (1 - t) = t$ <--- 1/RC. Acceptable.
That t = 0.001.

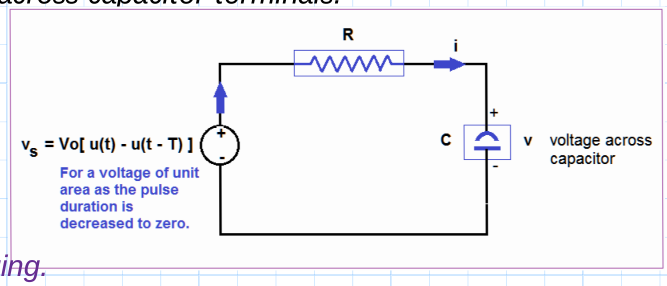
Now, if we set the LHS of the expression v(t) above equal to V_T?

$$V_T \;=\; 1-(1-t) \;=\; t \;=\; \frac{1}{RC}$$ <---This the result used in this solution.

Continuing now with the solution:
Focus on the 2 equations from section 7.10, in the form there, with the 3 conditions provided below: *Note V_T is voltage across capacitor terminals.*

$$v(t) := V_T \cdot e^{\frac{-(t-T)}{R \cdot C}} \qquad \text{where } t>T$$

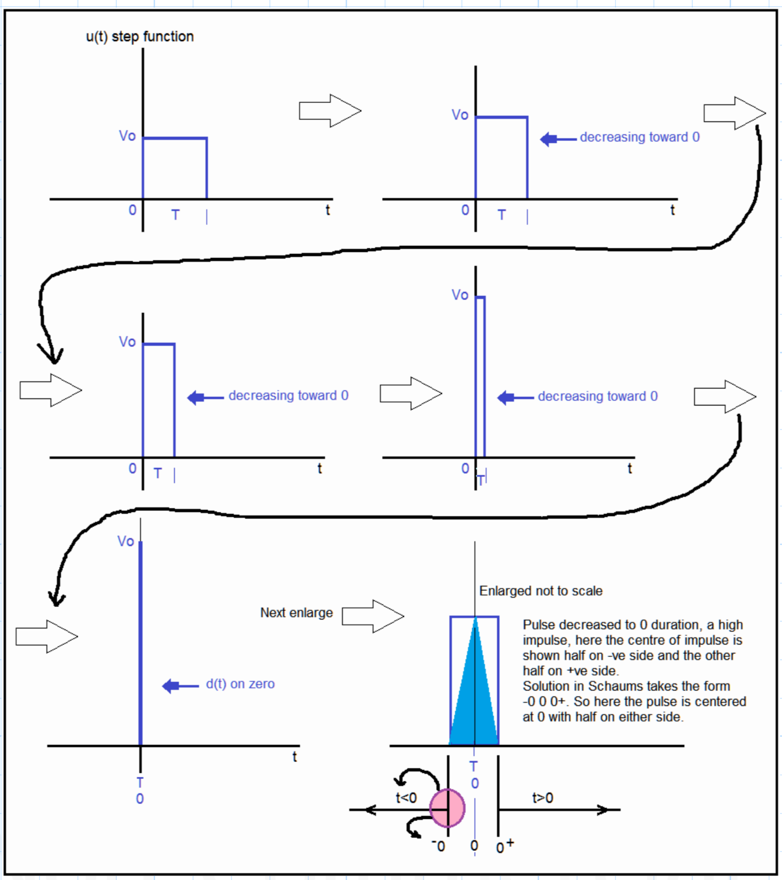
$$i(t) := -\left(\frac{V_T}{R}\right) \cdot e^{-\frac{(t-T)}{R \cdot C}} \qquad \text{where } t>T$$



i(t) is negative ONLY when C is discharging.

In our work so far, we were working with two time t conditions we add the 3rd:
1). 0 < t <T
2). t > T
3). now we bring in t <0

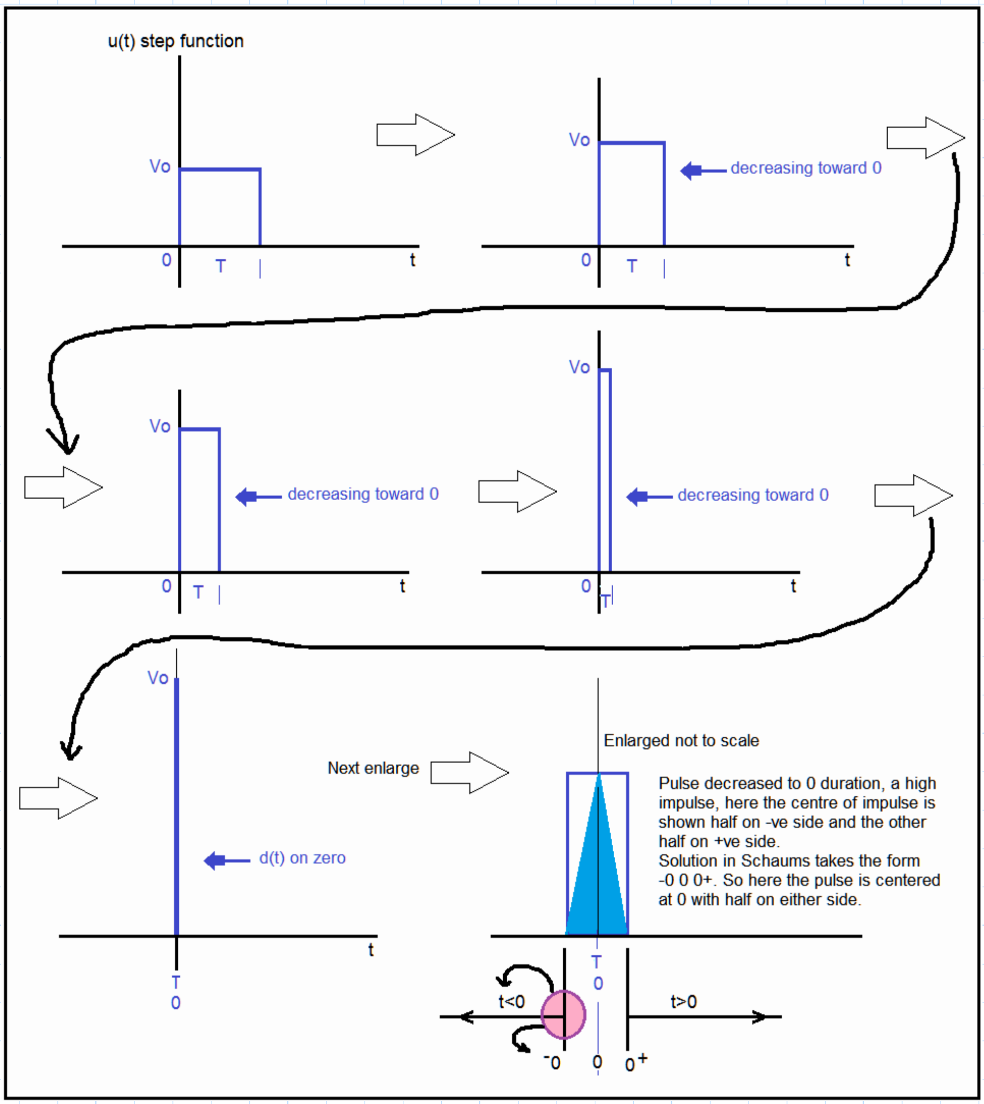
We use 'h' to denote the circuit's 'impulse response':



I resort to these sketches to base my solution to match the engineer-authors solution.
Whilst trying to keep to the understanding of the unit and impulse function.

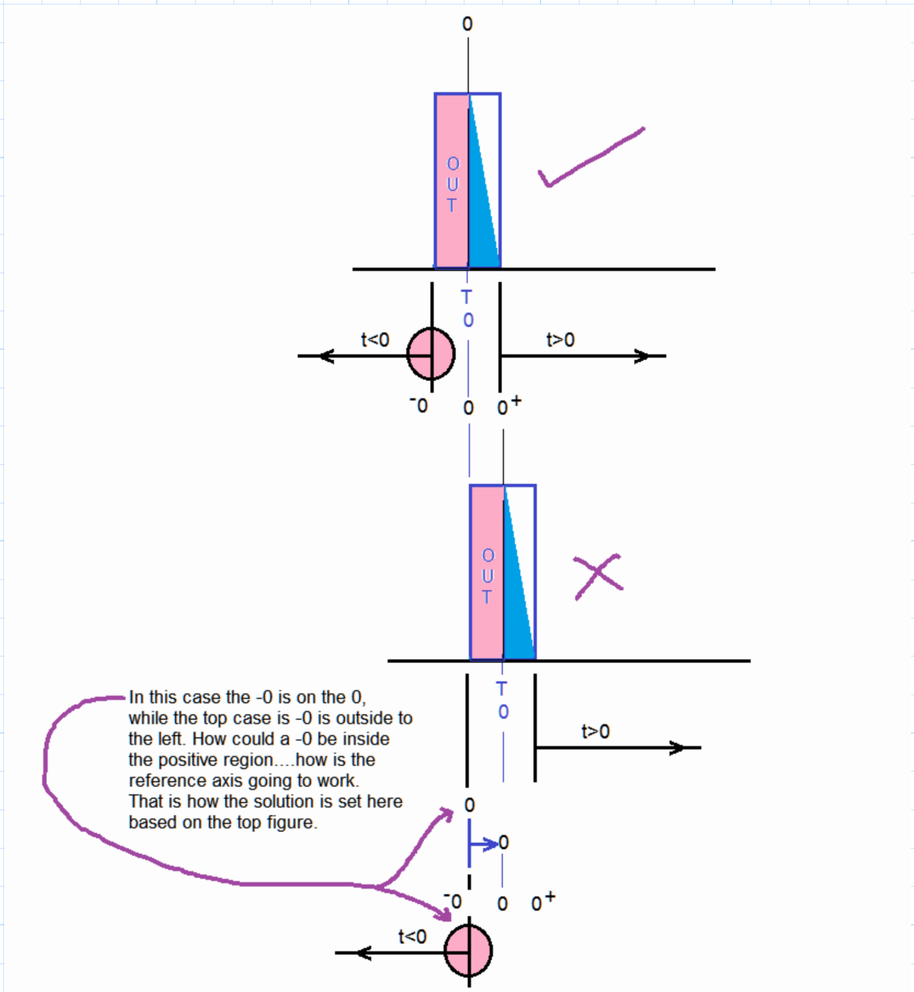
Time periods identified are:

1. t<0
2. -0
3. 0
4. 0+
5. t>0



You may have a better way of doing the bottom right detail or sketch.
Here the impulse is shown with a rising/inclining slope and declining slope.

Next sketch I move in closer to the solution's outcome. Not the steps or the calculations, rather what the solution looks like theory wise based on the capacitor voltage, and circuit current. Here, hopefully the timing of the 5 identifed on previous page are reflected to match the solution. Again, you may have a better presentation and accurate in comparison to mine.

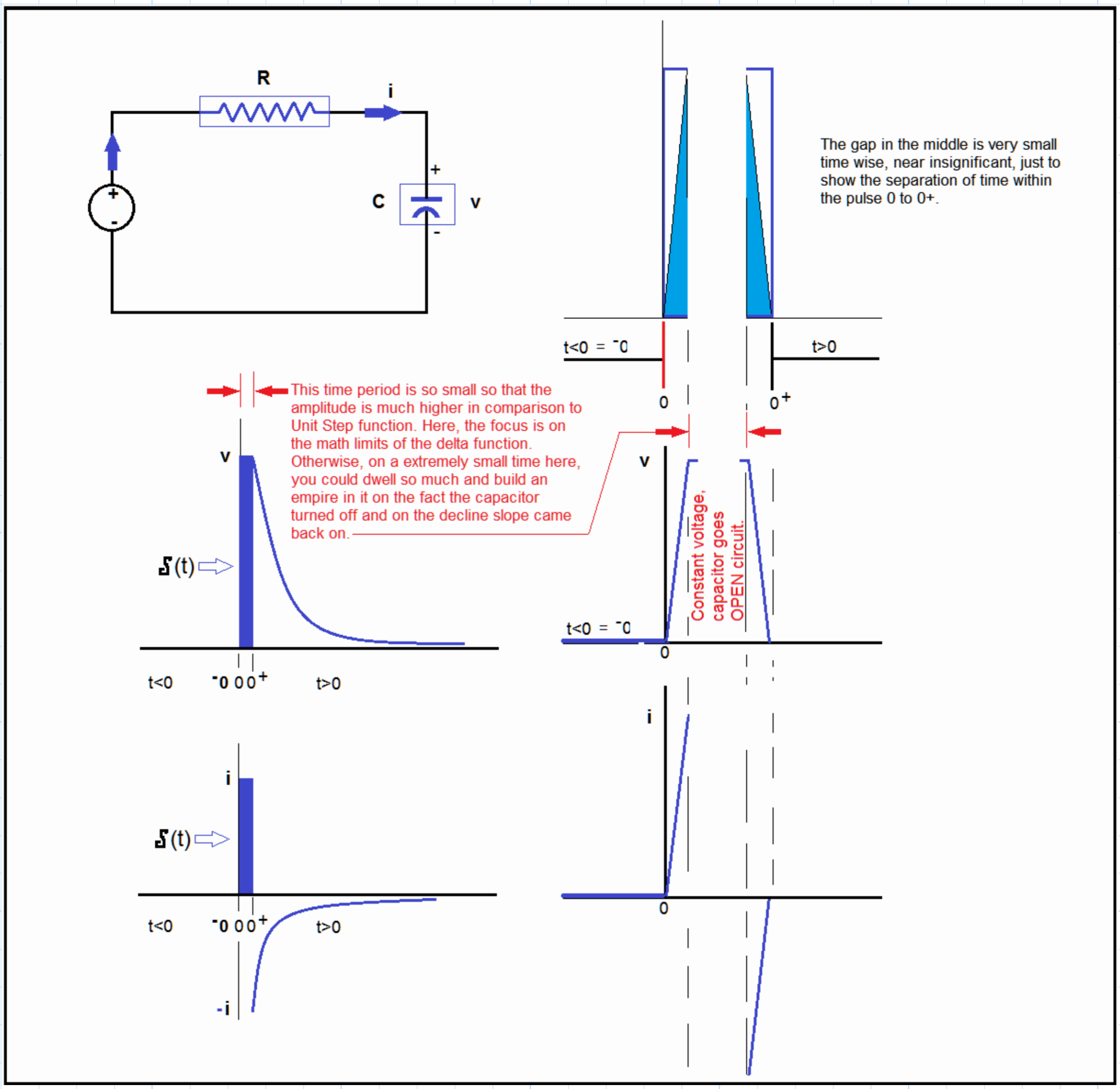


The figure above is on the discussion where does -0 belong.

To the right of 0 inside the positive t time or to the left in the negative t time?

Here I placed -0 in the left side in the -ve t time. You may disagree, but the point is that if -0 is on 0 as in the bottom sketch above, it may reflect as a unit impulse, but here the engineer-authors make it 0 for the unit impulse. So I placed a check mark and wrong mark above.

Next figure tries to show the circuit results matching to the impulse function.
You may have corrections here.



The sketches/details above serve to match the theory and solution's outcome.
The problem is broken into 3 time range:

1). t<0:

This has the time at point -0 coming into contact with t<0, which -0 is a zoomed in to a larger scale, while t<0 is merely moving to the time before the switch was closed or just -20s -1000s,...time before getting close to t=0 but closer to t=-0 in comparison to t=-0. So later we say -t OR t<0 = -0.

2). -0 0 0+:

This here trying to say all that is at point t=0, with a spread to -0 and 0+, and I have included incline decline slopes to fit the explanation.

3). t>0:

This is when 0+ meets t>0....where t maybe 1 s 20s to as far away as 1000s.

So next to the steps in the solution. Try to....hope its acceptable to you and others. If not you do better.

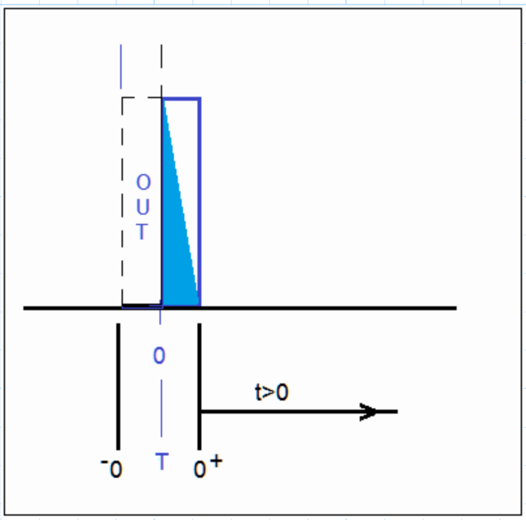
For t<0:

The circuit was energised at time t=0 and the unit step function turned ON. Prior to t=0, ie t<0 the u(t) is OFF. Therefore, voltage across the capacitor was zero, current thru the circuit was zero at t <0. The h_v response 0V and 0A.

$h_v := 0$ and $h_i := 0$

For -0 < t <0+:
with t at t=0 and u(t) goes positive then d(t) rises high since the duration is decreased to 0.

-0 < t :
This half is t=-0 to T=0. On the negative side. Where t=-0 and T=0 <--centre. Here we have a rising slope that comes to 1 on the y-axis at t=0=T. Here we have the impulse function = 1 = unit 1.



<----The solution in pictorial form in figure to the left.

Lets work it thru the expression/equation steps:

Lets set $t^{-0} = -0$ <---t $t^{0} = 0$ <---T

$$hv\left(0'^{-}'\right) = V_T \cdot e^{\frac{-(-0-0)}{R \cdot C}}$$

$$= V_T \cdot e^{\left(\frac{0+0}{R \cdot C}\right)}$$

$$= V_T \cdot e^{(0)}$$

$= V_T$ <--WRONG. Must be 0.

Remember: $V_T = 1-(1-t) = t = \dfrac{1}{RC}$ <---(1/RC)>>1 and aproximately 0? Ok. Can do.

$V_T = t = 0$ Set t=0 or at t=0.
Making a fit in the solution process/method.

So, for v(-0) : 0

Maybe good for now till proven otherwise. *Will not surprise me/us.*

Next for the hi(-0):

Lets tackle -(t-T) we have t=-0, and T on 0:

$$-(t-T) = -((-0)-0) \quad = -(-0) \quad = 0$$

$$hi\left(0^{'-'}\right) = -\left(\frac{V_T}{R}\right)\cdot e^{-\frac{(-t-T)}{R\cdot C}} = -\left(\frac{V_T}{R}\right)\cdot e^{\frac{(0)}{R\cdot C}} = -\left(\frac{V_T}{R}\right)\cdot e^{0} = -\left(\frac{V_T}{R}\right)$$

Apply the same done for hv(0-), set t=0:

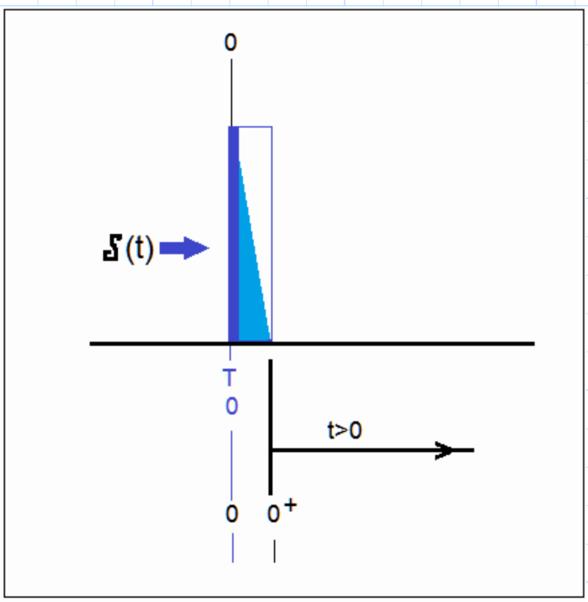
$$V_T = 1-(1-t) \quad = t \quad = \left(\frac{1}{R\cdot C}\right) \quad = 0$$ Approximately equal 0.

$$hi\left(0^{'-'}\right) = -\left(\frac{V_T}{R}\right) \quad = -\left(\frac{0}{R}\right) \quad = 0$$ Fits! It worked!

Other wise just make the case thru the pictorial provided before, off the centre to the left everything is zero. Simple. But since we attacked the problem in the ' -0 0 0+ ', we tried to evaluate leading to 0. Realistically hv and hi for e^(-0) is e^t<0 which are 0.

Next we move to the other half the right side (0+).

t < 0+ :



Ray of hope.
Sign of help.

We see a spike surge...a column rising (dark blue in colour) its the Unit Impulse function at t=0. Finally.

Can this be wrong? No.
Theory and definition of d(t) here it has to be unit value on the y-axis.

So next we have to show this for hv(0+) and hi(0+).

Here our t=0+ is in the positive side and t is approaching 0, ie T--->0, so same as t=0, on the positive side. t=0=0+.

Lets set $t^{0+'} = 0$ <---t $\quad t^{0+'} = T$ <---T

Remember T=1 in our early discussion of the solution.

$$hv\left(0^{'+'}\right) = V_T \cdot e^{\frac{-(0-T)}{R\cdot C}} = V_T \cdot e^{-\left(\frac{0}{R\cdot C}\right)+\left(\frac{T}{R\cdot C}\right)} = V_T \cdot e^{\left(\frac{T}{R\cdot C}\right)} = V_T \cdot e^{\left(\frac{1}{R\cdot C}\right)T}$$

$$hv\left(0^{'+'}\right) = V_T \cdot e^{\left(\frac{1}{R\cdot C}\right)1} = V_T \cdot e^{\left(\frac{1}{R\cdot C}\right)}$$

Next t = (1/RC).
Was discussed in example 7.11.
Substitute t for (1/RC).

$$hv\left(0^{'+'}\right) = V_T \cdot e^{(t)}$$

Also showed V_T = (1/RC) substitute that in next.

$$hv\left(0^{'+'}\right) = \left(\frac{1}{R\cdot C}\right) e^{(t)}$$

Next the exponent power t set it equal to 0+, since our time t = 0+ is saying t=0 in the positive side of -0 0 0+, 0<--->0+ is the positive side.

$$hv\left(0^{'+'}\right) = \left(\frac{1}{R\cdot C}\right) e^{(0+')} = \left(\frac{1}{R\cdot C}\right) e^{(0)} = \left(\frac{1}{R\cdot C}\right)$$

<---Thats the solution here.

Next for current i in the time t < 0+:

Current at t = 0+, as lim T-->0, exponent e^-(t-T/RC) = e^-(0-0/RC) = e^0 = 1.
Substitute 1/RC for V_T, results in 1/(RRC).

Similar to how the v(t=+0) was evaluated. No -VE sign for i(t) below because we do not know exactly when the capacitor is discharging. Negative only when discharging.

$$i(t) := \left(\frac{V_T}{R}\right) \cdot e^{-\frac{(0-0)}{R\cdot C}} = \frac{\left(\frac{1}{RC}\right)}{R} \text{ Amps.}$$

t=(1/RC)
(1/RC) = d(t) the impulse d(t).

Discussion:
At time t=0+ is at time t=0, this is when d(t) is maximum. We know time t wise on the x-axis the current(+ve/-ve) will be maximum at t=0 and then gradually decrease to 0.
What is that maximum value of i represented here:
V_T = (1/RC) which RC is the time constant (tau) of the series RC circuit.
Now, V_T = (1/tau). Also the maximum value of V_T = (1/RC) we got that from our earlier discussion, so V_T = (1/tau); (1/RC) = 1/tau; tau = RC, which is saying RC=RC. This is when the voltage is maximum so obviously the current will be maximum at this point because i = V/R. PLUS we know hv(0+) = 1/RC we just calculated. Thats the maximum. So we may reason or stand to logic, (1/RC) here in terms of peak is d(t). Dividing that by R gives the current hi(0+). The d(t) represents the impulse voltage value, which when divided by R gives hi(0+). We are NOT saying that the d(t) itself is the current value at the peak nor is current d(t).

$$i(t) = \frac{1}{R\cdot(RC)} = \frac{\delta(t)}{R}$$

Resulting with the expressions for hv and hi below:

$$0 < h_v < \left(\frac{1}{RC}\right) \qquad \text{and} \qquad h_i = \frac{\delta(t)}{R}$$

Next for t>0, which looks easy because things are in the +ve time direction, lets see how this goes.

t>0:

For the next condtion t>0, we realise why these equations are applicable, in this Series RC Circuit, because they represent that time past t-T which is past the period T, or past t=0+. Which is also saying past the impulse.

$$v(t) := V_T \cdot e^{\frac{-(t-T)}{R\cdot C}}$$

where t>T...same as past the impulse;
here t-T is shown as t, its also as t>0+

$$i(t) := -\left(\frac{V_T}{R}\right)\cdot e^{-\frac{(t-T)}{R\cdot C}}$$

where t>T, so t-T = >T, here t represents +t, t-T, and >0+.

Differentiate hv(t) and hi(t), replace t for (t - T), and make the substitution for V_T:

$$h_v(t) := V_T \cdot e^{\frac{-(t-T)}{R\cdot C}} \qquad h_i(t) := \left(\frac{V_T}{R}\right)\cdot e^{-\frac{(t-T)}{R\cdot C}}$$

<--No -ve sign for current response hi(t).

Got it-->
$$h_v(t) := \left(\frac{1}{R\cdot C}\right)\cdot e^{\frac{-(t)}{R\cdot C}} \qquad h_i(t) := \left(\frac{1}{R\cdot(R\cdot C)}\right)\cdot e^{-\frac{(t)}{R\cdot C}}$$

$$h_i(t) := -\left(\frac{1}{R^2\cdot C}\right)\cdot e^{-\frac{(t)}{R\cdot C}}$$

<--Got it. This was differentiated.

Lets place the results in a table form, so we can see the progress, and represent also the step function turning into impulse function in the final solution where applicable and possible.

t<0:

$h_v := 0$ and $h_i := 0$ <--Row 1

$0^{-} < t < 0^{+}$:

$0 < h_v < \left(\frac{1}{RC}\right)$ and $h_i = \frac{\delta(t)}{R}$ <--Row 2

t>0:

$h_v(t) := \left(\frac{1}{R \cdot C}\right) \cdot e^{\frac{-(t)}{R \cdot C}}$ $h_i(t) := -\left(\frac{1}{R^2 \cdot C}\right) \cdot e^{-\frac{(t)}{R \cdot C}}$ <--Row 3

Can we add-up the rows to represent the final solution for hv and hi? Yes. May be a problem if we had a value for hv and hi for t<0 but since here our step function equal zero we do not have any values for them. But if we did, we may add them in, taking their +ve and -ve values into consideration.

Row 1 is dead. Comes down to row 2 and 3. In row 2 we have the beginning for the voltage hv but the hv in row 3 captures row 2 when the exponent's power t =0. Yes Yes! Agree that.

Next, hi current response, needs looking into thats the figure on the next page - its an attempt.

hi goes from a impulse at t=0 then declines with the step function in t>0, (T was set at 0, T=0, should not impact the solution), so we see a flow from impulse the dark blue column to the form of step function the negative slope settling to 0.

We add them both for the total area. Maybe Yes? Good for now! Continued next page.

First the figure then the completion of solution.

Look over the figure below you may or surely do better, its not so the accuracy more the 'flow of right and wrong information' flowing in the mind. Maybe what they call a discusion.

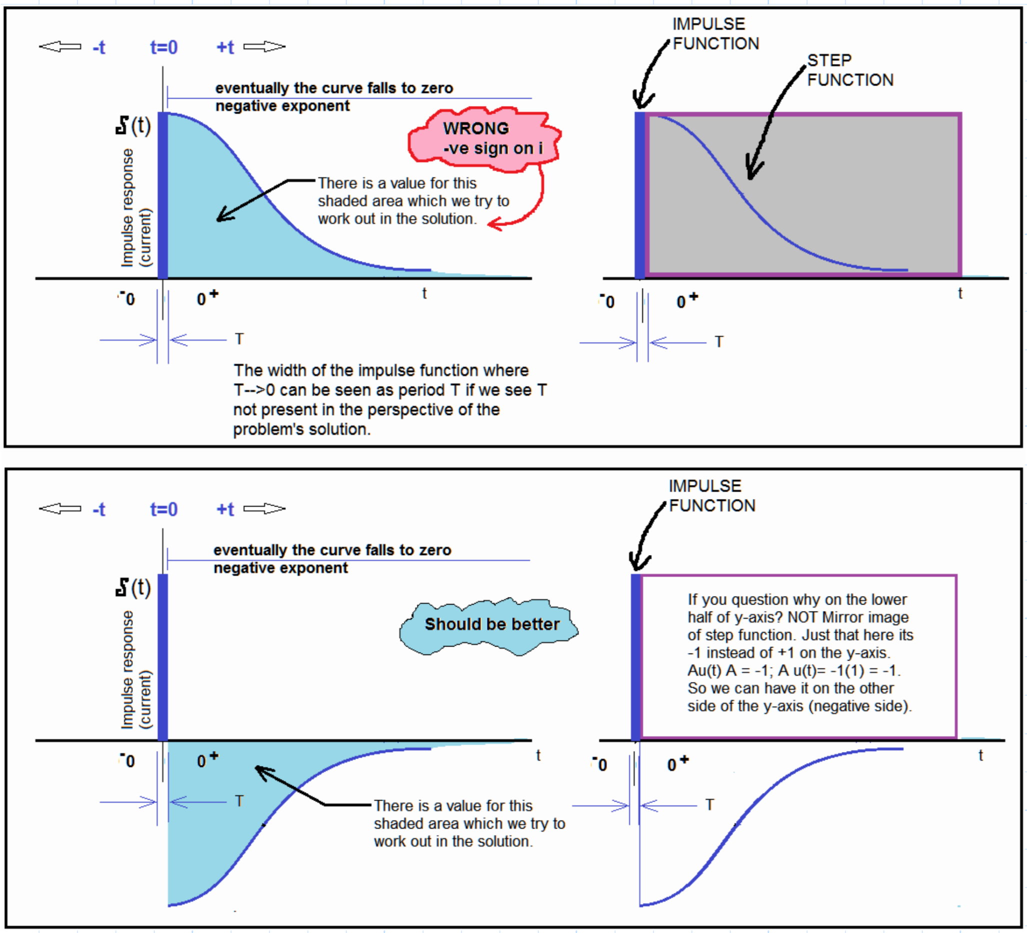


Figure to the left has a top and bottom figures, the bottom here said to be the better one, you may present a correct one, if there is not one found suitable here. It was not easy to do the sketch to capture the engineer authors solution.

hi goes from delta(t) thru u(t), so we add them both, knowing u(t) has a negative sign.

hv is seen thru the final time condition t>0.

$$h_i = \frac{\delta(t)}{R} + \left(-\left(\frac{1}{R^2 \cdot C}\right) \cdot e^{-\frac{(t)}{R \cdot C}} \right)$$

$$h_v = \left(\frac{1}{R \cdot C}\right) \cdot e^{\frac{-(t)}{R \cdot C}} \quad \text{and} \quad h_i = \frac{\delta(t)}{R} - \left(\frac{1}{R^2 \cdot C}\right) \cdot e^{-\frac{(t)}{R \cdot C}}$$

Answers.

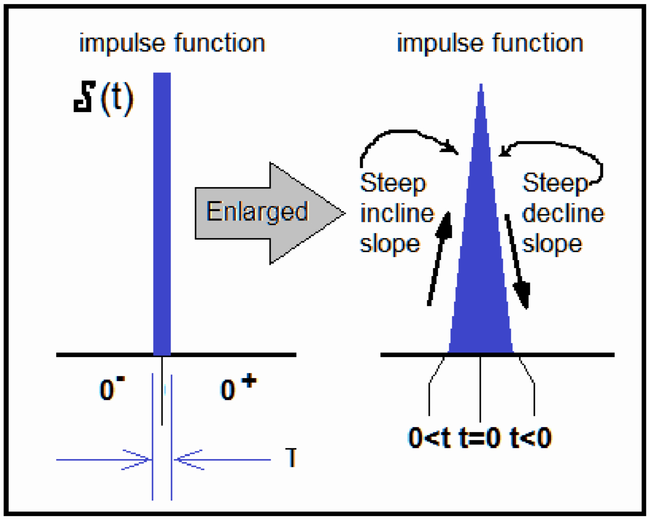
Both the hv and hi started in a form of? u(t), where t=0 OR t>0 = 1, dependent on when the intent was to go from 0 to 1, it maybe at t0 go 1 OR at t0 go 0. So we multiply hv and hi by u(t) to associate the case that these expression are in state '1' of u(t). Multiplying by u(t) merely multiplies by 1 when its ON state (1-state). You may explain it better. Same answer as above but we know here its under u(t) condition.

$$h_v = \left(\frac{1}{R \cdot C}\right) \cdot e^{\frac{-(t)}{R \cdot C}} \cdot u(t) \quad \text{and} \quad h_i = \frac{\delta(t)}{R} - \left(\frac{1}{R^2 \cdot C}\right) \cdot e^{-\frac{(t)}{R \cdot C}} \cdot u(t)$$

Answers. Schaums.

Example 7.12 was not easy for me. Some steps/process of the solution in Schaums are left out leaving us to work thru.
Most these examples are NOT meant or intended Right The 1st Time.

Discussion: Can an impulse function create a dv/dt?



The figure on the left, my thoughts, is that the vertical rectangular shape on the left is more so that is clearly understood and help fit the theory closer in terms of the area method. Going from rectangular to higher height column.

Capacitor will see a rise, slope on the left rising, then the peak for a moment, then the declining slope.

Slope is there.

We discussed we don't go from 0 to 1 rather 0...0.1...0.50...0.75...0.95...0.99 to 1.00.

So a slope is there, and we can say a capacitor experiences a steep rise in voltage, dv/dt - the impulse function. Similalry for current a di/dt.

So, first lets not get it in our minds, its constant vertical rise, there is no slope.

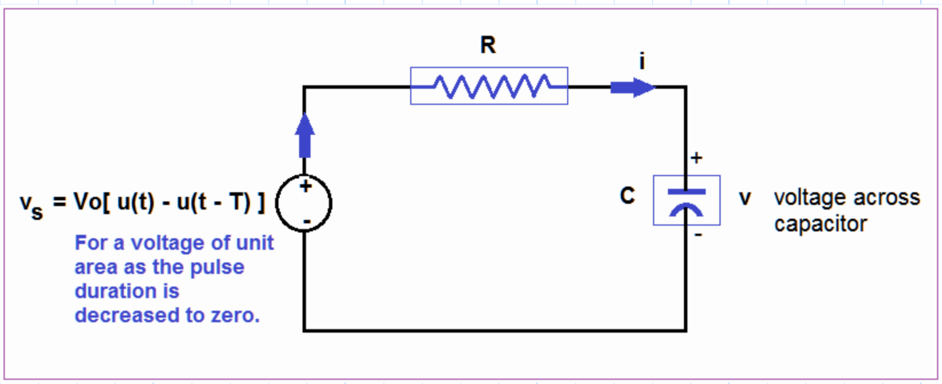
So far we have worked a RC circuit, where C has dv/dt applicable to it, and this circuit solution progressed from t<0, t=0, t>0. It was not clearly written in wording if such was the case that there was no slope. If there was none the capacitor would not had gotten charged. *i = C (dv/dt) and i (current) is charge per time.....so there must be a current to create voltage (potential) across the capacitor plates/terminals. <---Check your textbook.*

Thats was the discussion.

Next too, 7.13 and 7.14, are simpler in the group of three, 7.12, 7.13 and 7.14. Example problem 7.12 was difficult, lenghty, and may not be your suitable answer. You may yet have a better solution. Will not surprise me.

Example 7.13:

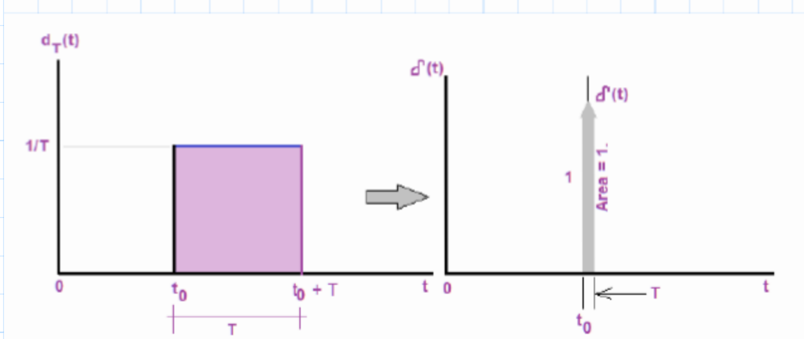
Find the impulse responses of the RC circuit in the same circuit as in example 7.12, series RC circuit, by taking the derivatives of its unit step responses.



Solution:

Page 156 Schaums: 'A unit impulse may be considered the derivatives of its unit step. Based on the propeties of linear differential equations with constant coefficients, we can take the derivative of the step response to find the impulse response.'

In our Part 1B notes page 15 example 6.18 unit delta/impulse sifting property has the figure below with some discussion on the derivative.



<---Something like this.
Notice on the left side its dT(t) the derivative. Right side is the delta function.

In section 7.3 'Establishing a DC Voltage across a Capacitor' we arrived to v and i expressions provided below. These are the expressions we will work with. These were provided under the u(t) condition. *We sense we need the derivatives of these equations.*

$$v(t) = V_0 \cdot \left(1 - e^{\frac{-t}{R \cdot C}}\right) \cdot u(t) \quad \text{and} \quad i(t) = \left(\frac{V_0}{R}\right) \cdot e^{\frac{-t}{R \cdot C}} \cdot u(t)$$

Because we are not given the amplitude of v and i, which is dependent on Vo here. The other parameters like R and C are circuit components, whilst the voltage wave form impacts the response magnitude. So we set Vo = 1, for a u(t) condition that merely says Vo u(t) = Vo (ON=1 when t0=0 or t=0).

Derivatives of v and i with Vo set to 1:
v:

$$v(t) = 1\left(1-e^{\frac{-t}{R\cdot C}}\right) \quad \text{--->} \quad \frac{dv}{dt} = -\left(\frac{1}{R\cdot C}\right)\cdot -e^{-\left(\frac{t}{R\cdot C}\right)}$$

$$\frac{dv}{dt} = \left(\frac{1}{R\cdot C}\right)\cdot e^{-\left(\frac{t}{R\cdot C}\right)}\cdot u(t)$$

<--- the u(t) condition makes known circuit is energised/ON under unit step conditions.

What we just arrived at was the derivative, i.e. t or T approaching the limit of 0. We were told the derivative of the step function may lead to the impulse function, which is based on the limit of t approaching 0. Usually, the time t here is positive and t is moving toward 0. -t 0 +t . We are taking the limit of t approaching 0 from the positive side (+t). So obviously the derivative is going to manage the solution for the time t=0 to t>0 its on the right side of 0. This takes care of the condition t>0. Agreed? Maybe for now! Should.

Question: Why it does not apply at -0.
The original function evaluated at t=0 would provide for the condition (-0) = 0.
This does not arise. u(t) is on at t=0. t=-0 or t<0 is not appearing as a unit step. The -0 0 0+ does not apply because -0 0 is out its zero, only 0 0+ for unit step when it picks up at t=0. The function or any function f(t) at t=0 would give the function value at t=0. For t=-1 we plug-in t=-1 in the function f(t). So how do we say that (-t) = (t=0) here? The capacitor or inductor gains energy from time -t to t=0 this is the time span before the circuit switch was closed at t=0. What does it matter if it was time t=-20 seconds before the switch was closed compared to t=-1....-0.0001.....almost zero and finally 0?
Not so much here because at near t=0 coming from the -ve t to t=0 what we want to evaluate is the maximum energy the capacitor or inductor had gained. And that happens just before the switch is closed at t=0. Agreed? Should. Try again? Yes.

So why is that at t=0 considered an impulse function? Why would it be seen as so narrow in time and high in amplitude? The Mathematical property of the exponent. At e^0 = 1. So the function's other parameters will result in maximum. We get a maximum of the step function, that should mean something! Thats All. Sad but its true. Some functions may have a high spike/surge/maximum here so it should be given consideration in the sum of all things.

Next we evaluate v(t) at t=0.

$$v(0) = 1\left(1-e^{\frac{-0}{R\cdot C}}\right) = 1\left(1-e^{-0}\right) = 1\,(1-1) = 1\,(0) = 0$$

Here we got nothing, just had nothing to contribute to a impulse function.
so the answer is just the derivative term.

$$h_v = \frac{dv}{dt} = \left(\frac{1}{R\cdot C}\right)\cdot e^{\frac{-(t)}{R\cdot C}}\cdot u(t)$$ Answer.

Above hv same as hv of the previous example 7.12.

i:

$$i(t) = \left(\frac{1}{R}\right) e^{\frac{-t}{R\cdot C}} \quad \text{--->} \quad \frac{di}{dt} = -\left(\frac{1}{R\cdot R\cdot C}\right)\cdot e^{-\left(\frac{t}{R\cdot C}\right)}$$

$$\frac{di}{dt} = -\left(\frac{1}{R^2\cdot C}\right)\cdot e^{-\left(\frac{t}{R\cdot C}\right)}\cdot u(t)$$

For t=0:

$$i(0) = \left(\frac{1}{R}\right) e^{\frac{-0}{R\cdot C}} = \left(\frac{1}{R}\right) e^{0} = \left(\frac{1}{R}\right)$$

This need be shown as a impuse function d(t).

$$i(0) = \left(\frac{1}{R}\right)\delta(t)$$

Because e^0 is max for the current would be the equivalent of delta function.

Now we add the two terms for t= and t>0:

$$\frac{di}{dt} = \frac{\delta(t)}{R} - \left(\frac{1}{R^2\cdot C}\right)\cdot e^{-\frac{(t)}{R\cdot C}}\cdot u(t)$$ Answers.

Comments: Example 7.13 needs looking deeper to get the core/crux/deep meaning of the impulse function is a derivative of the unit step PLUS don't forget to leave the f(t) at t=0 term! I don't know if that can be done by numerical evaluation without a discussion, somewhere there was a break in the flow that required a discussion be presented. We know that when these types of situation like when the 'limit of t approches 0' there is a numerical gap created and to procced with some reasoning/logical thought, that requires some insight be plugged into the solution. These are tough to solve because we have to go from automatic to manual drive. Usually do NOT happen at work you are better of making a guess, thats a joke.

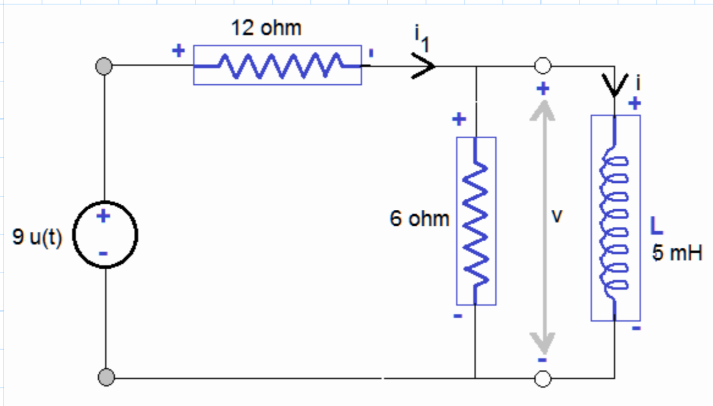
Joke: If bankers or business men thought like this in their problem solving....when t approaches 0....who is going to belief them? Who is going to buy or make the sale? You smell elements of deceit....cheating...trickery....untrust worthy implications.... So maybe thats why they stick to simple interest. Do you speak the banking language?

One more similar example 7.14, in the Schaums notes for section 7.11 page 156, it was writen '.......take the limit of the response to a narrow pulse, to be called the limit approach,....' So we were warned there was that 'limit approach' !

Example 7.14 (Impulse response of RC and RL circuits):

Find the impulse responses hi(t), hv(t), and hi1(t) of the RL circuit by taking the derivatives of its unit step responses. Refer to example 7.5 circuit.

Circuit below from same example 7.5.



<---Solved on page 29 of Part 2-A. Answers posted here for i1, i, and v. And are shown on the figure here.

$$V_{input} = 9 \cdot u(t)$$

Solution:

$$i = 0.75 \cdot \left(1 - e^{-800\,t}\right) \cdot u(t)$$ From Example 7.5

$$v = 3 \cdot e^{-800\,t} \cdot u(t)$$ From Example 7.5

$$i1 = \left(\frac{1}{4}\right) \cdot \left(3 - e^{-800\,t}\right) \cdot u(t)$$ From Example 7.5

All the above variables are unit step 'responses' caused/generated by the input voltage 9u(t).

As in the previous example, our first reaction, or more accurately my first reaction was to go ahead and take the derivative of each expression. Lets wait first, lets look at what we get when we differentiate? From our past exprience we came to know the impulse function can be obtained by taking the derivative of the step function, and also we had to do some 'limit approach', and make sense of what we got. So, ok, lets start with the derivatives, looks like there is no other option and I will have to cross the hurdle/river/ocean when I get there.

$$i = 0.75\cdot\left(1-e^{-800\,t}\right)\cdot u(t)$$

place the u(t) aside in the differentiation its a signal/waveform condition, but make the i response 1u(t) instead of 9u(t). Divide by 9.

$$i = \left(\frac{1}{9}\right)\cdot 0.75\cdot\left(1-e^{-800\,t}\right)\cdot u(t)$$

$$\frac{di}{dt} = (-800)\cdot\left(\frac{1}{9}\right)\cdot(0.75)\,\left(-e^{-800\,t}\right)$$

$$(800)\cdot(0.75) = 600$$

$$\frac{1}{9}\cdot 600 = \frac{200}{3}$$

$$\frac{di}{dt} = \frac{200}{3}\left(e^{-800\,t}\right)\cdot u(t)$$

Now place the u(t)....so this derivative we know comes on when t>0 and its input amplitude is 1. This we got (di/dt) is the impulse response of that input 1u(t) into the circuit with resistors and capacitor.

Problem?...hurdle.
Now when t goes from -0 (ie -t) to 0 we see a rise, steep slope this created an impulse at t=0 based on the original function. We had not gotten this yet.
What we got so far was the di/dt was for t>0 and starting at t=0, meaning starting at t=0 its going t>0 ie t positive, the downward slope. We got the impulse function here from di/dt of the unit step function - downward slope. This side, declining the exponential will decay with e^(-t). The inclining slope generates at =0 in the original equation an impulse function, here the definition is t approaches 0. Lets look at it like this, the impulse (d(t)) on the inclining slope is when t goes from negative into t=0, then the way out of t=0 to t is positive is the declining slope we get from differentiating the original equation. This goes back to assisting further an explanation for the previous example. DIY? Do It Yourself. *Timing wise bad joke.*

$$i_0 = 0.75\cdot\left(1-e^{-800\,t}\right)\cdot u(t)$$

Substitute for t=0

$$= 0.75\cdot\left(1-e^{-800\,(0)}\right) = 0.75\cdot(1-1) = 0$$

So no contribution here.

Now the solution is:

$$\frac{di}{dt} = \frac{200}{3}\left(e^{-800\,t}\right)\cdot u(t) + 0\,\delta(t)$$

That impulse function $\delta(t)$

$$h_i = \frac{200}{3}\left(e^{-800\,t}\right)\cdot u(t)$$

Answer.

Next.

This time lets start with the original equation for t=0 and remember it has to be for unit impulse too so divide by the 9 for 9u(t).

$$v = 3 \cdot e^{-800\,t} \cdot u(t)$$ Divide by 9 later after solving the expression.

$$v_0 = 3 \cdot e^{-800\,(0)} = 3 \cdot e^{-0} = 3$$ Thats the impulse at t=0, it need be for a unit step input, divide by 9.

$$hv_0 = \frac{3}{9}\delta(t) = \frac{1}{3}\delta(t)$$

$$v = 3 \cdot e^{-800\,t} \cdot u(t)$$ Divide by 9, then carry on.

$$v = \frac{3}{9} \cdot e^{-800\,t}$$

$$\frac{dv}{dt} = (-800) \cdot \frac{1}{3} \cdot e^{-800\,t}$$ Place in u(t)

$$\frac{dv}{dt} = (-800) \cdot \frac{1}{3} \cdot e^{-800\,t} \cdot u(t)$$

$$hv = (-800) \cdot \frac{1}{3} \cdot e^{-800\,t} + \frac{1}{3}\delta(t)$$ Answer. We got complete impulse response of that input of 1u(t) into the circuit with resistors and capacitor.

Next.

$$i1 = \left(\frac{1}{4}\right) \cdot \left(3 - e^{-800\,t}\right) \cdot u(t)$$

Solve for t=0

$$= \left(\frac{1}{4}\right) \cdot \left(3 - e^{-800\,(0)}\right) = \left(\frac{1}{4}\right) \cdot (3-1) = \left(\frac{2}{4}\right) = \frac{1}{2}$$ Next divide by 9

$$hi_0 = \left(\frac{1}{9}\right) \cdot \left(\frac{1}{2}\right) = \frac{1}{18} \cdot \delta\ t$$

Continued next page.

Next continue with the differentiation:

$$i1 = \left(\frac{1}{4}\right)\cdot\left(3 - e^{-800\,t}\right)\cdot u(t)$$

$$i1 = \left(\frac{1}{9}\right)\cdot\left(\frac{1}{4}\right)\cdot\left(3 - e^{-800\,t}\right)$$

$$i1 = \left(\frac{1}{9}\right)\cdot\left(\frac{1}{4}\right)\cdot\left(3 - e^{-800\,t}\right)$$

$$\frac{di1}{dt} = (-800)\,\left(\frac{1}{9}\right)\cdot\left(\frac{1}{4}\right)\cdot -e^{-800\,t}$$

$$\frac{di1}{dt} = (200)\,\left(\frac{1}{9}\right)\cdot\left(e^{-800\,t}\right)$$

$$\frac{di1}{dt} = \left(\frac{200}{9}\right)\cdot\left(e^{-800\,t}\right)\cdot u(t)$$

Putting the terms together:

$$hi_1 = \left(\frac{200}{9}\right)\cdot\left(e^{-800\,t}\right)\cdot u(t) + \left(\frac{1}{18}\right)\delta(t)$$ Answer.

Comments: First term does not need the impulse abbreviation d(t), only the second. Why? Because the 2nd term was a function calculation at time t=0.
So specifically here there is no ambiguity. Manner of speaking. The 1st term was differentiated for t>0, here the unit step function was carried to a higher level, increased in coefficient magnitude/value usually is, +ve or -ve. The derivative expression carries with it the unit step spread for t>0 not at one specific point t=0 where we see a distinct impulse (column rise). You may have a better explanation.

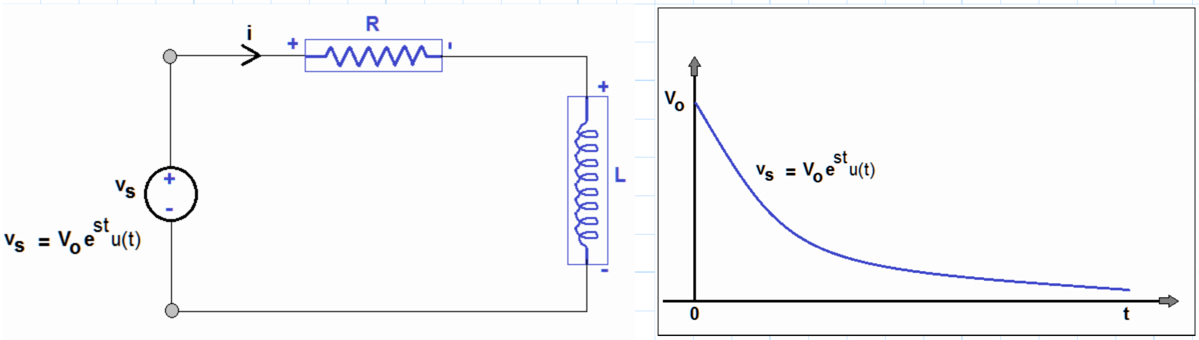
What you seen in 7.12, 13, and 14 were after several re-visits.

7.12 Summary of Step and Impulse Responses in RC and RL circuits.

You may find this in your textbook(s). It shows the circuit, next columns it shows the response equations for unit step response and unit impulse response. Example 7.12 was for series RC impulse response. Schaums Outline page 157 has series and parallel circuits for RC and RL, and for each the step and impulse responses.
I added it in at last page. May need it in the future and its in the file. You are welcome.

7.13 Response of RC and RL Circuits to Sudden Exponential Excitations.

The first order differential equation of a RL circuit with a sudden exponential voltage source v_s = Vo e^(st) u(t) is shown below, obtained through by applying KVL, with the circuit. The circuit below is at rest for t<0.



$$v_s = R \cdot i + L \cdot \left(\frac{di}{dt}\right)$$

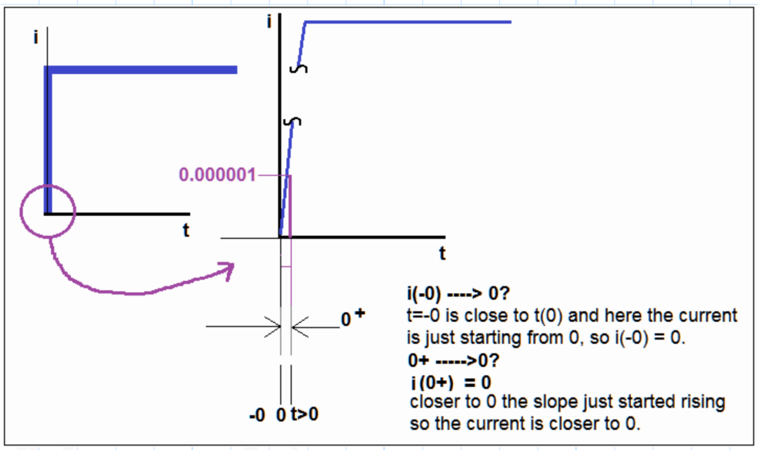
$$v_s = V_o e^{st} \, u(t)$$

$$R \cdot i + L \cdot \left(\frac{di}{dt}\right) = V_o e^{st} \, u(t)$$ <--- First equation for this section.

The objective here is to get to an expression that solves for the current. We are given the voltage source equation, and its curve is shown in the figure above to the right of the circuit.

Solution for t>0: $i(t) = i_h(t) + i_p(t)$ <---natural and particular solutions

$i(0+') = 0$



The figure on the left is trying to show, a small time after t=0, where t=0+ the current is 0; i(0+) = 0.

Here the rising slope is starting and the beginning time of t, the current is building up and approximatley 0.

Inductor dependent current, di/dt.
And
Capacitor dependent voltage, dv/dt.

Refer to 7.4 'Source Free RL circuit': Initial condition: $i(0) = A = I_0$

$$i(t) = I_0 e^{-\left(\frac{Rt}{L}\right)}$$

What was just presented (section 7.4) is stated below for the purpose of this section.

The natural response i_h(t) is the solution of Ri + L(di/dt) = 0.
The case with a 0 forcing function. Following an argument similar to section 7.4 we obtain:

$$i_h(t) = A \cdot e^{-\left(\frac{Rt}{L}\right)}$$

The solution for i_h(t) above satisfies the equation:

$$R \cdot i + L\left(\frac{di}{dt}\right) = 0$$

The forced response i_p(t) is a function which satisfies, the first equation provided in this section, for time t>0. The ONLY such function is:

$$i_p(t) = I_0 e^{st}$$

The solution for i_p(t) above satisfies the first equation:

First equation---> $R \cdot i + L \cdot \left(\frac{di}{dt}\right) = V_o e^{st}\, u(t)$

RHS NOT equal 0.

After substituting i_p(t) in the equation above, Io = Vo / (R + Ls).

$$R\left(I_0 e^{st}\right) + L\left(\frac{di}{dt}\right) = V_o e^{st}$$

$$\frac{di_p}{dt} = sI_{0e}{}^{st}$$

$$R\left(I_0 e^{st}\right) + L\left(sI_{0e}{}^{st}\right) = V_o e^{st}$$ divide by e^st

$$R\left(I_0\right) + L\left(sI_0\right) = V_o$$

$$I_0 (R + Ls) = V_o$$

$$I_0 = \frac{V_o}{(R + Ls)}$$ So this result verifies the statement provided above.

The first equation was set equal to zero, and had the condition i(0+) = 0.

$$R \cdot i + L\left(\frac{di}{dt}\right) = 0 \quad \text{equation}$$

$$i(0+') = 0 \quad \text{condition}$$

$$i_h(t) = A \cdot e^{-\left(\frac{Rt}{L}\right)}$$ <--- i_h(t) solution form.

Now to solve this equation set it equal to 0 we set A = -Vo/(R + Ls).
Only difference is the negative sign, which we will see why because its the only case that will make the expression equal 0.

$$i_h(t) = -\left(\frac{V_o}{R+Ls}\right) \cdot e^{-\left(\frac{Rt}{L}\right)}$$

$$\frac{di_h}{dt} = \left(\frac{R}{L}\right)\left(\frac{V_o}{R+Ls}\right) e^{-\left(\frac{Rt}{L}\right)}$$

Next substitute i-h(t) and di_h/dt in the equation: $R \cdot i + L\left(\frac{di}{dt}\right) = 0$

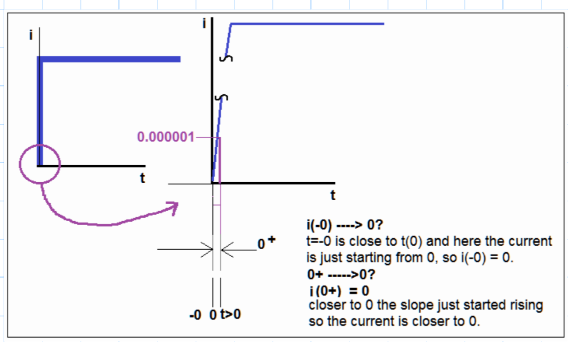
$$R\left(-\left(\frac{V_o}{R+Ls}\right) \cdot e^{-\left(\frac{Rt}{L}\right)}\right) + L\left(\left(\frac{R}{L}\right)\left(\frac{V_o}{R+Ls}\right) e^{-\left(\frac{Rt}{L}\right)}\right) = 0$$

multiply by e^(Rt/L)

$$R\left(-\left(\frac{V_o}{R+Ls}\right)\right) + L\left(\left(\frac{R}{L}\right)\left(\frac{V_o}{R+Ls}\right)\right) = 0$$

$$-\left(\frac{R \cdot V_o}{R+Ls}\right) + \left(\frac{R \cdot V_o}{R+Ls}\right) = 0$$ Verified. So we saw why it had to be a negative sign.

We can say boundary condition i(0+) is satisfied.
Because here the equation was set equal to 0,
now this may be seen as (0+)--->0,
and that becomes i(0+) = 0.



The full solution becomes:

$$i(t) = i_h(t) + i_p(t)$$

$$= -\left(\frac{V_o}{R+Ls}\right) \cdot e^{-\left(\frac{Rt}{L}\right)} + \left(\frac{V_o}{R+Ls}\right) \cdot e^{-(st)}$$

$$i(t) = \left(\frac{V_o}{R + Ls}\right) \cdot \left(e^{st} - e^{-\left(\frac{Rt}{L}\right)}\right)$$ <---This is the complete response.

This response works or is fullfilled under the condition the voltage source is unit step function, comes on when t=0 and t>0.
Next we adjust for unit step condition.

Therefore expression we seek: $$i(t) = \left(\frac{V_o}{R + Ls}\right) \cdot \left(e^{st} - e^{-\left(\frac{Rt}{L}\right)}\right) \cdot u(t)$$

This is solved.

Next we have a 'what if case', a special case, for the same process above.

$$R \cdot i + L\left(\frac{di}{dt}\right) = 0$$ <----Forcing function here equal 0.
Force 0 results with natural response i_h(t).

$$R \cdot i + L\left(\frac{di}{dt}\right) = V_0 e^{st} \cdot u(t)$$ <----Forcing function here on the RHS, NOT equal 0.
Force exist, results with forced response i_p(t).

Problem arises when 'what if the forcing function exponent is the same as that of the natural response i_h(t) where s =-R/L'? *Verify these notes yourself from here onwards.*

$$V_0 e^{st} \cdot u(t) \quad \text{--->} \quad V_0 e^{\left(\frac{-R}{L}\right)t} \cdot u(t)$$ <---Changed from s to (-R/L).
Forcing function RHS.

$$i_h(t) \quad \text{--->} \quad A \cdot e^{\left(\frac{-R}{L}\right)t}$$ <---i_h(t) natural response
unchanged

This impacts? The forced response i_p(t).

On the right side of the arrows we see the exponent's power term is the same (-R/L).
This will NOT work for the solution we went just thru for i_p(t).
Little confusing because expected to impact the natural response.
Our complete solution is the addition of natural and forced response;
i(t) = i_h(t) + i_p(t).

Lets try to show why this form of i_h(t) will not work for the solution in the same steps previously completed.

$$i_p(t) = I_0 \cdot e^{\left(\frac{-R}{L}\right)t} \qquad \frac{di}{dt} = \left(\frac{-R}{L}\right) I_0 e^{\left(\frac{-R}{L}\right)t}$$

Re-do this part, substituting i_p(t) in the equation below Ri + L(di/dt) = Vo_e^(-R/L)t.

$$R\left(I_0 \cdot e^{\left(\frac{-R}{L}\right)t}\right) + L\left[\left(\frac{-R}{L}\right) I_0 e^{\left(\frac{-R}{L}\right)t}\right] = V_0 e^{\left(\frac{-R}{L}\right)t}$$ Forcing function RHS.

$$R \cdot \left(I_0 \cdot e^{\left(\frac{-R}{L}\right)t}\right) - R \cdot \left(I_0 e^{\left(\frac{-R}{L}\right)t}\right) = V_0 e^{\left(\frac{-R}{L}\right)t}$$ next divide by e^(-R/L)t

$$R \cdot (I_0) - R \cdot (I_0) = V_0$$

$$0 = V_0$$ <---0 is a problem. We cannot form an expression for Io.
Lets say we found the problem!

We see a problem on the LHS, we have a forcing function on the RHS,
and we do not have a solution for Io on the LHS.
We see original term Io_e^(-R/L)t will not work.
All we did was change s to -(R/L) in the same form.
Now for the solution we have a new form for i_p(t) = Io_t_e^(-R/L)t.
Next we go thru it with the new i_p(t) form of solution.

$$i_p(t) = I_0 \cdot t \cdot e^{\left(\frac{-R}{L}\right)t}$$

Differentiation by multiplication:

$$u = t \qquad \frac{du}{dt} = 1$$

$$v = e^{\left(\frac{-R}{L}\right)t} \qquad \frac{dv}{dt} = \left(\frac{-R}{L}\right) e^{\left(\frac{-R}{L}\right)t}$$

$$\frac{d(uv)}{dt} = (t) \cdot \left(\frac{-R}{L}\right) e^{\left(\frac{-R}{L}\right)t} + e^{\left(\frac{-R}{L}\right)t}(1)$$

$$= e^{\left(\frac{-R}{L}\right)t} \cdot \left(1 - \frac{R \cdot t}{L}\right)$$

$$\frac{i_p(t)}{dt} = I_0\left[e^{\left(\frac{-R}{L}\right)t} \cdot \left(1 - \frac{R \cdot t}{L}\right)\right]$$ <---Its a mess in comparison so
we substitute this expression.

$$R I_0\left(t \cdot e^{\left(\frac{-R}{L}\right)t}\right) + L I_0\left[e^{\left(\frac{-R}{L}\right)t} \cdot \left(1 - \frac{R \cdot t}{L}\right)\right] = V_0 e^{\left(\frac{-R}{L}\right)t}$$

$$R I_0 \left(t \cdot e^{\left(\frac{-R}{L}\right) t} \right) + L I_0 \left(e^{\left(\frac{-R}{L}\right) t} \cdot \left(1 - \frac{R \cdot t}{L} \right) \right) = V_o e^{\left(\frac{-R}{L}\right) t}$$ Next expand it further.

$$R I_0 \left(t \cdot e^{\left(\frac{-R}{L}\right) t} \right) + L I_0 \left(e^{\left(\frac{-R}{L}\right) t} \right) - L I_0 \left(\frac{R \cdot t}{L} \cdot e^{\left(\frac{-R}{L}\right) t} \right) = V_o e^{\left(\frac{-R}{L}\right) t}$$

divide by e^(-R/L)t

$$R I_0 (t) + L I_0 - L I_0 \left(\frac{R \cdot t}{L} \right) = V_o$$

$$R I_0 t + L I_0 - I_0 R t = V_o$$

$$I_0 (Rt + L\_Rt) = V_o$$

$$I_0 (L) = V_o$$

$$I_0 = \frac{V_0}{L}$$ <--- As Schaums has it.

We worked on the forced response, the natural response is the same.

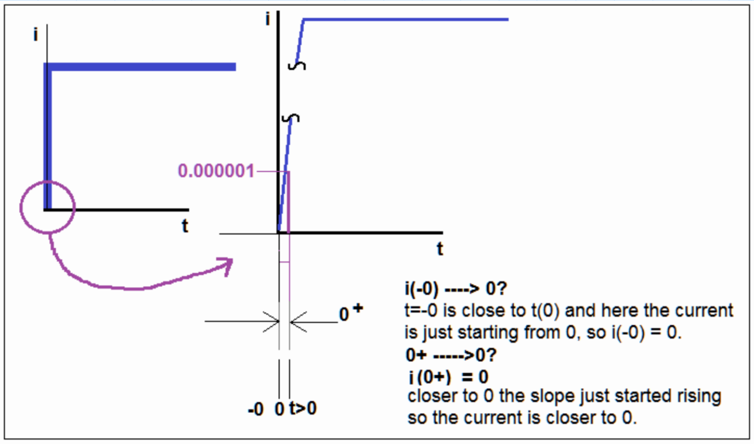
$$i_h(t) = A \cdot e^{\left(-\frac{R}{L}\right) t} \qquad i_p(t) = I_0 t e^{\left(\frac{-R}{L}\right) t}$$

$$i(t) = i_p(t) + i_h(t)$$

$$i(t) = (I_0 t + A) \cdot e^{\left(\frac{-R}{L}\right) t}$$ <---This is the complete solution under the new condition of the forcing function's exponent the same as the natural response (-R/L)t.

$$i(t) = \left(\frac{V_0 t}{L} + A \right) \cdot e^{\left(\frac{-R}{L}\right) t} \cdot u(t)$$ <---Substitute for Io, and place u(t)

Can we solve for A? We should have values of Vo, voltage peak, and L inductor value, and A we do not have an idea about. Would initial condition on time t<0, t=0, and t>0 do it? Instead we use -0, 0, and 0+ because the function is unit function on the source so the concentration is at point t=0. Remember, we can do initial condition because A is multplied to an exponential term e^(-R/L)t, the time t we take to the limit t approaches 0, so here t is around -0,0, and 0+.



Again, this time trying to show the intial condition on -0 is 0.

The theory on this began with circuit was at rest for t<0. So we know i(-0) = 0.

$$i(-0) = i(0) = 0 = \left(\frac{V_0(0)}{L} + A\right) \cdot e^{\left(\frac{-R}{L}\right) 0}$$

$$0 = (0 + A) \cdot 1$$

$$A = 0$$

Now a somewhat improved with inital condition i(-0) = 0, the expression for i(t) is updated for A=0.

$$i(t) = \left(I_0 t + 0\right) \cdot e^{\left(\frac{-R}{L}\right) t}$$

$$i(t) = I_0 t \cdot e^{\left(\frac{-R}{L}\right) t} \cdot u(t)$$

Expression we were seeking--->

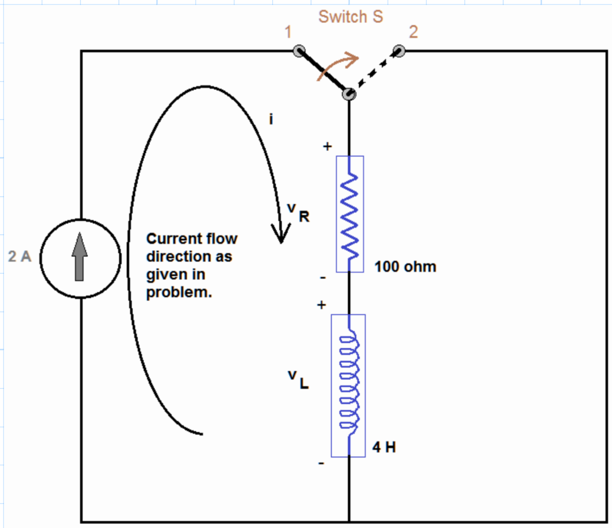
$$i(t) = \left(\frac{V_0}{L}\right) t \cdot e^{\left(\frac{-R}{L}\right) t} \cdot u(t)$$

Next RL circuit solved problems/examples.

Using the equations just achieved.

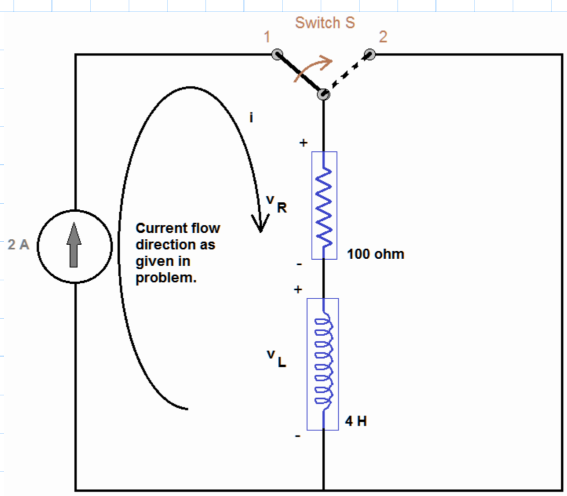
Solved Problem 7.4 and 7.5 RL Circuit.

7.4: Switch in the RL circuit shown below (left) is moved from position 1 to 2 at t=0. Obtain vR and vL with polarities as indicated.



Solution:

Circuit above has a constant current source.
The circuit to the right shows the current flow direction when switch is in position 1.
The switch when it moves to position 2, the current direction and flow is in the right side circuit. Flowing out of the the L positive polarity. +ve convention.
Figure below shows the current direction in the left and right, position 1 and 2 of switch; currents are in opposite directions.
Left circuit energises the inductor L, then goes open and closes to position 2, now the inductor L circuit discharges (passes transient current i) into the circuit in the right side.
We want to solve for vR and vL.



Current source is constant at 2A.

Time t for the circuit on the left is t<0. It energised inductor fully, then switched-off.
Immediately next at time t=0 the switch closed contact to position 2.
Now the inductor is discharging current into the RL circuit on right, in time t>0.

So we need a decaying expression for current in the circuit on the right side for t>0.

$i = I_0 \cdot e^{\frac{-Rt}{L}}$ <---Inductor current decaying since no source in circuit.

$t < 0 \quad I_0 = 2$ A.

$R := 100 \qquad L := 4 \qquad \frac{R}{L} = 25$

$i = 2 \cdot e^{-25 \cdot t}$ A. This is the current in the left side circuit.

$v_R = R \cdot i$
$= 100 \cdot 2 \cdot e^{-25 \cdot t}$
$= 200 \cdot e^{-25 \cdot t}$ V. Answer.

Right side circuit inductor fully energised and now releasing crurrent into the resistor. Inductor is the current source, and the circuit voltage can be measured by the resistor voltage. The resistor voltage in the expression above is in relation to time t in the exponent term. Eventually this voltage vR is going to settle to 0 because of the inductor behaviour.

We have vR, with respect to the polarity in the right side circuit.
vL is the opposite polarity of vR in the right circuit.
vL is the voltage source.

$v_L = -vR$

$= -200 \cdot e^{-25 \cdot t}$ V. Answer.

Continuing with Solved Problem 7.5 for the same circuit and component values.
For the transient problem in 7.4 obtain pR and pL.

$pR = v \cdot i = \left(200 \cdot e^{-25 \cdot t}\right) \cdot \left(2 \cdot e^{-25 \cdot t}\right) = 400 \cdot e^{-50 \cdot t}$ W. Answer.

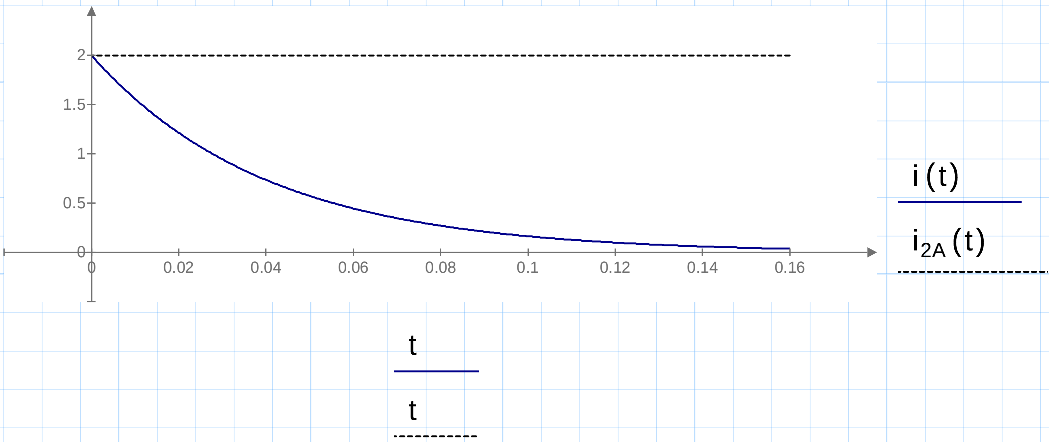
$pL = v \cdot i = \left(-200 \cdot e^{-25 \cdot t}\right) \cdot \left(2 \cdot e^{-25 \cdot t}\right) = -400 \cdot e^{-50 \cdot t}$ W. Answer.

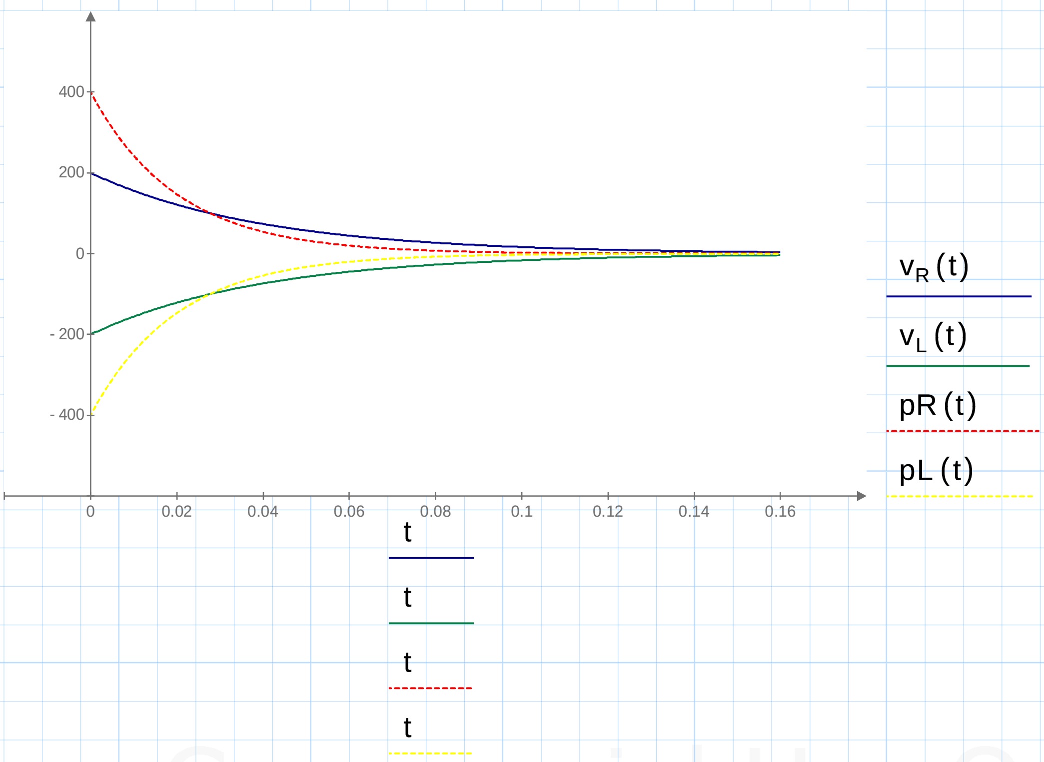
Discussion: Power is -ve for the inductor this is correct since its sending power out to the resistor, resistors are known for absorbing power. So, the signs are correct in that perspective. We do NOT find the power at a specific point of time, we have a general expression for t>0. These can be plotted, provided next page.

$\tau_{RL} := \frac{L}{R} = 0.04$ s. Circuit time constant. **clear** (t)

$i(t) := 2 \cdot e^{-25 \cdot t}$ $\quad v_R(t) := 200 \cdot e^{-25 \cdot t}$ $\quad v_L(t) := -200 \cdot e^{-25 \cdot t}$

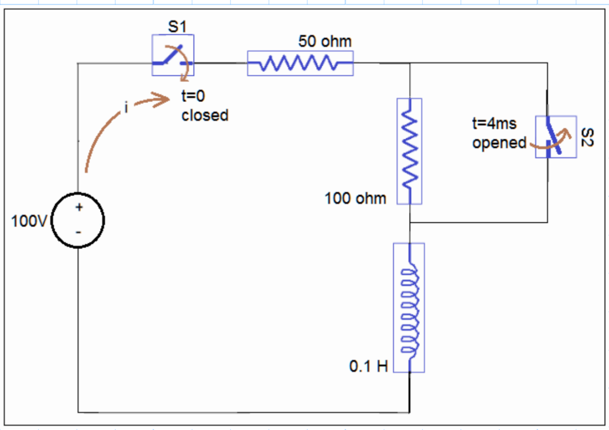
$i_{2A}(t) := 2$ $\quad pR(t) := 400 \cdot e^{-50 \cdot t}$ $\quad pL(t) := -400 \cdot e^{-50 \cdot t}$





Solved Problem 7.14 (RL circuit):

The switch S1 is closed at t=0. Switch S2 is opened at t = 4 ms.
Obtain i for t>0.



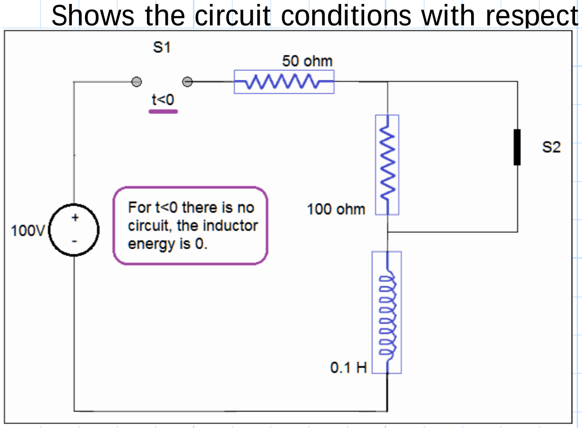
$R1 := 50$ ohms

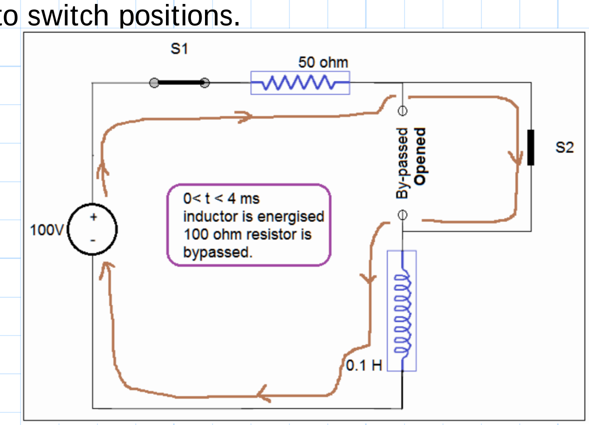
$R2 := 100$ ohms

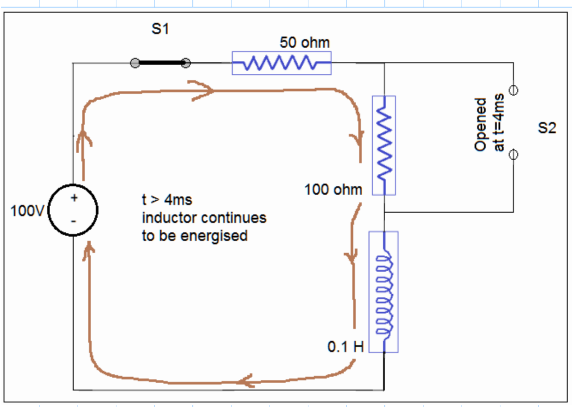
$L := 0.1$ H.

Solution:

Figures below from left to right then to bottom row.
Shows the circuit conditions with respect to switch positions.







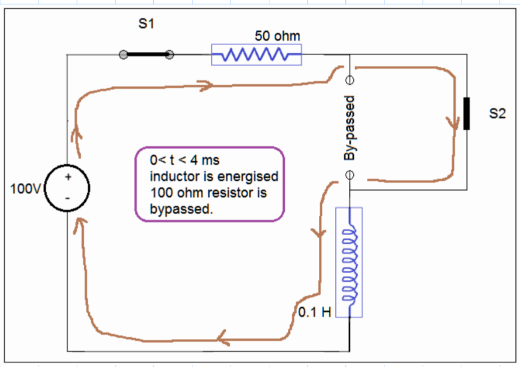
Note 1: Inductor terminal points are brought together, shorted, no resistance seen across them. Under this condition, terminals brought to one point, obviously we cannot take the voltage of one point to mean the voltage across the inductor. But from that point to a reference point in the circuit there is voltage measured. This I am saying so you don't fall for a trap thinking there is no voltage but there is current. That is wrong. Shorted also says that its more to a pure conductor with almost no resistance. Now then if no resistance no V=IR, but in ac that resistance is called impedance jwC, which is in the ac RLC analysis. Back to its closer to a pure conductor with low resistance.

t<0:

Maybe we get ahead this time, this by making the statement i=0 for t<0.
the switch is closed at t=0. We were not given any ifnromation on the circuit before t=0. The inductor does not hold its energy forever its dissipated somewhere over time.

$i(t<0) \;=\; 0$

0 < t < 4 ms:



Circuit resistance:

$R_{04} := 50$

$L := 0.1$

$\tau_{04} := \left(\frac{L}{R_{04}}\right) = 0.002$ s time constant.

$\tau_{04} \cdot 1000 = 2$ <--- in ms.

Inductor is a short circuit to dc current. *Voltage exist over the inductor its not a matter where the inductor +ve and -ve points are shorted, current passes thru, it does not have voltage, it does have voltage that can be measured.* See note 1 on previous page.
We may calculate the dc current in the circuit.

$V_s := 100 \qquad i_{04} = \frac{V_s}{R_{04}} = 2$

This 2 A fits for Io. Not saying the shorted inductor is not behaving like an inductor, because at this shorted point, we see the current characteristic based on the inductor current expression using the exponential term.

Current i starts from 0 and rises to 4 ms.

$i \;=\; \left(\frac{V}{R}\right) \cdot \left(1 - e^{\frac{t}{\tau_{04}}}\right)$ tau_04 = 0.002s or 2 ms

$i \;=\; 2 \cdot \left(1 - e^{\frac{t}{2}}\right)$ A. 2 is in ms for tau, (0<= t <=4 ms) Answer.

Since we continue with the circuit pass t=4 ms,
we need to know the current at 4 ms.

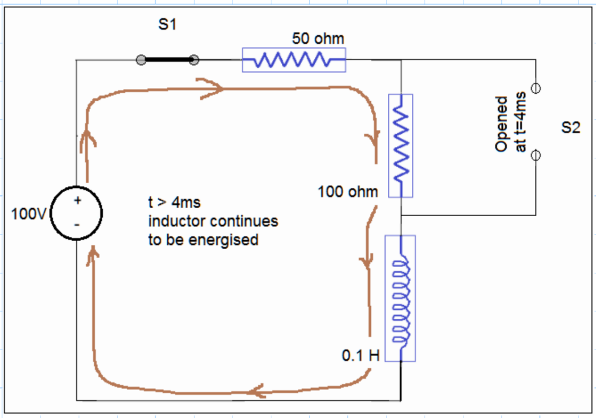
$i(0.004) \;=\; 2 \cdot \left(1 - e^{-\left(\frac{0.004}{0.002}\right)}\right) = 1.729$ A. Answer.

Next for the time starting at t = 4 ms. This is when switch 2 is opened.
The time of concern is t=4 and t>4 ms. Circuit changed when S2 opened.

t >= 4 ms:

Two conditions apply: 1. current from the old circuit with S2 closed at t=4 ms
2. current from the new circuit with S2 opened at t=>4ms.

When S2 closed the voltage supply energised the inductor with resistance 100 ohm, when S2 opened the resistance increased to 150 ohm, and the voltage source continued to energise the inductor. So, maybe this need to be captured in time t>=4 ms.



Circuit resistance:

$R_{4\_5} := 50 + 100 = 150$

$L := 0.1$

$\tau_{\_eql\_grt\_4} := \left(\frac{L}{R_{4\_5}}\right) = 0.000667$

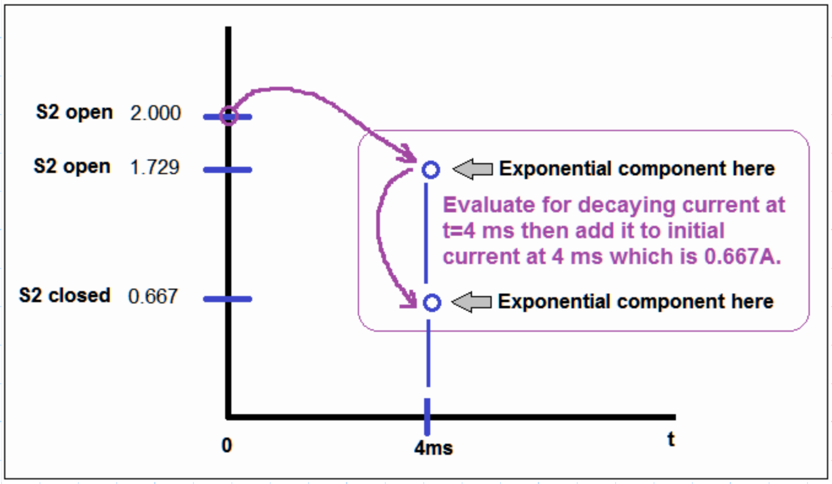
Subscript _eql_grt_4 stands for time t>=4

$= \left(\frac{2}{3}\right) \cdot 10^{-3}$ ms - in fraction form.

Current in the new circuit:

$V_s := 100$ $\quad i_{4\_5} = \frac{V_s}{R_{4\_5}} = 0.667$ A. Answer.

Our current dropped in the new circuit when S2 opened because of increase in current.



<----This looks like the plan, you may explain it better. At 4 ms we have 2 conditions based on switch S2.

$$i = \left(\frac{V}{R}\right)\cdot\left(1-e^{\frac{t}{\tau_{04}}}\right)$$

<---(Initial - the decaying exponential form of current):
This form **'initial - final'**.
Our circuit is past starting point, where this did not so much matter in terms of position in time, now we are looking at time t=4ms as the starting point.
So our initial current would be from when S2 was open.
Starts at 1.729 and decays to 0.667A.
Our time expression for the exponent need to be adjusted for t0=4, so we use the form **-(t-t0)** which here it should be -(t - 4) ---> e-^(t - t0).

$$i_{\_eql\_grt\_4} = (1.729-0.667)\cdot\left(e^{-\frac{(t-4)}{\frac{2}{3}}}\right)+0.667$$

$$= (1.062)\cdot\left(e^{-(t-4)\left(\frac{3}{2}\right)}\right)+0.667$$

$$= (1.062)\cdot\left(e^{-\left(\frac{3t}{2}\right)}\cdot e^{6}\right)+0.667$$ <--- splitting up the exponent term.

$1.062\cdot e^{6} = 428.441$ numerical evaluation.

$$= 428.441\cdot e^{-\left(\frac{3t}{2}\right)}+0.667$$ A. time t in ms, this is for t>=4ms. Answer.
Huge spike in the current.
The time constant in the S2 open condition is much smaller in comparison to S2 closed.

That was how Schaums solved it.
Try to do a plot in the next page.
We have 2 time constants, the smaller of which may decide on the time t axis.

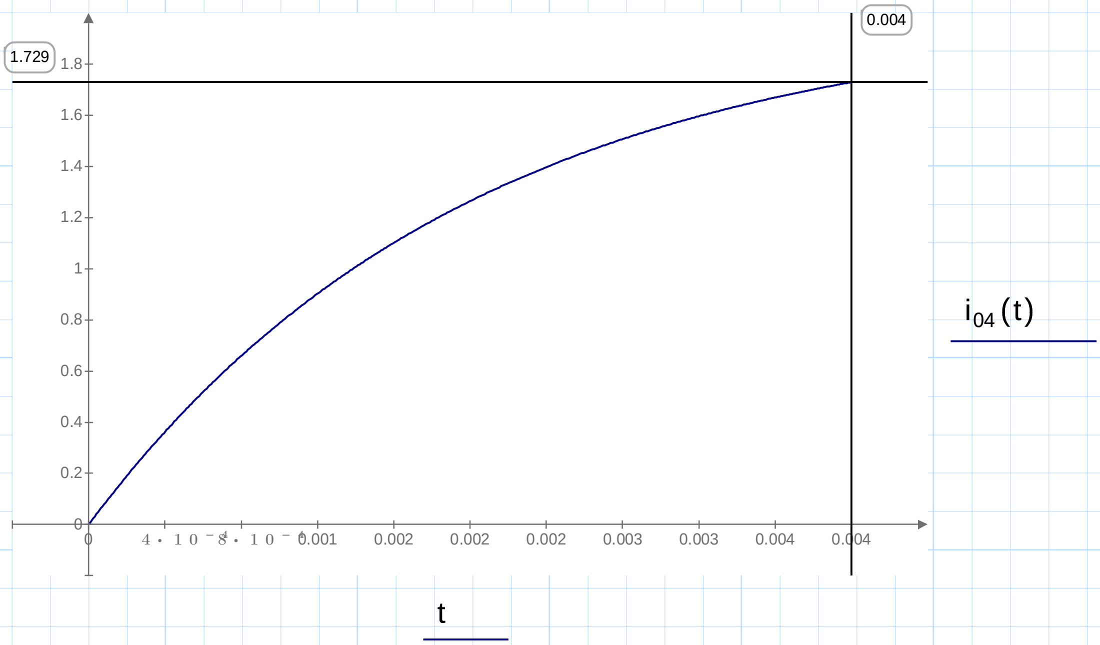
$\tau_{04} = 0.002$ $\tau_{\_eql\_grt\_4} = 0.000667$

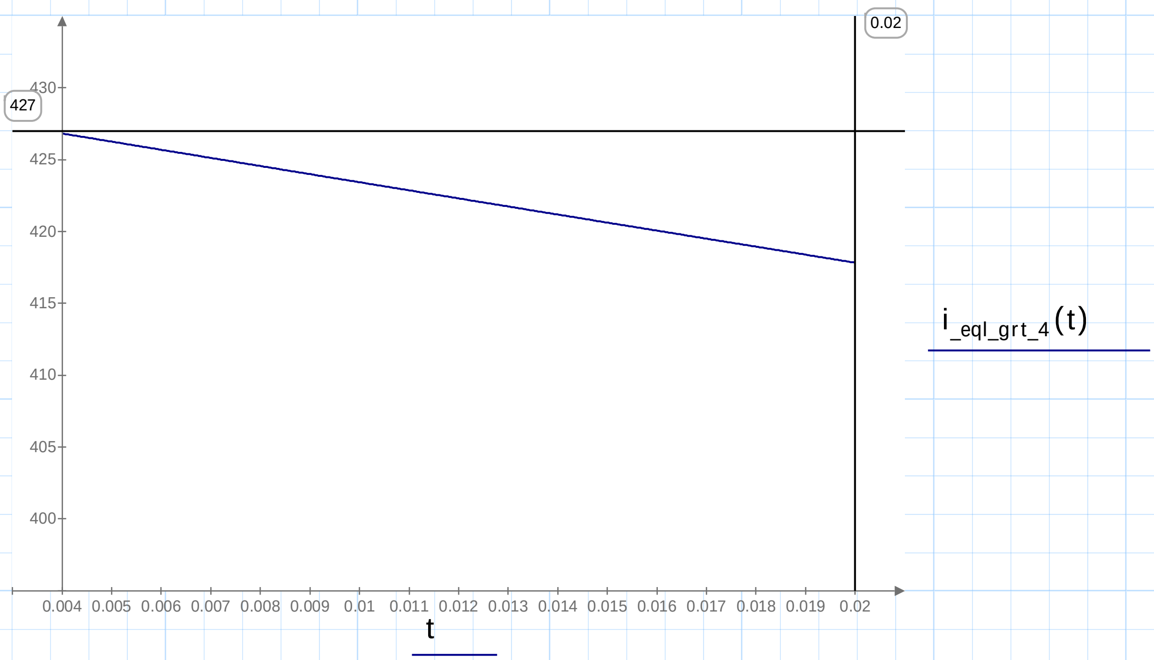
clear (t)

$$i_{04}(t) := 2\cdot\left(1-e^{-\left(\frac{t}{0.002}\right)}\right)$$

$$i_{\_eql\_grt\_4}(t) := 428.441\cdot e^{-(1.333\cdot t)}+0.667$$

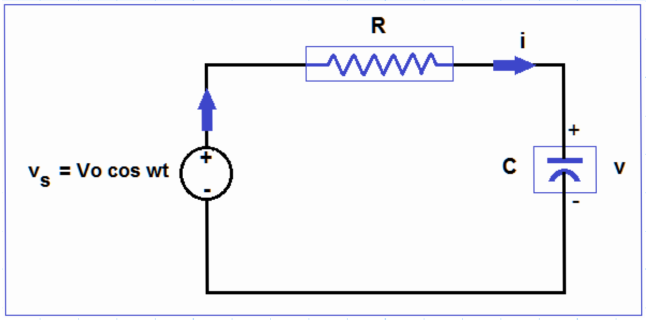
First plot is normal, peaks to 1.729A at 4 ms.
Second plot picks-up at 4 ms but starts at aproximately 429 A with a decay.
Huge spike from 1.729 to 427A.



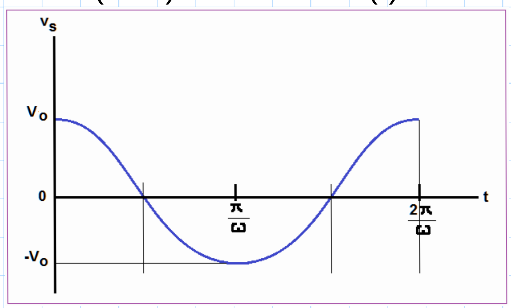


7.14: Response of RC and RL Circuits to Sudden Sinusoidal Excitations.

Because its sinusoidal the first thing that comes to mind is this is that important one.
Yes, for the power industry.
Could there maybe limitations to it? Most everyting has conditions so we see.
We keep to the series RL circuit as Schaums does, your text book may have RC.



The forcing function will be Vo cos *wt*.
KVL:
Ri + L(di/dt) = Vo cos *wt*.
Next with the u(t) for the condition t>=0.
Ri + L(di/dt) = Vo cos *wt* u(t).



$$Ri + L\left(\frac{di}{dt}\right) = V_0 \cos(\omega t) \cdot u(t)$$

The complete solution will be similar: $i(t) = i_h + i_p$

Where: $i_h(t) = A \cdot e^{-\left(\frac{R}{L}\right)t}$

$i_p(t) = I_0 \cdot \cos(\omega t - \theta)$ <---Current response we show the phase angle theta.

Discussion:

We are given the solutions, these come from the differential equations course work. They have a book full, so over the history of DE there a specific forms used in specific industry environments. We are not going to start a DE course here but most you may have seen some of its basics. Laplace Transform comes from the? DE course work.

I am having a little problem here why i_h(t) is not in the sinusoidal form rather exponential. This is where I said may be some mis-perfection here.

In a sudden surge case, sinusoidal will not spike up or down like exponential, the shape of sinusoidal is smoother in comparison. So the natural response of the solution here takes the exponential form where there can be a steep slope in a short time.

Cosine and Sine revolve around 0 1 0 -1 (cosine), and 1 0 -1 0 (sine). The amplitude A or here we have placed Io for the magnitude of the curve, the sinusoids curve has limitations compared to the steep spike of exponential.

Lets insert i_p(t) and di_p(t)/dt in the KVL expression to solve for Io.

$$i_p(t) = I_0 \cdot \cos(\omega t - \theta)$$

$$\frac{di_p}{dt} = ?$$

$$u = \omega t \qquad \frac{du}{dt} = \omega$$

$$y = \cos(u) \qquad \frac{dy}{du} = -\sin(u)$$

$$\left(\frac{du}{dt}\right)\left(\frac{dy}{du}\right) = \frac{dy}{dt} = \omega \cdot -\sin(u) = -\omega \cdot \sin(\omega t)$$

$$\frac{di_p}{dt} = -\omega \cdot \sin(\omega t)$$

$$R\left(I_0 \cdot \cos(\omega t - \theta)\right) + L\left(-\omega \cdot \sin(\omega t)\right) = V_0 \cos(\omega t)$$

$$R I_0 \cos(\omega t) - R I_0 \theta - \omega \cdot L \cdot \sin(\omega t) = V_0 \cos(\omega t)$$ <---Here.

STOP---> NOW go to Page 39 of 'Part 1B Input Output Waveform Circuiting Prerequisites To Laplace Transforms Electric Circuits'.
? Almost There!...may need changes but should be same, results are same here.
The derivation for Io and Theta were provided there.

$$I_0 = \frac{V_0}{\sqrt{R^2 + L^2 \cdot \omega^2}}$$

$$\theta = \tan^{-1}\left(\frac{L\,\omega}{R}\right)$$

Now the i_p(t) form of solution after substitution takes:

$$i_p(t) = I_0 \cdot \cos(\omega t - \theta) = \left(\frac{V_0}{\sqrt{R^2 + L^2 \cdot \omega^2}}\right) \cos\left(\omega t - \tan^{-1}\left(\frac{L\,\omega}{R}\right)\right)$$

$$i(t) = i_h + i_p$$

$$i(t) = A \cdot e^{-\left(\frac{R}{L}\right) t} + \left(\frac{V_0}{\sqrt{R^2 + L^2 \cdot \omega^2}}\right) \cos\left(\omega t - \tan^{-1}\left(\frac{L\,\omega}{R}\right)\right)$$ Next solve for A.

Next for the initial condition, i(0+) = 0.
0+ is just past t=0, and here the response of i(t) is near 0 or approximately 0.

$$i(0+') = A \cdot e^{-\left(\frac{R}{L}\right) t} + I_0 \cdot \cos(\omega t - \theta) = 0$$

$$A \cdot e^{-\left(\frac{R}{L}\right) 0} + I_0 \cdot \cos(\omega 0 - \theta) = 0$$

$$A + I_0 \cdot \cos(-\theta) = 0$$

Remember how cos(-theta) used to get fixed? See below.

$\sin(-30) = 56.61$ **deg** $\quad$ $\cos(-30) = 8.838$ **deg**
$\sin(30) = -56.61$ **deg** $\quad$ $\cos(30) = 8.838$ **deg**

Works for cosine on the right above not sine, cos(-theta) becomes cos(theta).

$$A + I_0 \cdot \cos(\theta) = 0$$

$$A = -I_0 \cdot \cos(\theta)$$

If that is what A equals in the form of the i_p(t) which is sinusoidal, then the final or complete solution changes to:

$$i(t) = -I_0 \cdot \cos(\theta) \cdot e^{-\left(\frac{R}{L}\right) t} + I_0 \cdot \cos(\omega t - \theta)$$

$$i(t) = I_0 \cdot \cos(\omega t - \theta) - I_0 \cdot \cos(\theta) \cdot e^{-\left(\frac{R}{L}\right) t}$$

$$i(t) = I_0 \cdot \left(\cos(\omega t - \theta) - \cos(\theta) \cdot e^{-\left(\frac{R}{L}\right) t}\right)$$

<---- Complete response.
Thru the initial condition, solving for A.
For t=0, the exponential term equal 1.
The sinusoidal expression in the exponential term.

Substituting the previous expressions provides i(t) below:

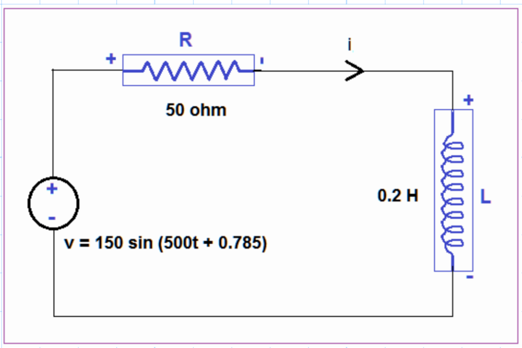
$$i(t) = \left(\frac{V_0}{\sqrt{R^2 + L^2 \cdot \omega^2}}\right) \cos\left(\omega t - \tan^{-1}\left(\frac{L\,\omega}{R}\right)\right) - I_0 \cdot \cos(\theta) \cdot e^{-\left(\frac{R}{L}\right) t}$$

Solved Problem 7.19 - Sinusoidal RL circuit:

A series RL circuit with R = 50 ohm and L = 0.2 H,
has a sinusoidal voltage v = 150 sin (500t + 0.785) V,
applied at t=0.

Obtain the current for t>0.

Solution:



t>0:
KVL: Ri + L (di/dt) = v

$$v = 150 \cdot \sin(500 \cdot t + 0.785) \quad V.$$

$$50\,(i) + 0.2\left(\frac{di}{dt}\right) = v$$

$$50\,(i) + \left(\frac{1}{5}\right)\left(\frac{di}{dt}\right) = v$$

$$250\,(i) + \left(\frac{di}{dt}\right) = 5 \cdot v \quad \text{Multiply by 5.}$$

$$250\,(i) + \left(\frac{di}{dt}\right) = 750 \cdot \sin(500 \cdot t + 0.785) \quad V.$$

$$\left(\frac{di}{dt}\right) + 250\,(i) = 750 \cdot \sin(500 \cdot t + 0.785) \quad V. \quad \text{Re-arranged.}$$

This expression or equation above is a differential equation.
It has 2 parts to its solution; natural and forced.
Here the natural response, i_c(t) OR i_h(t), is called complementary as usually they are in differential equation course, this when the RHS = zero.
The other solution the forced response, RHS equal the voltage expression above. Its called the particular solution or forced solution; i_p(t).

$$i(t) = i_c(t) + i_p(t)$$

Next, lets try to solve the differential equation.
USING the notes we had in responsce of RL circuit to sudden sinusoidal excitation.

$$i_h(t) = A \cdot e^{-\left(\frac{R}{L}\right)t} \quad \text{<---natural response form}$$

$$i_h(t) = A \cdot e^{-\left(\frac{50}{0.2}\right) t}$$

$$= A \cdot e^{-250\,t}$$

$$\frac{d(i\_h(t))}{dt} = -250\ A \cdot e^{-250\,t}$$

$$\left(\frac{di}{dt}\right) + 250\ (i) = 0$$ RHS = 0; for natural response function.

$$-250\ A \cdot e^{-250\,t} + 250\ \left(A \cdot e^{-250\,t}\right) = 0$$ Proved it equal zero, but no solution for i_c(t) or i_h(t) till we solve for A.

Initial condition t(0+) = 0

$$-250\ A \cdot e^{0} + 250\ \left(A \cdot e^{0}\right) = 0$$

$$-250\ A + 250\ A = 0$$

$$250\ A = 250\ A$$

That can only happen when A =1? When A=2, thats 500 = 500. So what is A?
So for the present time A is not determined yet.

$$i_h(t) = k \cdot e^{-250\,t}$$ <--- Schaums shows k instead of A, k being an unknown constant or real number.

Lets move on to solve for the forced response; RHS is the voltage function.

$$v = 150 \cdot \sin(500 \cdot t + 0.785)$$ V. <--- With this forcing function the form of solution for the forced response needs to fit/ match the forcing function.

$$i_p(t) = A \cdot \cos(500\ t) + B \cdot \sin(500\ t)$$ <---Schaums more suitable form for i_p(t). This is the way of the DE coursework. I/We have to apply our intuition, guess work, build skills, etc. Not easy.

$$\frac{di_p(t)}{dt} = A \cdot \cos(500\ t) + B \cdot \sin(500\ t)$$

$$= -500\ A \cdot \sin(500\ t) + 500\ B \cdot \cos(500\ t)$$

Next we substitute into the KVL equation.

$$\left(\frac{di}{dt}\right) + 250\,(i) = 750 \cdot \sin(500 \cdot t + 0.785) \quad \text{V.}$$ <---Expression early part of the solution.

$$(-500\,A \cdot \sin(500\,t) + 500\,B \cdot \cos(500\,t)) + 250\,(A \cdot \cos(500\,t) + B \cdot \sin(500\,t))$$
$$= 750 \cdot \sin(500 \cdot t + 0.785)$$

RHS rework sum of sine term:

$$\sin(500 \cdot t + 0.785) = \sin(500\,t)\,\cos(0.785) + \cos(500\,t) \cdot \sin(0.785)$$

$$\cos(0.785) = 0.707 \qquad \sin(0.785) = 0.707$$

$$= 0.707 \cdot \sin(500\,t) + (0.707) \cdot \cos(500\,t)$$

$$750 \cdot \sin(500 \cdot t + 0.785) = 750 \cdot (0.707 \cdot \sin(500\,t) + (0.707) \cdot \cos(500\,t))$$
$$= 530.3\,\sin(500\,t) + 530.3\,\cos(500\,t)$$
$$= 530.3\,(\sin(500\,t) + \cos(500\,t))$$

Now back to our expression with the updated RHS above.

$$(-500\,A \cdot \sin(500\,t) + 500\,B \cdot \cos(500\,t)) + 250\,A \cdot \cos(500\,t) + 250 \cdot B \cdot \sin(500\,t)$$
$$= 530.3\,(\sin(500\,t) + \cos(500\,t))$$

Re-arranging for like-terms; RHS split for sine and cosine term:

$$-500\,A \cdot \sin(500\,t) + 250\,B \cdot \sin(500\,t) = 530.3\,(\sin(500\,t)) \qquad \text{Equation 1}$$
$$250 \cdot A \cdot \cos(500\,t) + 500\,B \cdot \cos(500\,t) = 530.3\,(\cos(500\,t)) \qquad \text{Equation 2}$$

Divide equation 1 by sin500t, and equation 2 by cos500t.

$$-500\,A + 250\,B = 530.3$$
$$250\,A + 500\,B = 530.3$$

Next we solve using matrix.

$$LHS := \begin{bmatrix} -500 & 250 \\ 250 & 500 \end{bmatrix} \qquad RHS := \begin{bmatrix} 530.3 \\ 530.3 \end{bmatrix} \qquad LHS^{-1} = \begin{bmatrix} -0.002 & 8 \cdot 10^{-4} \\ 8 \cdot 10^{-4} & 0.002 \end{bmatrix}$$

$$(LHS^{-1}) \cdot RHS = \begin{bmatrix} -0.42424 \\ 1.27272 \end{bmatrix} \qquad A = -0.4242 \qquad B = 1.2727$$

Substitute these values of A and B into the equation for i_p(t):

$$i_p(t) = A \cdot \cos(500\,t) + B \cdot \sin(500\,t)$$

$A = -0.4242 \qquad B = 1.2727$

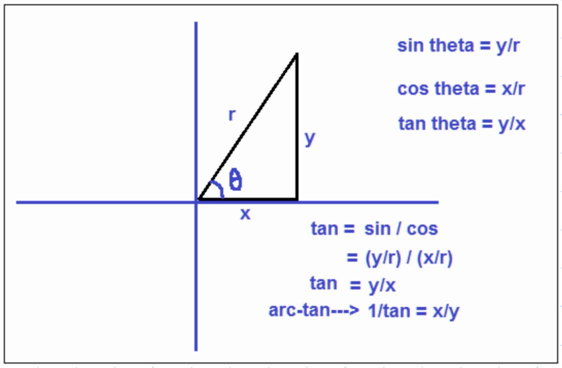
$$i_p(t) = -0.424 \cdot \cos(500\,t) + 1.273 \cdot \sin(500\,t)$$

The expression above can now be further condensed, made useful, in terms of 'electrical engineering applications', using the Pythoras theorem.

Cosine term will be the x-axis length-vector, and the sine term the y-axis length-vector. The magnitude of this will provide the vector length.

$$r_{xy} = \sqrt{\left(1.273^2\right) + (-0.423)^2} = 1.341$$

The angle it makes with the x-axis:
Sine: y/r and Cosine x/r,
so Sine / Cosine = (y/r) / (x/r)
= (y/r) x (r/x) = y/x = tan



$$\theta = \operatorname{atan}\left(\frac{-0.423}{1.273}\right) = -0.321$$

Phase angle = -0.321 in radian.

$$i_p(t) := 1.341 \cdot \sin(500\,t - 0.321) \quad A.$$

The 500t in the sin term remains because thats the fundamental frequency, w = 2 Pi f, same as the voltage which was given at the start of the problem.

Returning to the complete solution:

$$i(t) = i_h(t) + i_p(t)$$

$$= k \cdot e^{-250\,t} + 1.341 \cdot \sin(500\,t - 0.321)$$

To solve for k in the exponent term we may use the initial condition, now since we have a full expression for i(t).
i(0+) = 0 = i(0).

$= \quad i_h(t) + i_p(t)$

$i(0) \quad = \quad k \cdot e^{-250 \cdot 0} + 1.341 \cdot \sin(500\ (0) - 0.321)$

$0 \quad = \quad k \cdot e^{-0} + 1.341 \cdot \sin(-0.321)$

$0 \quad = \quad k + 1.341 \cdot \sin(-0.321)$

$1.341 \cdot \sin(-0.321) = -0.423$

$0 \quad = \quad k - 0.423$

$k \quad = \quad 0.423$

Magic! Maybe, so thru the initial condition we were able to solve for k.
Schaums value for k = 0.425, thats because of the decimal values carried thru the solution, all answers matched. So now we plug-in k in the expression, for the complete solution.

$i(t) \quad = \quad i_h(t) + i_p(t)$ OR $i_c(t) + i_p(t)$

$i(t) := 0.423 \cdot e^{-250\ t} + 1.341 \cdot \sin(500\ t - 0.321)$ A. Answer.

Next plot the forcing function (voltage) and the response (current).

$R := 50 \qquad L := 0.2 \qquad \tau_{RL} := \frac{L}{R} = 0.004$ s

$\tau_{RL2} := 2 \cdot \tau_{RL} = 0.008 \qquad \tau_{RL3} := 3 \cdot \tau_{RL} = 0.012 \qquad \tau_{RL4} := 4 \cdot \tau_{RL} = 0.016$

$\tau_{RL5} := 5 \cdot \tau_{RL} = 0.02$

What the voltage and current is expected at time t=0.

$v(0) \quad = \quad 150 \cdot \sin(500 \cdot 0 + 0.785) = 106.024$

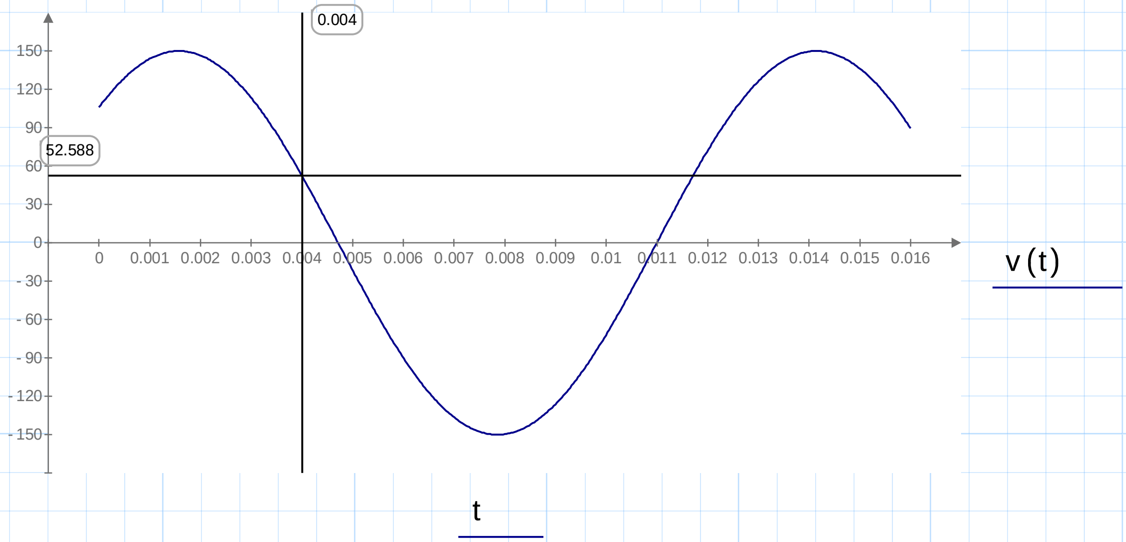
$i(0) \quad = \quad 0.423 \cdot e^{0} + 1.341 \cdot \sin(500\ (0) - 0.321) = -0.000106$ Good as zero.

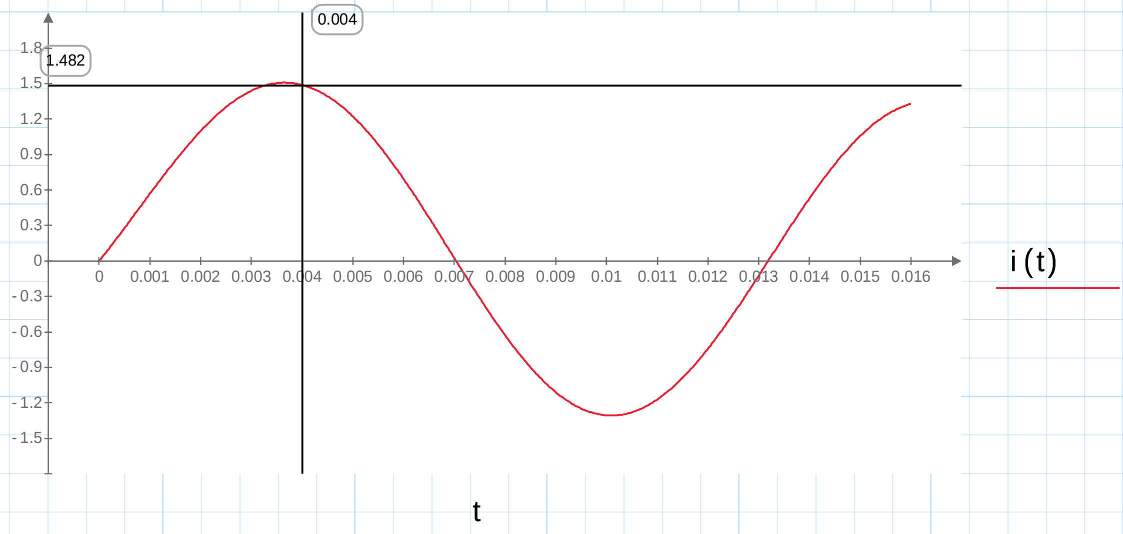
clear (t)

$v(t) := 150 \cdot \sin(500 \cdot t + 0.785)$

$i(t) := 0.423 \cdot e^{-250\ t} + 1.341 \cdot \sin(500\ t - 0.321)$

Plots of v(t) and i(t):



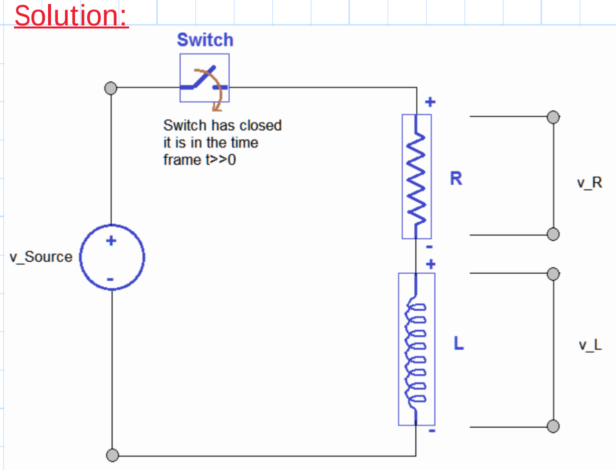


Looks like the voltage v appears first at t=0 before current because current is 0. Maybe said current is lagging. Verify with your local engineer. Input source is a sinusoidal source, the output or more suitable the word 'response' here which is the current i(t) also shows a sinusoidal waveform with an exponential term added-in for the complete expression. You may, should, say the plots help in understanding the solution. *"What good is the math, when we dont understand the math results with respect to the circuit response or the circuits operation" - Karl Bogha. Maybe in a textbook soon. So we say dont walk away from the final answer till we have a better understanding of what is going on with the circuit. Review the plots, note the component sizes, the amplitudes on the plots, the waveform, current and voltage at nodes,......much there including the time contants.*

REVISIT Example 6.24 Damped Sinusoid: (From Part 1B - page 31-32):
Relevant here for the sinusiod response - Full Completion.

The current i = Io_e^-at_cos(wt).
This current passes through a series RL circuit.
a). Find v_RL, voltage across the resistor and inductor combination.
b). Compute v_RL for Io=3A, a=2, w=40 rad/s, R=5ohm, and L=0.1H.
Sketch i as a function of time.

Solution:



Series RL circuit.
Asumption here the switch was closed and the circuit is in analysed in t>0. The voltage source is not removed from the circuit so that tells me the analysis is in time t>0, and also no initial conditions were given.

$$i = I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$$

a).

$$vR = R \cdot i = R \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$$

$$vL = L \cdot \left(\frac{di}{dt}\right)$$

$$\left(\frac{di}{dt}\right) = \left(-a \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)\right) + \left(I_0 \cdot e^{-a \cdot t} \cdot \omega \cdot -\sin(\omega \cdot t)\right)$$

$$= -I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

$$vL = -L \cdot I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

Voltage across R and L combined ?

$$vRL = vR + vL$$

$$= R \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t) - L \cdot I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

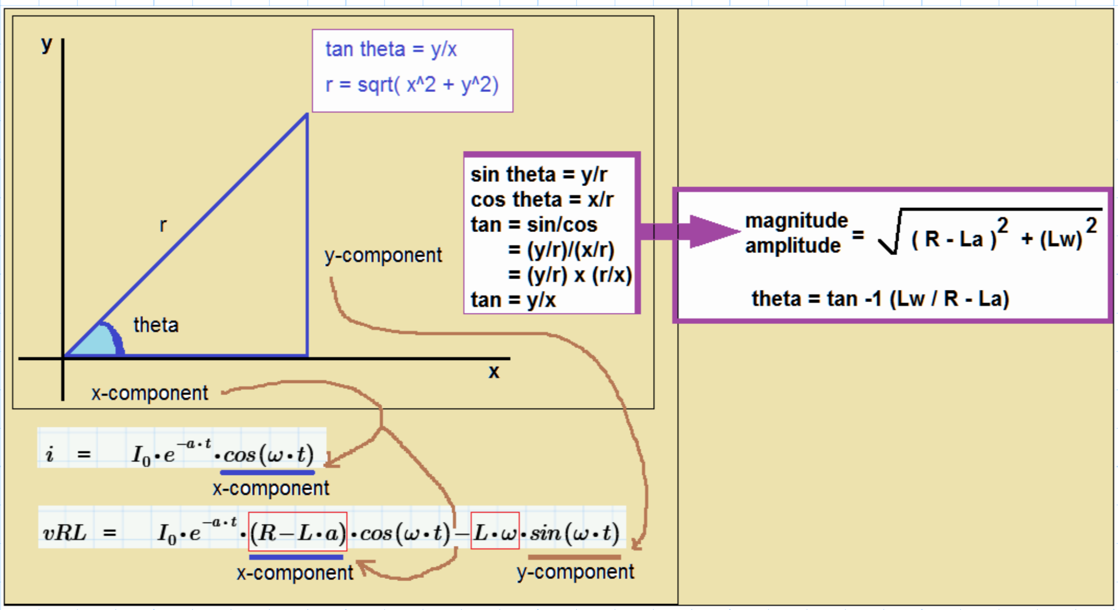
$$vRL = I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$ Answer.

Next we place both the current and voltage across RL, to study the mathematical form they are in. We just seen one sinusoidal forcing function and response in solved problem 7.19. Its steps may repeat for another problem, it may provide the direction on how to proceed with this solution in repsect to the x and y component.

$$i = I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$$ <--- sinusoidal form for current

$$vRL = I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$ <---similar response form, here a combination of cosine and sine.

Coming from Part 1, it may be pre-mature to jump to conclusion that the solution was as it was readily available, in the same steps in the previous example. After example 6.24 in Part 1 (Waveform) we did some derivation work on the expressions, which revealed the triangle/ Pythores theorem was applicable. Now, having completed section 7.14's example, we may proceed to apply the similar steps here. It may not work hopefully it will for that damped sinusoid solution.



Applying the coefficients for the solution - as shown in figure above.

We use the coefficients of the cos and sin terms in the expression vRL, so solve for the voltage amplitude Vo and phase angle theta.

Loosely speaking not directly related to this solution, in example 7.19, Schaums used the phrase *'The method of undetermined coefficients for obtaining i_p(t).....'*
So pull out your maths book and look further on the relevance of coefficients.
Remember in example 7.6 we used coefficients, and that was a small complex circuit. The next step uses the information provided in the last figure.

$$V_0 = I_0 \cdot \sqrt{(R - L \cdot a)^2 + L^2 \cdot \omega^2}$$

$$\theta = \operatorname{atan}\left(\frac{L \cdot \omega}{(R - L \cdot a)}\right)$$

Satisfied so far? Some consolation does come from the previous example which had similar steps. Continuing with the remaining solution as done in Part 1 B.

$I_0 := 3 \quad a := 2 \quad \omega := 40 \quad R := 5 \quad L := 0.1$

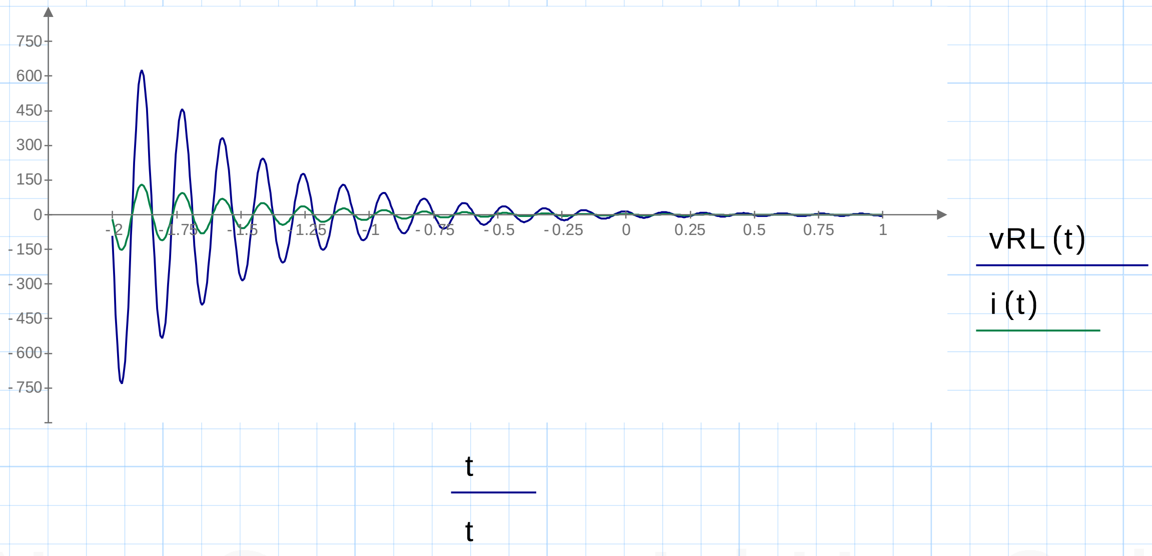
clear (t)

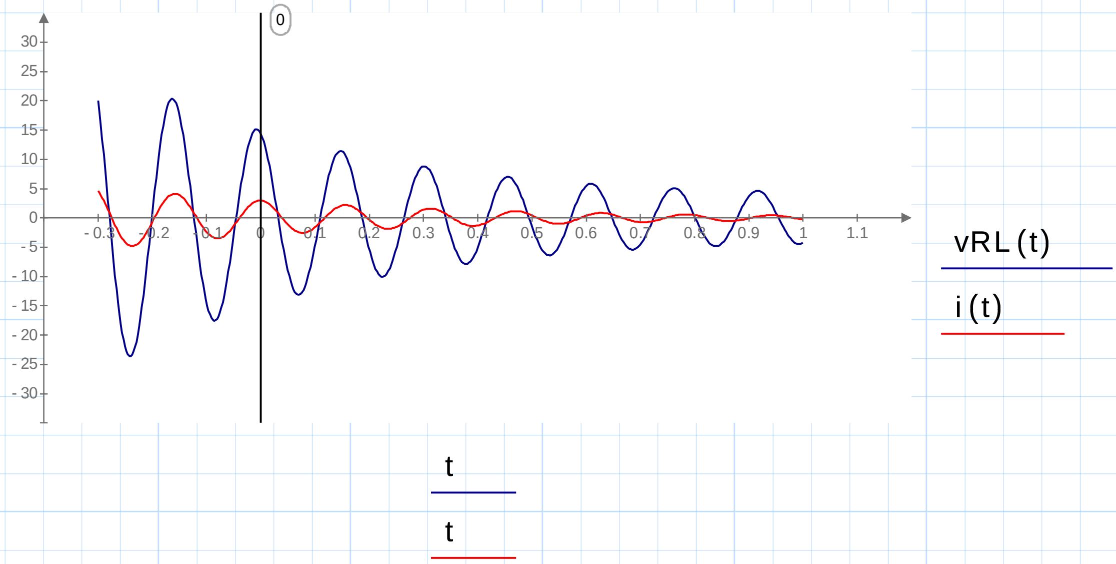
Substitute in for vRL(t) and i(t):

$$vRL(t) := I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$

$i(t) := I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$ <--- Expression for current we set earlier. Answer.

The general plots for voltage and current applying the values given, followed by a zoomed in plot.





b).

$I_0 := 3 \quad a := 2 \quad \omega := 40 \quad R := 5 \quad L := 0.1$

clear (t)

Substitute in for vRL(t) and i(t):

$vRL(t) := 3 \cdot e^{-2 \cdot t} \cdot (5 - (0.1) \cdot 2) \cdot \cos(40 \cdot t) - 0.1 \cdot 40 \cdot \sin(40 \cdot t)$

$vRL(t) := 14.4 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t) - 4 \cdot \sin(40 \cdot t)$

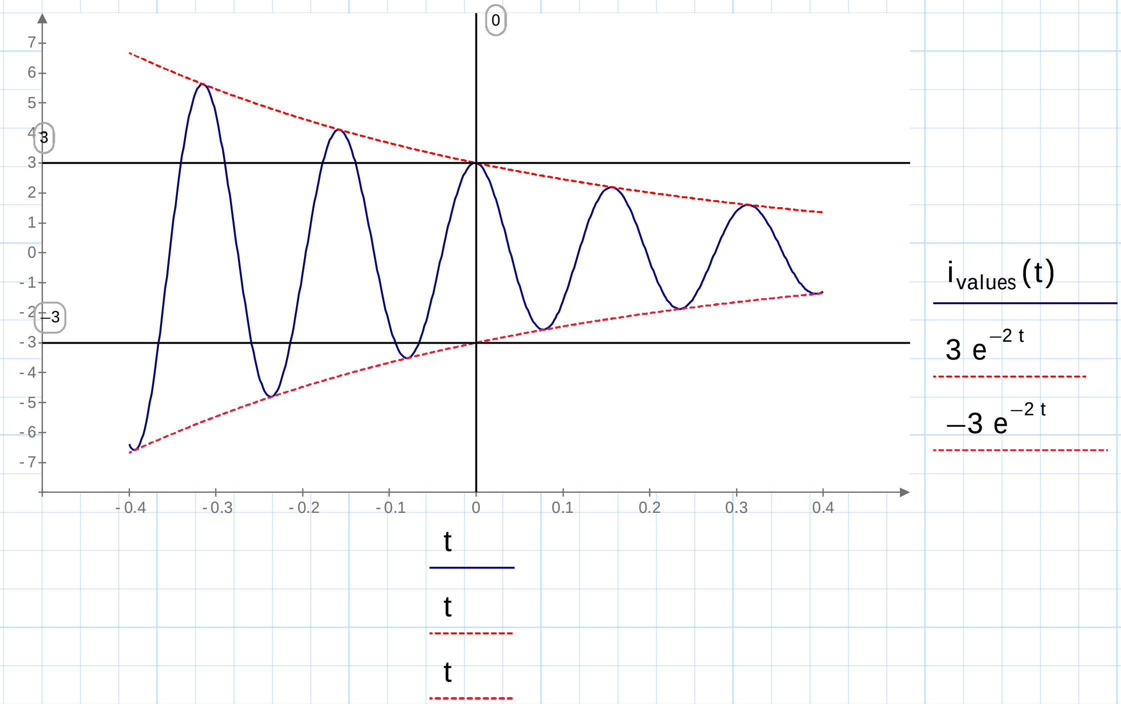
$V_0 := I_0 \cdot \sqrt{(R - L \cdot a)^2 + L^2 \cdot \omega^2}$ Equation 1 $V_0 = 18.745$ V Answer.

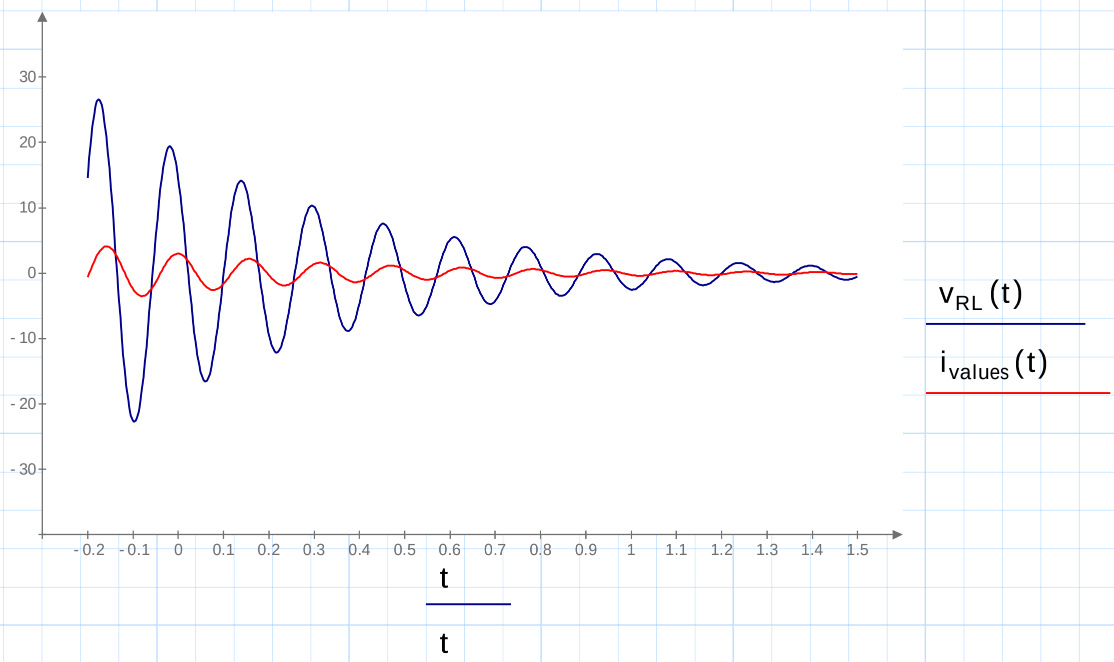
$\theta := \operatorname{atan}\left(\frac{L \cdot \omega}{(R - L \cdot a)}\right)$ Equation 2 $\theta = 39.806$ **deg** phase angle Answer.

$v_{RL}(t) := 18.75 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t + 39.8 \, \mathbf{deg})$

$i_{values}(t) := 3 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t)$ <--- Expression for current with values.

Plot below of current i as a function of time. With envelope 3e^-2t in dashed red.
The lower plot has voltage and current on a larger time range.





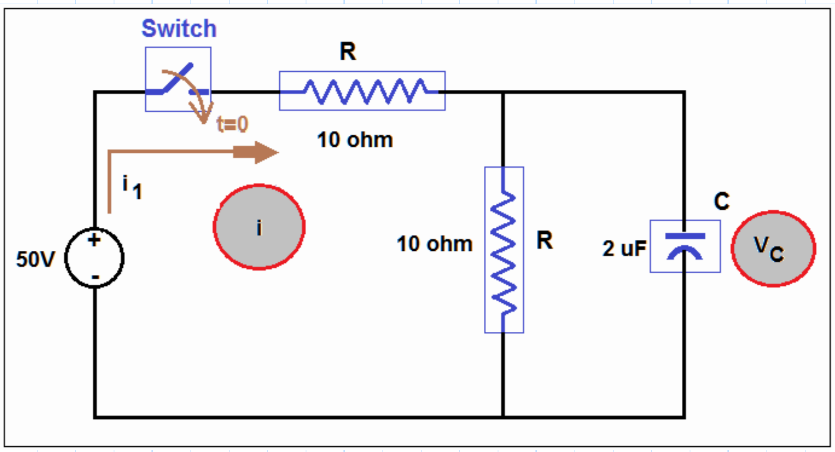
Completion of example 6.24 Damped Sinusoid.

Finished 2 sinusoidal circuits, will leave it there.
Had a few RL circuits recently, next work on a general RC circuit.
In the electrical installation-construction industry; Inductors relate more to transformers, while Capacitors more of storage devices.
Lots of applications they have in many areas...countless areas.

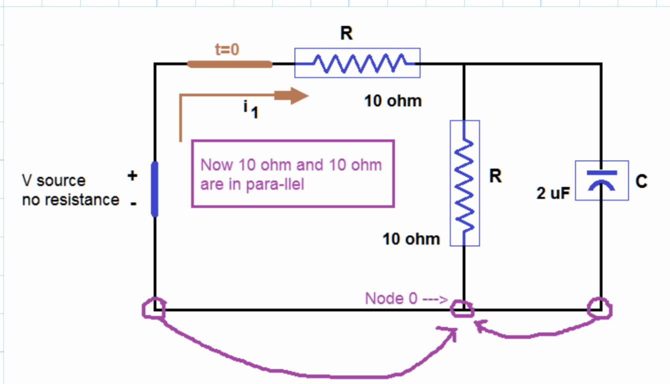
Example 7.17 RC Circuit:

The switch in circuit below is closed at t = 0.

Obtain the current i, and capacitor voltage v_c for t>0.



Solution:



<---Circuit without a voltage source can be called a voltage free circuit, thus the response of the components maybe called their natural response, same for the resistors too.

Natural Response:
Voltage source removed.
Two resistors 10 ohm are now in parallel.

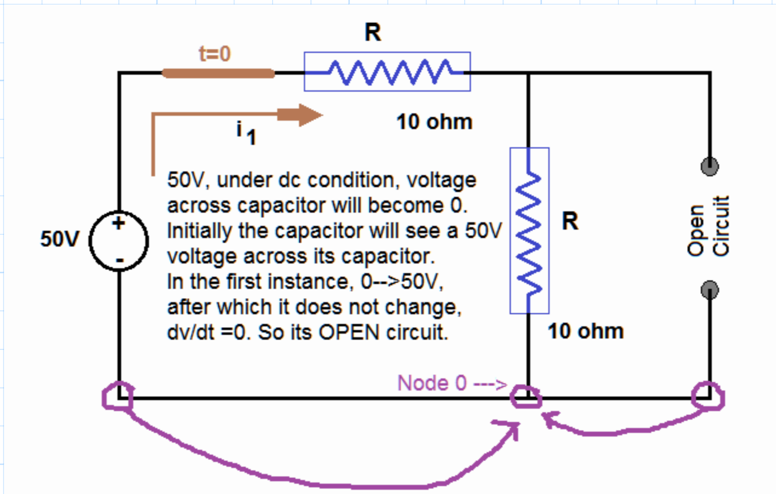
$R_{eq} := \frac{10 \cdot 10}{10 + 10} = 5$ $\quad C := 2 \cdot 10^{6}$ $\quad$ Circuit time constant: $\tau_{RC} := R_{eq} \cdot C = 1 \cdot 10^{7}$

-0< t <0+:

Trying to say t just before 0, 0-, and just after 0, 0+.
You may say that is t=0. It is but here its focusing on switch t closes at t=0. So just before is -0, yes, and just after is 0+, yes.

v(0+) = v(0-) = 0V = v_C(0). <---This is whats called 'by continuity of charge'.
(Just after t=0 i.e. 0+ the capacitor voltage is close to 0, so we can say voltage of capacitor at 0+ its the same as at -0).

When t---> Infinity:

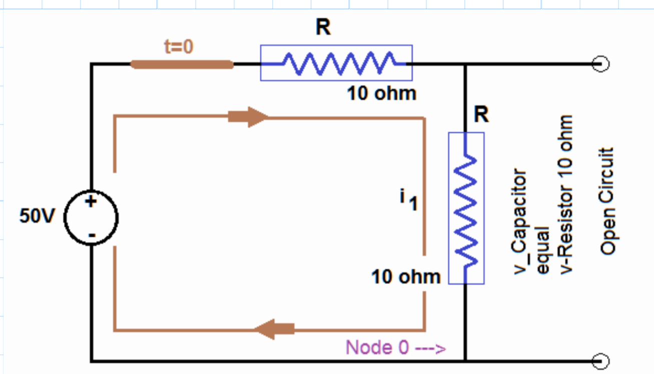


Capacitor becomes open circuit. The voltage source symbol is 50 dc.

Now leaving the 20 ohm in series with the 50 volts.

Current for this condition can be calculated.

$$i_{infinity} = \frac{50}{(10+10)} = 2.5 \quad A.$$



Voltage across capacitor at time t--> infinity is:

$$v_{C\_infinity} := 2.5 \cdot 10 = 25 \quad V.$$

Summing all the different times, -0 to infinity, may result in the complete solution?
Lets see what that gives, because -0 is just the same as 0+, so that gets all the times in the t>0, what we are lookinmg for.

Summing them is not be the best of things, instead apply the technique studied in these notes, Chapter 7 in Schaums. What we have is initial and final values under initial and final conditions. We have a voltage expression for this in exponential form.

$$v = V_0 \cdot \left(1 - e^{\frac{-t}{RC}}\right)$$ RC circuit, for t> OR t=0.

End condition v_C is 25V, with the start condition v_C = 0V.
But this expression gives the decaying voltage across the capacitor!
Correct.
We got the voltage for the final condition t---> infinity, this will be Vo.
Initial condition v(-0) = v(0+) = 0 = v_C(0+).

$$v_C = 25 \cdot \left(1 - e^{\frac{-t}{10\ us}}\right)$$ V, tau in microseconds so time t in the expression is in microseconds. Answer.

The current sought is the sum of current in the capacitor and 10 ohm resistor:

Next on to the current in the Capacitor: $i_c = C \cdot \left(\frac{dv}{dt}\right)$

$$v = 25 \cdot \left(1 - e^{\frac{-t}{10\ us}}\right)$$

$$\frac{dv}{dt} = -\left(\frac{25}{10 \cdot 10^{-6}}\right) e^{\frac{-t}{10\ us}}$$

$$i_c = C \cdot \left(\frac{dv}{dt}\right)$$ Numerical calculation: $\left(\frac{25}{10 \cdot 10^{-6}}\right) \cdot 2 \cdot 10^{-6} = 5$

$$i_c = 5 \cdot e^{\frac{-t}{10\ us}}$$ A.

Current in the 10 ohm resistor:
10 ohm resistor is parallel to the 2 uF capacitor.

$$i_{10ohm} = \frac{v_C}{10\ ohm} = \frac{25 \cdot \left(1 - e^{\frac{-t}{10\ us}}\right)}{10} = 2.5 \cdot \left(1 - e^{\frac{-t}{10\ us}}\right)$$ A.

$$i = i_C + i_{10ohm} = 5 \cdot e^{\frac{-t}{10\ us}} + 2.5 \cdot \left(1 - e^{\frac{-t}{10\ us}}\right) = 5 \cdot e^{\frac{-t}{10\ us}} + 2.5 - 2.5\ e^{\frac{-t}{10\ us}}$$

$$i = 2.5 + 2.5\ e^{\frac{-t}{10\ us}} = 2.5\left(1 + e^{\frac{-t}{10\ us}}\right)$$ A. Answer. Was NOT simple, good example.

7.15 Summary of Forced Response in First Order Circuits:

Voltage equation with respect to time t: $v(t)$

Coefficient of voltage equation (external to v(t) not time based): a ---->Gets to something like this---> $a \cdot v(t)$

First order derivative of voltage equation: $\frac{dv(t)}{dt}$

Forcing function: (RHS of equation, and this is NOT equal to 0) $f(t)$

Discussion & Comments:

We have the voltage function v(t) and we have the forcing function f(t). Our solution is focussed to the forced response i_p(t). Before we jump into solving for the current response, our voltage v(t) has to be modified or selected or improved so that it meets the requirements of the solution process. For instance Io can be determined, A can be evaluated, satisfaction to conditions, and similar requirements - seen in previous examples. Example, when we solve for a quadratic equation the roots need to be satisfy the equation and the roots need to satisfy the behaviour of the variable it respresents.

We have f(t), we may need to modify v(t) so that we get the v_p(t) meeting math and circuit conditions.

Table coming up is on how to manage f(t) by applying the appropriate v_p (t).

Schaums: The forced response v_p(t) depends on the forcing function f(t). The table on next page summarises some useful pairs of forcing function and what should be guessed for v_p(t). The responses are obtained by substitution in the differential equation. By a weighted linear combination of the entries in the table and the appropriate time delay, the forced response to a new functions may be deduced. - Page 159, table page 160.

We have the differential equation:

$$\frac{dv(t)}{dt} + a \cdot v(t) = f(t)$$

$f(t)$	$v_p(t)$		
1	$\frac{1}{a}$		
t	$\left(\frac{t}{a}\right) - \left(\frac{1}{a^2}\right)$		
e^{st} provided s NOT equal to -ve a.	$\frac{e^{st}}{s+a}$		
e^{-at}	$t \cdot e^{-at}$		
$\cos(\omega t)$	$A\cos(\omega t - \theta)$	where	$A = \frac{1}{\sqrt{a^2 + \omega^2}}$ $\tan(\theta) = \frac{\omega}{a}$
$e^{-bt}\cos(\omega t)$	$Ae^{-bt}\cos(\omega t - \theta)$	where	$A = \frac{1}{\sqrt{(a-b)^2 + \omega^2}}$ $\tan(\theta) = \frac{\omega}{(a-b)}$

Verify with your textbook tables.

This is the end of Schaums Outline Chapter 7 at section 15.

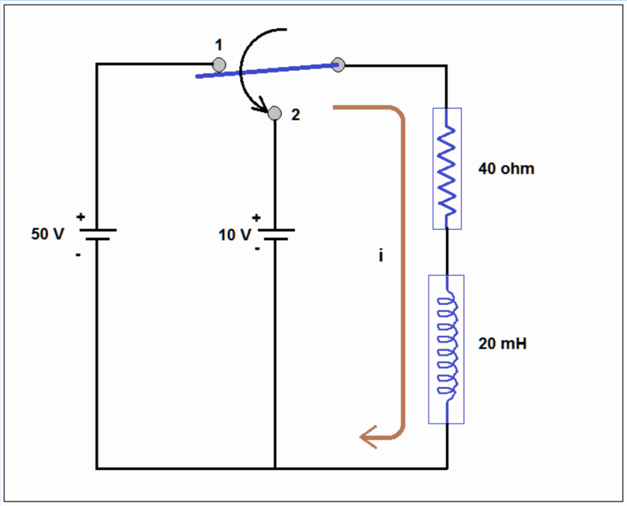
Followed by a few Solved Problems.

Two similar circuit layouts one a RL circuit the other RC circuit in solved problems 7.21 and 7.22.

Solved Problem 7.21: RL circuit.

The switch in the circuit below has been in position 1 for a long time.
It is moved to position 2 at t = 0.

Obtain the expression i for t>0.



$V1 := 50$

$V2 := 10$

$R1 := 40$

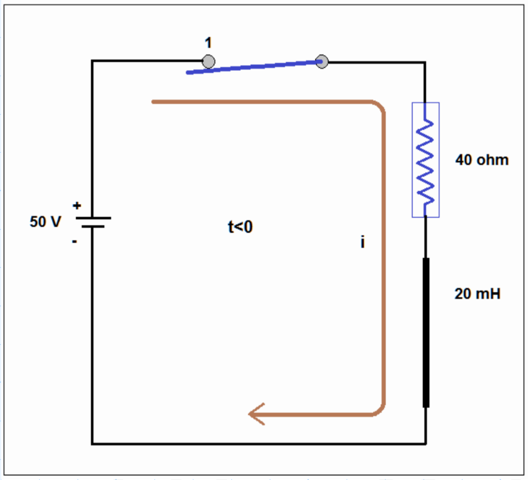
$L := 20 \cdot 10^{-3}$

Solution:

In this circuit we have 2 dc voltage sources, so we know the inductor is going to turn short circuit. DC voltage source will generate a dc current. Constant current will cause or make a short circuit of the inductor.
This is what this problem is about with respect to time t.

t<0 for a long time:

In this condition the inductor has shorted.
This is switch in position 1.
see figure to the right.



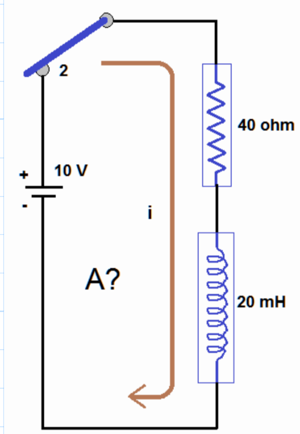
$$i_1(-0) \quad = \quad \frac{V1}{R1} = 1.25$$

t=0:

Here we have the change over centred at t=0.
-0 --> 0 ---> 0+.

i(-0) = 1.25A.
i(-0) = i(0) = 1.25A.
Initially at t=0 when contact is made to position 2, the inductor will experience the presence of dc voltage across it, for one instance, this will energise the inductor.
Then, the continous dc voltage appears.
Therefore,

i(0+) = i(0) which i(0) = i(-0).
Engineer authors saying i(-0) = i(0+).
Seem to have t=0 missing but its the same as saying t(-0) = t(0), so we just drop the intermediate point t(0) and reach to the end point t(0+). This becomes t(-0) = t(0+) but its NOT the time rather the current at these points in time. Which is i(-0) = i(0+). Hence, same saying i(-0) = i(0+).
It gave the impression t=0 dont matter. Matter of speaking it maybe allocated for the switch making contact, and that time instant is devoted to contact. Thats how it is in EVERY textbook.
If you asked.



$$i(-0) \quad = \quad i(0+') \quad = \quad 1.25 \;\; A.$$

Next we move to t>0.

t>0:

The inductor has become a short circuit.

$$i_2(t>0) \quad = \quad \frac{V2}{R1} = 0.25 \;\; A.$$

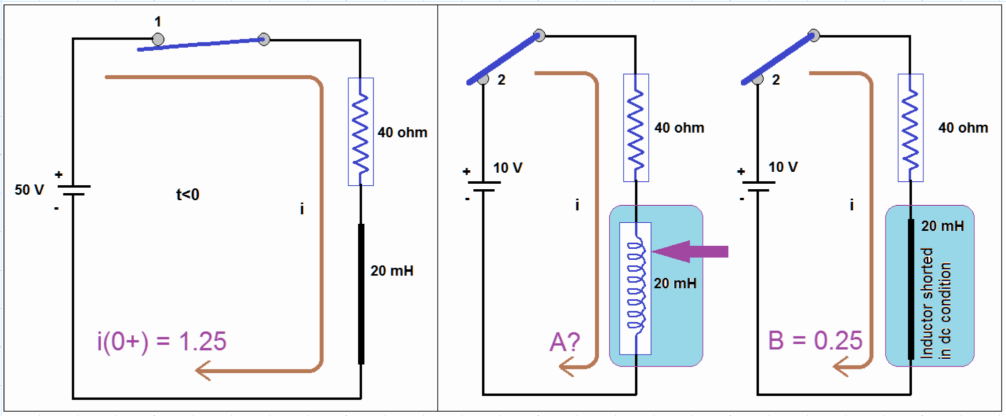
Lets sequence the values so far:

$$i(-0) \;\; ---> \;\; i(0) \;\; ---> \;\; i(0+') \;\; ---> \;\; i(t>0)$$

1.25   1.25   1.25   0.25

We have a 1 A drop from 1.25 to 0.25 where did it go?
This is what we call A in the figure above.

Lets form an expression to solve for A next.



The situation in the figure above is to show surely the inductor played a role in the sequence of current changes - refer to the purple text in figure.

The initial presence of current thru the inductor would create a voltage across it for a fraction of a second - L(di/dt). See Short talk below.

After which it turns to a short circuit. Other wise whats the role of the inductor in the circuit? Test our understanding of the subject matter? Maybe.

Let B = i(t>0):

Short talk: Voltage source being dc should not create a dv/dt, its non varying. But when it sees the voltage the first time across it, it may have a knee jerk experience OR reaction OR inertial reaction OR energy disruption. Only when it realises its a constant dc then it turns to be a short circuit. Takes time to get to realise its a constant dc.

$$B = i_2(t>0)$$

$$A = i(0+') - B$$

$$= 1.25 - 0.25$$

$$A = 1.00$$ <-----Can this be the inductor current that is a decaying current component in the circuit?
Decaying because at t>0 the current has dropped from 1.25 to 0.25A.
Yes.
This decaying portion would have to be in exponential form, with the circuit time constant. See section 7.5.

Inductor circuit is in switch position 2.

$$\tau_{RC} := \frac{L}{R1} = 5 \cdot 10^{-4} \qquad \text{In fraction form} = \frac{1}{2000} = 5 \cdot 10^{-4} \qquad \frac{1}{\tau_{RC}} = 2000$$

$$i_h(t) = Ae^{-\left(\frac{R}{L}\right)t} \qquad i_p(t) = \frac{V_0}{R}$$ Review section 7.5.

$$i(t) = Ae^{-\left(\frac{R}{L}\right)t} + \frac{V_o}{R}$$ From section 7.5. Our A = 1.0 and B = Vo/R = 0.25.

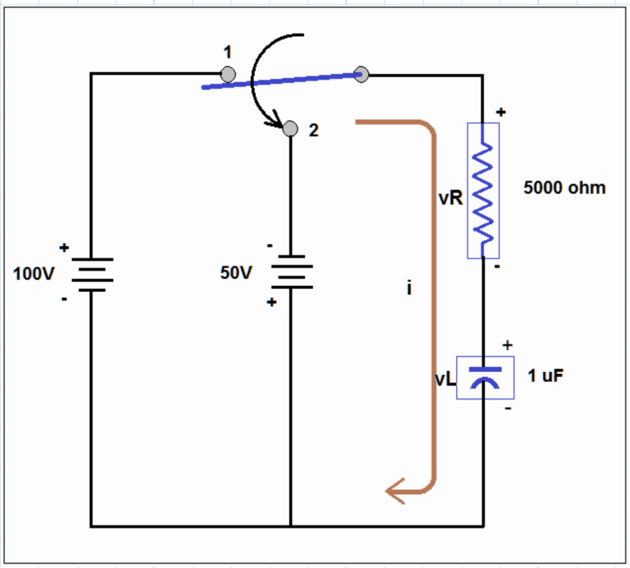
$$i(t>0) = 1.0\,e^{-2000\,t} + 0.25$$ A. Answer.

Math been easy here, showing the reasoning may been the objective. Good example.

Solved Problem 7.22 (RC circuit):

The switch in the circuit shown below is moved from position 1 to 2 at t=0.

Find vC and vR, for t>0.



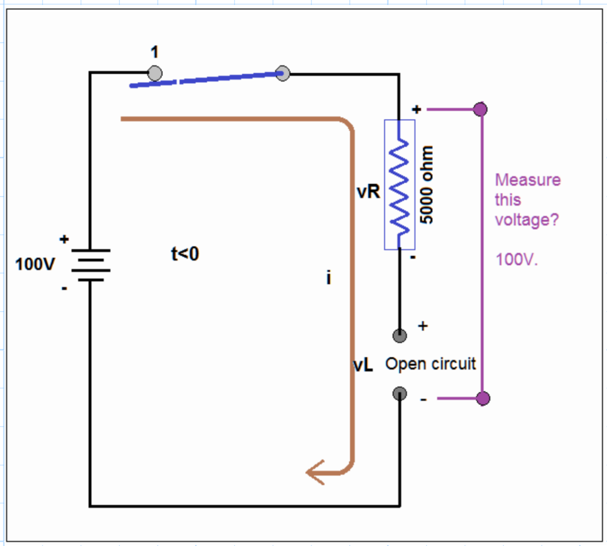
$R1 := 5000$

$C := 1 \cdot 10^{-6}$

$V1 := 100$

$V2 := 50$

Solution:
Careful review shows its similar to previous problem. So we attempt with the same steps. Instead of current we seek voltage, thats primarily because its a capacitor where v_C = C(dv/dt).



t<0 for a long time:

In this condition the capacitor has opened circuit. This was switch in position 1. See figure to the right.

Yes, there is no current flow in the circuit because capacitor has become open circuit, but the voltage across both R and C can be measured and thats is 100V.

$v(-0) = 100 \quad V$

Discussion: If the word 'measured' was not used then it may been difficult to arrive to the solution for most of us. There are some who have an intuitive understanding....they got it first time maybe. But it would be hard, in terms of the problem statement, I/We would been looking for an equation to solve for 100V.

t=0:

Here we have the change over centred at t=0.
-0 --> 0 ---> 0+.

v(-0) = 100V.
v(-0) = v(0) = 100V <---Instant switch closed on position 2.
Next just very near past t=0, i.e. 0+?
Here that voltage would have to start dropping toward 50V but initially its at 0+ its near 100V which we may say is equal to 100V.

$$v(0+') = v(0) = 100 \quad V.$$

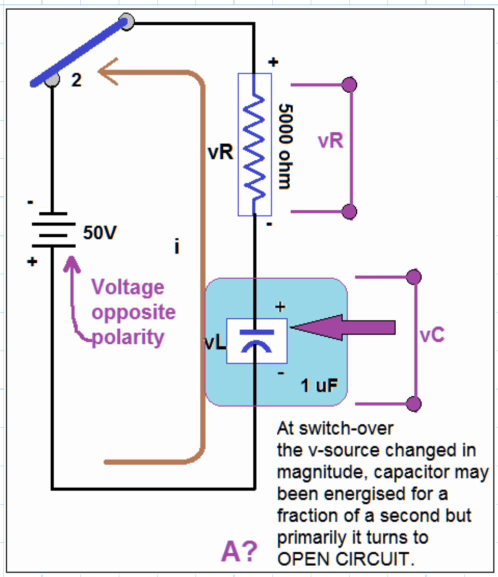
So now we can say before change over from position 1 to 2 and after, that instant, the voltage is the same.

$$v(-0) = v(0+') = 100 \quad V.$$

Next we move to t>0.

t>0:

Circuit voltage in position 2 is 50V. Polarity is opposite to 100V. The Capacitor has become an open circuit. We can as before measure the voltage from one end of the capacitor to the other end of the resistor. This equals -50V. Figure to right.



$$v_2(t>0) = -50 \quad V.$$

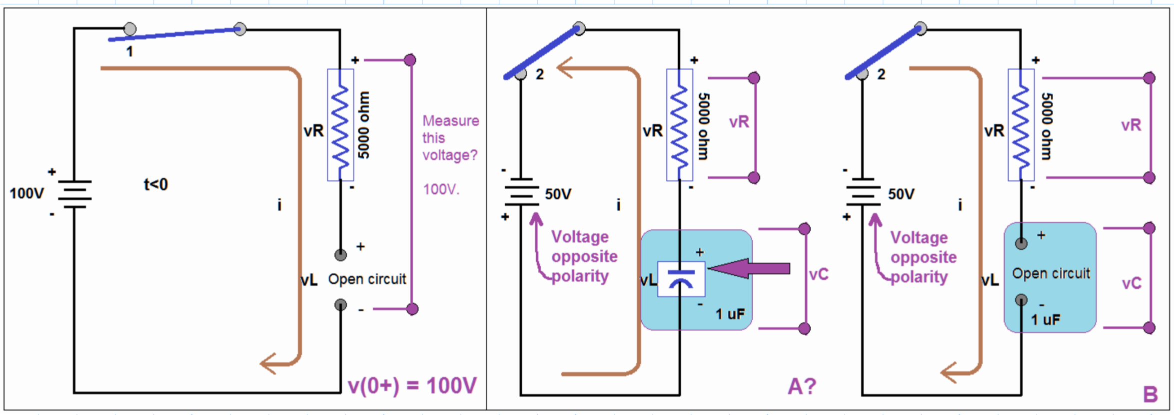
Lets sequence the values so far:

$$v(-0) \;\text{--->}\; v(0) \;\text{--->}\; v(0+') \;\text{--->}\; v(t>0)$$

100 100 -50 -50

We have 150V drop from 100 to -50 where did it go?
100 - (-50) = 150V missing?
This is what we call A in the figure above.

Lets form an expression to solve for A next.



The situation in the figure above is that surely the capacitor played a role in the sequence of voltage changes.

The initial presence of current thru the capacitor would create a voltage across it for a fraction of a second - C(dv/dt). Though its constant the first instance it sees a value it reacts, knee-jerk reaction. *I am saying.* But it must turn to open circuit.

Other wise whats the role of the capacitor in the circuit? Test our understanding of the subject matter? Maybe.

Let B = v(t>0):

$B \;=\; v_2(t>0) \;=\; -50$

$A \;=\; v(0+') - B$

$\;\;\;\;= 100 - (-50)$

$A \;=\; 150$ <-----Can this be the capacitor voltage that is a decaying voltage component in the circuit? Decaying because at t>0 the voltage has dropped from 100 to -50V. Yes. This decaying portion would have to be in exponential form with circuit time constant. See section 7.3.

Capacitor voltage decays a 150V, +100 t0 -50V; 150V decay - This is what A is here. Capacitor circuit is in switch position 2. See section 7.3 for details.

$\tau_{RC} := R1 \cdot C = 0.005$ In fraction form = $\frac{1}{200} = 0.005$ $\frac{1}{\tau_{RC}} = 200$

$v_C(t) = v_p(t) + v_h(t) = V_0 + Ae^{\frac{-t}{\tau_{RC}}} = -50 + 150\,e^{-200\,t} = 150\,e^{-200\,t} - 50$ Answer.

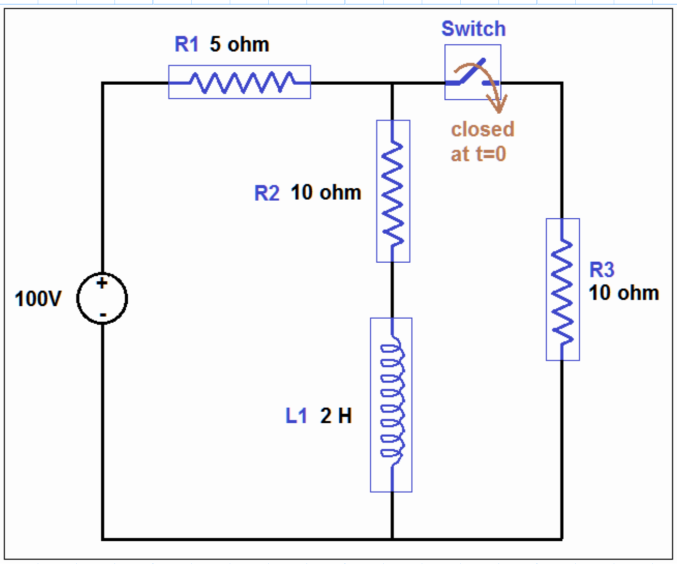
From the last circuit with switch at position 2, t>0, we do a KVL equation:

$v\_50V + v_C + v_R = 0$ $\quad 50 + \left(150\,e^{-200\,t} - 50\right) + v_R = 0$ $\quad v_R = -150\,e^{-200\,t}$ Answer.

Repeated math been easy here, showing the reasoning may been the objective. Good example. Please go thru to verify the way of the logic reasoning thinking....you surely can do better.

Supplementary Problem 7.39 RL Circuit:

Write simultaneous differential equations for the circuit shown below.
Solve for i1 and i2.
Switch is closed at t = 0.
After having been open for an extended period of time.
Hint: Problem can be solved by applying initial and final conditions to general solutions as was done in Solved Problerm 7-17.



$R1 := 5$ Ohm

$R2 := 10$ Ohm

$R3 := 10$ Ohm

$L1 := 2$ H.

$V_0 := 100$ V

Solution:

t>0:

KVL loop equation for i1:

$$5\,(i1 + i2) + 10\,(i1) + 2\left(\frac{di1}{dt}\right) = 100$$

$$15\ i1 + 5\ i2 + 2\left(\frac{di1}{dt}\right) = 100 \quad \text{Eq 1}$$

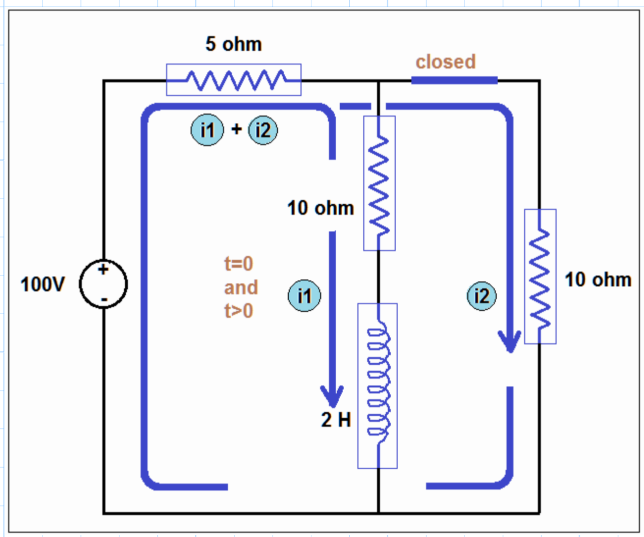
KVL loop equation for i2:

$$5\,(i1 + i2) + 10\,(i2) = 100$$

$$5\ i1 + 15\ i2 = 100 \quad \text{Eq 2}$$

$$i2 = \frac{100 - 5 \cdot i1}{15}$$

Substitute into Eq 1.



$$15\ i1 + 5\left(\frac{100 - 5 \cdot i1}{15}\right) + 2\left(\frac{di1}{dt}\right) = 100$$

$$225\ i1 + 500 - 25 \cdot i1 + 30\left(\frac{di1}{dt}\right) = 1500$$

$$200\ i1 + 30\left(\frac{di1}{dt}\right) = 1000 \qquad \text{divide by 30}$$

$$6.667 \cdot i1 + \left(\frac{di1}{dt}\right) = 33.333$$

Now the analysis is in time t>0. Here at t = infinity, dt = infinity,
di1/dt approximately equal 0. The equation now at t(infinty):

$$6.667 \cdot i1 = 33.333$$

$$i1\,(t > 0) = \frac{33.333}{6.6667} = 5 \text{ A.}$$

Susbtitue i1(t>0) to solve for i2(t>0):

$$i2\,(t > 0) = \frac{100 - 5 \cdot (5)}{15} = \frac{75}{15} = 5 \quad \text{A.}$$

Alternate method to calculate the same above:

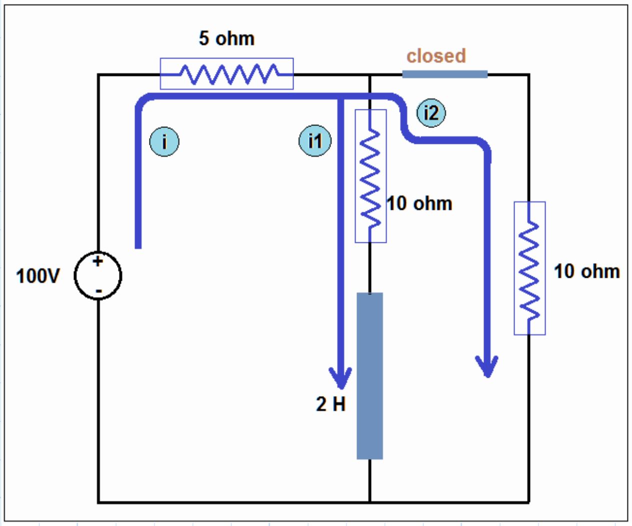
Next the circuit total resistance:

$$R_T := R1 + \left(\frac{R2 \cdot R3}{R2 + R3}\right)$$

$$R_T = 10$$

Circuit total current:

$$i_T := \frac{V_0}{R_T} = 10 \quad \text{A.}$$



i1 and i2 are equally divided by the 10 ohm parallel resistors.
Which can be obtained thru current division.

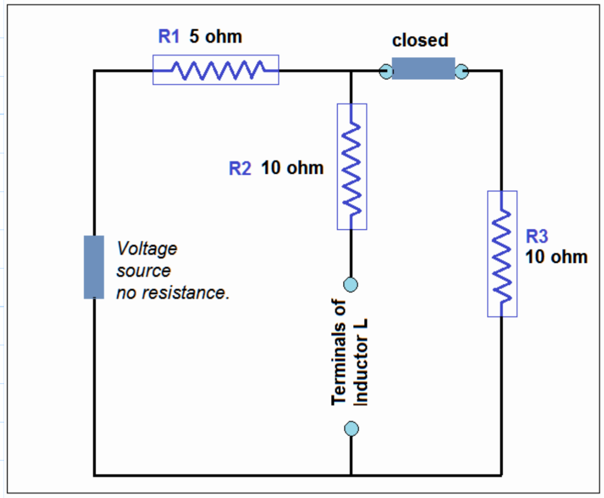
$$i1 \quad = \left(i_T\right)\cdot\left(\frac{R3}{R2+R3}\right) = 5 \qquad A.$$

and

$$i2 \quad = \left(i_T\right)\cdot\left(\frac{R2}{R2+R3}\right) = 5 \qquad A.$$

Circuit time constant:

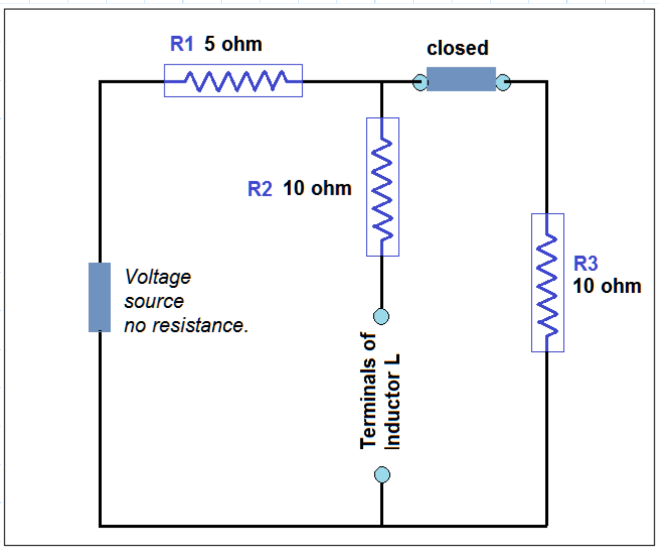
Calculate resistance seen from the terminals of the inductor



R1 and R3 parallel

$$R13 := \frac{R1\cdot R3}{R1+R3} = 3.333$$

Progressing to circuit below.



R13 in series to R2:

$R_{eq} \quad = \quad R123 \quad = \quad R13 + R2 = 13.333$

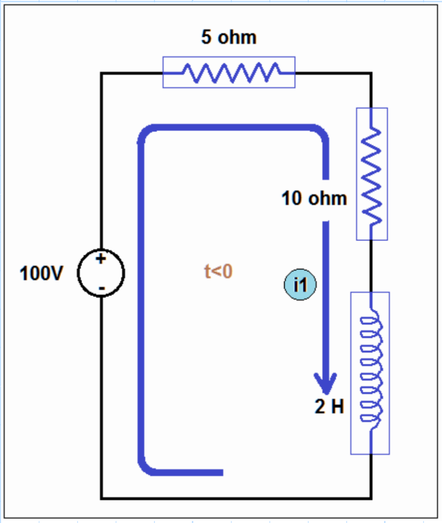
$R_{eq\_seen\_L1} := 13.333$

$\tau_{RL} := \dfrac{L1}{(R_{eq\_seen\_L1})} = 0.15$ Circuit time constant.

$\dfrac{1}{\tau_{RL}} = 6.667$

Next calculate i1(t<0):

t<0:
Switch open for long period.
Inductor L1 is short circuited (Vo = 100V dc).



$i1\,(t<0) = \dfrac{V_0}{(R1 + R2)} = 6.6667$

$i1\,(t<0) = i\,(-0)$

$i1\,(-0) = i\,(0+')$

$i1\,(0+') = 6.6667$

Now solve for complete current response for i1(t):

$i1\,(-0)$ ----> $i1\,(0+')$ ----> $i1\,(t>0)$

6.6667 ----> 6.6667 ----> 5.0 A.

As in the recent other example for RL circuit, we have some current decaying from 6.667 to 5.0A. We set out final current as B=5.0A.
The decaying current as A, and the initial current i1(0+) = 6.6667A.

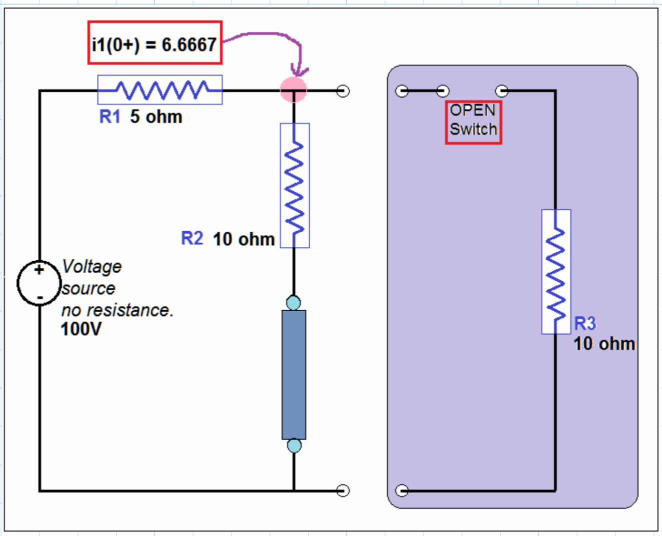
$i1_0 := 6.6667 \qquad B := 5.0$

$A \quad = \quad i1_0 - B = 1.667$

$i1\,(t) \quad = \quad Ae^{\left(\frac{-R}{L}\right)t} + \dfrac{V_0}{R} \quad = \quad A \cdot e^{\frac{t}{\tau_{RL}}} + B = 1.67\, e^{-6.67\,t} + 5$ A. Answer.

Next we tackle i2(t). It requires some re-thinking into the problem. Times like this look over or re-start from the beginning, pull out the similar steps, look for the places where there is missing values for current at specific nodes.
To solve for i1(t), i1(0+) played a critical role, how do we find i2(0+)?
Yes, so some back-tracking some re-thinking some review need be done.

When the switch closed that was the only oppurtunity for current to enter the R3 branch.
So lets look at that to get to i2(0+).

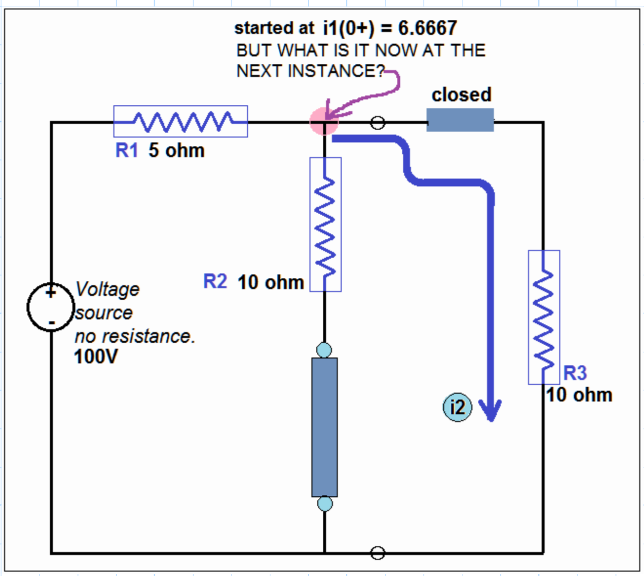


The circuit just before closing the switch. The current i1(0) appears at the node.

When the switch closes, this current will run thru the R3 resistor, because there is resistance in this branch, the current has to reduce from 6.6667 to some value.
We mean the current splits at node, and portion of it flows thru R3. This current maybe lower or higher depending on resistor value.

See next figure below.

Circuit above total resistance = $R12 := R1 + R2 = 15$ ohm



So our next step is to adjust 6.6667 as it flows into the R3 circuit branch.
Solved!

With 15 ohms at i1(0+) we see 6.6667 A.

Seems simple enough we seek a percentage of i1(0+), this will be the current entering or seen at (0+), i.e. i2(0+). Lower than 6.6667 as the resistance is increasing by 10 ohm. Apply current division NOT simple percentage. Here R2 impacts the current in branch of R3.

$$i2(0+') \quad = \quad \left(\frac{R2}{R12}\right)\cdot 6.6667 = 4.444$$

We have the final settling current for i2(t>0) = 5A.

Now solve for complete current response for i2(t):

$i1(0+)$ ----> $i2(0+')$ ----> $i2(t>0)$

6.6667 ----> 4.4445 ----> 5.0 A.

We have some current decaying from 4.4445 to 5.0A.
Final current is B=5.0A.
For i2(t) the decaying current A and the initial current i2(0+) = 4.4445A.
Same circuit time constant because its the same circuit when switch closed.

$i2_0 := 4.4445 \qquad B := 5.0$

$A \;=\; i2_0 - B = -0.5555$

$$i2(t) \;=\; Ae^{\left(\frac{-R}{L}\right)t} + \frac{V_0}{R} \;=\; A\cdot e^{\frac{t}{\tau_{RL}}} + B \;=\; -0.555\, e^{-6.67\,t} + 5A.$$ Answer.

We have both the correct answers. This is one of the latter supplementary problems, no solution provided. Answers provided. It required going over the study notes, examples, textbook, and circuit solving skills to get to the solution. This solution will help solve other similar problems.

One last solved problem, which is the last solved problem in chapter 7.
This solved problem will help in the signals and system course.

Most higher level electrical courses do NOT always have problems that have circuit component values.

One such class is the signals and systems course. So this next example reflects that environment.

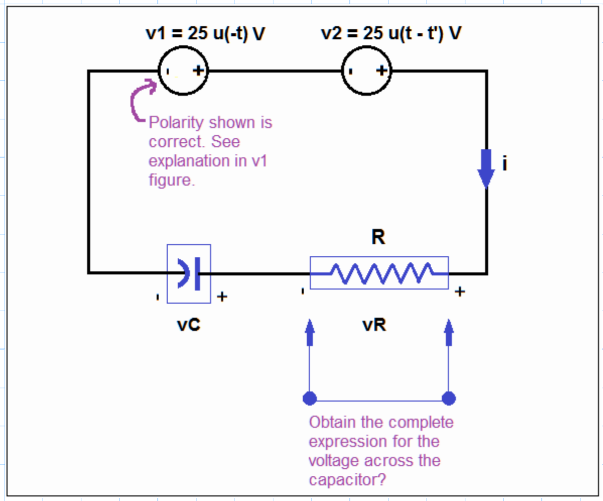
Solved Problem 7.24 RC circuit:

A series RC circuit, with R= 5 k ohm and C = 20uF, has 2 voltage sources in series.

They are:

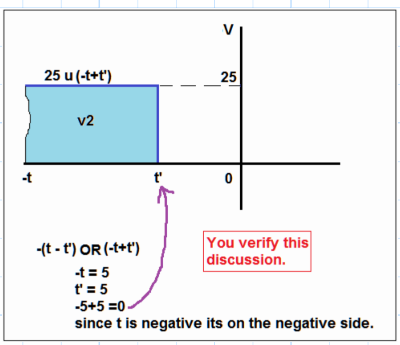
$$v1 = 25 \cdot u(-t) \quad V$$
$$v2 = 25 \cdot u(t - t') \quad V$$

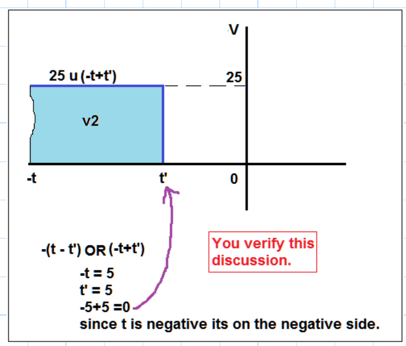
Obtain the complete expression for the voltage across the capacitor and make a sketch, if **t'** is positive.



Solution:

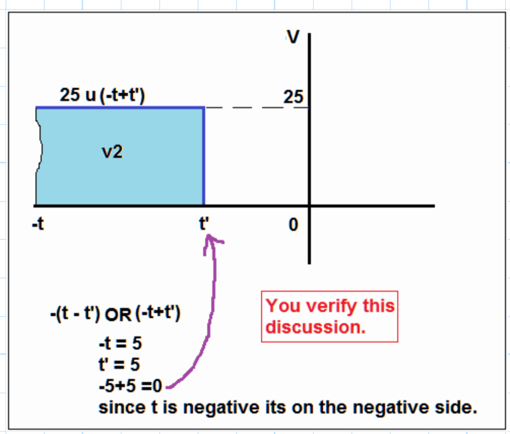
Lets do a sketch on the voltage v1 and v2.
v1 = 25u(-t).... its a unit step with an amplitude of 25 but it turns on positive when t is negative.
v2 = 25u(t - t') the t' may be T, usually T is set for period, so this where u(t-t') is on the positive side of t. Also when t=0, since t=t' will make (t'-t') = 0.





Figures above, v1 to the left and v2 to the right.

Discussion figure--->

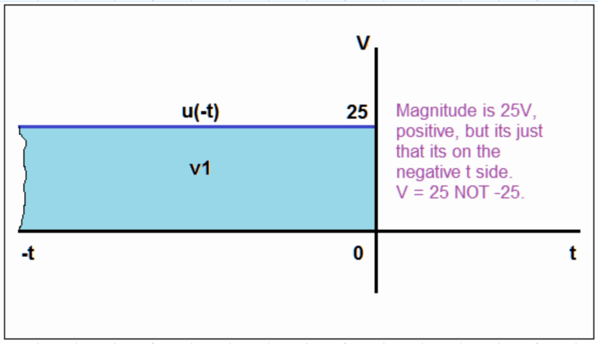


Discussion: It could be **-**(t - t') this becomes (-t+t'), either way would make it -t+t'.

Here lets say t' = 5, then at some point t=-5, we have (-5)+5 =0 here it goes positive but in the -ve t direction.

t < 0:

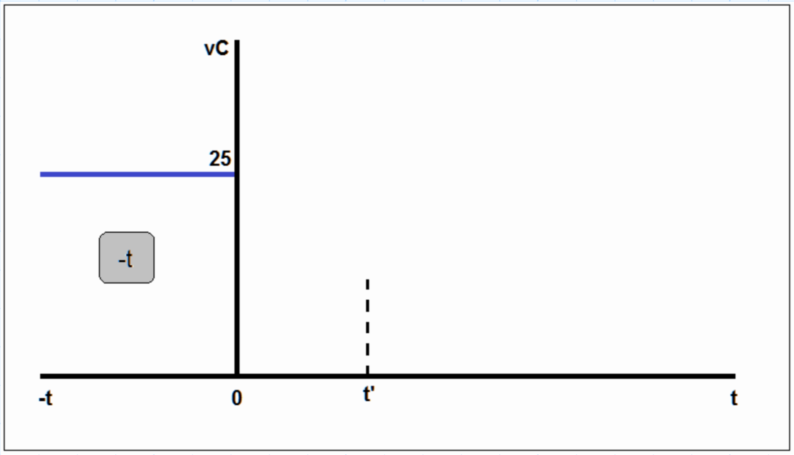
v1 supplies 25V when t<0.
Remember, -0 0 0+.
So -t can be -0, ending up as 0+.
Here v1 can supply the capacitor only when t--->-0.



$$v_{C\_less\_0} = 25 \cdot e^{-10 \cdot t}$$

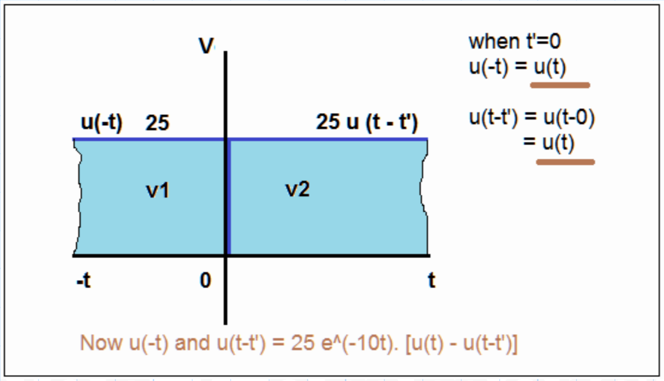
v2 does not come on to supply capacitor.

Therefore voltage across capacitor is 25V. Resistor will experience a voltage across it too, its a series circuit, would this cause a voltage drop across the resistor so that the maximum across the capacitor is less than 25V? No. The capacitor gets its potential rise up to the maximum of the source. Which here we say is 25V. Tricky.
But the maximum the capacitor can experience is 25V. Schaums indicates 25V for -t.



Plot thus far.
What the capacitor voltage waveform looks like.
All rough plots not software generated, sketched.

t = 0 :



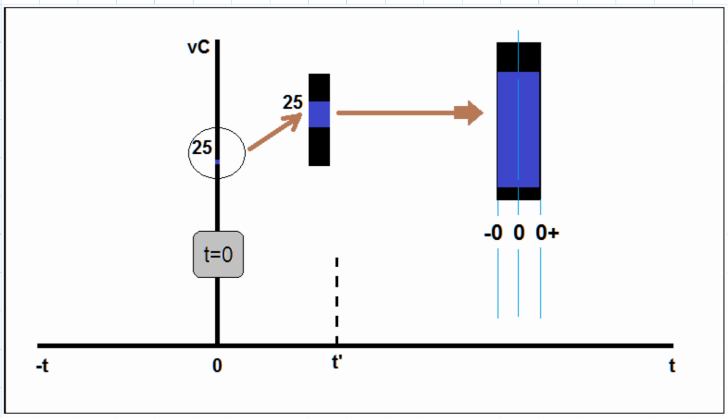
At t=0 v1 supplies the voltage.
For v1, -0 -->0 --> 0+ is the case 25(-t) can be ON at t=0.
Not past t>0.
v2 can come on if t' = 0, then 0 - 0' = 0, and 25(t-t') comes on.
So its good for both v1 and v2 supplying capacitor at t'=0.

$$v_{C\_t'\_eq\_0} = 25 \cdot e^{-10 \cdot (t-t')} = 25 \cdot e^{-10 \cdot (t-0)} = 25 \cdot e^{-10 \cdot (t)}$$

$$= 25 \cdot e^{-10 \cdot (t)} \cdot (u(-t) - u(t-t')) = 25 \cdot e^{-10 \cdot (t)} \cdot (u(t) - u(t-t'))$$

Here the step functions were included to show the exponential expression applied to both conditions.

Negative sign, u(-t) taken out because t=0 , becomes u(t). Specific to the case.



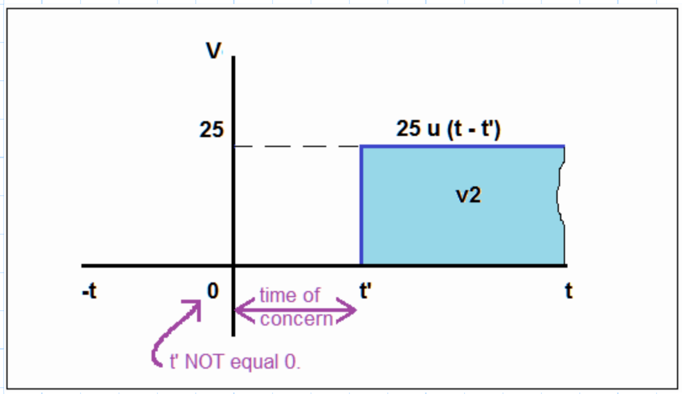
Plot at t=0.
Contribution can be from both functions.
This need to be resolved.

0 < t < t' :

Depending on where t' is located in t>0 thats where v2 starts supplying voltage to capacitor.

2 cases:
1. t close to 0 aproximately t=0 but this just got covered in t=0.

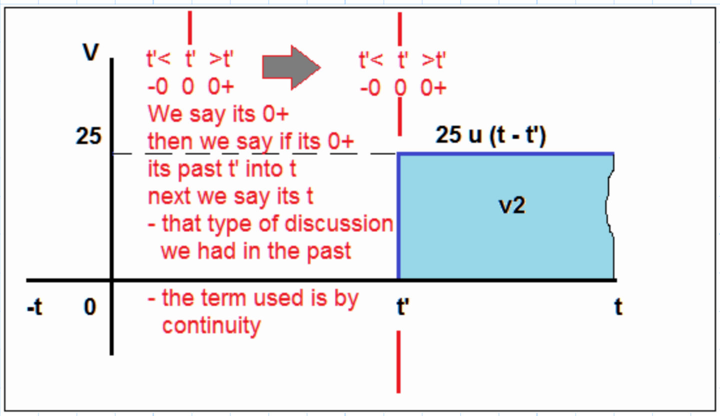
2. t close to t' aproximately t=t'.
This maybe the case to foucs.



Discussion: Here the capacitor voltage starts decaying because there is no voltage source. The voltage is maintained across the capacitor terminal to the value of 25V, but when there is no voltage source, no constant voltage to maintain it, vC begins to drop/ decay. Decays from 25V toward 0V. Dependent on where t' is, that decides if the capacitor voltage decays to 0 or not. If t' is close to 0 then the time may be too short to decay to 0 because the next stage comes in for the voltage source v2 when t = t'.

$R1 := 5000$ $\quad C1 := 20 \cdot 10^{-6}$ $\quad \tau_{RC} := R1 \cdot C1 = 0.1$ $\quad \frac{1}{\tau_{RC}} = 10$

When t is close to t' then v2 cannot supply the capacitor. It has to be t = t' minimum.
But when we see t' spread like -0 0 0+, we say t'(-0) = vC(-0) and by continuity t'(0+) = cV(0+) now here t'(0+) had crossed over t'.
Now v2 can supply the capacitor.

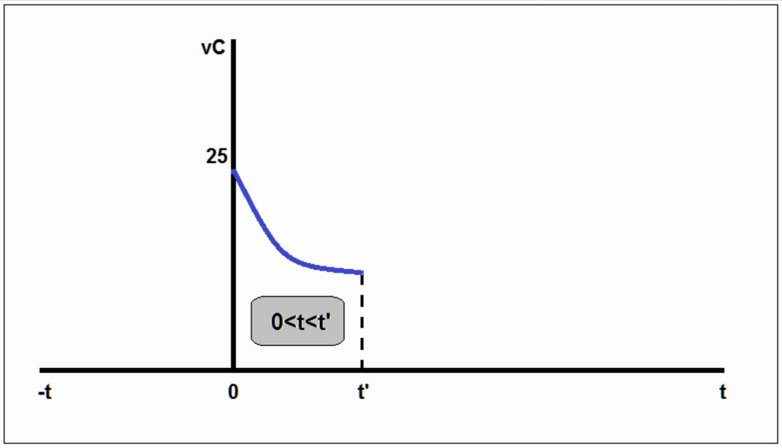


<--- t'(+) crossed in to t which makes t>t', and v2 can supply voltage to capacitor.

So based on that, -0 0 0+, t is applied in the exponential expression instead of t'. Otherwise v2 will not be able to supply the capacitor.

$v_{c\_t'\_eq\_t} = 25 \cdot e^{-10 \cdot t}$ V. <---Voltage decay value at t'(0+) = t;
close enough to t = t',
which is saying same as if on the dot of t.

Decaying from 25V down toward 0...when [0 < t < t'(0+)].
Schaums uses t instead of t' in the solution. Here I attempted to provide the reason for it. You may do better if it is different.



Plot for this interval.

First a short exercise on exponential provided below for the next time frame.

$e^2 = 7.389$ $\quad e^3 = 20.086$

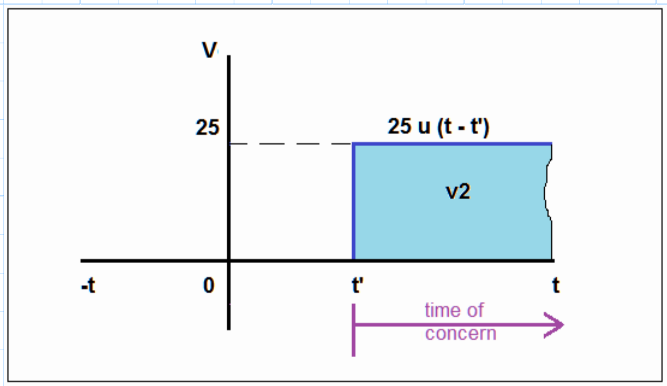
$\left(e^2 \cdot e^3\right) = 148.413$ $\quad e^{(2+3)} = 148.413$ <----We use this property next.

$e^{(2 \cdot 3)} = e^6 = 403.429$ <---Wrong! Not multiplied.

t > = t' :

At t' and past t':
When t=t' we have v2 supplying 25V to the capacitor.

BUT, since the capacitor voltage dropped to some value in the time 0<t<t', the voltage v2 supplies now has to raise the capacitor voltage from that value, v_c_t', up to 25V.



1). We have t=t' which we just did this resulting with vC(t'=t) = 25 e ^-10t.

$$v_{c\_t'} = 25 \cdot e^{-10 \cdot t} \text{ V.}$$ <---Covered.

2). We have t>t' which can be represented by (t - t') provided t >t' but NOT yet infinity.
For this the expression will be:

$$v_{C\_t\_eq\_grtr\_t'} = 25 \cdot e^{-10 \cdot (t - t')}$$

$$v_{C\_t\_eq\_grtr\_t'} = 25 \cdot \left(e^{-10 \cdot (t)} - e^{10\,(t')}\right) \text{ V.}$$

3) We have t way past t' where t is reaching infinity, which merely is saying time has adequately passed far enough whereby the capacitor is fully charged to 25V.
Here we have:

$$v_{C\_t\_infinity} = 25 \text{ V.}$$ No longer needing an exponential term its a flat constant 25V.

Now lets add them up for t=t' and t>t' and t---> infinity:

$$v_{c\_t'} = 25 \cdot e^{-10 \cdot t} \text{ V.}$$

$$v_{C\_t\_eq\_grtr\_t'} = 25 \cdot \left(e^{-10 \cdot (t)} - e^{10\,(t')}\right) \text{ V.}$$

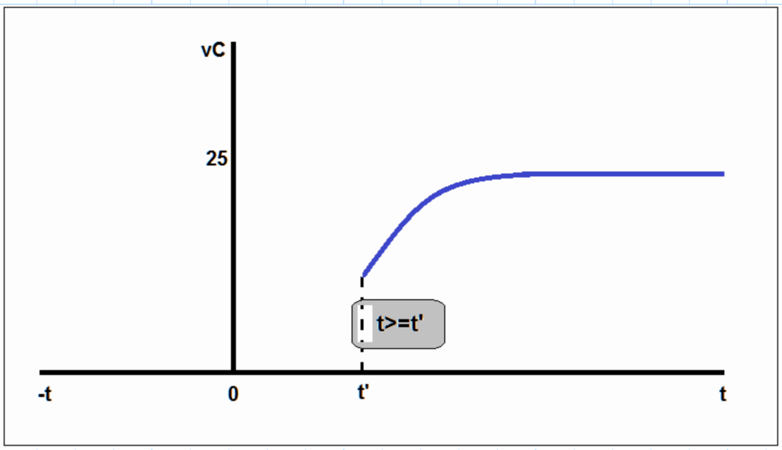
$$v_{C\_t\_infinity} = 25 \text{ V.}$$

$$v_{C\_t\_eq\_0\_grtr\_t'} = v_{C\_t\_infinity} + v_{C\_t\_eq\_grtr\_t'} + v_{c\_t'}$$

$$= 25 + \left(25 \cdot \left(e^{-10 \cdot (t)} - e^{10\,(t')}\right)\right) + \left(25 \cdot e^{-10 \cdot t}\right)$$

$$= 25 \cdot \left(1 + \left(e^{-10 \cdot t} - e^{10\,t'}\right) + e^{-10 \cdot t}\right)$$

$$= 25 \cdot \left(1 - \left(e^{10\,t'} - 1\right) \cdot e^{-10 \cdot t'}\right)$$ <---Schaums solution.



Plot of the last segment.

Next we sum all the expressions for the capacitor voltage:
Listing the expressions for vC, for t<0, t=0, and t>0:

t < 0:

$$v_C = 25 \cdot u(-t)$$

t = 0 :

$$v_C = 25 \cdot e^{-10 \cdot (t)} \cdot (u(t) - u(t - t'))$$

0 < t < t' :

$$v_{c\_t'} = 25 \cdot e^{-10 \cdot t'}$$ <-----This term was added in to the next.

t > = t' :

$$v_C = 25 \cdot \left(1 - \left(e^{10\,t'} - 1\right) \cdot e^{-10 \cdot t'}\right) \cdot u(t - t')$$

Sum all terms:

$$v_C = 25 \cdot u(-t) + 25 \cdot e^{-10 \cdot (t)} \cdot (u(t) - u(t-t')) + 25 \cdot \left(1 - \left(e^{10\,t'} - 1\right) \cdot e^{-10 \cdot t'}\right) \cdot u(t-t')$$

Answer. (Schaums).

$$v_C = 25 \cdot \left(u(-t) + \left(e^{-10 \cdot (t)} \cdot (u(t) - u(t-t'))\right) + \left(1 - \left(e^{10\,t'} - 1\right) \cdot e^{-10 \cdot t'}\right) \cdot u(t-t')\right)$$

Factor out 25.

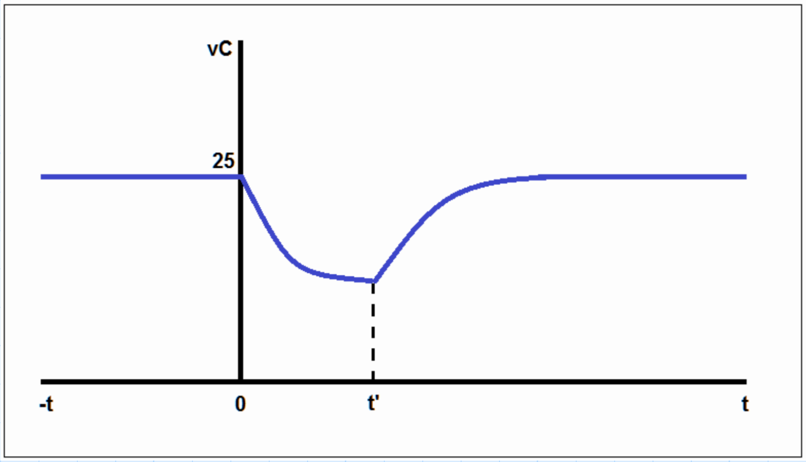


Figure above is the same Schaums provided for the final solution.

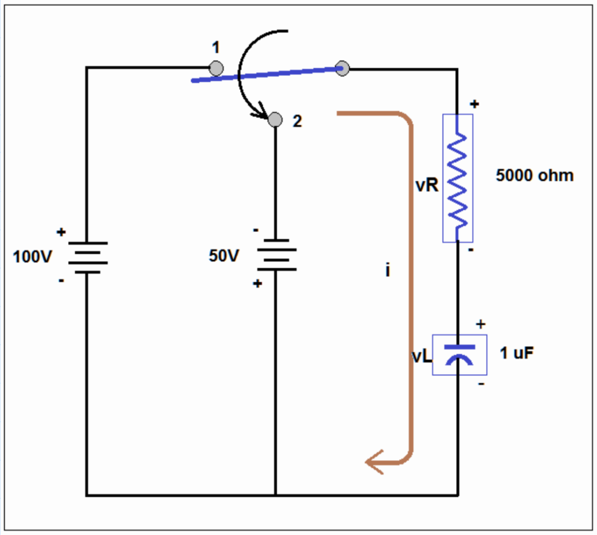
Comments:

This was a challenging solved example.
Every step of the way I worked to make my solution's reasoning fit Schaums solution.
It required 'Deep Learning' <---thats a joke but by some definitions it may be.
The solution which I can see was accurate based on Schaums, my reasoning may not hit the mark every time, so you check and correct/improve on that.

We completed solved Problem 7.22 it was a RC circuit.
The following problem is 7.23 on the same circuit it required we produce the energy function for the circuit.
This may be relevant in your course work now in the capacitor energy side of things.
A little problematic it is in this case.

Solved Problem 7.23 (Energy function):

Obtain the energy function for the circuit of problem 7.22.



We found vR:

$v_R(t) := -150 \cdot e^{-200 \cdot t}$ V.

We found vC:

$v_C(t) := 150 \cdot e^{-200 \cdot t} - 50$ V.

$C := 1 \cdot 10^{-6}$ F

$R := 5000$ Ohms

Capacitor:

$$w_C = \left(\frac{1}{2}\right) C \cdot v_C^2 = \left(\frac{1}{2}\right) C \cdot \left(150 \cdot e^{-200 \cdot t} - 50\right)^2$$

$$= \left(\frac{1}{2}\right) C \cdot \left(\left(150 \cdot e^{-200 \cdot t} - 50\right)\left(150 \cdot e^{-200 \cdot t} - 50\right)\right)$$

Expand the left side term:

Note: Not raising the power;
(e^3)^2 = e^6.
Multiplying: e^2 x e^3
=e^(2+3)
= e^5.

$$\left(150 \cdot e^{-200 \cdot t}\right)\left(150 \cdot e^{-200 \cdot t}\right) = 2.25 \cdot 10^4 \cdot e^{-400 \cdot 2\,t}$$

$$(-50) \cdot \left(150 \cdot e^{-200 \cdot t}\right) = -7.5 \cdot 10^3 \cdot e^{-200 \cdot t}$$

$$2.25 \cdot 10^4 \cdot e^{-400 \cdot 2\,t} - 15 \cdot 10^3 \cdot e^{-200 \cdot t} + 2500$$

$$22500 \cdot e^{-400 \cdot 2\,t} - 15000 \cdot e^{-200 \cdot t} + 2500$$ divide by 2.5x10^-3

$$9 \cdot e^{-400 \cdot 2\,t} - 6 \cdot e^{-200 \cdot t} + 1$$

$$\left(3\,e^{-200\,t} - 1\right)^2$$ Factored, Exponent: 2t = t+t, t^2 = t+t.

Evaluate the front term:

$\left(\frac{1}{2}\right) C = 0.5 \cdot 10^{-6}$ $\quad w_C = \left(\frac{1}{2}\right) C \cdot v_C^{\,2} = 0.5 \cdot 10^{-6} \cdot \left(3\, e^{200\, t} - 1\right)^2$ J

Expression in uJ:

$w_C(t) := 0.5 \cdot \left(3\, e^{-200 \cdot t} - 1\right)^2$ uJ My Answer.

Schaums Answer: $w_C(t) := 1.25 \cdot \left(3\, e^{-200 \cdot t} - 1\right)^2$ mJ Schaums Answer <---.

Comments: *I got the squared term to the left. 0.5 instead of 1.25.*
Units uJ instead of mJ. Here may be an error in Schaums Answer? No?
You should be able to get the correct answer having got some hints
here provided Schaums Answer is correct.
I once got to Schaums answer without the square on the last term and
the correct units mJ. I knew exponents were tricky I gave it another go.
I leave the final decision with you and your local engineer/instructor.
Condition as in 7.22 at time t=0 switch position to 2. The v expression for
R and C seems to have t<0 taken into account for t=0 in 7.22. Should not need
working on energy for t<0 when v=100V. Tricky.

Resistor:

$p = v \cdot i \quad i = \frac{v}{R} \quad p = v \cdot \frac{v}{R} = \frac{v^2}{R}$

$v_R(t) = \int_0^t \left(\frac{v_R^{\,2}}{R}\right) dt$ Over a period of time t.

$v_R(t) = \frac{1}{R} \int_0^t \left(v_R^{\,2}\right) dt$ Do we need to take the integral over time t?
No we have t in general not t=3s specific.

$v_R = -150 \cdot e^{-200 \cdot t}$

$v_R^{\,2} = \left(-150 \cdot e^{-200 \cdot t}\right)\left(-150 \cdot e^{-200 \cdot t}\right)$

$v_R^{\,2} = 22.5 \cdot 10^3 \left(e^{-400 \cdot 2\, t}\right)$

$w_R = \frac{v_R^{\,2}}{R} = \frac{22.5 \cdot 10^3\ e^{-400 \cdot 2\, t}}{5000} = 4.5\, e^{-400 \cdot 2\, t}$ My Answer.

$w_R = 11.25 \cdot \left(1 - e^{-400\, t}\right)$ Schaums Answer <---.

Good example for you to fix Schaums answers are correct. Unless a typing error.
Check with your local engineer/instructor.

Table Section 7.12:

The table for step and input response of RL and RC circuits provided on last page. It was left out for you to get it from your textbook, its provided here from Schaums page 157 in section 7.12. SEE NEXT PAGE.

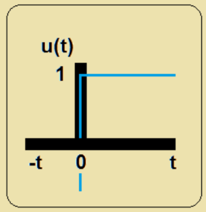
End of Chapter 7 First-Order Circuits (Schaums).

Next to Chapter 8 Higher-Order Circuits and Complex Frequency.

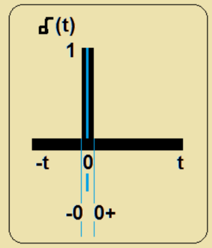
For all any any errors and omissions apologies in advance.

Summary of Step and Impulse Responses in RC and RL Circuits.

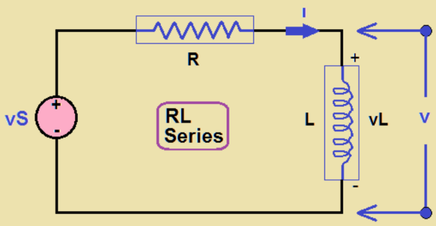
Step and Impulse Response In RL Circuit



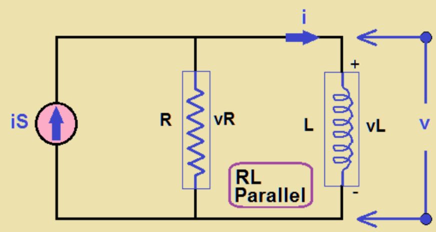
Unit Step Response



Unit Impulse Response

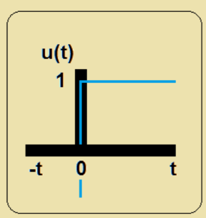


Unit Step Response	Unit Impulse Response
$vS = u(t)$	$vS = \delta(t)$
$v = e^{-Rt/L} u(t)$	$hv = (R/L)\, e^{-Rt/L} u(t) + \delta(t)$
$i = (1/R)(1 - e^{-Rt/L})u(t)$	$hi = -(1/L)\, e^{-Rt/L} u(t)$

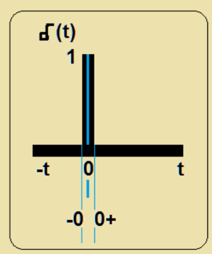


Unit Step Response	Unit Impulse Response
$iS = u(t)$	$iS = \delta(t)$
$v = R\, e^{-Rt/L} u(t)$	$hv = -(R^2/L)\, e^{-Rt/L} u(t) + R\,\delta(t)$
$i = (1 - e^{-Rt/L})u(t)$	$hi = (R/L)\, e^{-Rt/L} u(t)$

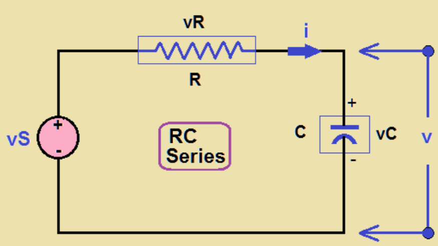
Step and Impulse Response In RC Circuit



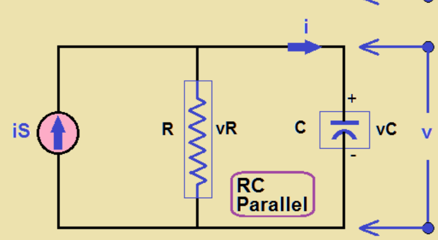
Unit Step Response



Unit Impulse Response



Unit Step Response	Unit Impulse Response
$vS = u(t)$	$vS = \delta(t)$
$v = (1 - e^{-t/RC})\, u(t)$	$hv = (1/RC)\, e^{-t/RC} u(t)$
$i = (1/R)\, e^{-t/RC} u(t)$	$hi = -(1/R^2C)\, e^{-t/RC} u(t) + (1/R)\,\delta(t)$



Unit Step Response	Unit Impulse Response
$iS = u(t)$	$iS = \delta(t)$
$v = R(1 - e^{-t/RC})\, u(t)$	$hv = (1/C)\, e^{-t/RC}\, u(t)$
$i = e^{-t/RC}\, u(t)$	$hi = -(1/RC)\, e^{-t/RC} u(t) + \delta(t)$

End.