Introduction to
<u>Basics</u> of Quantum Mechanics for

Solar Engineering Physics and PV Power Plants.

A set of solved example problems.
From chapters 2, 3, 4, and 5 of the book titled ‘Quantum Mechanics A Textbook for Undergraduates’. Mahesh C. Jain. PHI. 2007.

Entered by: Karl S. Bogha. Year: 2019.
ME Studies, University of Canterbury. New Zealand.

Any errors and omissions apologies in advance.

Constants:

$h := 6.63 \cdot 10^{-34}$ J/s h is the 'quantum of energy'.
$c := 3 \cdot 10^{8}$ ms^-1
$eV := 1.6 \cdot 10^{-19}$ J
$e := 1.6 \cdot 10^{-19}$ J

Problem 2.1

Show that Planck's law reduces to Wein's law in the short wavelength limit and Rayleigh-Jean's law in the long wavelength limit.

Solution:

See Chapter 3 Planck's Theory in Physics textbook by SN Ghoshal.
Page 43. Both short and long wavelength solved.
OR other Modern Physics textbooks.

Problem 2.2

Find the number of photons emitted per second by a 40W source of monochrome light of wavelength 6000 A (Angstrom - unit of length for measuring wavelength)

Solution:

nhv = E n is number of photons emitted per second
n = E/(hv)
n = E(lamda)/(hc)
v = c/(lamda), so 1/v = (lamda/c)

$\lambda := 6000 \cdot 10^{-10}$ Angstrom
$E := 40$ Watts
$h := 6.63 \cdot 10^{-34}$
$c := 3 \cdot 10^{8}$

$$n := \frac{(E \cdot \lambda)}{(h \cdot c)} = 1.207 \cdot 10^{20}$$

$n = 1.207 \cdot 10^{20}$ Number of photons per second. Ans.

Problem 2.3

The work function of a photosensitive surface is 3.45eV.
Will photoemission occur if a photon of energy 3.8eV is incident on the surface?
If yes, find in joules the maximum kinetic energy of the photoelectron.

Solution:

Energy of the photon is more than the work function of the surface, (net energy), photoemission will occur:

Work function of a metal (or material) is the minimum amount of energy required to remove an electron from its surface.

$WorkFunctionSurface := 3.2\ eV$

$PhotonEnergy := 3.8\ eV$

$KE\_of\_photoelectron := PhotonEnergy - WorkFunctionSurface$

$KE\_of\_photoelectron = 9.6 \cdot 10^{-20}$ Ans.

Problem 2.4

The work function of a metal is 3.45 eV.
What is the maximum wavelength of a photon that can eject an electron from the metal.

Solution:

v_o is the tresh hold frequency
h v_o = WF (work function)
v_o = c/lamda_o
now hc/lamda_o = WF
so lamda_o = hc/WF

$WF := 3.45 \cdot \left(1.6 \cdot 10^{-19}\right)$

$\lambda\_ := \frac{(h \cdot c)}{WF} = 3.603 \cdot 10^{-7}$ meter

$Angstrom\_unit := 1 \cdot 10^{-10}$ measure of Angstrom

$\lambda\_ := 3603\ Angstrom$ Ans.

Problem 2.5

A metal of work function 3.0eV is illuminated by light of a wavelength 3000 Angstorm. Calculate
(a). tresh hold frequency
(b). maximum energy of photoeletrons
(c). stopping potential

Solution:

a).
v_o is the tresh hold frequency
h v_o = WF (work function)
v_o = WF/h

$WF := 3.0\ eV \qquad eV = 1.6 \cdot 10^{-19}$

$$v\_o := \frac{WF}{h} = 7.24 \cdot 10^{14}$$ Hz. Ans.

b).
v = c/lamda

$c := 0$ clear
$c := 3 \cdot 10^{8}$ m/s

$\lambda := 3000 \cdot 10^{-10}$

$c = 3 \cdot 10^{8}$ m/s speed of light

$$v := \frac{c}{\lambda} = 1 \cdot 10^{15}$$ Hertz (1/s) frequency of incident radiation. Ans.

c).
Maximum energy E = h(v - v_o)

$E_{photon} := h \cdot (v - v\_o)$

$E_{photon} = 1.83 \cdot 10^{-19}$ J.

Energy in electron volt eV:

$$E_{photon\_eV} := \frac{E_{photon}}{eV} = 1.144$$ eV Ans.

d).

Stopping potential Vo = Emax/e = (h/e)(v - v_o)

$V_o := \left(\frac{h}{e}\right) \cdot (v - v\_o)$ $V_o = 1.144$ Volt Ans.

OR

$V_{o1} := \frac{E_{photon}}{eV} = 1.144$ Volt Ans.

Problem 2.6

Find the frequency of the light which ejects from a metal surface electrons, fully stopped by a retarding potential of 3V.
The photoelectric effect begins in this metal at a frequency of 6x10^14 s^-1.
Find the work function for this metal.

Solution:

Threshold frequency $v\_o := 6 \cdot 10^{14}$

WF = h v_o
$WF := h \cdot v\_o = 3.978 \cdot 10^{-19}$

$WF_{eV} := \frac{WF}{eV} = 2.486$ eV

eV_o = hv - hv_o

v = (eV_o + h_v_o)/h

retarding potential = 3eV $eV = 1.6 \cdot 10^{-19}$

$eV\_o := 3 \cdot eV = 4.8 \cdot 10^{-19}$

$v := \frac{(eV\_o + h \cdot v\_o)}{(h)}$

$v = 1.324 \cdot 10^{15}$ Hz Ans.

Problem 2.7

Work function of Na (Sodium) is 2.3eV.
Does sodium show photoelectric emission for light of wavelength 6800 Armstrong?
h = 6.63 x 10^-34 Js.

Solution:

$\lambda := 6800 \cdot Angstrom \qquad \lambda = \left(6.8 \cdot 10^{-7}\right) m \qquad c := 3 \cdot 10^{8} \frac{m}{s} \qquad h := 6.63 \cdot 10^{-34} J \cdot s$

$eV = 1.6 \cdot 10^{-19} \qquad eV := 1.6 \cdot 10^{-19} J$

Energy of incident photon = hc/(lamda)

Energy of incident photon = hc/lamda (ie at this wavelength)

$$E_{photon} := \frac{(h \cdot c)}{\lambda} = \left(2.925 \cdot 10^{-19}\right) J$$

$$E_{photon\_eV} := \frac{E_{photon}}{eV} = 1.828 \quad eV$$ Ans.

Energy of incident photon 1.83 eV is less than 2.3 eV,
No photoelectric emission is possible with the given light (wavelength).

Problem 2.8

Light of wave length 3500 Angstrom is incident on two metals A and B.
Which metal will yield photoelectrons if their work functions are 4.2 eV and 1.9 eV respectively.

The work function of the metal is 0.1 eV.

Solution:

$\lambda := 3500 \cdot 10^{-10}$ Angstrom
$h := 6.6 \cdot 10^{-34}$ J.s $\qquad c := 3 \cdot 10^{8}$ m/s $\qquad eV := 1.6 \cdot 10^{-19}$

$$E := \frac{(h \cdot c)}{\lambda} = 5.657 \cdot 10^{-19} \qquad E\_ev := \frac{E}{eV} = 3.536 \quad eV$$

E_ev > 1.9eV of B, this light wavelength 3500 Angstom will generate photoelectron. Ans.
E_ev < 4.2 eV of A, this light wavelength 3500 Angstom will NOT generate photoelectron.

Problem 2.9

Calculate the maximum kinetic energy of a photoelectron (in eV) emitted on shining light of wavelength 6.2 x 10^-6m on a metal surface.

The work function of the metal surface is 0.1 eV.

Solution:

$\lambda := 6.2 \cdot 10^{-6}$ m $W := 0.1$ eV $h := 6.6 \cdot 10^{-34}$ J.s $c := 3 \cdot 10^{8}$ m/s

Maximum kinetic energy of a photoelectron is given by:
Emax = hv - W = (hc/lamda) - W

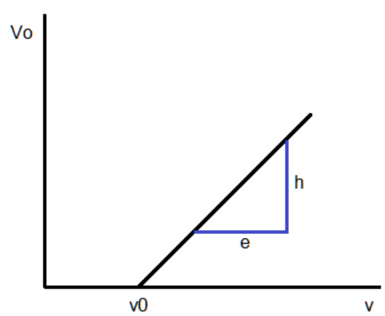
$$Emax := \frac{(h \cdot c)}{(\lambda)\ (eV)} - (W) = 0.1$$ eV Ans.

Problem 2.10

In an experiment on photoelecric effect, the slope of the cut-off voltage versus frequency of incident light graph is found to be 4.12x10^-15 Vs. Given e = 1.6 x10^-19 C.

Estimate the value of Planck's constant.

Solution:



v0: treshold frequency
slope of graph: h/e.
Planck's constant can be determined by measurig the slope of the graph

eVo = hv - W
OR
Vo = (h/e)v - (W/e)

Slope of the Vo-v curve = h/e

h = slope x e

$slope := 4.12 \cdot 10^{-15}$ Vs
$e := 1.6 \cdot 10^{-19}$ J
Planck's constant h?
$h := slope \cdot e = 6.592 \cdot 10^{-34}$ Js Ans.

Problem 2.11

What should be the frequency of the incident radiation to eject electrons of maximum speed 10^6 m/s from potassium metal?

Work function of potassium is 2.26 eV

Solution:

$W\_potassium := 2.26 \cdot eV = 3.616 \cdot 10^{-19}$ J (convert eV to Joules).

Maximum kinetic energy (1/2) m v^2= hv - W
hv = (1/2) m v^2 + W
v =((1/2) m v^2 + W) / h

$m\_potassium := (9 \cdot 10^{-31})$ mass of potassium electron kg

$max\_speed\_potassium\_electron := (10^{6})$ m/s

$$e\_max\_speed := \left(\frac{1}{2}\right) \cdot (m\_potassium) \cdot (max\_speed\_potassium\_electron)^{2}$$

$$e\_max\_speed = 4.5 \cdot 10^{-19}$$

$$hv\_potassium := e\_max\_speed + W\_potassium = 8.116 \cdot 10^{-19}$$

v_frequency

$$frequency\_v := \frac{(hv\_potassium)}{(h)} = 1.231 \cdot 10^{15}$$ Hz Ans.

Comment: An incandescent lamp frequency is 50 or 60 Hz, (just under 10^2 ie 100 Hz) and this potassium electron's frequency is at 10^15 Hz. This is several millions times higher, 10^13, than the visible light we see from an incandescent lamp!

Problem 2.12

(a).

A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Angstrom. For light of wavelength 3000 Angstrom, the stopping potential is 1.85 V. Find the value of Planck's constant.

(b).

At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Angstrom but the intensity of light is increased two times, will photoelectric current be obtained? Give reasons for your answer.

Solution:

a).

(hc/lamda-1) = eV1 + W ....equation 1.

(hc/lamda-2) = eV2 + W ....equation 2.

W is the same for equation 1 and 2 because its the same metal surface.

Subtracting 1 from 2

hc ((1/lamda-2) - (1/lamda-1)) = e(V2 - V1)

Rearrainging for h:

h = (e (V2 - V1)) / c ((1/lamda-1) - (1/lamda-2))

$eV = 1.6 \cdot 10^{-19}$ $\quad V1 := 0.82$ $\quad V2 := 1.85$ $\quad \lambda 1 := 4000 \cdot 10^{-10}$ $\quad \lambda 2 := 3000 \cdot 10^{-10}$

$c := 3 \cdot 10^{8}$

$A := eV \cdot (V2 - V1) = 1.648 \cdot 10^{-19}$

$$B := c \cdot \left(\left(\frac{1}{\lambda 2} \right) - \left(\frac{1}{\lambda 1} \right) \right) = 2.5 \cdot 10^{14}$$

$h := \frac{A}{B} = 6.592 \cdot 10^{-34}$ Js. Ans.

b). No. When the intensity is increased the stopping potential does not increase because the stopping potential depends only on the wavelength of light NOT its intensity. Ans.

Problem 2.13

Light of wavelength 4560 Angstrom and power 1 mW is incident on a caesium surface.
Calculate the photoelectric current, assuming a quantum efficiency of 0.5%.
Work function of Cesium = 1.93 eV; h = 6.62 x 10^-34 Js.

Solution:

$W\_cesium := 1.93$ eV
$Power\_cesium := 1 \cdot 10^{-3}$ mW
$e := 1.6 \cdot 10^{-19}$ J
$h := 6.62 \cdot 10^{-34}$ J
$c := 3 \cdot 10^{8}$ m/s
$\lambda := 4560 \cdot 10^{-10}$

Energy of one photon = hv = (hc)/(lamda).

$$Energy\_1photon := \frac{(h \cdot c)}{(\lambda)} = 4.355 \cdot 10^{-19} \quad J$$

Number of photons incident on the surface of cesium per second

$$No\_photons := \frac{Power\_cesium}{Energy\_1photon} = 2.296 \cdot 10^{15}$$

Since only 0.5% of the photons incident on the cesium surface release electrons, the number of electrons released per second is

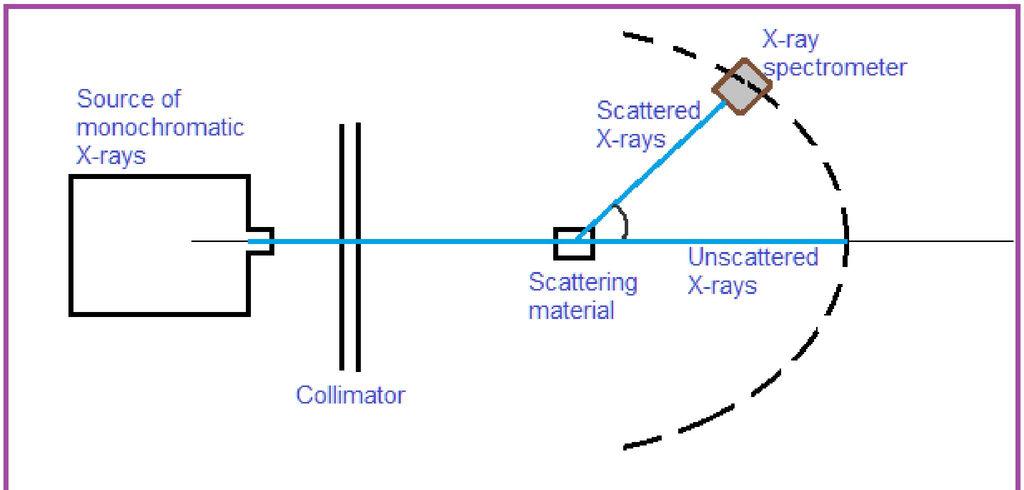
$$No\_electrons := No\_photons \cdot \left(\frac{0.5}{100}\right) = 1.148 \cdot 10^{13}$$

Finally we reach to where we can now calculate the photoelectric current, ne:

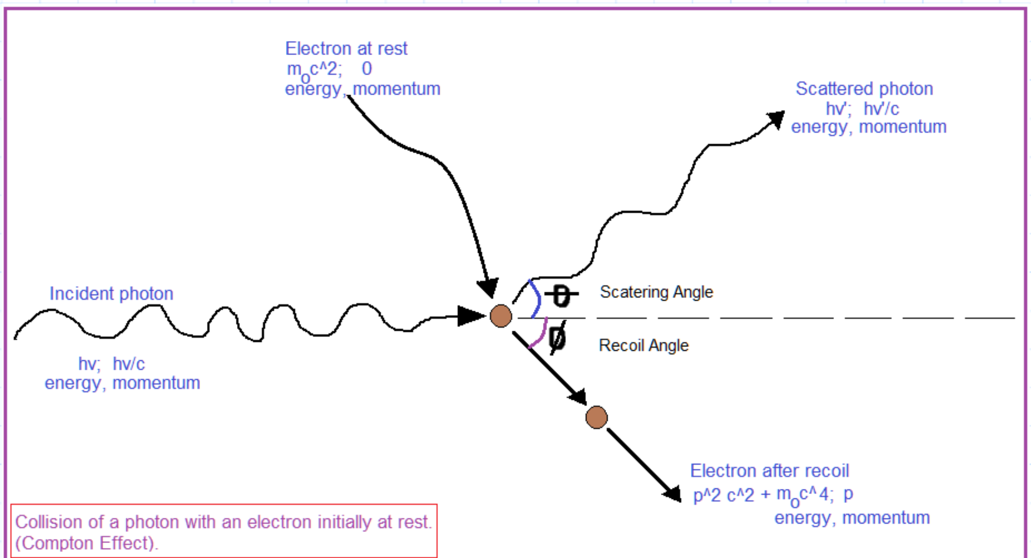
$$Photoelectric\_current := No\_electrons \cdot e = 1.837 \cdot 10^{-6}$$ Amps. Ans.

Problems 2.14 thru 2.19 relate to Compton Effect and Kinetic Energy of the Recoil Electron. See Figure below.

Refer to Mahesh C. Jain OR other Modern Physics textbooks for formulas and deriviations.



Experimental arrangement for observing Compton scattering.



Collision of a photon with an electron initially at rest. (Compton Effect).

$$\Delta\lambda = \frac{2h}{m_0 c}\sin^2\frac{\theta}{2}$$

$$\cot\phi = \frac{v - v'\cos\theta}{v'\sin\theta}$$

$$\cot\phi = (1+\alpha)\tan\frac{\theta}{2}$$

$$\alpha = \frac{hv}{m_0 c^2}$$

$$E = hv\,\frac{2\alpha\cos^2\phi}{(1+\alpha)^2 - \alpha^2\cos^2\phi}$$

The quantity $(h/m_0 c)$ is called the Compton wavelength of the electron. Its value is 0.0242 Angstrom. This quantity is NOT the wavelength of the eletron. It is the shift in wavelength of a photon scattered off an electron at 90 degrees to the initial direction. If m_0 is replaced by the mass of the atom, change in wavelength is negligible because an atom is many thousand times heavier than an electron.

Problem 2.14

X-Rays of wavelength 2.0 Angstrom are scattered from a carbon block.
The scattered photons are observed at right angles to the direction of the incident beam.

Calculate:
a). The wavelength of the scattered photon.

b). The energy of the recoil electron.

c). The angle at which the recoil electron appears.

Given: Rest mass of an electron m_o = 9.1 x 10^-31 kg, c = 3 x 10^8 m/s, and
h = 6.6 x 10^-34 Js.

Solution:
a).
λ and λ' are the wavelength of theincident and scattered photon, and
θ the scattering angle, then

delta lamda = lamda' - lamda =(h/m_o c)(1 - cos (theta)).

$\lambda := 2.0 \cdot 10^{-10}$ *Its already in Angstrom; ie 10^-10.*

$h := 6.6 \cdot 10^{-34}$ J
$c := 3 \cdot 10^{8}$ m/s
$m_0 := 9.1 \cdot 10^{-31}$ kg
$\theta := 90\ \textit{deg}$

$$\Delta\lambda := \left(\frac{h}{m_0 \cdot c}\right)(1 - \cos(\theta)) = 2.418 \cdot 10^{-12}$$

$$\frac{\Delta\lambda}{10^{-10}} = 0.024 \quad \text{Angstrom.}$$

lamda' = lamda + delta lamda = (h/m_o c)(1 - cos (theta)).

$\lambda' := \lambda + \Delta\lambda = 2.024 \cdot 10^{-10}$ Ans.

b). Neglecting the binding energy of the electron, its recoil energy is given by

E = h(v - v') where v is the frequency
= hc ((1/lamda) - (1/lamda'))

$$E := (h \cdot c) \cdot \left(\left(\frac{1}{\lambda}\right) - \left(\frac{1}{\lambda'}\right)\right) = 1.182 \cdot 10^{-17} \quad \text{J Ans.}$$

c). The angle theta at which the recoil electron appears is given by

cot (phi) = (1/sin(theta) ((v/v') - cos(theta))
= (1/sin(theta) ((lamda'/lamda) - cos(theta))

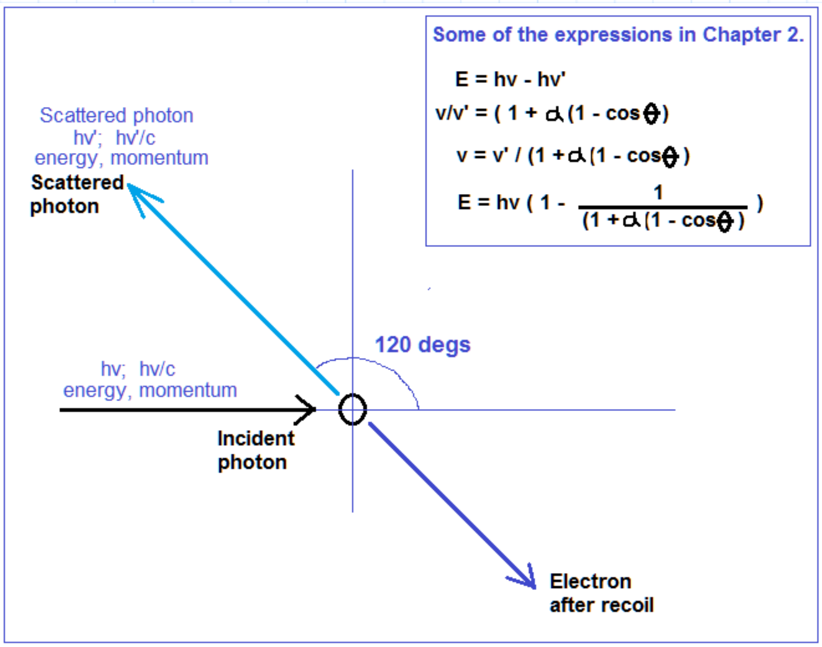
$$\cot(\phi) := \left(\frac{1}{\sin(\theta)}\right) \cdot \left(\left(\frac{\lambda'}{\lambda}\right) - \cos(\theta)\right) = 1.012$$

$\phi := \text{acot}(1.012) = 44.658\ \mathit{deg}$ Ans.

Problem 2.15

A photon of energy 0.9 MeV is scattered through 120 degs by a free electron.
Calculate the energy of the scattered photon.

Solution:



We desire to calculate hv'.

$$\alpha := \frac{(h \cdot v)}{\left(m_0 \cdot c^2\right)}$$

v' = (v)/(1 + (alpha)(1 - cos(theta))
Multiply by h both sides
hv' = (hv)/(1 + (alpha)(1 - cos(theta))
Applying trig identity
hv' = (hv)/(1 + 2(alpha)sin^2(theta/2))

Given hv = 0.9 MeV.

$h := 6.6 \cdot 10^{-34}$ J
$c := 3 \cdot 10^{8}$ m/s
$m_0 := 9.1 \cdot 10^{-31}$ kg
$\theta := 120\ deg$

$$\alpha := \frac{(h \cdot v)}{\left(m_0 \cdot c^2\right)}$$

$a1 := 0.9 \cdot 10^{6}$ eV

$$a2 := \frac{\left(m_0 \cdot c^2\right)}{(eV)} = 5.119 \cdot 10^{5} \quad eV$$

$$b1 := \sin(60\ deg)^2 = 0.75$$

hv' = (hv)/(1 + 2(alpha)sin^2(theta/2))
= (a1)/(1 + 2 (a1/a2)(b1)

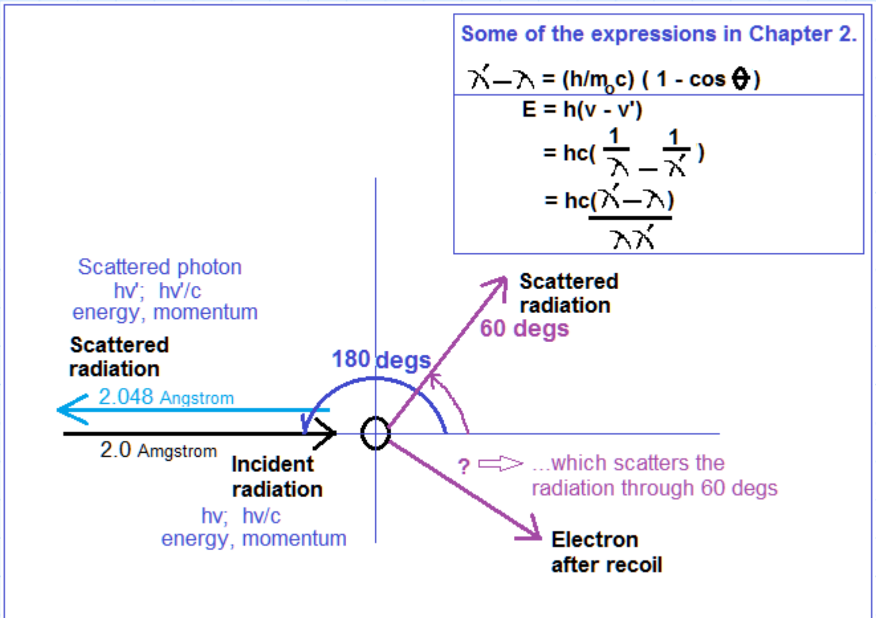
$$hv' := \frac{a1}{\left(1 + 2\left(\frac{a1}{a2}\right) \cdot b1\right)} = 2.474 \cdot 10^{5}$$

$hv' := 0.2744 \cdot 10^{6}$ OR 0.2474 MeV Ans.

Problem 2.16

In a Compton scattering experiment, the incident radiation has wavelength 2.000 Angstrom while the wavelength of the radiation scattered through 180 degs is 2.048 Angstrom.
Calculate:
a). the wavelength of the radiation scattered at an angle of 60 degs to the direction of incidence
b). energy of the recoil electron which scatters the radiation through 60 degs.

Solution:



*Please note this figure may be incorrect.
You verify.*

(lamda)' - (lamda) = (h/m_o c)(1 - cos(theta))
We know lamda and theta, so we can calculate (h/m-o c), here theta is 180 degs that is the radiation at 180 degs. At 180 degs we have a specific radiation 2.048, and at 0 deg (incident) we have a specific radiation 2.048. Solving for (h/m_o c) then we proceed to 0 degs incidence and 60 degs scattered radiation.

$$\lambda'\_minus\_\lambda := (2.048 - 2.0)\cdot 10^{-10} = 4.8\cdot 10^{-12}$$

$$a1 := (1 - \cos(180\,deg)) = 2$$

$$h\_div\_m_o c := \frac{\lambda'\_minus\_\lambda}{a1} = 2.4\cdot 10^{-12}\ \text{m}$$

a). When theta = 60 degs to the incident radiation
(lamda)' = (lamda) + (h/m_o c)(1 - cos(theta)) by rearranging

$$\lambda' := 2.000\cdot 10^{-10} + (h\_div\_m_o c)\cdot(1 - \cos(60\,deg)) = 2.012\cdot 10^{-10}\ \text{m}$$ Ans.

OR Ans is 2.012 Angstrom.

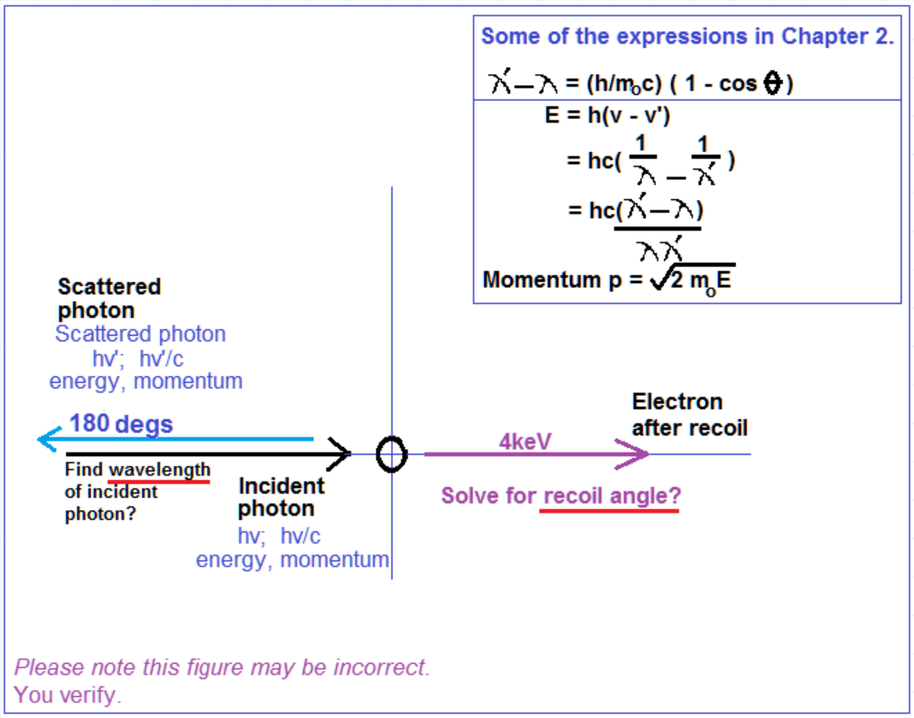
b). We know both the wavelenghts; lamda' and lamda. we can plug these in to the hv - hv' expression where we substitute lamda for v. Where v = 1/lamda.
Therefore, hv - hv' = hc ((1/lamda') - (1/lamda))

$$E\_recoil\_60degs := (h \cdot c) \cdot \left(\left(\frac{1}{\lambda} \right) - \left(\frac{1}{\lambda'} \right) \right) = 5.905 \cdot 10^{-18} \text{ J}$$ Ans.

Problem 2.17

In a Compton scattering experiment, the X-ray photon is scattered at an angle of 180 degs and the electron recoils with an energy of 4keV.
Calculate the wavelength of the incident photon.

Solution:



Kinetic energy of the recoil electron in Joules:

$e\_recoil\_E := 4 \cdot 10^{3}$ eV $\qquad m_0 := 9.1 \cdot 10^{-31}$ kg mass of electron

$KE\_e\_recoil := e\_recoil\_E \cdot eV = 6.4 \cdot 10^{-16}$ J.

Momentum p of electron:

$p := \sqrt{(2 \cdot m_0 \cdot KE\_e\_recoil)} = 3.413 \cdot 10^{-23}$ kg m/s

Let lamda be the wavelength of the incident photon and
lamda' be the wavelength of the scattered photon.
Conservation of energy gives
(hc/lamda) - (hc/lamda') = KE of recoil electron = 6.4*10^-16 J ......(I)

Exactly where is the recoil electron's direction?

Use the cot(phi) = (1 + alpha) tan (theta/2) tp solve for angle psi.

$h := 6.3 \cdot 10^{-34}$ J
$c := 3 \cdot 10^{8}$ m/s
$m_0 := 9.1 \cdot 10^{-31}$ kg
$\theta := 180\ deg \qquad \theta 1 := \frac{\theta}{2} = 90\ deg$

$$\alpha := \frac{(h \cdot c)}{\left(m_0 \cdot c^2\right)} = 2.308 \cdot 10^{-12}$$

$a1 := (1.0 + \alpha) \qquad a1 = 1$ Approximately equal 1 the decimal value 2.418x10^-12is almost 0.

$a2 := \tan(\theta 1) = 1.633 \cdot 10^{16}$ Approximately infinity which is logical for tan 90 deg - invalid.

Therefore a1 x a2 = 1.0 approximately.

Cot(phi) = 1, acot(1) = ? deg Not valid.
There are no table values of acot(1), a discussion or inspection may lead to conclude the angle phi is 0 deg. The angle phi is the angle of the recoil electron, the scattered photon is 180 deg ie the angle theta. So we say the recoil electron is in the same direction as the incident photon - 0 degs.
Scattered and recoil are 180 degrees apart. See figure above.

Applying the law of conservation of momentum in the direction of the incident as positive:
recoil + scattered = incident:
p cos(phi) + (h/lamda')cos(theta) = h/(lamda)....Eq 2.17-1

$b1 := \cos(0\ deg) = 1$ We calculated p prior, therefore p x (cos(0) = p x 1
$p\cos\theta := p \cdot b1 = 3.413 \cdot 10^{-23}$
$b2 := \cos(180\ deg) = -1$ as expected -ve sign since scattered is in opposite direction to incident.

Now after rearranging Eq 2.17-1
(h/lamda) + (h/lamda') = 3.413*10^-23
Multiply both sides by c.....this now gives the conservation of momentum expression
(hc/lamda) + (hc/lamda') = 3.413*10^-23 (c)

$LHS\_m := p \cdot c = 1.024 \cdot 10^{-14}$

(hc/lamda) + (hc/lamda') = 1.024*10^-14 ....(II)

Adding (I) and (II)

LHS = 2(hc/lamda).

$RHS := KE\_e\_recoil + LHS\_m = 1.08787 \cdot 10^{-14}$

Therefore lamda = (2hc)/RHS

$\lambda := \frac{(2 \cdot h \cdot c)}{(RHS)} = 3.4747 \cdot 10^{-11}$ m

$\lambda := 0.3474 \cdot 10^{-10}$ Angstrom. Ans.

Problem 2.18

a). What is the maximum kinetic energy that can be imparted to a free electron by a photon of initial frequency v?
b). Is it possible for the photon to transfer all its energy to the electron?

Solution:

a). Kinetic energy of a recoil electron is

$$E = (hv) \frac{(\alpha)(1 - \cos(\theta))}{1 + (\alpha)(1 - \cos(\theta))}$$

What is the impact of cos(theta)? If angle theta is 180 degs the cos(180 deg) = -1.
This value -1 increases the value of E it makes E maximum.
Emax when theta = 180 degs. Correct.

$$E_{max} = (hv) \frac{(\alpha)(1 - (-1))}{1 + (\alpha)(1 - (-1))}$$

$$E_{max} = (hv) \frac{(\alpha)(2)}{1 + (\alpha)(2)}$$

$$E_{max} = (hv)\left(\frac{2(\alpha)}{1 + 2(\alpha)}\right)$$ Ans.

b).

For any real value alpha takes the RHS's left most term (2alpha/(1+2 alpha) is less than 1. Which makes the 2nd term in the RHS term less than 1.0.
Hence, Emax < hv since hv is multiplied by the factor less than 1.0.

Emax < hv.
Since the maximum energy on the LHS is less than hv, the free electron cannot receive the maximum energy from the photon i.e. hv rather less than hv.

So, a photon cannot transfer all its energy to a free electron. Thanks! Ans.

Note: The Compton effect (delta)lamda = (h/m_o c) (1 - cos(theta)) here too we see when angle theta is 180 degs, cos(180) = -1, then (1 - cos(180)) = 1 - (-1) = 2.
(delta) lamda = lamda' - lamda = 2(h/m_o c). The maximum Compton shift.

Problem 2.19

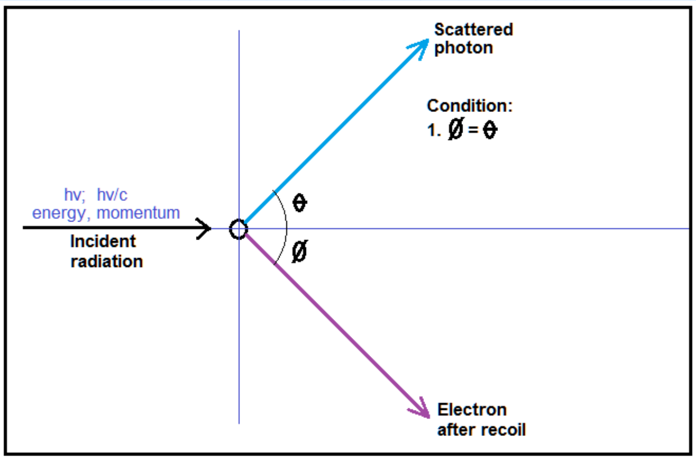
Gamma ray photons of energy 1.02 MeV are scattered from electrons which are initially at rest.

a). Find the angle for symmetric scattering (i.e. theta = phi) at this energy.

b). What is the energy of the scattered photon for this case?

Solution:

a).



Relation between theta and psi:

cot (psi) = (1 + alpha)(tan (theta/2))

Set angle **phi** = angle **theta**.

Rewriting in sin and cos terms, and substitute angle pis for theta.

cos (theta)/sin(theta) = (1 + alpha) (sin (theta/2)/cos(theta/2))

We want to solve for angle theta.

The physicist Mahesh C. Jain reduced the expression above using trig identities and re-arranging to:

Sin(theta/2) = 1 / (Sqrt(2(alpha + 2) *.....you may verify this yourself*

Evaluating alpha:

alpha = hv/m_0 c^2

Numerator = 1.02 MeV
Denominator = half the numerator = 0.51 MeV

$$\alpha := \frac{1.02}{0.51} = 2$$

$$\sin(\text{theta\_div\_2}) := \frac{1}{\sqrt{(2\cdot(2+2))}}$$

$$\sin(\text{theta\_div\_2}) := \frac{1}{\sqrt{(8)}} = 0.354$$

$\theta := 2\cdot(\operatorname{asin}(0.354)) = 41.464\ \textbf{deg}$ Ans. *You may verify this yourself.*

b).

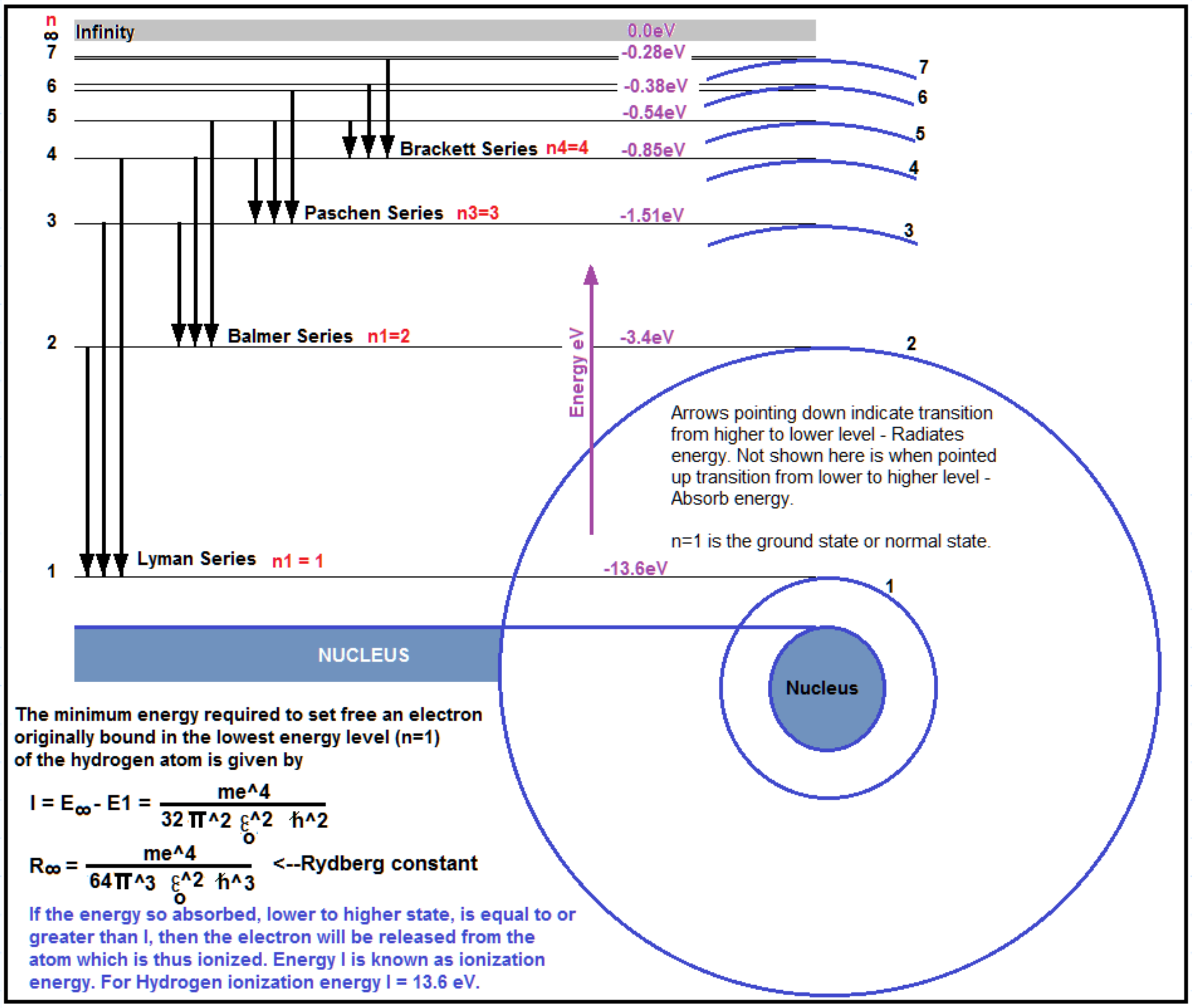
Energy of the scattered photon:

hv' = hv / (1 + alpha(1 - cos(theta))

$$hv' := \frac{(1.02)}{(1+\alpha\cdot(1-\cos(\theta)))} = 0.679 \quad \text{MeV}$$ Ans.

End of file.

Bohr's model is essential for understanding the continuing chapters in modern/atomic physics. Loosely though there were later found flaws and replaced by quantum mechanic's explanation(s). It is essential for the student to understand Bohr's atomic model before progressing to more suitable theories. Bohr proposed 3 postulates which were in contradiction to laws of classical mechanics and electromagnetic theories. This subject matter may be found in any current UG modern physics textbook.Calculations are good, its just the assumption or background on postulates is off.



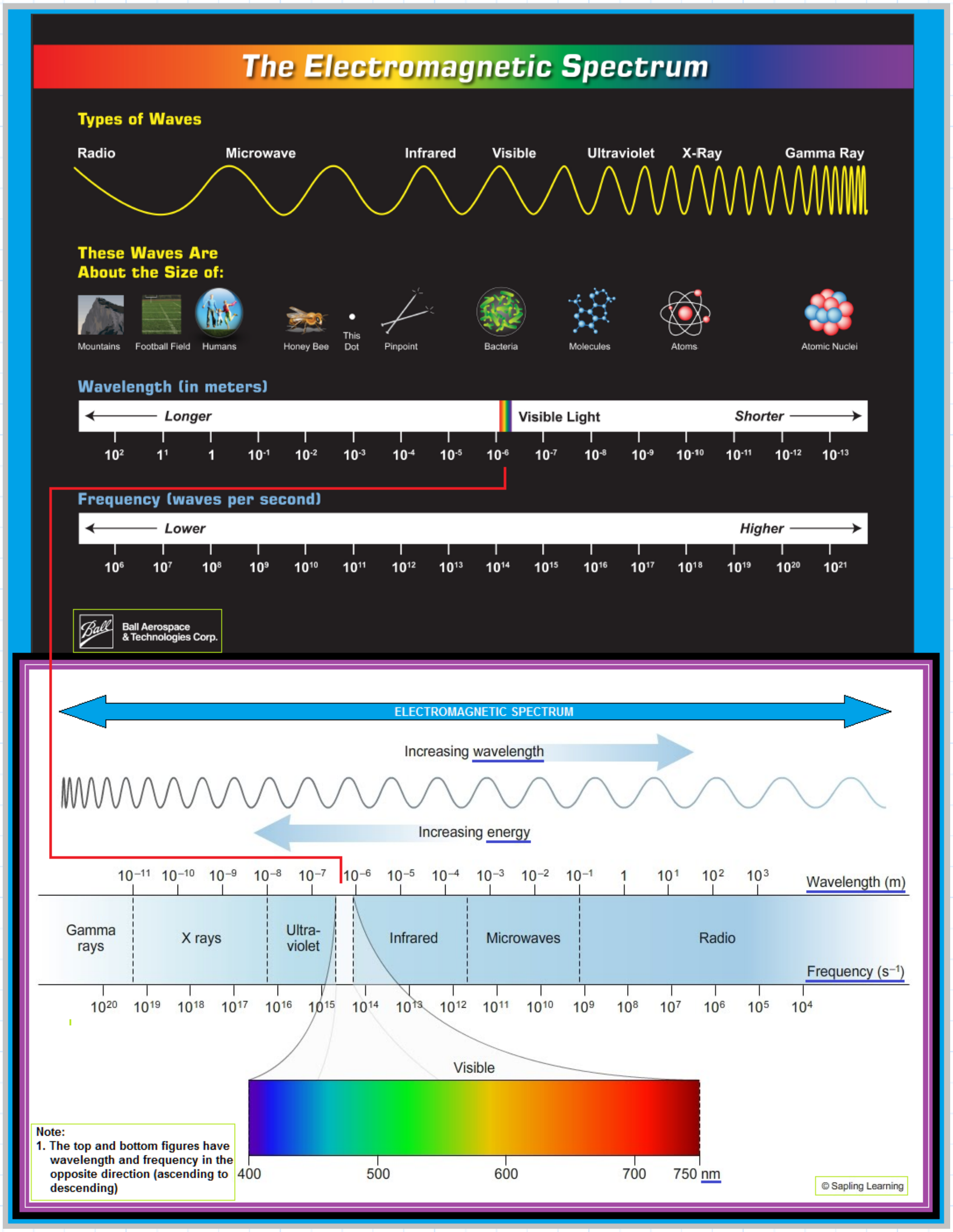
Resource: Modern Physic (Atomic) by S.N. Ghosal.

Figure: Energy level of Hydrogen.

Bohr's theories and explanations are not all knew as shown in the figure above. You can related to them. You should have some understanding of the equations in the figure above, these would be found in most UG modern physics or quantum mechanics textbook.

The contents of the subject matter here leads to or generated the electromagnetic spectrum. The spectrum we have seen in many ocassions in Physics, and Electrical Engineering courses. The parameters here are wavelength, and frequency. Part I we had Plank's contant h, wavelength (lamda), and frequency (v), eventually leading to the spectrum.

Quantum Mechanics A Textbook for Undergraduates. Mahesh C. Jain. PHI. Department of Physics, Hindu College University of Delhi.
To Support Studies In: Modern Physics by S.N. Ghosal.
Purpose: Quantum Mechanics for 'Power Plant Engineering' Studies.
Exercise by: K S Bogha. Part II Structure of the Hydrogen Atom and The Bohr Model.



Figures above of the electromagnetic spectrum; source Ball Aerospace Tech... and Sapling Learning.
Note the order of frequency-wavelength are in the opposite direction in the two figures.

Constants:

$h := 6.63 \cdot 10^{-34}$ Js
$c := 3 \cdot 10^{8}$ m/s
$eV := 1.6 \cdot 10^{-19}$ J
$e := 1.6 \cdot 10^{-19}$ J

Problem 3.1

The energy of an exited hydrogen atom is -3.4eV.
Calculate the angular momentum of the electron according to Bohr theory.

Solution:

Brief notes:

Bohr's radius of Hydrogen a_0 = (4(pi)(epsilon_0)(h/2pi))/ (me^2)).

Energy in terms of Bohr's radius a_0, En = -((2pi/h) Z^2) / (2 m a_0 n^2).

En = -13.6 Z^2 / n^2 eV
for Hydrogen n = 1, and Z = 1, therefore En1_Hydrogen = -13.6 eV.
similarly for the other series relative to n and Z.

Angular momentum = (nh/2 pi).

Energy level of the hydrogen atom in the nth orbit : En = -13.6 /n^2 eV.
n^2 = -13.6 eV/(-3.4) eV

$$n\_squared := \frac{-13.6}{-3.4} = 4$$

$$n := \sqrt{n\_squared} = 2$$

$$Angular\_momentum := \frac{(n \cdot h)}{(2 \cdot \pi)} = 2.11 \cdot 10^{-34}$$ Js Ans.

Problem 3.2

The energy of the ground state of hydrogen atom is -13.6eV.
Find the energy of the photon emitted in the transition from n=4 to n=2.

Solution:

The photon is falling from n=4 to n=2, higher energy to lower energy, it is radiating/ emitting energy. Initial state ni = 4(higher energy), final state nf = 2(lower energy).

Energy emitted = E_ground state x ((1/ni-lower Energy^2) - (1/nf-higher Energy^2))

$ni := 4$ initial state - higher energy

$nf := 2$ final state - lower energy

$E_{1\_H} := -13.6$ eV

$$E_{4\_2} := E_{1\_H} \cdot \left(\left(\frac{1}{ni^2} \right) - \left(\frac{1}{nf^2} \right) \right) = 2.55$$ eV +ve sign radiated or emitted energy Ans.

Lets reverse the direction.
The atom transitions from n = 2 to n =4.
The atom will absorb energy because its travelling from lower energy state 2 to higher energy state 4.
Here we reverse the order of the expression for ni and nf.

$$E_{2\_4} := E_{1\_H} \cdot \left(\left(\frac{1}{nf^2} \right) - \left(\frac{1}{ni^2} \right) \right) = -2.55$$ eV -ve sign energy absorbed Ans.

Comments: You verify on the sign convention.
Its not exactly like Kirchchoff's Law in electric circuit's convention.
Lets say if we set the convention, radiated outward is +ve sign, then absorbed would be -ve sign. Provided the ni and nf line up with the order of energy direction movement.

Problem 3.3

The H-alpha line of Balmer series is obtained from the transition n = 3 (energy = -1.5eV) to n = 2 (energy = -3.4eV).

Calculate the wavelength for this line.

Solution:

Comment: Why is the word 'line' used in the question? Its just like the spectrum chart those lines, obtained from laboratory experiements.

General expression:
E2 > E1 energy level n= 2 is higher than 1

hv = E2 - E1
frequency v = c/lamda

(hc/lamda) = E2 - E1

wavelength lamda = hc / (E2 - E1)

In our problem we go from E3 to E2 (higher to lower - radiating energy)

$E3 := -1.5$ eV
$E2 := -3.4$ eV

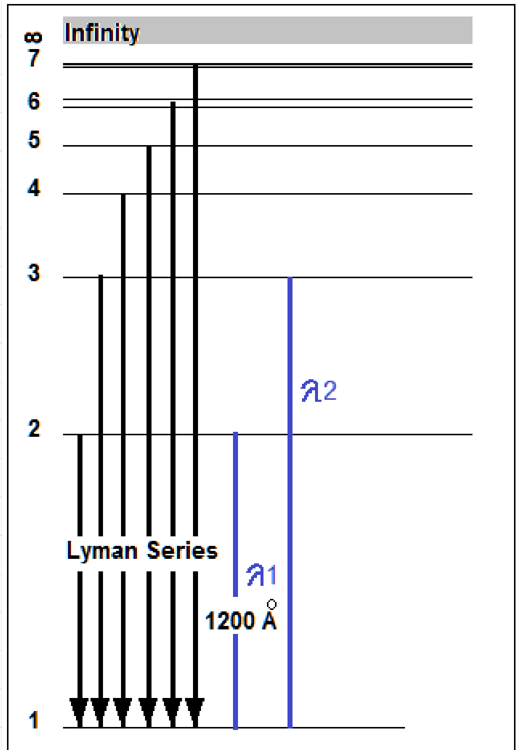
$$\lambda\_H_{\alpha\_3\_2} := \frac{(h \cdot c)}{(E3 - E2) \cdot eV} = 6.543 \cdot 10^{-7} \text{ m.}$$

$\lambda\_H_{\alpha\_3\_2} := 6543 \cdot 10^{-10}$
$\lambda\_H_{\alpha\_3\_2} := 6543$ ***Angstrom*** Ans.

Problem 3.4

The first line of the Lyman series in the hydrogen spectrum has the wavelength 1200 Angstrom.

Calculate the wavelength of the second line.



Hydrogen spectrum

Solution:

Lyman series (ultraviolet region:

1/lamda = R((1/1^2) - (1/n^2))
Lyman series starts at 1 for the first term,
for the second term n = 2,3,4,...

So moving up to the next
line is n = 2, from 1 to n=2.

1/lamda1 = R ((1/1) - (1/2^2))
= R (1 - (1/4)
= R (3/4).

Next to the line up from 2 is 3.

1/lamda2 = R ((1/1) - (1/3^2))
= R (1 - (1/9)
= R (8/9).

If we divide (1/lamda 1) by (1/lamda2) it would give us the value of (1/lamda2) through proportioning.

(1/lamda1) / (1/lamda 2) = lamda 2 / lamda 1. Correct.

We know the value of lamda1 equal 1200 Angstrom we then multipy to it to get the wavelength of lamda2.

(1/lamda2) / (1/lamda1) R(3/4) / R(8/9)

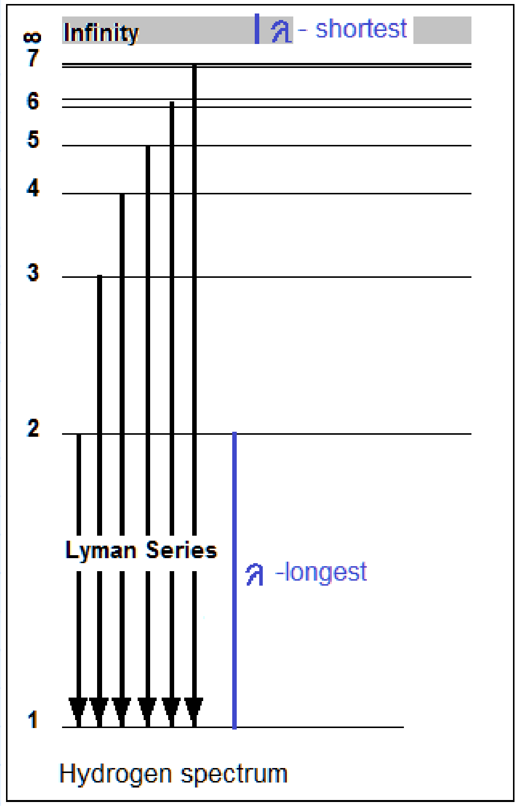
= 27/32.

lamda2 = (27/32) x 1200 = 1012.5 Angstrom Ans.

Problem 3.5

Find the longest and shortest wavelengths of the Lyman series.
Given Rydberg constant = 1.097 x 10^7 m^-1.

Solution:



For the lyman series: (1/lamda) = R(1 - (1/n^2))

Longest wavelength is between n=2 and n=1
which is actually n = 2.

1/lamda_long = R ((1/1) - (1/2^2))
= R (1 - (1/4)
= R (3/4).

$R := 1.097 \cdot 10^{7}$ 1/m (m^-1).

$One\_div\_\lambda\_long := R \cdot \left(\frac{3}{4}\right) = 8.228 \cdot 10^{6}$

$\lambda\_long := \frac{1}{One\_div\_\lambda\_long} = 1.215 \cdot 10^{-7}$

$\lambda\_long = 1215\ Angstrom$ Ans.

Shortest wavelength is between n=infinity
and n=1 which is actually n = infinity.

1/lamda_short = R ((1/1) - (1/infinity^2))
= R (1 - (0))
= R.

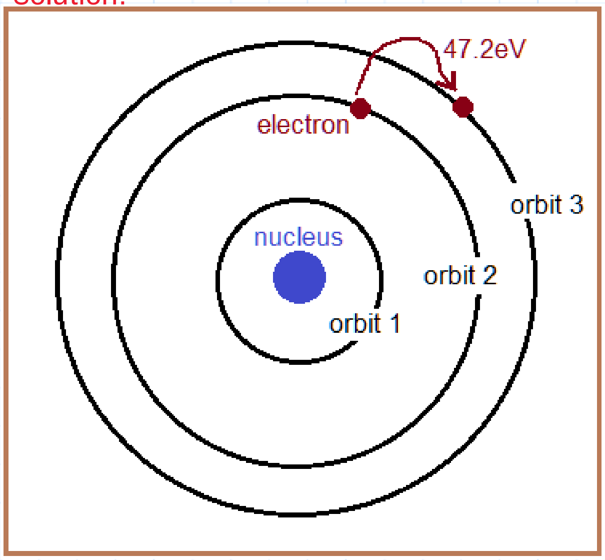
$\lambda\_short := \frac{1}{R} = 9.116 \cdot 10^{-8}$

$\lambda\_short = 911.6\ Angstrom$ Ans.

Problem 3.6

A hydrogen like atom has one electron revolving around a stationary nucleus.
The energy required to excite the electron from the second orbit to the third orbit is 47.2 eV.
What is the atomic number of the atom.

Solution:



En = (-2.2x10^-18)(Z^2/n^2) eV
this becomes
En = (-13.6)(Z^2/n^2) eV

Then for frequency and wavelength o the radiation in the transition from n2 to n1:

En = -13.6(Z^2) ((1/ni^2) - (1/(nf^2)) eV
It is -13.6 in the expression, though it was made positive 13.6 as per textbook. However the formulae/expression in textbook is -13.6.
Z is the atomic number.
Electron is transitioning from 2 to 3, moving up or outward, this requires absorbtion of energy. This is the 47.2eV absorbed i.e. excitation energy. Here we set it as negative.

Discussion: The convention we use is -ve for absorbed. The original expression was for radiated energy where ni<nf (number 2 <3), here its absorbed the opposite direction. So we reverse it to make nf be placed at ni ie at the front. So the resulting sign worked out correctly for the answer. This worked with 47.2eV set as positive.
Since, as we set 47.2 eV negative it worked out correctly with the original expression also. You verify.
En = -13.6(Z^2) ((1/nf^2) - (1/(ni^2)) eV....alternate expression

-47.2 eV = - 13.6 (Z^2) ((1/ni^2) - (1/nf^2))

$ni := 2 \qquad nf := 3$

$$RHS := -13.6 \cdot \left(\left(\frac{1}{ni^2} \right) - \left(\frac{1}{nf^2} \right) \right) = -1.8889 \quad Z^2 \text{ eV}$$

$$Z\_squared := \frac{-47.2}{-1.8889} = 24.988$$

$$Z := \sqrt{Z\_squared} = 4.999$$

Alternate with nf = 3 placed in front.
47.2 eV = - 13.6 (Z^2) ((1/nf^2) - (1/ni^2))

$$RHS\_alt := -13.6 \cdot \left(\left(\frac{1}{nf^2} \right) - \left(\frac{1}{ni^2} \right) \right)$$

$RHS\_alt = 1.889$

$$Z\_squared_{alt} := \frac{47.2}{1.8889} = 24.988$$

$Z := 5$ Atomic number of the atom at a whole number. Ans.

Problem 3.7

Which state of the triply ionized Beryllium has the same orbital radius as that of the ground state of hydrogen?

Compare the energies of the two states.

Solution:

Triply ionised Beryllium: Be+++.
Gaining 3 positive ions.
Z = 4.
n?
Hydrogen H: Z = 1.
n = 1.

Expression for the radius of the nth bohr orbit
rn = (4pi epsilon_0 ((h/2pi)^2)n^2) / (Z(e^2)m)

integer n is the quantum number, substituting epsilon_0, (h/2pi), e, and m for their values into the expression rn results in

rn = 0.53 (n^2/Z) Angstrom

Because the state of Be+++ that has the same orbital radius as H would have their radius equal. The radius is dependent on Z. Since we are comparing two similar equations the constant term 0.53 is cancelled.

((n^2)/Z) Be+++ = ((n^2)/Z) H

((n^2)/4) = ((1^2)/1) = 1

((n^2) = 4

n = 2.

n Be+++ = 2 (2nd orbit) Ans.

We compare the energies of the 2 states by having one divide the other, or how many times Hydrogen goes into the energy of Be+++.

En proportional to: -13.6(Z^2/ n^2)...in comparison -13.6 cancelled out.
En proportional to: (Z^2/ n^2)
En_Be+++ / En_H = ((4^2)/(2^2)) / ((1^2)/(1^2))
= 4 / 1
= 4 Ans.

Problem 3.8

Which state of the doubly ionised Lithium has the same energy as the ground state of hydrogen state energy of the Hydrogen atom?

Hint: Compare the orbital radii of the two states.

Solution:

Doubly ionised Lithium: Li^{++}.
Gaining 2 positive ions.
Z = 3.
n?

Hydrogen H: Z = 1.
n = 1.

rn = 0.53 (n^2/Z) Angstrom

rn = (n^2/Z) Angstrom. When comparing two elements the constant 0.53 cancelled out.

((n^2)/3) Li++ = ((n^2)/Z) H
Lets try n = 3
((3^2)/3) = ((1^2)/1) = 1
3 = 1. Ans.

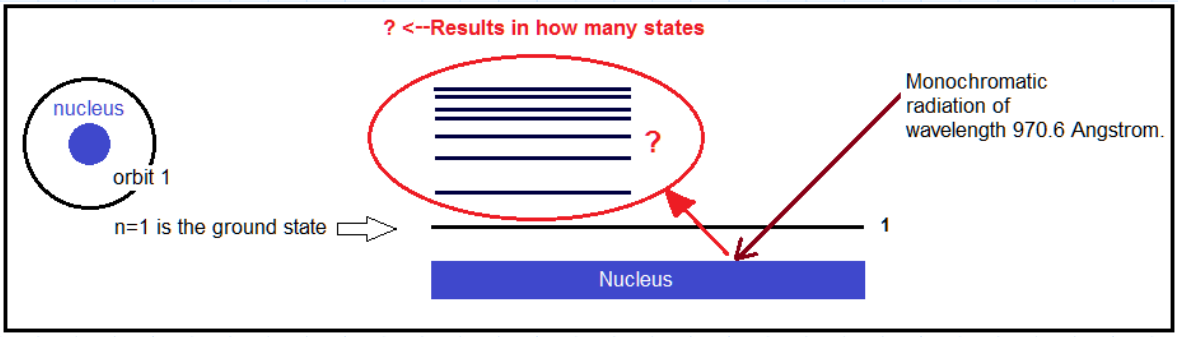
When n = 3 for Li, n = 1 for Hydrogen.
The n = 3 state of Li++ has the same energy as the the n = 1 state of H.

Problem 3.9

Hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 970.6 Angstrom.

How many different wavelengths are possible in the resulting emission spectrum? Find the longest wavelength amongst these.

Solution:



Energy of the radiation quantum (hv):

Since we are given the wavelength of the excitor source we have to use lamda expression for hv, hc/lamda to calculate E.

$\lambda := 970.6 \cdot 10^{-10}$ m^-1

$h := 6.6 \cdot 10^{-34}$ Js

$E := \frac{(h \cdot c)}{\lambda} = 2.04 \cdot 10^{-18}$ J

$E\_eV := \frac{E}{eV} = 12.75$ eV energy of the monochromatic radiation.

$E\_Hydrogen := -13.6$ eV in the unexcited state at n = 1.

Final or resulting energy after the excitation energy is struck on the Hydrogen atom:

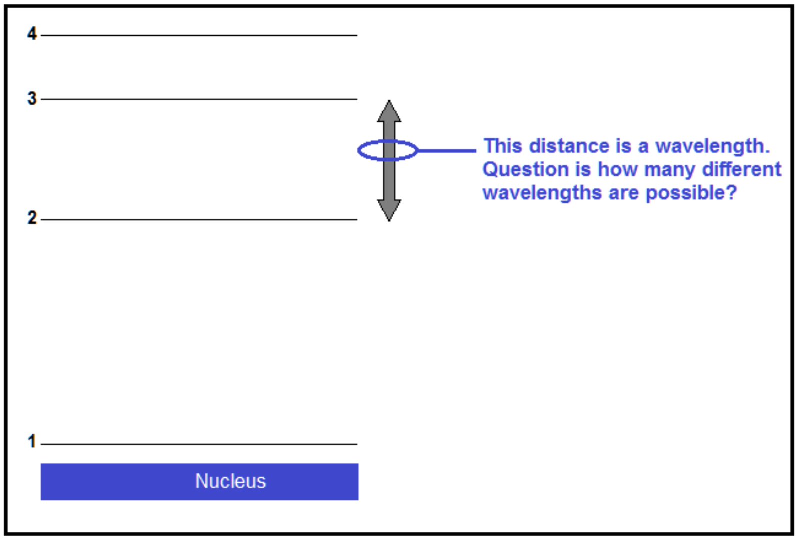
$En\_final := E\_Hydrogen + E\_eV = -0.85$ eV

Remember: This formula **En = -13.6 / n^2 eV** for Hydrogen at state n = 1 can produce the spread of lines, or states. The variable 'n' should solve for the core of our problem. This is the spread of states the Hydrogen atom generates and each state has its wave length. *Usually comes to realisation as we solve the difficulty!*

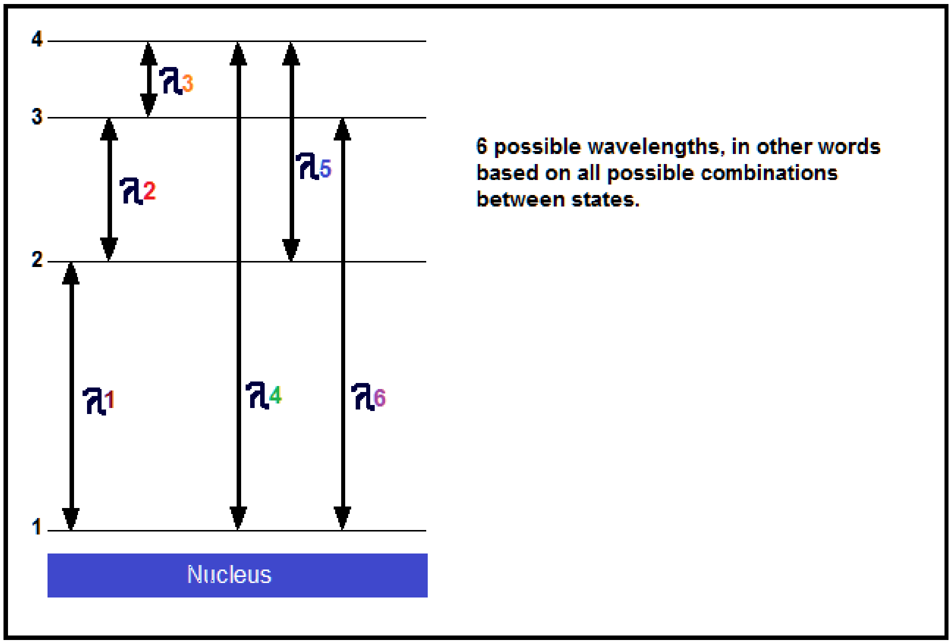
En = -13.6 / n^2 eV
En_final = (-13.6 eV) / n^2
n = Sqrt((-13.6 eV/ - 0.85 eV)

$$n := \sqrt{\left(\frac{-13.6}{-0.85}\right)} = 4$$

We know there are 4 states achievable, n=4 indicates the inclusion of state 1, so we have 1, 2, 3, and 4 states.



6 possible wavelengths, shown in figure below. Ans.



How do we find the longest wavelength among the six wavelengths?

Returning to the expression

hc/lamda = E = En
lamda = hc/En
En is the value at the line n, the wavelength is across 2 energy state lines.
Therefore applicable to us in this difficulty is the value of energy across 2 states corresponding to the wavelength. Correct!

En4 - En3 for example would be lamda 3 in the figure above.
So here to find lamda 3, **lamda 3 = (hc)/(En4 - En3).** Correct. Just so happens its the smallest energy difference.

The value of **lamda gets larger** with a **smaller energy difference (En+1 - En)**. Correct Again!

So if we know the smallest energy difference of the 6 possible energy differences, then by inserting this energy difference value into the expression, solving for lamda we should get the longest wavelength among the 6.

We know from the Hydrogen energy states values given in textbooks, here provided on page 1, in this case it is between En4 and En3.
However we can also easily calculate this for each Hydrogen state
En = -13.6eV/n^2.

$$E4 := \frac{-13.6}{4^2} = -0.85 \quad eV$$

$$E3 := \frac{-13.6}{3^2} = -1.5111 \quad eV$$

Now calculating from lamda 3 as shown in the figure on previous page.

$$\lambda 3 = \frac{(h \cdot c)}{(E4 - E3) \cdot eV} = 1.872 \cdot 10^{-6}$$

m^-1 do not forget to include eV in the denominator so we get the result correctly in m^-1.

$\lambda 3 = 18720$ *Angstrom* Ans.

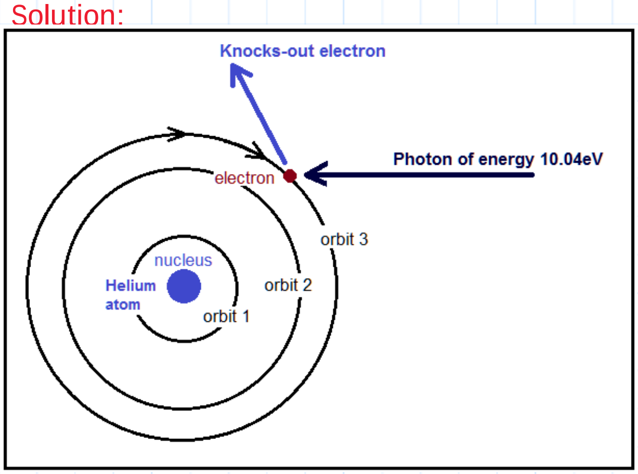
Comments:

1. This became a simple solution to a problem where it was first difficult to visualise.
2. As we progressed to the answers the simple equations were able to reveal much more use compared to prior questions OR than we first came to know.
3. You may have your own comment here.
4. Quantum Mechanics maybe difficult due to the lack of visualisation of the learner, for me its that plus its a difficult subject matter when it comes to Schrodinger Equations Hamiltonian .....we need solved problems that guide our studies. This usually found in some good Engineering textbooks. That is how I identify a good engineering textbook.

Problem 3.10

In a singly-ionized Helium atom the electron is in the third orbit.
A photon of energy 10.04 eV knocks out the electron.
Calculate the stopping potential of the electron.
The ionization energy of Hydrogen atom is 13.6 eV.

Solution:



Discussion:
Knocks-out electron?
Current is a flow of electron.
We have photon knocking out 1 electron in the 3rd orbit of an ionised Hydrogen atom.

Is this not an introduction to Solar PV cells?
I say it is. It takes a lot of photons and electrons to get an appreciable electric current, and that is found in many solar (PV)panels.

Atom: He
Z: 2
Ionization: 1, atom is positively charged He+ (say nucleus is +pos charged).
Electron: 3rd orbit is found 1 electron (electron is negatively charged)

Helium ionization energy is found based on comparision to Hydrogen atom ionisation energy.

E(He) = (Z^2) HeV

$Z_{He} := 2 \qquad H_{ionization\_energy} := 13.6 \ \text{eV}$

$He_{ionization\_energy} := H_{ionization\_energy} \cdot \left(Z_{He}{}^{2}\right) = 54.4$ eV at ground state (for He+)

Note: n = 1 ground state is first orbit
n = 2 is second orbit
n = 3 is third orbit
n = 4 is the fourth orbit and so on ....

Energy of He electron in the 1st orbit is when n = 1:
He_n equal 1 = -(He+)/n^2

$n_1 := 1$

$$En\_He_{n\_equal\_1} := \frac{-\left(He_{ionization\_energy}\right)}{\left(n_1\right)^2} = -54.4 \ \text{eV}$$

Energy of He electron in the 3rd orbit is when n = 3:

$n_3 := 3$

$$En\_He_{n\_equal\_3} := \frac{\left(En\_He_{n\_equal\_1}\right)}{\left(n_3\right)^2} = -6.044 \quad eV$$

Energy of the incident photon is 10.04 eV. This is a positive value (10.04 eV).

$En_{photon} := 10.04 \ \ eV$

Energy of the knocked out election equal the energy of the photon that struck the electron in the 3rd orbit subtracted the energy of the He electron in the 3rd oribt.

The phrase above may not come across as technical. How about if we were to say the conservation of energy or something momentum like?

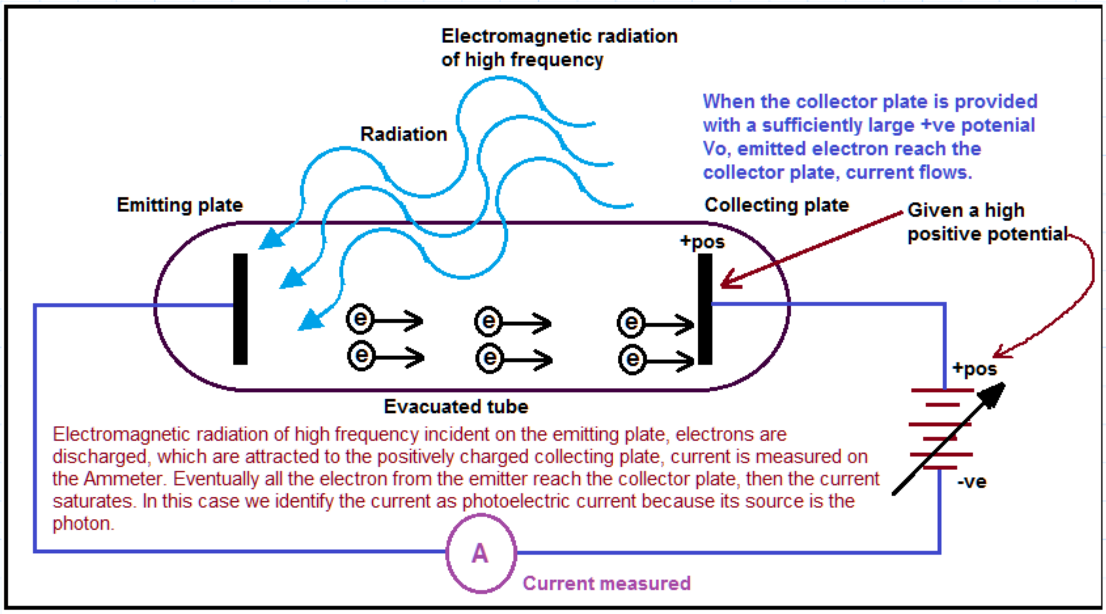
Resultant energy of electron 3rd orbit = Energy of photon + Energy of He electron 3rd orbit

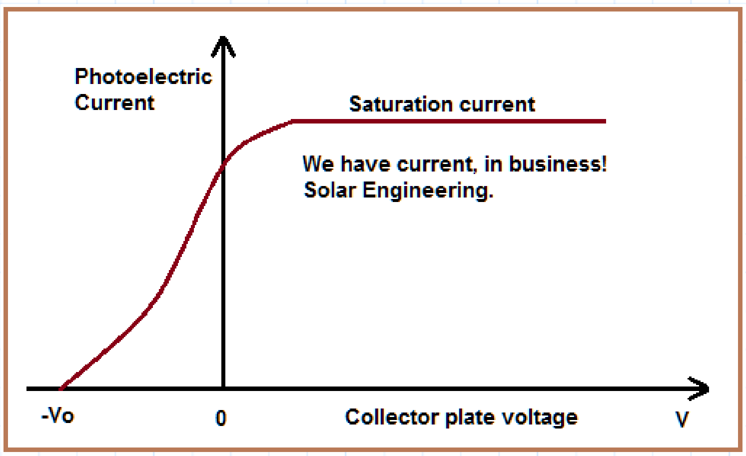
$$En\_He\_e\_3rd\_Orbit := En_{photon} + En\_He_{n\_equal\_3} = 3.9956 \quad eV$$

$En\_He\_e\_3rd\_Orbit := 4.0$ eV approximately to 1 decimal place.

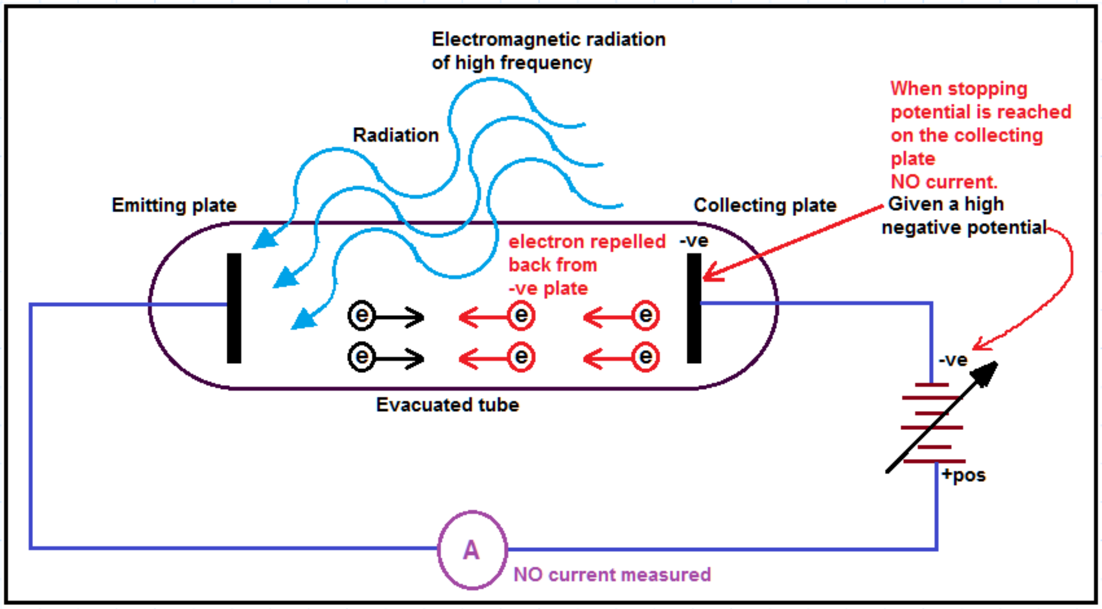
Therefore the stopping potential is equal to 4.0 eV Ans.

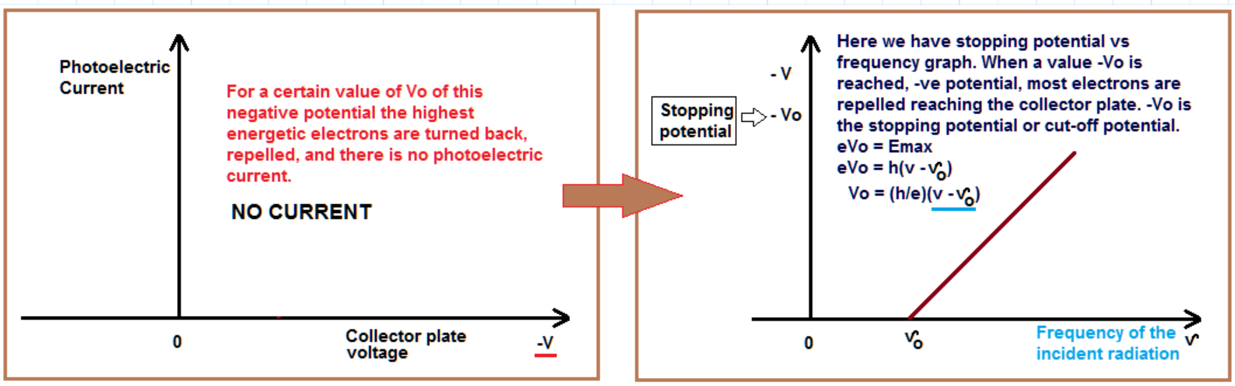
What is the stopping potential?
Could the stopping potential the energy required to add to the Helium electron in the 3rd orbit to prevent it from getting knocked out by the photon with 10.04 eV energy?
See the figures below, close, it has to do with the collector plate voltate!
Heinrich Hertz did that experiment, it was followed through by Einstein's theory on photons. Stopping potential also known as cut-off potential which is more often used in electrical engineering.





Next page stopping voltage explained. Refer to quantum mechanics textbook for detailed explanation for photoelectric current results obtained through this experiement using the positively charged collector plate.





That was the explanation for stopping voltage, the voltage reached when no flow of current is possible, in this case the electron would not flow, electron is prevented from flowing if the electron is given the additional electron voltage of 4 eV. It would have to be negative voltage -4eV.

If you have a better explanation you are welcome to provide.

Comment: Could the electron voltage 4eV be seen or rationalised as an insulator voltage because it prevents the flow of electron? Funny, just a thought! Another thought maybe applicable in high speed switching operations in power electronic or photo-electronics.

Problem 3.11

The ionization of a Hydrogen-like atom is 4 Rydberg.
Find the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state.

1 Rydberg = 2.2 x 10^-18 J.
h = 6.6 x 10^-34 Js.

Solution:

$R := 2.2 \cdot 10^{-18}$ J

H-like atom ionisation energy = 4 x Rydberg.

$E_{H\_like} := 4 \cdot R = 8.8 \cdot 10^{-18}$ J

Energy of the electron in the ground state (n=1) is equal to E_H_like but with a negative sign.

$E1_{H\_like} := -E_{H\_like} = -8.8 \cdot 10^{-18}$ J

Energy of the electron in the 2nd orbit (n=2) is equal to:

$n2 := 2$

$$E2_{H\_like} := \frac{-E_{H\_like}}{n2^2} = -2.2 \cdot 10^{-18} \quad J$$

Similar to a previous example problem using the energy between 2 states to compute the wavelength. Radiation emitted is (E2 - E1).

(hc)/lamda = E2 - E1, therefore lamda (wavelength) = (hc)/(E2 - E1).

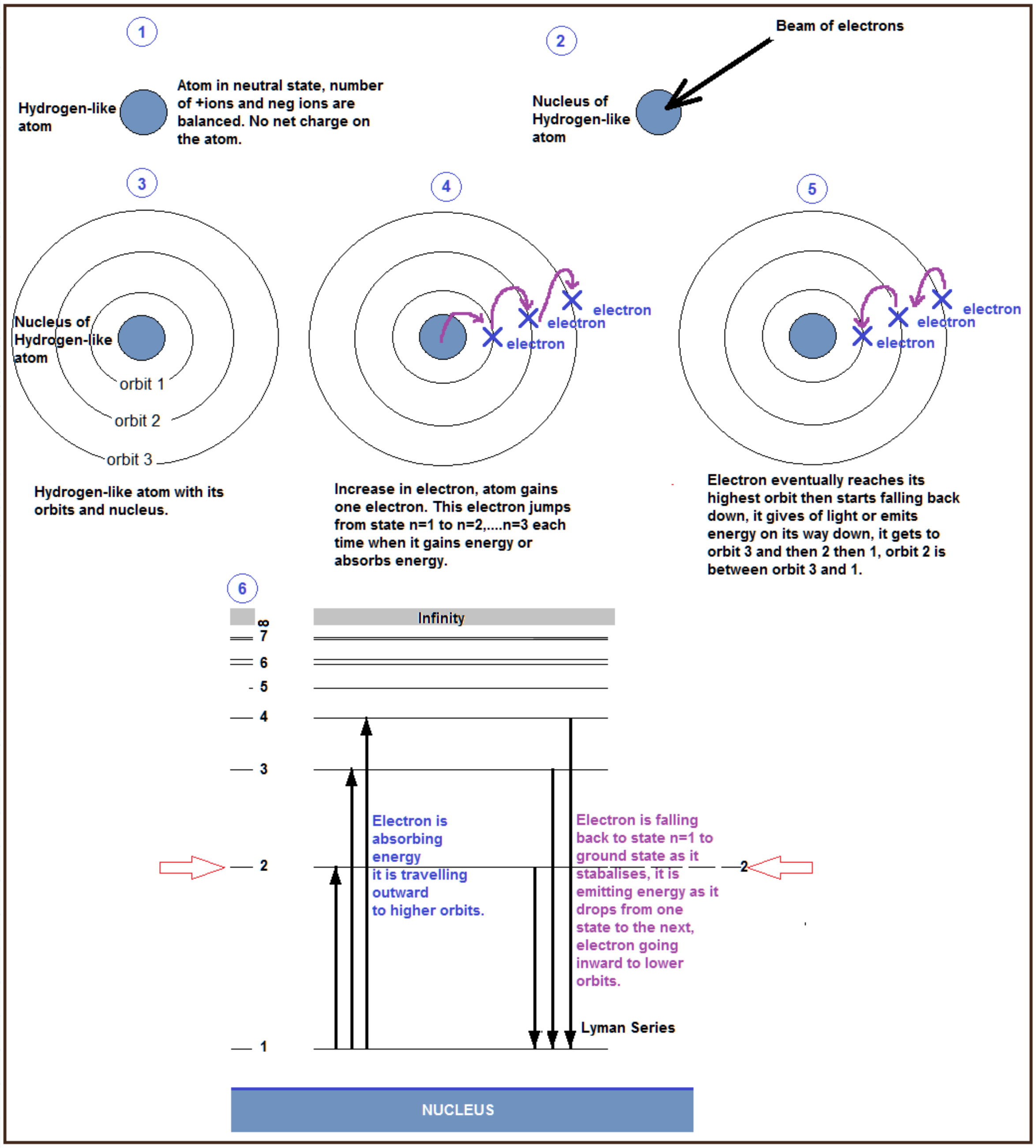
$$\lambda := \frac{(h \cdot c)}{\left(E2_{H\_like} - E1_{H\_like}\right)} = 3 \cdot 10^{-8} \quad m\text{^-1.}$$

$\lambda := 300 \cdot 10^{-10}$ ***Angstrom*** Ans.

Problem 3.12

A beam of electrons bombards a sample of Hydrogen.
Through what potential difference must the beam be accelerated if the second line of Lyman series is to be emited?
1 Rydberg = 2.2 x 10^-18 J.
h = 6.6 x 10^-34 Js.

Solution:



The figure above may assist in understanding the problem. If you have a better idea or figure you are welcome to present it. *Any errors and omission in this set whole of example problems, apologies in advance.*

Lyman series of the Hydrogen atom starts with the orbit n=1.

Energy is absorbed on the when going from n=1 to higher orbits.
Energy is emitted when going from higher orbits to n= 1 ie to the ground state or normal state. This problem statement is concerned about energy emitted.

We start with bombarding an atom that is not ionised. Lets say in its neutral state. It gains energy and starts sending the electron outward, energy absorbed. When it reaches it highest state it then falls back to ground state. The Lyman series, has n = 2 between n=3 and n=1.

If we find the potenial difference between state n=3 and n = 1 this would provide us the energy that is bound between n=3 and n=1.

The problem statement ask what must the 'beam' be accelerated.....there is only one beam that which bombards the sample of Hydrogen.

Lets say acceleration occurs when the electron progress from lower to higher states, gaining energy from the beam the electron accelerates. So if we calculate the energy bound, eV, betwen states 3 and 1 that may provide us the minimum potential difference. We are NOT computing acceleration of electrons, rate of change of speed/ velocity of electrons, rather just the potential difference. Little simpler!

Previous Comments: You verify on the sign convention.
Its not exactly like Kirchchoff's Law in electric circuit's convention.
Lets say if we set the convention, radiated outward is +ve sign, then absorbed would be -ve sign. Provided the ni and nf line up with the order of energy direction movement.

Binding energy of the atom is n=1 state:

$En1 := 13.6$ eV.

Binding energy of the atom in n=3 state:

$En3 := \frac{En1}{(3^2)} = 1.511$ eV.

Energy required to leap or jump from n=1 to n=3 state:

$E\_pdiff\_n1\_n3 := En1 - En3 = 12.089$ eV. Ans.

On the way back from n=3 to n=1, the 2nd state or line of the Lyman series would be emitted. We say the energy gap is the same between n1 to n3 as calulcated to between n3 to n1. Not easy. Love This Problem. You may verify.

Problem 3.13

The Rydberg constant for Hydrogen is 1.09678 x 10^-7 m^-1.
The Rydberg constant for ionised Helium is 1.09722 x 10^-7 m^-1.
Mass of the Helium nucleus is 4X that of Hydrogen nucleus.
Calculate the ratio of the electron mass to the proton mass.

Solution:

Background notes: The nucleus when compared to the electron has infinite mass, it is so much larger than the electron in orbit. At times we assume because the mass of the nucleus is so large compared to the electon that the nucleus is stationary and only the electron is in orbital motion. Situationnow is because there are some details about the 'spectra' which depend on the actual mass of the nucleus. However, in actuality or reality or real situation is that the 'nucleus and the electron' rotate or move about their same centre of mass and both have the same angular velocity. So we make some adjustments on the electron mass in comparison to the nucleus mass, this then provides for a corrective to the infinite nuclear mass to a 'finite' nuclear mass.

Replace the electron mass m by the reduced mass u (mu) of the electron-nucleus system.

u = mM / (m + M)....m mass of electron and M mass of nucleus.
m+M is the total mass - denominator.
What should the numerator be?
If both the masses together were the same,
ie electron (m) + nucleus (M), it would be 1 x 1 = 1.
If the electron was a percentage smaller than nucleus, then we could say m x M.
where m is a fraction of M, example 0.0001 x 1.0 = 0.0001 results with the same mass of electon. It is the mass of electron (numerator), to 'nucleus and electron' in denominator we seek. This expression provides a fix or remedy.

v (frequency) = 1/(lamda) = R(infinity) (Z^2) ((1/n1^2) - (1/n2^2))

R(infinity) = (m/(4 pi c (h'^3))) (e^2/(4 pi epsilon_o)^2.
h' = h/2pi.
R(infinity) is the Rydberg contant = 1.09737 x 10^-7 m^-1.

Now correcting for the Rydberg constant for Hydrogen
R_H = R(infinity) / (1 + (m/M_H)   M_H mass of Hydrogen nucleus.
Now R_H = 1.09678 x 10^-7 is closer to an empirical value 1.096776 x 10^-7 that it is closer to R(infinity) 1.09737 x 10^-7.
Look up in modern physics or quantum mechanics textbook on the full notes on this

.... continuing with solution.

R_H = (1 + (m/M_H))
R_He = (1 + (m/M_He))
R_He nucleus mass = 4 X M_H

R_He = (1 + (m/4M_H))

R_H / R_He = (1 + (m/M_H)) / (1 + (m/4M_H)).

Reduces to

m/M_H = (R_He - R_H) / (R_H - (R_He/4))

$R_H := 1.09678 \cdot 10^{-7}$ $\quad R_{He} := 1.09722 \cdot 10^{-7}$

$$m\_div\_M_H := \frac{(R_{He} - R_H)}{\left(R_H - \left(\frac{R_{He}}{4}\right)\right)} = 5.35 \cdot 10^{-4}$$

Does not show a comparsion clearly, lets do some division by numerator.

$(R_{He} - R_H) = 4.4 \cdot 10^{-11}$ $\quad$ Numerator := 0.00044

$\left(R_H - \left(\frac{R_{He}}{4}\right)\right) = 8.225 \cdot 10^{-8}$ $\quad$ Denominator := 0.822

$$New\_Numerator := \frac{Numerator}{Numerator} = 1$$

$$New\_Denominator := \frac{Denominator}{Numerator} = 1868$$ Whole number.

The ratio of the mass of electron to proton is 1 / 1868. Ans.

Any errors or omissions apologies in advance.
These introductory level solved problems help grasp a complex subject at UG level.
Hope this helps further progress in the science and engineering of solar engineering.

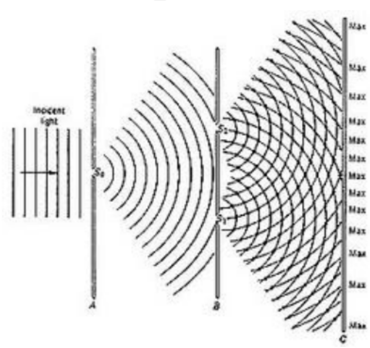
Textbook: Modern (Atomic) Physics Vol I.
Author:S.N. Ghosal
Publisher: S. Chand.

Chapter 9: Wave particle duality; Hisenberg's uncertainty principle.

The solved example problems in the textbook 'Quantum Mechanics: A textbook for Undergraduates' by Mahesh C. Jain are used for UG subject matter comprehension.

April 8, 1999 • *Phys. Rev. Focus* 3, 21

Focus: The de Broglie Wavelength of a Packet of Light



Texas Christian University

Double Slit. Unlike this typical interference experiment where plane waves fall on a double slit, physicists have been able to observe the interference pattern from a wave packet consisting of just two photons, measuring the effective wavelength in the process. **Show Less**

Source: April 8, 1999. Phy. Rev. Focus 3,21.

Example problems here are primarily de Broglies Hypothesis related.

Comments: Chapter 9, Ghosal textbook, contents are excellent in understanding the background for therory, its mathematics, it includes derivations, figures, and explanations. You may find this in similar textbooks today.

Constants:

$h := 6.63 \cdot 10^{-34}$ Js
$c := 3 \cdot 10^{8}$ m/s
$eV := 1.6 \cdot 10^{-19}$ J
$e := 1.6 \cdot 10^{-19}$ J
$m_{neutron} := 1.675 \cdot 10^{-27}$ kg
$m_{electron} := \left(9.1 \cdot 10^{-31}\right)$ kg

Problem 4.1

Find the de Broglie wavelength of electrons accelerated through a potential difference of 100V.

Solution:

Wavelength lamda = h / Sqrt(2 m e V)
OR
= (h / Sqrt(2 m e)) (1/(Sqrt(V)) Angstrom
= 12.3 (1/Sqrt(V)) Angstrom

$V := 100$ Volt

$$\lambda := \frac{12.3}{\sqrt{V}}$$ Angstrom

$\lambda = 1.23$ Angstrom Ans.

Problem 4.2

Find the de Broglie wavelength of electrons moving with a kinetic energy of 100 eV.

Solution:

Wavelength lamda = h / (Sqrt(2 m K))

$$\lambda := \frac{(h)}{\sqrt{\left(2 \cdot m_{electron} \cdot 100 \cdot eV\right)}} = 1.229 \cdot 10^{-10}$$

$\lambda := 1.229$ *Angstrom* Ans.

Problem 4.3

What should be the kinetic energy of a neutron in eV so that its associated de Broglie wavelength is 1.4 x 10^-10 m?

Mass of neutron = 1.675 x 10^-27 kg.

Solution:
Wavelength lamda = h / (Sqrt(2 m K)) rearraing for K (kinetic energy)
KE or K = h^2 / (2 m lamda^2)

$\lambda := 1.4 \cdot 10^{-10}$ m

$$KE := \frac{(h^2)}{2 \cdot m_{neutron} \cdot \lambda^2}$$

$KE = 6.695 \cdot 10^{-21}$ J

$KE := \frac{KE}{eV} = 0.0418$ eV

$KE := 4.18 \cdot 10^{-2}$ eV Ans.

Problem 4.4

An electron in a hydrogen-like atom, is in an excited state.
It has a total energy of -3.4 eV.
Calculate (a) the kinetic energy
(b) the de Broglie wavelength of the electron.

Solution:
a). Hydrogen like atom,...similar to the energy level diagram of Hydrogen.
n = 1, En1-H = - 13.6 eV -ve sign indicating energy absorbed travelling downward what was reffered in part II as energy direction (movement) the usual direction is downward, absorbing energy.
Here this H-like atom has a total value of -3.4 eV.
This -3.4eV may be considred in the positive sign if its seen like this;
KE or K = - (total energy of H-like electron E)
= - (-3.4 eV)
= 3.4 eV Ans.

Comment: *You may ask what is the purpose of this exercise? It may be just to show how to interpret the KE's direction another way. You may say why should be KE be of an opposite sign to the total energy given? Its in an excited state, it was given excitation, its initial energy may been +ve, the excitation created a change it got -ve energy, finally we interpter the final energy as positive. Something to consider before labelling a final answer to any of QMs problems.Ok,now look what happens in (b) part of solution, if it were negative -3.4 the answer would be a complex number - 6.663 x 10^-10.*

b). Applying de Broglie equation for wavelength

Wavelength lamda = h / (Sqrt(2 m K))

$K := 3.4\ eV$ note the postive sing used here, otherwise sqrt of a -ve number returns a complex number. Unit eV included in expression.

$$\lambda := \frac{(h)}{\sqrt{(2 \cdot m_{electron} \cdot K)}} = 6.663 \cdot 10^{-10}$$

$\lambda := 6.663$ ***Angstrom*** Ans.

Problem 4.5

Find the kinetic energy of a neutron in electron-volt if its de Broglie wavelength is 1.0 Angstrom.

Mass of neutron = 1.674 x 10^-27 kg; h = 6.60 x 10^-34 Js.

Solution:

$m_{neutron} := 1.674 \cdot 10^{-27}$ kg

$h := 6.60 \cdot 10^{-34}$ Js

$\lambda := 1 \cdot 10^{-10}$ Angstrom

$$KE_{neutron} := \frac{\left(\frac{(h^2)}{2 \cdot m_{neutron} \cdot \lambda^2}\right)}{eV} = 0.081 \quad eV$$

$KE_{neutron} := 8.1 \cdot 10^{-2}$ eV Ans.

Problem 4.6

A ball of mass 10 g is moving with a speed of 1 m/s.
Calculate the de Broglie wavelength associated with it.
Can the effect of this wavelength be observed experimentally?

Solution:

Wavelength lamda = h / (mv) ...refer to modern physics textbook or qm textbook for this formula/excpression..

$m\_ball := 10 \cdot 10^{-3}$ kg .....10 g = 10/1000 g = 0.01 = 10 *10^-3.

$v := 1.0$ m/s

$$\lambda := \frac{h}{(m\_ball \cdot v)} = 6.6 \cdot 10^{-32}$$ m Ans.

The de Broglie wavelength was calculated above, its a very very small length, its not visible. So if we were to conduct an experiement on this ball to directly see effect of this wavelength would be unrealistic. The wavelenth is so much smaller than the ball of mass 10 gram/s dimensions. The effect cannot be observed experimentally. Ans.

Input OR Comment OR Joke: *If this ball was rolling at a constant speed on a flat table, could a highly charged electron beam aimed at it at the same speed of 1 m/s, running along side, reveal any pattern on a photographic plate? Like in the Thompson experiement. The ball be made of a material type that would permit electron flow through it. Maybe a pattern on the plate may reveal something. But to set it up moving with a speed of 1m/s may be a hurdle. This may not work if both the beam and ball were moving at the same speed then there is no speed detected.*

Problem 4.7

Calculate the de Broglie wavelength of thermal neutrons at 27 deg C.
Given Boltzmann constant k = 1.38 x 10^-23 J/K.

Solution:

$m_{neutron} := 1.67 \cdot 10^{-27}$ kg
$T := 27$ C
$T_K := T + 273 = 300$ K.

$k_{Boltzman} := 1.38 \cdot 10^{-23}$ J/K.

$KE\_thermal\_neutron := k_{Boltzman} \cdot T_K = 4.14 \cdot 10^{-21}$

$$\lambda := \frac{(h)}{\sqrt{2 \cdot m_{neutron} \cdot KE\_thermal\_neutron}} = 1.775 \cdot 10^{-10}$$

$\lambda := 1.78$ ***Angstrom*** Ans.

Problem 4.8

A proton and deutron have the same kinetic energy.
Which of the two has longer de Broglie wavelength?

Solution:

Deutron is twice, 2X, the mass of proton.
This data maybe of interest in solving this example problem.

m-p: mass of proton
m-d: mass of deutron
m-d = 2 x (m-p)

lamda-p : h/ Sqrt (2 m-p K)

lamda_d : h/ Sqrt (2 (2 X m-d) K)

lamda_p / lamda_d = h / Sqrt (2 m-p K) Sqrt (2 (2 X m-d) K) / h

lamda_p / lamda_d = Sqrt(2) Ans.

Problem 4.9

Show that the de Broglie wavelength of an electron is equal to its compton wavelength when its speed is c/sqrt(2), c being the speed of light.

Solution:

Compton wavelength = h/ ((m_0) c).
Is known as the Compton wavelength of the electron.
de Broglie wavelength = h / p = h /mv = h / Sqrt (2 m K).

The expressions which are smilar among the two are
Compton --> h/ ((m_0) c) and h /mv <-- Broglie

Without going into the details on the background theory, and mathematical details, the equation

E = mc^2 = (m_o (c^2)) / Sqrt(1 - (v^2/c^2))

m_0 is the rest mass of a particle and v its velocity, and
m = m_0 / Sqrt(1 - (v^2/c^2))
is obtained through Einstein's work in change of mass of a body with velocity. This can be studied in any advanced Physics or most Modern Physics textbook. *One textbook referenced here is SN Ghosal Modern Physics.*

lamda = h / (mv) ......de Broglie
m = m_0 / Sqrt(1 - (v^2/c^2))
= (h/m_0 v) [Sqrt(1 - (v^2/c^2)]

substitute v for c multiply by (c/v)...keeping things the same

lamda = (h/m_0 c) [(1 - (v^2/c^2))^(1/2)] (c/v) = h/(mv) ....Correct.

substitute v (velocity) = c/Sqrt(2).

lamda = (h/m_0 c) [(1 - (c/Sqrt(2)^2/c^2))^(1/2)] (c/(c/Sqrt(2))) = h/(mv)

lamda = (h/m_0 c) [(1 - ((c^2/2)/c^2))^(1/2)] (c/(c/Sqrt(2))) = h/(mv)

lamda = (h/m_0 c) [(1 - (1/2))^(1/2)] (Sqrt(2)) = h/(mv)

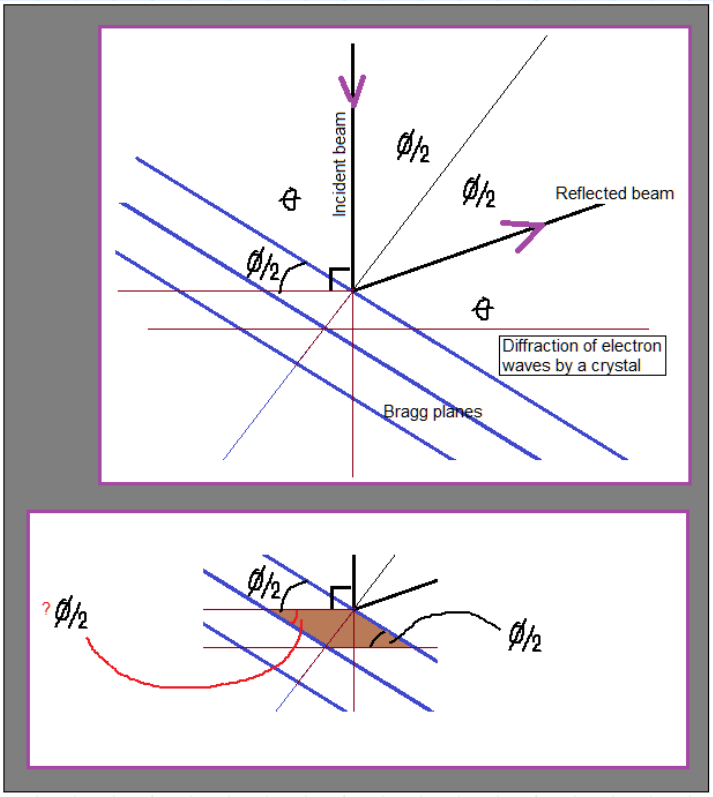
lamda = (h/m_0 c) [(1/2)^(1/2)] (Sqrt(2)) = h/(mv)
lamda = (h/m_0 c) [Sqrt(1/2)] (Sqrt(2)) = h/(mv)
lamda = (h/m_0 c) (1) = h/(mv)
lamda = (h/m_0 c) is the Compton wavelength of the electron. Ans.

Interesting proof. Is Sqrt(2) a magic number?
Sqrt(2) is often applied in engineering.

Problem 4.10

In a Davidson-Germer diffraction experiment electrons of kinetic energy 100 eV are scattered from a crystal. The first maximum in intensity occurs at theta = 10.0 deg.
(a). What is the spacing between the crystal planes?
(b). How many peaks will there be in the interference pattern?

Solution:



See Modern Physics textbooks for Davisson Germer experiment and Bragg planes for crystals.The formula provided here their deriviation are provided there.

Figure to the right shows the angles concerned in the equation.

Basic derivation for the equation to apply provided below.

a).
n (lamda) = 2 d sin (theta)
d is the spacing between Bragg planes, and n is an integer.
Angle theta and phi are shown in the figure above.

(theta) + (phi/2 + phi/2) + (theta) = 180 degs
or
(theta) + (phi) + (theta) = 180 degs

(theta) = (180 degs - phi) / 2
(theta) = 90 degs - (phi / 2)

Applying geometry, and trignometry
sin(phi/2) = d/D .....where D is the interatomic spacing.
d = D sin (phi/2)

n(lamda) = 2 D sin (phi/2) sin (90 - (phi/2))

n(lamda) = 2 D sin (phi/2) sin (90 - (phi/2))
= 2 d sin (theta)
n(lamda) = 2 D sin (phi/2) cos (phi/2)

trig identity cos (phi/2) = sin (90 - (phi/2))
trig identity sin 2(phi) = 2 sin(phi)cos(phi)
sin(phi) = 2 sin(phi/2)cos(phi/2)
subisituting into 2 D sin (phi/2) cos (phi/2) and the 2 cancels.

n(lamda) = D sin (phi).....this is the equation to be applied.

$V := 100$ V

$\lambda := \frac{12.3 \cdot 10^{-10}}{\sqrt{V}}$ expression for electron.

$\lambda = 1.23 \cdot 10^{-10}$

$\theta := 10\ deg$

$d := \frac{\lambda}{2 \cdot \sin(\theta)} = 3.542 \cdot 10^{-10}$

$d := 3.542$ Angstrom Ans.

b).
n (lamda) = 2 d sin (theta)

We want to solve for n, and the variable is sin(theta) for the maximum value of n sin(theta) has to be maximum.
Sin(theta) = 1 is a possible maximum when angle theta = 90 degs

now the expression becomes:

n (lamda) = 2 d sin (90)
n (lamda) = 2 d (1)
n = 2d/lamda
The number of peaks cannot be greater then the RHS so
n < OR = 2d/lamda

$n := \frac{2 \cdot d}{\lambda} = 5.759 \cdot 10^{10}$ Therefore the maximum value n can take is 5, it cannot be larger then the next integer 6.

$n := 5$ Ans.

Comment: *PV panel has a thickness and so do the PV cells, so could this be applied here? Yes. You ask your lecturer or specialist to verify.*

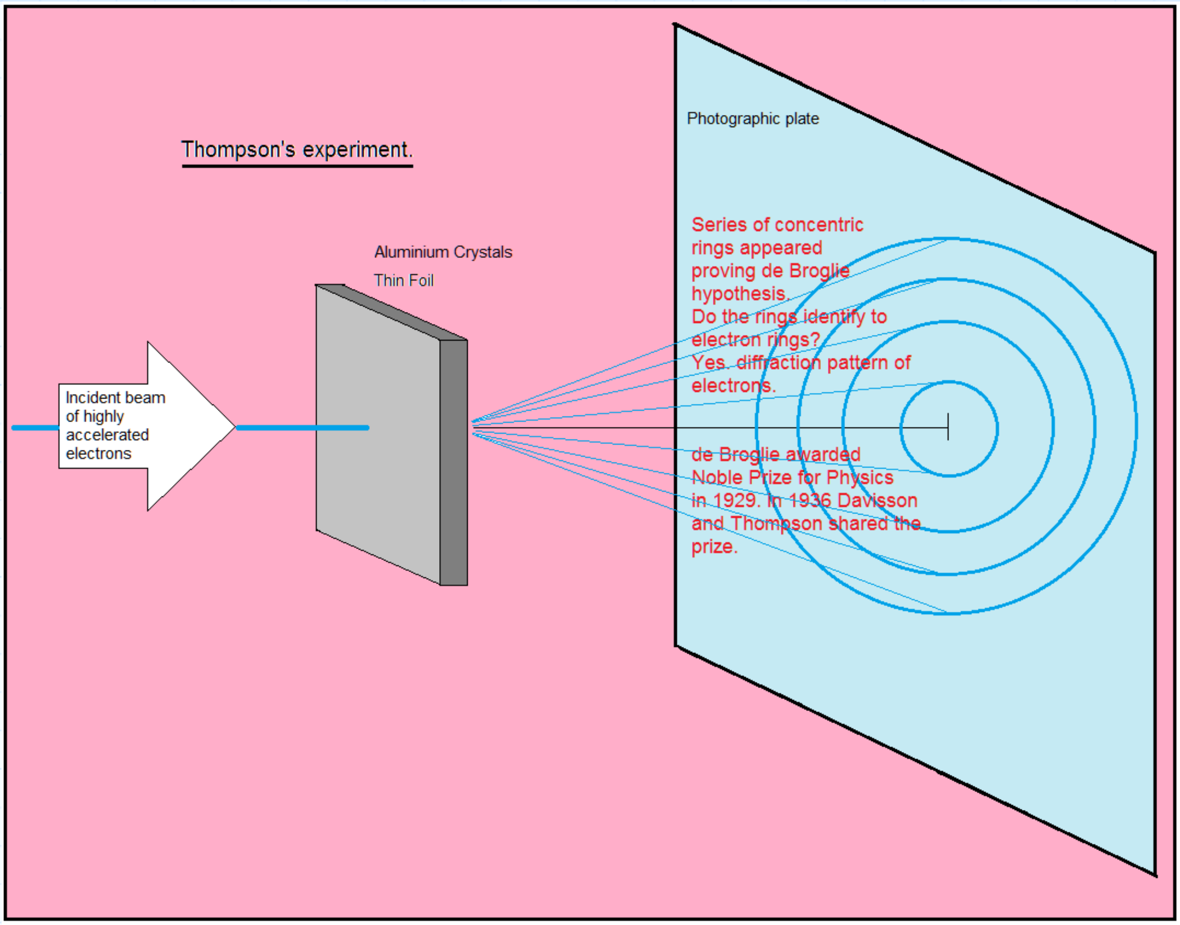
Problem 4.11

A narrow beam of electrons, accelerated through a potential difference of 30 kV, passes through a thin aluminium foil and produces a diffraction pattern on a photographic plate on the opposite side of the foil.

If the first diffraction ring is obtained at an angle of 59' 36" from the incident beam, calculate the grating space in the aluminium crystals.

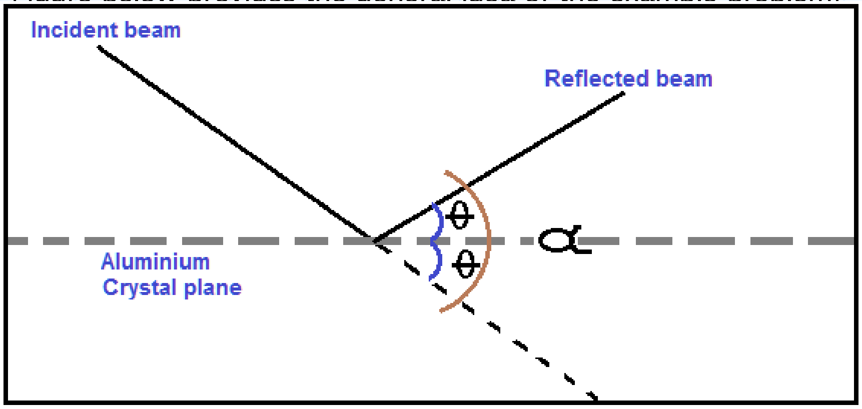
Solution:

Figure below is Thompson's Experiment - general idea.
You are welcome to put in your ideas on what this experiment portray(s). What I have said in the sketch may not be agreeable to you. For now to solve the example problem follow the equations derived!



FYI: *This experiment was done by the son, the father also a physicist was JJ Thompson, he (father) received a Nobel Prize for showed electron has a particle nature in 1906. The son GP Thompson received a Nobel Prize in 1937 shared with Davisson, showed the electron has a wave nature. You verify.*

Figure below provides the general idea of the example problem.



Angles in figure above,
Alpha = 2 Theta

We call alpha the angle of deviation
and theta the 'glancing angle'.

Angle alpha = 0 deg 59' 36"

$\alpha 1 := DMS(0, 59, 36) = 0.017\ rad$

$\alpha := 0.017\ rad = 0.974\ deg$

$\theta := \frac{\alpha}{2} = 0.487\ deg$ Angle theta = 29' 48"

The first diffraction ring is when n = 1

n (lamda) = 2 d sin (theta).

$n := 1$

d = (n lamda) / (2 d sin(theta))

The electron beam was accelerated through a potential difference of 30kV

From the wavelength, voltage expression for an electron...

lamda = 12.3 Angstrom / Sqrt(V) ...solve for wavelength lamda

$V := 30 \cdot 10^3$ V $\quad$ $Sqrt\_V := \sqrt{30 \cdot 10^3} = 173.205$

$\lambda := \frac{(12.3 \cdot 10^{-10})}{\sqrt{V}} = 7.101 \cdot 10^{-12}$

$Sin\_theta := \sin(\theta) = 0.0085$

$Sin\_theta := 0.0087$

$$d := \frac{n \cdot \lambda}{2 \cdot \sin(\theta)}$$

$d = 4.177 \cdot 10^{-10}$

$d := 4.177$ ***Angstrom*** Ans.

Part III basically a few examples using formulas that are short not highly involved. Introductory level.

Any errors and omissions apologies in advance.

Thus far the 3 part solar engineering tutorials and partially study material should or may be sufficient to take you to the start of Wave Packets and The Uncertainty Principle topics. This topics require a textbook to master the theories, mathematical deriviations, and related subject matter.

Requires you to have a Modern Physics textbook for the study material.

Textbook: Modern (Atomic) Physics Vol I.
Author:S.N. Ghosal
Publisher: S. Chand.

Chapter 9: Wave particle duality; Hisenberg's uncertainty principle.

The solved example problems in the textbook 'Quantum Mechanics: A textbook for Undergraduates' by Mahesh C. Jain are used for UG subject matter comprehension.

Chapter 5 of QM For UGs.
Mahesh C Jain.

Example solved problems here are mostly on the red text above (Mahesh).
This for me is the hardest part in the early material in an UG quantum mechanics course. The subject matter on wave mechanics. Problems here more on how to apply some relations.

There is no need for notes here because the subject matter is best studied in modern physics or quantum mechanics textbooks. Certainly in quantum mechanics textbooks.

The relationship between phase and group velocities need to be understood in theory, mathematical explanation and derivation. This relationship can be shown using Fourier Series-Transforms, OR Algebraic-Differentiation deriviations. So no doubt its a little involved.

Having understood the contents/subject matter, the application is mostly applying formulae and expressions in the example problems.

Any errors and omissions apologies in advance.

Constants:

$h := 6.63 \cdot 10^{-34}$	Js
$c := 3 \cdot 10^{8}$	m/s
$eV := 1.6 \cdot 10^{-19}$	J
$e := 1.6 \cdot 10^{-19}$	J
$m_{neutron} := 1.675 \cdot 10^{-27}$	kg
$m_{electron} := (9.1 \cdot 10^{-31})$	kg

Problem 5.1

A wave packet has the amplitude function

A(k) = 1/sqrt(epsilon), - epsilon/2 <= k <= epsilon/2
0, |k| > epsilon/2

Find the wave function psi(x) and hence verify the reciprocity relation
delta x/ delta k > 1. If you dont know Fourier dont worry just go thru the steps and the notes. Look up on the internet just get a general idea of its purpose.

Solution:

$i := \sqrt{-1}$ $\quad \varepsilon_0 := 8.84 \cdot 10^{-12}$ ....F/m farad per meter here its permitivity of air, just picked it so it does not show a red flag in the expression....error in software data entry.

$\varepsilon := 8.84$

psi(x) term is obtained by taking the Fourier transform of A(k):

$$\psi(x) := \left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot \int_{-\infty}^{\infty} A(k) \cdot e^{i \cdot k \cdot x}\, dx$$

$$\psi(x) := \left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot \int_{\frac{-\varepsilon}{2}}^{\frac{\varepsilon}{2}} \left(\frac{1}{\sqrt{\varepsilon}}\right) \cdot e^{i \cdot k \cdot x}\, dx$$

$$\psi(x) = \left(\frac{1}{\sqrt{2 \cdot \pi \cdot \varepsilon}}\right) \cdot \left(\frac{e^{i \cdot k \cdot x}}{i \cdot x}\right) \Bigg|_{-\frac{\varepsilon}{2}}^{\frac{\varepsilon}{2}}$$

$$\psi(x) := \left(\frac{1}{\sqrt{2 \cdot \pi \cdot \varepsilon}}\right) \cdot \left(\frac{2}{x}\right) \cdot \frac{\left(e^{i \cdot \varepsilon \cdot \frac{x}{2}} - e^{-i \cdot \varepsilon \cdot \frac{x}{2}}\right)}{(2) \cdot i}$$

Short Talk on the side:

The A(k) is function but more like the conditions on how works, where it values rests for non-zero and zero.

So by applying some vague or super accurate method we try to get a function on A(k) that takes the form of a wave. Thats all!

Which came first the math or the signal theory? Math. Fourier math came first and it was worthy to apply in signals which signals you know are waves.

Applying the Euler term and multiplying by 2/2:

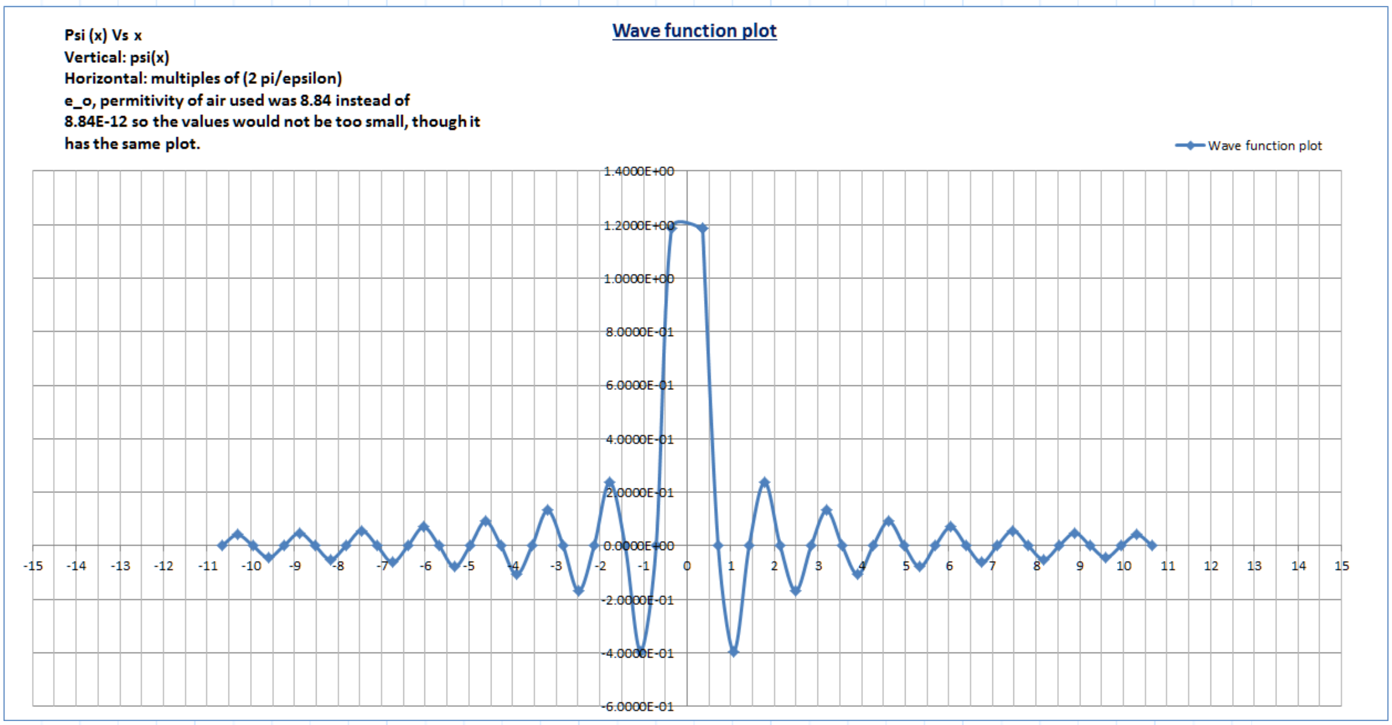
$$\sin\left(i\cdot\varepsilon\cdot\frac{x}{2}\right) = (2)\cdot\left(e^{i\cdot\varepsilon\cdot\frac{x}{2}} - e^{-i\cdot\varepsilon\cdot\frac{x}{2}}\right)$$

$$\sin\left(i\cdot\varepsilon\cdot\frac{x}{2}\right) = (2)\cdot\left(e^{i\cdot\varepsilon\cdot\frac{x}{2}} - e^{-i\cdot\varepsilon\cdot\frac{x}{2}}\right)$$

$$\psi(x) := \left(\sqrt{\frac{2}{\pi\cdot\varepsilon}}\right)\cdot\left(\frac{\sin\left(\varepsilon\cdot\frac{x}{2}\right)}{x}\right)$$

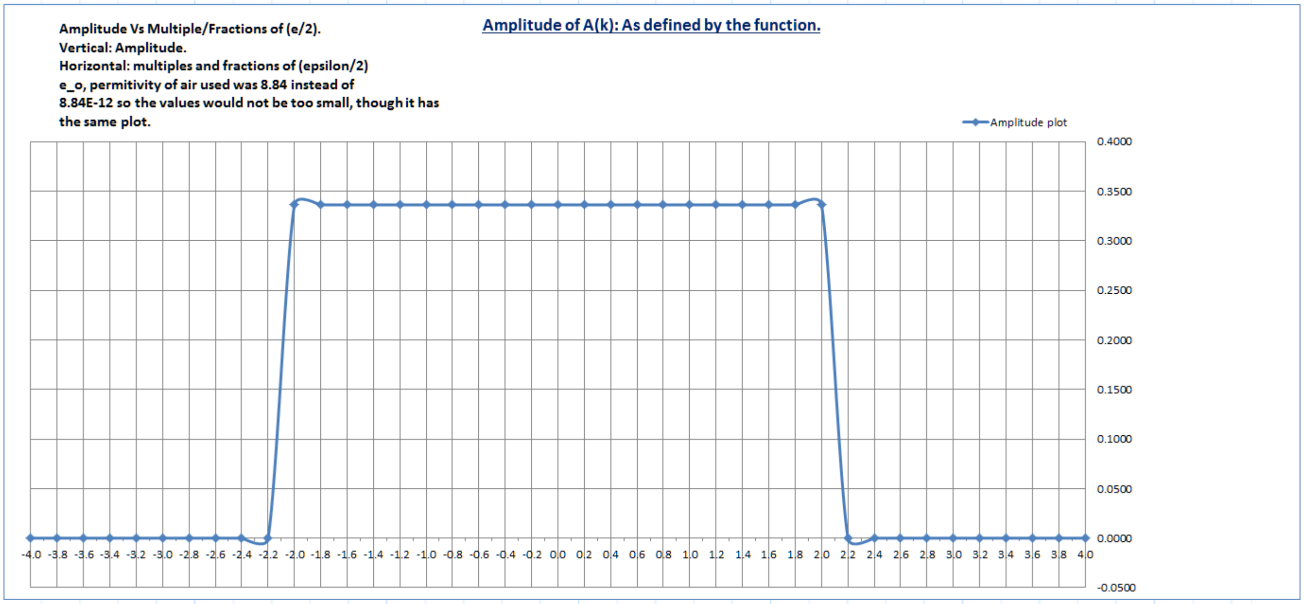
Ans. More like a reduced term.
This is the wave function psi(x).

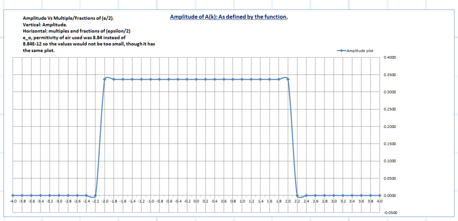
Graph of psi(x) shown below. The cental peak of psi(x) is zero at x = +/- (2pi)/epsilon.
This graph was done in Excel.



Important general property of wave packets: delta(x) is the spatial extent of the wave packet, and delta(k) is its wave number range, then when
(delta (x))(delta (k)) >=1, reciprocity relation exist.

The graph of the amplitude A(K), is between (-e/2) and (e/2). In Excel the edges are not exactly squared off. Could be done in Mathcad, instead Excel used for plot and tabulation.





Width of the centre peak is (4 pi)/epsilon.
Shown to the right.

So delta(x) = 4 pi/epsilon

Returning to the amplitude graph,
width of A(k)
delta(k) = e (epsilon)

Therefore,

(delta (x)) (delta(k))
= (4 pi / epsilon) x (epsilon)
= 4 pi.
= > or = 1
This is the reciprocity relation.

From this example, maybe we see it maybe we dont! Here it is note below.

Mahesh Jain, Chapter 5 page 74: It is impossible to make the widths delta(x) and delta (k) small. The smaller the spatial extent of a wave packet, the larger is the range of wave numbers in its Fourier decomposition, and vice versa. This general feature of wave packets has very deep implications in QM in the form of Hisenberg's uncertainty principle.

So smaller delta(x) larger delta(K), and larger delta(x) smaller delta(k). We show its product is greater than or equal 1 then it has reciprocity.

Reciprocity in mathematics is variable A multiplied by B results in 1.
If A = 3/4 then B has to equal 4/3.

Here delta(x) x delta(k) = 1 is a reciprocity
plus we extend it to values greater than 1.

'In QM a particle, electron-neutron, is represented by a wave packet.
The wave packet is made up of multiple waves, each wave has a phase velocity.
Each packet has one group velocity.
So can we determine where the particle represented by a wave packet is going to hit within a given area of choice? No. You may be lucky sometimes but in an empirical sense No.
That No is part of a foundation on the Uncertainty Principle.' - My Wording.

Mahesh puts it as 'the position of the particle is indeterminate within the width of the wave packet. Similarly the momentum of the particle is indeterminate within the region where the momentum wave function psi(p) is non zero.

Momentum is mass x velocity, if the velocity is unable to precisely locate where the particle will land/strike on the choice area, no less can the momentum.

We ask why is it so critical to know where the particle lands?
Take the solar panels for example, an easy case, we want the photon to land on the electron hit it hard throw the electron out of orbit and start flowing. So in this perspective knowing the location and precision of strike or landing may be important. We dont make ONE solar panel (PV) the size of a football field! You agree? Goal!

Returning to reciprocity, the x is distance or coordinate in the x-axis, k the space wave function, here delta(x) x delta(k) > or = 1.

We know h' multiplied by k results in p
h' = h/(2 pi)
h'k = p
k = p / h'

substitute k into the reciprocity expression,
delta(x) delta(p) > or = h.

Ok. But does this give us any better precision than before? **No.**

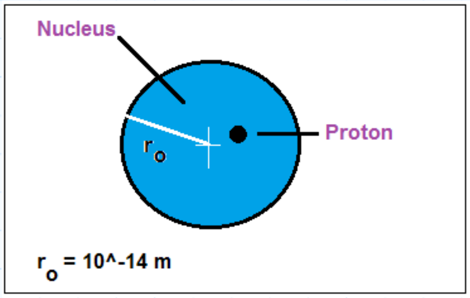
However it extends our lack of precision and turns into an understanding of uncertainty. **Hisenberg's uncertainty principle states: It is not possible to specify both the position and momentum of a particle simultaneously with artibrary precision; the product of uncertainties in the position and momentum is always greater than a quantity of order h' (ie h/(2 pi)) - Mahesh.**

Problem 5.2

Calculate the uncertainty in the momentum of a proton confined in a nucleus of radius 10^-14 m.

From this result, estimate the kinetic energy of the proton.

Solution:



Having read and UNDERSTOOD the theory of uncertainty principle and the several short formulae/expressions derived with their assumptions/conditions we proceed with the example solution. Applying these formulas is easy it may require some creative visualisation on the the out come of the solution. So work on this.

Uncertainty principle for delta p = h(slant)/r_o
where r_o is the radius of the electronic orbit.

Constants:

$h := 6.63 \cdot 10^{-34}$ Js
$c := 3 \cdot 10^{8}$ m/s
$eV := 1.6 \cdot 10^{-19}$ J
$e := 1.6 \cdot 10^{-19}$ J
$m_{neutron} := 1.675 \cdot 10^{-27}$ kg $\quad m_{proton} := m_{neutron}$
$m_{electron} := (9.1 \cdot 10^{-31})$ kg

$h' := \frac{h}{(2 \cdot \pi)} = 1.055 \cdot 10^{-34}$ $\quad r_0 := 10^{-14}$ m

$\Delta p := \frac{h'}{r_0}$

$\Delta p = 1.055 \cdot 10^{-20}$ kg m/s. This is the uncertainty in the momentum NOT momentum.

Momentum p to be of order of (delta) p ....as taught in the theory part......

then the E or KE of the proton is given by: E = (p^2)/2m
= aprox to h' / (2 m r_o^2)
m is the mass of proton here in this solution same as neutron.

$$\text{KE Or E} = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_o r}$$

$$\triangle r \approx r$$

$$p \approx \triangle p \approx \frac{\hbar}{r}$$

$$\text{KE Or E} = \frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_o r}$$

$$\frac{dE}{dr} = 0$$

$$E = \frac{p^2}{2m} \approx \frac{\hbar^2}{2mr_o^2}$$

<---Some key expressions provided to the right, refer to any QM textbook.

$$KE := \left(\frac{h'^2}{2 \cdot m_{proton} \cdot r_0^{\,2}}\right) = 3.324 \cdot 10^{-14} \quad J$$

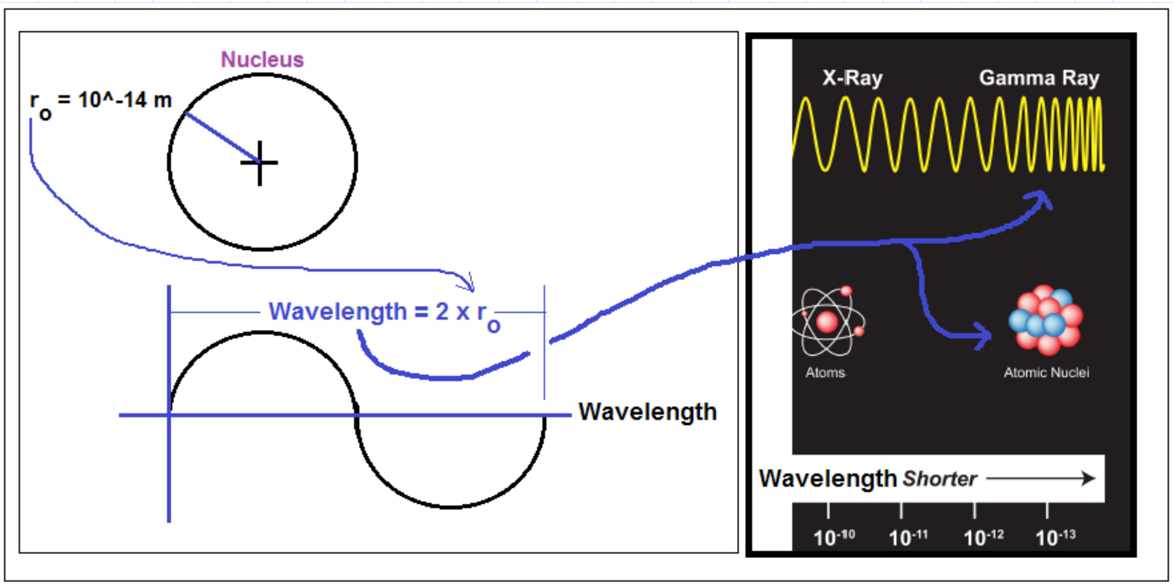
Lets make the result in eV:

$$KE\_eV := \frac{KE}{eV} = 2.077 \cdot 10^{5} \quad eV$$

KE_ev = 0.2077 MeV Ans.

For me information in figure below is more than FYI (For Your Information).
I am not a Physics major hence relating wavelength to the spectrum is not typical work for me, for most electrical engineers in communications it is!

So the wavelength is around the Gamma Ray range...which our problem is close too.

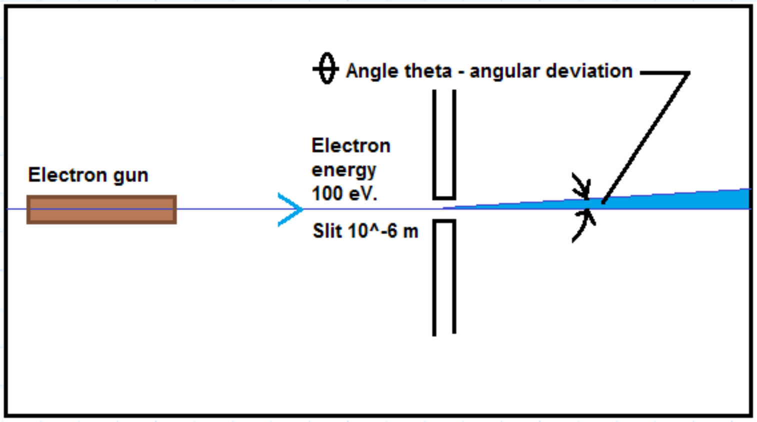


Problem 5.3

An electron of energy 100 eV is passed through a slit of width 10^-6 m.

Estimate the uncertainty introduced in the angle of emergence.

Solution:



Key variables:

$E_{electron} := 100 \cdot eV$

$\Delta x := 10^{-6}$ m

$h' = 1.055 \cdot 10^{-34}$

Energy E of a free particle of mass m and momentum p: E = p^2/2m
p = Sqrt(2mE).

The momentum of the electron is:

$p_{electron} := \sqrt{2 \cdot m_{electron} \cdot E_{electron}}$ $\quad p_{electron} = 5.396 \cdot 10^{-24}$ kg m/s

Uncertainty in the electron momentum:

$\Delta p := \frac{h'}{\Delta x} = 1.055 \cdot 10^{-28}$ kg m/s.

Uncertainty in the angle of emergence:

Some reasoning required here. We have the uncertainty in the electron momentum, we have the electron momentum. So would uncertainty divided by the certain electron momentum calculated (p_electron) provide a deviation? Yes. A degree of deviation from the certainty. This would be in radians. Not in degrees since we do not have any degree given in the problem to use directly or indirectly. Obviously Radians from the reading of the problem!....em?

$\Delta\theta := \frac{\Delta p}{p_{electron}} = 2 \cdot 10^{-5}$ Radians. $\quad One\_Radian := \frac{(60 \cdot 180)}{\pi} = 3437.75$ Min of Arc.

$One\_Radian := \frac{(60 \cdot 60 \cdot 180)}{\pi} = 206264.81$ Seconds of Arc.

$\Delta\theta_{ARC} := \Delta\theta \cdot One\_Radian = 4.033$ Seconds of Arc. Ans.

Problem 5.4

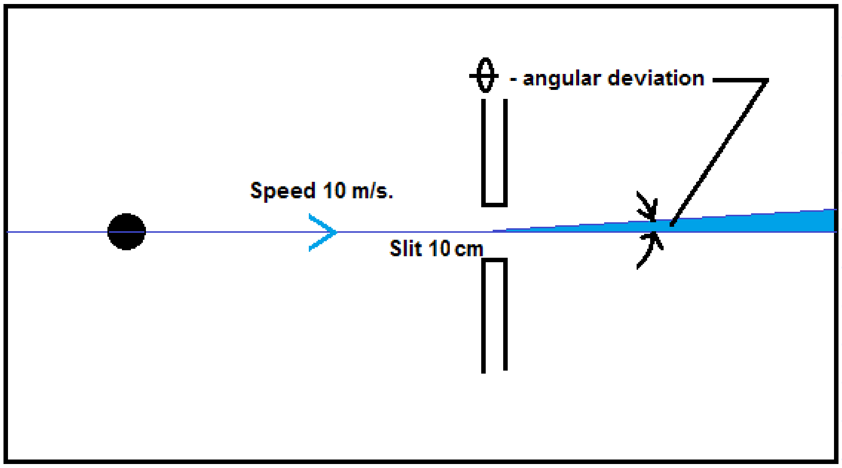
Revisit Problem 5.3.

A lead ball of mass 0.2 g.
Thrown with a speed of 10 m/s.
Through a slit of radius 1.0 cm i.e. 0.1 m.

Estimate the uncertainty introduced in the angle of emergence.

Comment(s): Problem is more to the physical world size.

Solution:



Key variables:

$\Delta x := 0.01$ m (10 cm)

$v := 10.0$ m/s

$m_{ball} := 0.2 \cdot 10^{-3}$ kg

$h' = 1.055 \cdot 10^{-34}$

Energy E of a free particle of mass m and momentum p: E = p^2/2m
p = Sqrt(2mE).

The momentum of the 0.2 g ball:

$p\_ball := m_{ball} \cdot v = 0.002$ kg m/s

Uncertainty in the electron momentum:

$\Delta p := \frac{h'}{\Delta x} = 1.055 \cdot 10^{-32}$ kg m/s.

<u>Uncertainty in the angle of emergence:</u>

$\Delta \theta := \frac{\Delta p}{p\_ball} = 5.28 \cdot 10^{-30}$ Radians.

$One\_Radian := \frac{(60 \cdot 60 \cdot 180)}{\pi} = 206264.81$ Seconds of Arc.

$\Delta \theta_{ARC} := \Delta \theta \cdot One\_Radian = 1.088 \cdot 10^{-24}$ Seconds of Arc. Ans.

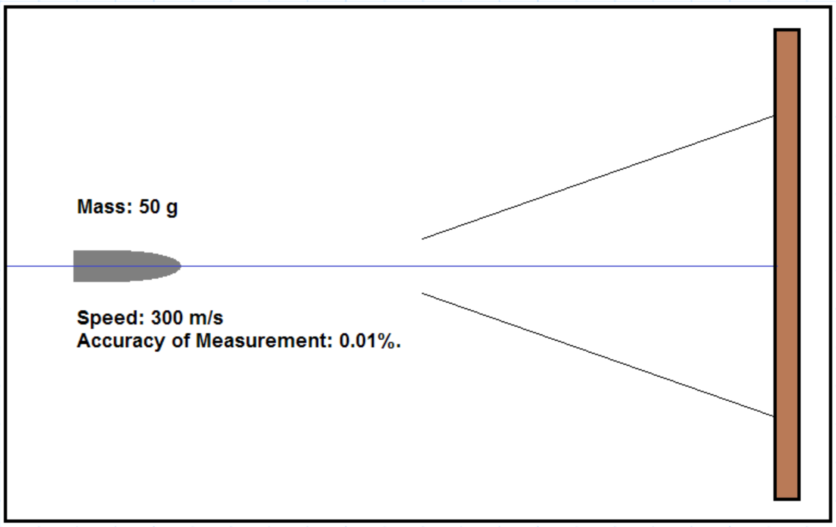
Problem 5.5

Speed of a bullet of mass 50 g is measured to be 300 m/s.
Measured at an accuracy of 0.01%.

What accuracy can we locate the position of the bullet?

Comment(s): A slight twist from the past 2 problems, to find the location (delta)x.

Solution:



Key variables:

$v := 300.0$ m/s

$v_{accuracy} := 0.01\%$ Percent

$m_{bullet} := 50 \cdot 10^{-3}$

$h' = 1.055 \cdot 10^{-34}$

Uncertainty in momentum (delta)p = (delta) mv = m (delta) v

$v_{adjusted} := v \cdot v_{accuracy} = 0.03$ m/s.

$\Delta p := m_{bullet} \cdot v_{adjusted} = 0.002$ kg m/s

(delta)p (delta)x = h' .....h'=h/(2 pi).....and (delta)x is what we seek.

$\Delta x := \frac{h'}{\Delta p} = 7.035 \cdot 10^{-32}$ m Ans. This is the accuracy to locate the position.
It is remarkably small compared to the other real world physical values in the problem.

Problem 5.6

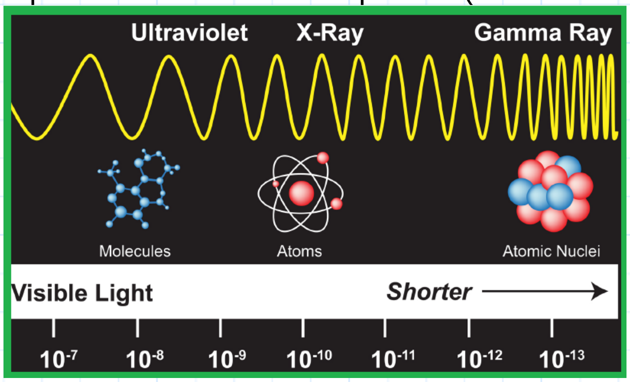
The lifetime of a nucleus in an excited state is 10^-12s.

Calculate the probable uncertainty in the energy and frequency of a gamma ray photon emitted by it.

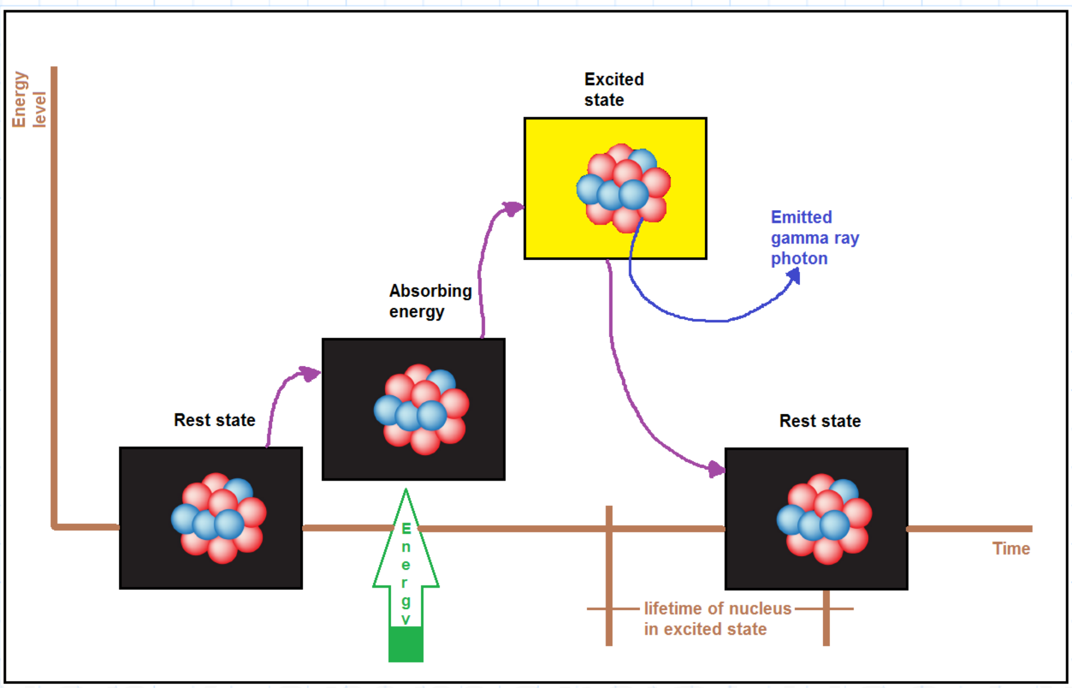
Comment(s): This can be an energy problem in nuclear power plants.

Solution:

The figures-sketches below may not be accurate for the problem's solution, it merely tries to give a sequence of events and location of the gamma ray in the spectrum chart relative to particle (molecule-atom-nuclei).



<---Figure from Ball Aerospace Technologies Corporation.



Uncertainty relation between enery-time is:

(delta)E (delta)t approximatley equal h'

(delta)E the uncertainty in energy approximately equal (h'/delta t)

$\Delta t := 10^{-12}$ seconds

$\Delta E := \frac{h'}{\Delta t} = 1.055 \cdot 10^{-22}$ J Ans.

Now, lets try to calculate similalry the uncertainty in frequency.

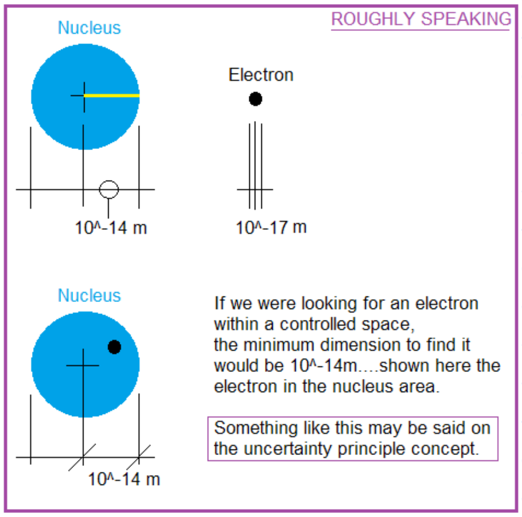
(h)(delta v) approximately equal (delta)E the uncertainty in energy.
Note its h not h'.

$\Delta v := \frac{\Delta E}{h} = 1.592 \cdot 10^{11}$ Hz. Ans.

Problem 5.7

Using the uncertainty principle, show that an alpha particle can exist inside a nucleus.

Solution:



Radius of a nucleus is 10^-14 m.
A particle, electron proton......something really small....including alpha particle, if were to exist in the nucleus, then the uncertainty in its position must be of this order:

(delta)x approximately equal 10^-14 m.

(delta)x ~ 10^-14 m.

$\Delta x := 10^{-14}$ m. Δx= r the radius

Note: The expression is (delta)p = h'/r.

The uncertainty in the momentum of the particle, as shown previouly, must be of the order of:

(delta)p (delta) x = h'

(delta)p = h'/(delta)x.

$$\Delta p_{alpha} := \frac{h'}{\Delta x} = 1.055 \cdot 10^{-20} \text{ kg m/s.}$$

Momentum of the particle is at the minimum of this order, i.e. 10^-20.
Notice the order is much lower than (delta)x!

There's no need to open up a Physics textbook on what exacly an alpha particle is, Encyclopedia Britianica is here.

Enclycopedia Britianica: Alpha particle, positively charged particle, identical to the nucleus of the helium-4 atom, spontaneously emitted by some radioactive substances, consisting of two protons and two neutrons bound together, thus having a mass of four units and a positive charge of two. Discovered and named (1899) by Ernest Rutherford, alpha particles were used by him and coworkers in experiments to probe the structure of atoms in thin metallic foils. This work resulted in the first concept of the atom as a tiny planetary system with negatively charged particles (electrons) orbiting around a positively charged nucleus (1909–11). Later, Patrick Blackett bombarded nitrogen with alpha particles, changing it to oxygen, in the first artificially produced nuclear transmutation (1925). Today, alpha particles are produced for use as projectiles in nuclear research by ionization—i.e., by stripping both electrons from helium atoms—and then accelerating the now positively charged particle to high energies.

Rest mass of an alpha particle is roughly 4 times the mass of a proton.

$m_{proton} := 1.67 \cdot 10^{-27}$ kg.

$m_{alpha} := 4 \cdot m_{proton} = 6.68 \cdot 10^{-27}$ kg.

The speed of the alpa particle thus would be?
From the momentum relation (delta)p = mv, where v = (delta) p/m.

$$v_{alpha} := \frac{\Delta p_{alpha}}{m_{alpha}} = 1.58 \cdot 10^{6} \quad \text{m/s}$$

The speed does not seem very high...because the order of the speed of light is 10^8, and the alpha particle is at 10^6, just 10^3 difference.

What does this say?

Its in the non-relativistic area. So we use the non-relativistic expression for KE.

$$KE_{alpha} := \frac{(\Delta p_{alpha})^2}{2 \cdot m_{alpha}} = 8.334 \cdot 10^{-15} \quad \text{J}$$

KE in eV:

$$KE_{alpha\_eV} := \frac{KE_{alpha}}{eV} = 5.209 \cdot 10^{4}$$

$KE_{alpha\_eV} := 52.09 \cdot 10^{3}$ keV. Ans.

How do we go further? To proof the GROUND STATE energy of the helium atom?

Since its the GROUND STATE, we are saying the energy is at its lowest value. Obviously, then one way to proof is that at higher state the alpha particle energy would be greater then 52.09 x 10^3 keV. Such is the case, when energy carried by an alpha-particle emitted by nuclie it is much higher than this value. Now we can say alpha-particles can exist inside a nucleus.

Comment:

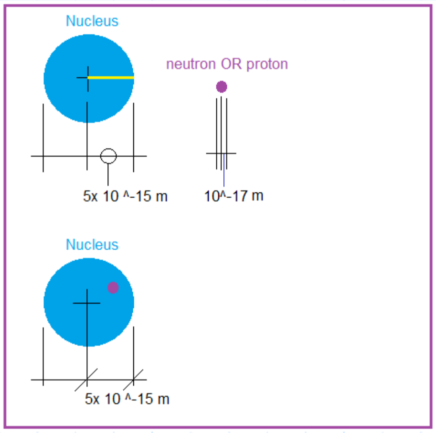
"Some of us may say such is not the best case of defence going to court, but such is the subject matter of quantum mechanics" - Karl Bogha.

A rather interesting example to had given the quantum oppurtunity to make that comment.

Problem 5.8

A nucleon (neutron or proton) is confined to a nucleus of radiuis 5x10^-15 m. Calculate the minimum possible values of the momentum and the kinetic energy of the nucleon.

Solution:



We read it similar to the previous problem, not exactly the same, but some toughts do lead us to the same equations.

Review again the previous problem. To get the minimum possible value what needs to be adjusted in the expression

(delta)p = h' / (delta)x?

delta(x) needs to be maximised or set maximum so (delta)p is minimum because h' is the same in the expression.

Notes: (delta)x (delta)k > = 1.

If (delta)x is made small, then delta(k) must get larger, then their multiplication is greater than or equal to 1.

So obviously we cannot make both widths small or both widths large.

Theory:

The smaller the spatial extent, (delta)x, of a wave packet, the larger the range of wave numbers, (delta)k, in its Fourier decomposition, and vice versa.

The exact statment and proof of the position momentum uncertainty relation: using root-mean square deviation results with

(delta)x (delta)p > = h'/2.

We know (delta)x (delta)k >=1, h'k = p, thus (delta)x (delta)p >= h'.
Then.......from applying..
..Schrodinger Equations....Schwarz inequality... (delta)x (delta)p > = h'/2.

Maximum uncertainty in the position of the nucleon(neutron or proton) is going to be at either side. So we multiply the radius by 2. Why? see above notes.

(delta)x (delta)p > = h'/2.
2 (delta)x (delta)p > = h'. The momentum is not going to be the same the adjustable is (delta)x, where it is made max resulting in minimum momentum p.

$r := 5 \cdot 10^{-15}$ m.

$\Delta x := r$

$\Delta x_{max} := 2 \cdot \Delta x = 1 \cdot 10^{-14}$ m.

According to the uncertainty principle, the minimum uncertainty thus in momentum of the particle is

$\Delta p_{min} := \dfrac{h'}{\Delta x_{max}} = 1.055 \cdot 10^{-20}$ kg m/s Ans.
The momentum cannot be less that this value.

Continuing with the minium kinetic energy KE:

$KE_{min} := \dfrac{\left(\Delta p_{min}{}^{2}\right)}{2 \cdot m_{neutron}} = 3.324 \cdot 10^{-14}$ J.

The answer in eV:

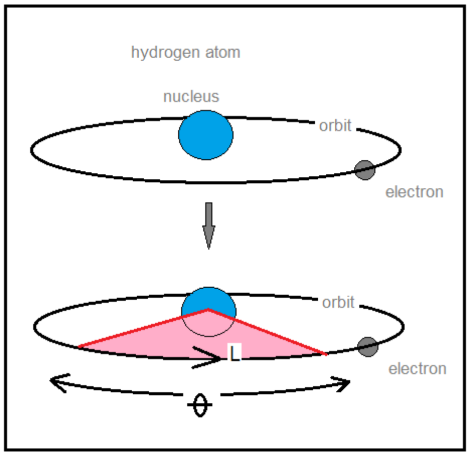
$KE_{min\_eV} := \dfrac{KE_{min}}{eV} = 2.077 \cdot 10^{5}$

$KE_{min\_eV} := 0.2077$ MeV Ans.

Problem 5.9

If the angular momentum of the electron in a hydrogen atom is known to be 2h' within 5% accuracy, show that its angular position in a perpendicular plane cannot be specified at all.

Solution:



Lets say the figure on the left roughly describes the problem.

We have linear, straight line, momentum:

(delta)x (delta)p = h'

and we got angular momentum:

(delta) angle theta (delta) angular momentum L = h'.

Uncertainty principle for angular momentum:

(delta)theta (delta)L approximately equal h'

L = 2h' with 5% accuracy

(delta)L = 2h' (5/100) = h'/10

(delta) theta = h' / (delta)L
= h' (10/h')
= 10 radians

1 radian = 2 pi, the angle obviously cannot be greater than 1 radian, we have 10 radian!

Conclusion: With L = 2h' the angular position in a perpendicular plane cannot be determined. Ans.

Problem 5.10

The average lifetime of an excited atomic state is 10^-8s.
If the wavelength of the spectral line associated with the transition from this state to the ground state is 6000 Angstrom, estimate the width of this line.

Solution:

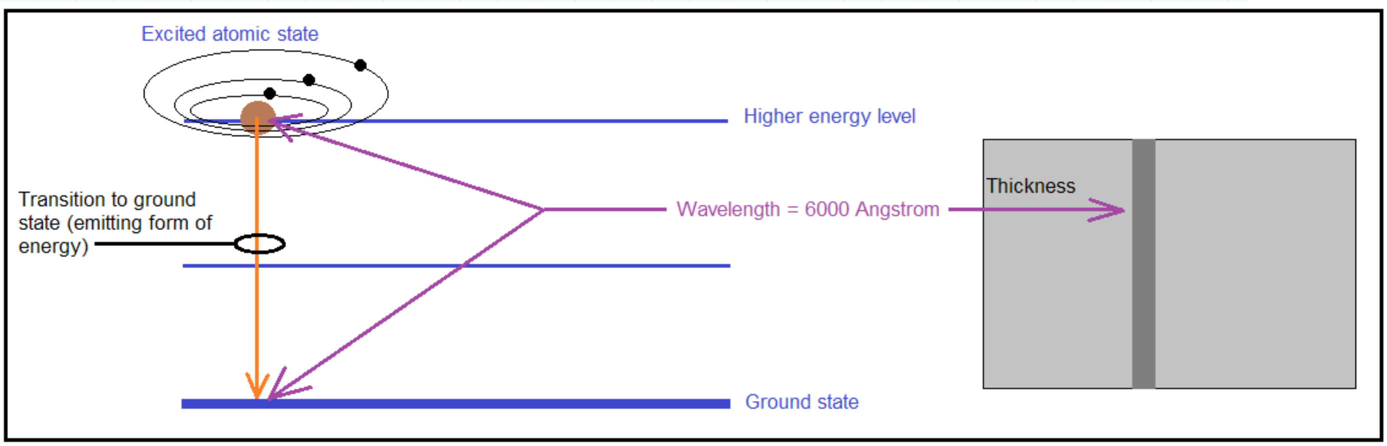
In this exercise I consider this an informative problem.
I hope it was the outcome from the subject matter studied through the QM textbook(s), because the contents is quite deep with many lengthy derivations.
If we can then turn around and do this short solution to the question for the line width, its a little accomplishment!

The width of the spectral line is what we seek. This is saying if there was an experiment it results with a spectral line, and that line's width is what we seek.

Not easy!

Lets see what Mahesh the Physicst-Lecturer presented for the solution.

Rough sketch provided in figure below it tries to capture the problem. May not be an accurate or correct sketch on identifying the thickness as shown. You are most welcome to correct it. Check the internet.......spectroscopy!



E = hv ...from our previous lessons.

v = 1/lamda

E = h/lamda

Taking the derivative on each side

(E)_d = hc/(lamda) d_lamda

becomes

E d_Energy = hc (1/lamda^2) d_lamda

d_Energy and D-lamda the differential delta

$$\Delta E = \frac{(hc)}{(\lambda)^2} \Delta\lambda$$

This the expression intended to show, was not easy writting it as shown above.

Clearly we see the (delta) lamda term represents the width of the spectral line of concern!

So that is what we need to solve for. We have seen similar expression in past solution of differential equation course, here its one of those examples.

The uncertainty principle of concern here is

(delta)E (delta)t = h'

substitute for (delta)E

(hc/lamda^2) (delta) lamda(delta)t = h'

(delta)lamda = (lamda^2) / (2 pi c (delta)t)

$\lambda := 6000 \cdot Angstrom \quad \lambda = (6 \cdot 10^{-7})\ m$

$\lambda := 6 \cdot 10^{-7}$ m

$\Delta t := 10^{-8}$ s

$c = 3 \cdot 10^{8}$ m/s

$\Delta\lambda := \frac{(\lambda^2)}{2 \cdot \pi \cdot c \cdot \Delta t} = 1.91 \cdot 10^{-14}$ m Ans.

This is an extremely thin thickness.
Its not the wavelength rather thickness.

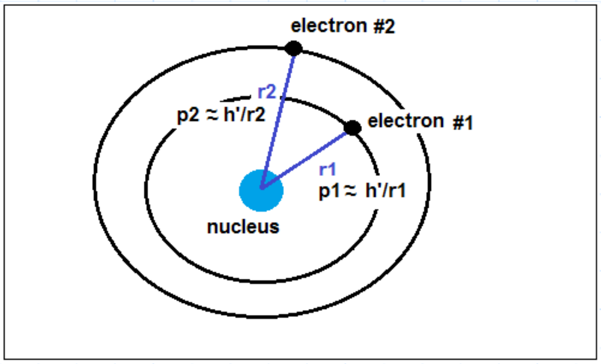
Problem 5.11

Using the uncertainty principle, estimate the ground state energy of the helium atom.

Solution:

When we hear Helium we are taught to comapre it to Hydrogen atom, they have some similarities.

Helium atom has 2 electrons.



Uncertainty principle using momentum and radius:

(delta) r (delta) p = h'

(delta) r1 (delta) p1 = h'
(delta) r2 (delta) p2 = h'

Kinetic energy of the system, composed of the nucleus and 2 electrons:

KE equal approximately (h'^2/2m) ((1/r1^2) + (1/r2^2))

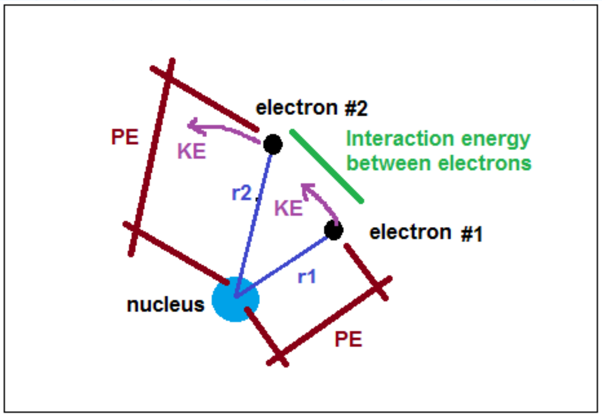
m the electron mass in the expression above.

PE of the interaction of 'electrons' with the 'nucleus of charge 2e' since we have 2 electrons is:

PE equal approximately -2 e^2 ((1/r1) + (1/r2)) ......from Physics 1st year college/university.

Now, the ORDER of separation 'between the electrons' is the 'sum of the order of r for each electron':
(r1 + r2)......uncertainty principle.

The interaction energy between the electrons is approximately:
(e^2) / (r1 + r2).



Total energy of the system is the sum of

1. KE
2. PE
3. Interaction energy between electrons

$$E \approx \underbrace{\frac{h'^2}{2m}\left(\frac{1}{r1^2} + \frac{1}{r2^2}\right)}_{KE} - \underbrace{2e^2\left(\frac{1}{r1^2} + \frac{1}{r2^2}\right)}_{PE} + \underbrace{\left(\frac{e^2}{r1 + r2}\right)}_{\text{Interaction energy between electrons}}$$

Ground state the energy E = 0.

So as usual we differentiate to set it as dE/d = 0
Since we have to variables r1 and r2 we will differentiate wrt to r1 and r2, giving us 2 equations, we solve for?
r1 and r2.

$$\frac{dE}{dr1} = \frac{-h'^2}{m\,r1^3} + \frac{2e^2}{r1^2} - \frac{e^2}{(r1 + r2)^2} = 0 \qquad \text{...Eq 1}$$

and

$$\frac{dE}{dr2} = \frac{-h'^2}{m\,r2^3} + \frac{2e^2}{r2^2} - \frac{e^2}{(r1 + r2)^2} = 0 \qquad \text{...Eq 2}$$

2 variables r1 and r2, and 2 equations...we can solve for the variables r1 and r2.
Lets not trouble ourselves with it, we'll assume the textbook has the correct solution.

You are welcome to solve it.

$$r1 = r2 = \frac{4}{7}\left(\frac{h'^2}{me^2}\right)$$

Substituting back r1 and r2 into the euqation for E, we have

$$E = -\frac{49}{16}\left(\frac{me^4}{h'^2}\right)$$

Solve for E above by substitution of values.

$m_{electron} = 9.1 \cdot 10^{-31}$ $\quad h' := 1.05 \cdot 10^{-34}$ $\quad eV = 1.6 \cdot 10^{-19}$

$$E := \left(\frac{-49}{16}\right) \cdot \left(\frac{(m_{electron}) \cdot (e^4)}{(h'^2)}\right) = -1.657 \cdot 10^{-37} \text{ J}$$

In eV:

$$E_{eV} := \frac{E}{eV} = -1.035 \cdot 10^{-18}$$

$E_{eV} := -10.35$ eV Ans.

End of Chapter 5 exercises from QM for UG by Mahesh C. Jain.

The next chapter is on the theory side of Schroedinger Equation.
No solved problems here.

Recommended textbook on Schroedinger Equation:
Quantum Mechanics DeMystified: A Self-Teaching Guide. David McMahon. 2006. McGrawHill. Chapters 1 and 2.