

3

Equivalent Digital Low Pass Filter (I[•]order)

Introduction.

This worksheet, developed using a common operational amplifier in an inverting configuration, begins with a brief summary of the main results of the circuit analysis, enriched with graphics and examples. Seven signals, among the most common, are generated and used as input of the amplifier or, once sampled, as input to the digital filter. The many algorithms to implement the corresponding digital filter are derived applying the z-transform. Two approximations are applied to each result to derive the corresponding difference equations. Thus one will see that, as the analog filter is effective, just is the digital one with the used approximations. After reviewing this worksheet, the reader just has to choose the algorithm and implement the firmware for the DSP.

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2) Digital first order low pass filter difference equations,	
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- 1) Sequence of the periodic response,
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- 3) Schematics,
- 4) Graphics,
- 5) Comparison of the Bode plots of the z and s transfer functions

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References

- Reference:C:\Users\Franc\new folder\programs.xmcd
- Reference:C:\Users\Franc\new folder\global data.xmcd
- Reference:C:\Users\Franc\new folder\Signals-Only List.xmcd
- Reference:C:\Users\Franc\new folder\Pulse Train Data.xmcd
- Reference:C:\Users\Franc\new folder\sawtooth pulse data.xmcd
- Reference:C:\Users\Franc\new folder\sawtooth pulse train data.xmcd
- Reference:C:\Users\Franc\new folder\FM data.xmcd
- Reference:C:\Users\Franc\new folder\PM data.xmcd

References

Definitions and a few necessary constants:

$$\text{Amplifier Gain: } A_3 = -10, \quad (3.1)$$

$$\text{Cut off frequency: } f_3 = 15.0 \cdot \text{kHz}, \quad (3.2)$$

$$\text{Cut off period: } T_3 = \frac{1}{f_3}, \quad T_3 = 66.67 \cdot \mu\text{s} \quad (3.3)$$

$$\text{Pass Band edge: } \omega_3 = 2 \cdot \pi \cdot f_3, \quad (3.4)$$

$$\text{time constant: } \tau_3 = \frac{1}{\omega_3} \quad (3.5)$$

$$\text{Quality factor: } Q_3 = 10.0, \quad (3.6)$$

$$\text{damping factor: } \zeta_3 = \frac{\omega_3}{2 \cdot Q_3}, \quad \zeta_3 = 4.71 \cdot \frac{\text{krad}}{\text{sec}}, \quad (\omega_3 = \frac{1}{\tau_3} = 2 \cdot \zeta_3 \cdot Q_3) \quad (3.7)$$

Defined in "global data.xmcd":

$$\text{Op. Amp. saturation voltage: } V_{\text{sat}} = 15 \cdot \text{V} \quad (3.8)$$

$$\text{Number of samples for the FFT: } N_{\text{gd}} = 256 \quad (3.9)$$

$$\text{Number of elements of a series: } N_{\text{gd}} = 50 \quad (3.10)$$

$$\text{An integer constant: } U_0 := U_{\dagger}, \quad (3.11)$$

$$k := k_{\dagger} \quad (\text{defined once for all in "global data"}) \quad (3.12)$$

The Bode diagrams will have an extension defined by a multiple $U_0 = 100$, of ω_s , freely chosen.

$$T_{\text{test}} := \frac{2 \cdot \pi \cdot \tau_3}{5} \quad (3.13)$$

$$\omega_{\text{test}} := \frac{2 \cdot \pi}{T_{\text{test}}} \quad f_{\text{test}} := \frac{1}{T_{\text{test}}} \quad \omega_{\text{test}} = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}} \quad (3.14)$$

$$V_i := V_{\text{pp}}$$

3.1 Analog Low Pass Filter (I^oorder)

It is the simple and well known analog low pass active filter (active inverting integrator) shown in the picture below:

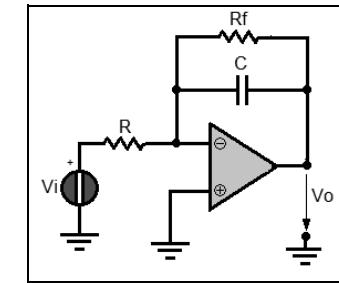


Fig.: (3.1.1)

The main results of the analysis are reported hereafter.

3.1.1) Transfer function

$$\text{The transfer function (ideal Op. Amp.) is: } W_{\text{lp}}(s) = -\frac{R_f}{R} \cdot \frac{1}{1 + s \cdot R_f \cdot C}. \quad (3.1.1.1)$$

The voltage gain is given by: $A_3 = -\frac{R_f}{R}$ so that the t. f. can be written:

$$W_{\text{lp}}(s) = \frac{A_3}{1 + s \cdot R_f \cdot C}, \quad (3.1.1.2)$$

collecting $R_f \cdot C$, results:

$$W_{\text{lp}}(s) = \frac{A_3}{R_f \cdot C \left[\frac{1}{(R_f \cdot C)} + s \right]}, \quad (3.1.1.3)$$

$$\text{placing: } \omega_3 = \frac{1}{R_f \cdot C} = \frac{1}{\tau_3} \quad \text{and} \quad \frac{A_3}{R_f \cdot C} = A_3 \cdot \omega_3 = -\frac{R_f}{R} \cdot \frac{1}{R_f \cdot C} = -\frac{1}{R \cdot C} = -\omega_3 \text{dB},$$

$$\text{and calling the angular frequency for which the voltage gain is 0dB: } \omega_{3\text{dB}} = \frac{1}{R \cdot C}, \quad (3.1.1.4)$$

results:

$$\omega_3 = 0.09 \cdot \frac{\text{Mrads}}{\text{sec}}$$

hence

$$\omega_{3\text{dB}} := -A_3 \cdot \omega_3, \quad (3.1.1.5)$$

whose numerical value is:

$$\omega_{3\text{dB}} = 0.94 \cdot \frac{\text{Mrads}}{\text{sec}},$$

the corresponding period is.

$$T_{3\text{dB}} := \frac{2 \cdot \pi}{\omega_{3\text{dB}}}. \quad (3.1.1.6)$$

$$\text{Finally, after the assumptions made, the transfer function can be written as: } W_{\text{lp}}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3} \quad (3.1.1.7)$$

$$\text{or: } W_{lp}(s) := \frac{-\omega_3 \text{dB}}{s + \omega_3}, \quad -A_3 = \frac{\omega_3 \text{dB}}{\omega_3}. \quad (3.1.1.8)$$

From the definition of transfer function, it is derived that:

$$V_o(s) = W_{lp}(s) \cdot V_i(s), \quad (3.1.2.1)$$

which, in the time domain, is a convolution product. Laplace antitransforming I have the exact system output, that is:

$$v_o(t) = -\omega_3 \text{dB} \cdot e^{-\frac{t}{\tau_3}} \cdot \int_0^t v_i(\sigma) \cdot e^{\frac{\sigma}{\tau_3}} d\sigma. \quad (3.1.2.2)$$

Output Approximation

The transfer function $W_{lp}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3}$ has a pole in $s = -\omega_3$,

collecting s, it can be written as: $W_{lp}(s) = \frac{A_3 \cdot \omega_3}{s \cdot \left(1 + \frac{\omega_3}{s}\right)}$.

Developing in a Maclaurin series the term $\frac{1}{\left(1 + \frac{\omega_3}{s}\right)}$,

$$\text{with the hypothesis that: } |x| = \left(\left|\frac{\omega_3}{s}\right|\right) \ll 1 \Rightarrow |\omega_3| \ll |s| = |j \cdot \omega| = \omega, \quad (3.1.2.3)$$

$$\omega_3 = \frac{1}{R_f \cdot C} = \frac{1}{\tau_3} \ll \frac{2 \cdot \pi}{T}, \quad (3.1.2.4)$$

or for $T \ll (2 \cdot \pi \cdot \tau_3)$, results :

$$\left(\frac{1}{1+x}\right) (\approx) ((1-x+x^2-x^3+x^4-x^5+\dots)). \quad (3.1.2.5)$$

Hence, for $\omega \gg \omega_3$ and in a first approximation, it can be written:

$$W_{lp}(s) \approx \left(\frac{A_3 \cdot \omega_3}{s}\right), \quad (3.1.2.6)$$

and for the system output:

$$V_o(s) = A_3 \cdot \omega_3 \cdot \frac{V_i(s)}{s} \quad (3.1.2.7)$$

anti transforming :

$$v_o(t) \approx \left(A_3 \cdot \omega_3 \cdot \int_0^t v_i(\sigma) d\sigma\right), \quad A_3 \cdot \omega_3 = -\frac{1}{R \cdot C} = -\omega_3 \text{dB}. \quad (3.1.2.8)$$

Ultimately the approximated temporal trend of the output signal is:

3.1.2) Graph of the impulse response

$$v_o(t) = -\omega_3 \text{dB} \int_0^t v_i(\sigma) d\sigma \quad (3.1.2.9)$$

Impulse response:

$$A_3 := A_3 \quad s := s \quad a := a \quad \omega_3 := \omega_3$$

$$\text{Dirac pulse response: } w(t) := \frac{A_3 \cdot \omega_3}{s + \omega_3} \text{ invlaplace, s, t } \rightarrow A_3 \cdot \omega_3 \cdot e^{-t \cdot \omega_3}, \quad A_3 = -10$$

$$\text{The impulse response is: } w(t) = A_3 \cdot \omega_3 \cdot e^{-\omega_3 \cdot t} \cdot \Phi(t), \quad A_3 \cdot \omega_3 = -942.48 \cdot \frac{\text{krad}}{\text{s}} \quad (3.1.2.10)$$

$$t := 0 \cdot \tau_3, 0 \cdot \tau_3 + \frac{10 \cdot \tau_3}{10000} \dots 10 \cdot \tau_3$$

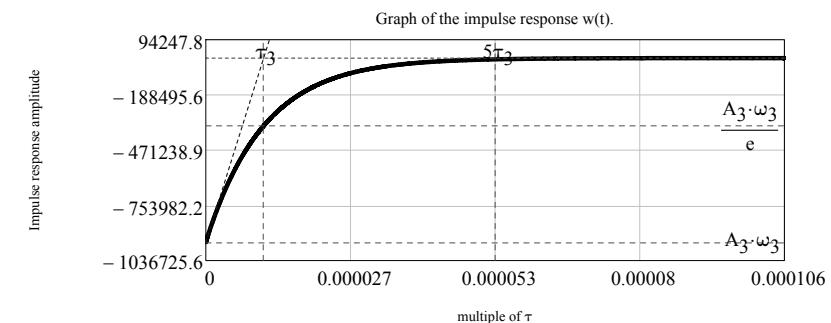


Fig.: (3.1.1.1)

3.1.3) Bode plots

$$\text{Equation of the oblique asymptote: } A_{dB}(\omega) := 20 \cdot \log(|\omega_3 \text{dB}| \cdot \sec) - 20 \cdot \log(\omega \cdot \sec)$$

$$\omega_3 = 0.09 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \omega := \frac{\omega_3}{U_0}, \frac{\omega_3}{U_0} + \frac{\omega_3 \text{dB} \cdot U_0 - \frac{\omega_3}{U_0}}{U_0^2} \dots U_0 \cdot \omega_3 \text{dB} \quad \omega_3 \text{dB} = 0.94 \cdot \frac{\text{Mrads}}{\text{sec}}$$

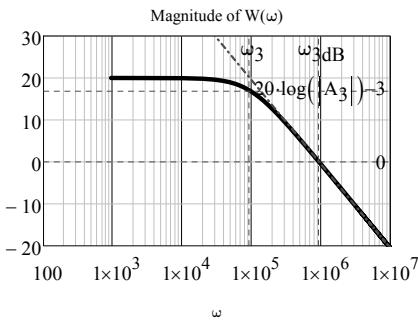


Fig.: (3.1.1.2)

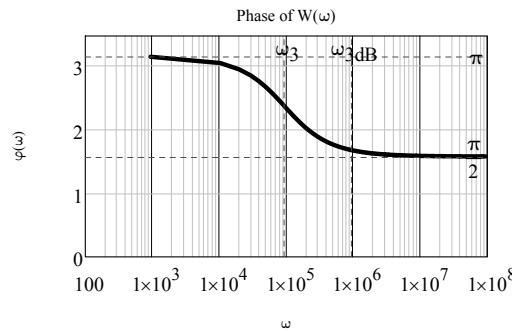


Fig.: (3.1.1.3)

Attenuation:

Asymptote equation: $B_{dB}(\omega) := -20 \cdot \log(|\omega_{3dB}| \cdot \text{sec}) + 20 \cdot \log(\omega \cdot \text{sec}) + 20 \cdot \log(|A_3|)$

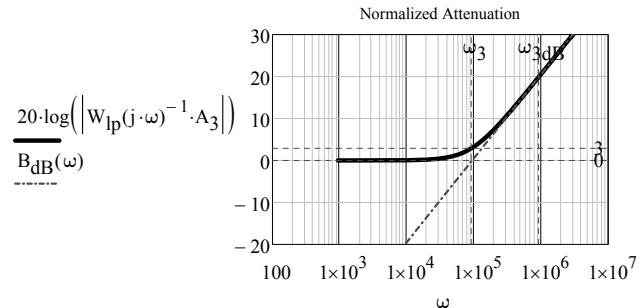


Fig.: (3.1.1.4)

3.2

ANALOG FILTER OUTPUT ANALYSIS

In this paragraph I shall calculate the filter response to various input signals.

Chosen period of the test signal, $T_{\text{test}} = 13333.33 \cdot \text{ns}$. At the corresponding frequency, the voltage gain of the filter is $20 \cdot \log(|W_{\text{lp}}(j \cdot \omega_{\text{test}})|) = 5.85 \cdot \text{dB}$.

3.2.1) Voltage step response $V_i = 5 \cdot \text{mV}$

$$A_3 := A_3 \quad s := s \quad a := a \quad \omega_3 := \omega_3 \quad V_i := V_i$$

$$\text{Time response to the step is: } y(t) := \frac{A_3 \cdot \omega_3 \cdot V_i}{s \cdot (s + \omega_3)} \text{ invlaplace, } s, t \rightarrow -A_3 \cdot V_i \cdot (e^{-t \cdot \omega_3} - 1)$$

$$\text{Hence, the step response is: } y_{\text{sr}}(t) := -A_3 \cdot V_i \cdot (e^{-t \cdot \omega_3} - 1) \cdot \Phi(t)$$

Tangent (drawn a segment only) at the origin:

$$y_{\text{as}}(t) := \begin{cases} A_3 \cdot V_i \cdot \omega_3 \cdot t & \text{if } 0 \leq t \leq \tau_3 \\ \text{break} & \text{otherwise} \end{cases} \quad y_{\text{as}}(t) := A_3 \cdot V_i \cdot \omega_3 \cdot t \cdot \text{rect1}(t, 0, \tau_3)$$

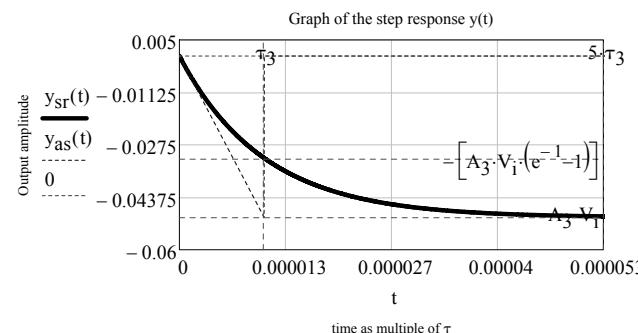


Fig.: (3.2.1.1)

Computation of the signal bandwidth as a periodic signal, with period $2T_3$.

Given the periodic signal:

$$y_{\text{srp}}(t) := \sum_{k=0}^{N_{\text{gd}}} [y_{\text{sr}}(t - k \cdot 2 \cdot T_3) \cdot [\Phi(t - k \cdot 2 \cdot T_3) - \Phi(t - (k+1) \cdot 2 \cdot T_3)]]$$

$$t := 0 \cdot T_3, 0 \cdot T_3 + \frac{10 \cdot T_3}{10000} \dots 10 \cdot T_3$$

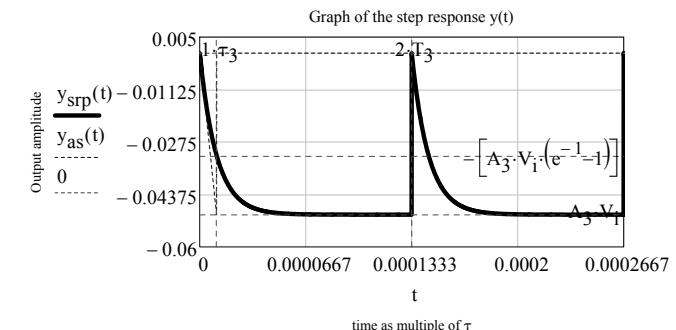


Fig.: (3.2.1.1')

$$\text{Dimensionless Output: } V_{\text{stpr}}(t) := \frac{y_{\text{srp}}(t)}{V}$$

Apply the following program to find among the other the signal bandwidth.

$$rtfs := rt_{\text{gd}} \quad Ustpf_s := \text{BCSA}(V_{\text{stpr}}, rtfs, N_{\text{gd}}, 0, 2 \cdot T_3)$$

Function parameters description:

BCSA(Adimensional signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for Bandwidth Calculation and Signal Analysis"

The function returns a matrix made of three columns.

The first column contains:

- pos. 0: relative error,
- pos. 1: bandwidth (dimensionless),
- pos. 2: the nth. harmonic number corresponding to the give relative error,
- pos. 3: temporary variable,
- pos. 4: Parseval,
- pos. 5: signal average,
- pos. 6: signal r.m.s..

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

$$V3 := \begin{cases} \text{return } Ustpf_s \text{ if IsString}(Ustpf_s) \\ Ustpf_s \text{ otherwise} \end{cases}$$

	0	1	2	3
0	0.1	-0.09	0	0
1	360000	0.01	0	0
2	49	0	0	0
3	0	0	0	0
4	0	0	0	0
5	-0.05	0	0	0
6	0.05	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	...

V3 =

$$\text{coefffa3}_{fs} := \text{Ustpf}_s^{(1)} \quad \text{coefffb3}_{fs} := \text{Ustpb}_s^{(2)}$$

$$\text{coefffa3}_{fs}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.09 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

$$\text{coefffb3}_{fs}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

$$N_{gd} = 50$$

$$\boxed{\text{Signal bandwidth: } B_{3fs} = 0.36 \cdot \text{MHz}}$$

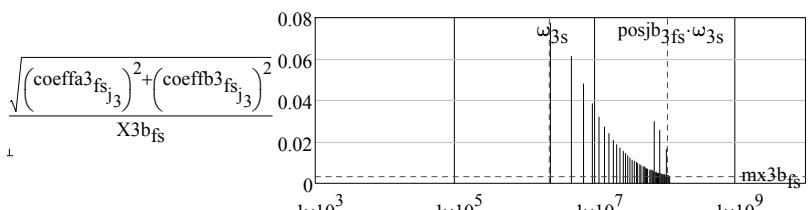


Fig.: (3.2.1.2)

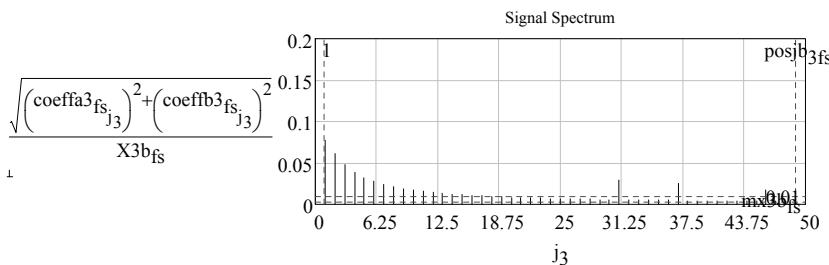


Fig.: (3.2.1.3)

$$\text{Signal bandwidth: } B_{3fs} = 0.36 \cdot \text{MHz}$$

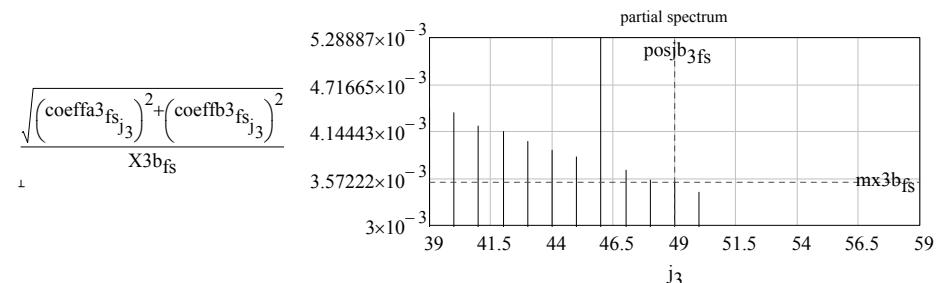


Fig.: (3.2.1.4)

$$\text{sampling frequency: } f_s := 2 \cdot B_{3fs} \quad f_s = 0 \cdot \text{GHz}$$

$$\text{sampling period: } T_{3s} := \frac{1}{f_s} \quad T_{3s} = 1388.89 \cdot \text{ns}$$

$$\text{sampling time step: } n_{3k} := \frac{k}{f_s}, \quad \frac{N_{0gd}}{f_s} \cdot \frac{1}{T_{3s}} = 256$$

$$n^{3T} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1.39 \cdot 10^{-6} & 2.78 \cdot 10^{-6} & 4.17 \cdot 10^{-6} & 5.56 \cdot 10^{-6} \end{bmatrix} \cdot \text{sec}$$

$$\text{Output sampling without considering Op Amp saturation: } ys_{rk} := \frac{y_{sr}(n_{3k})}{\text{volt}} \quad N_{0gd} = 256 \quad n_{3255} = 354.17 \cdot \mu\text{s}$$

$$f_{test} = 0.08 \cdot \text{MHz} \quad \frac{f_s}{f_{test}} = 9.6 \quad \omega_{3s} = 2.26 \cdot \frac{\text{Mrads}}{\text{sec}}$$

Fourier Transform of the test signal |Oy := fft(ysr)|

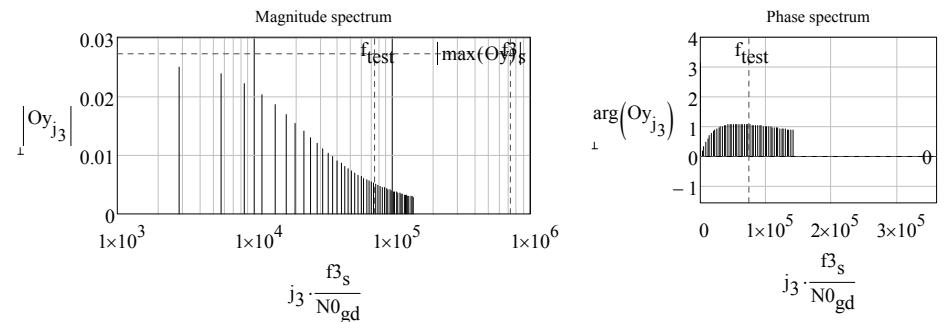


Fig.: (3.2.1.5)

Fig.: (3.2.1.6)

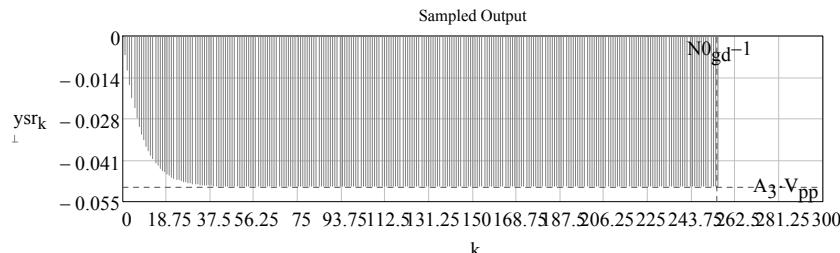


Fig.: (3.2.1.7)

Reconstruction of the output signal according to the Shannon sampling theorem

$$shstp(t) := \left[\sum_{n=0}^{N0gd-1} (ysr_n \cdot \text{sinc}(\omega_{3s} \cdot t - n \cdot \pi)) \right]$$

$N0gd-1 = 255$

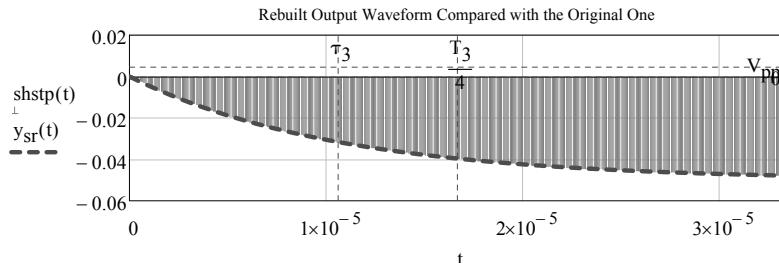


Fig.:3.2.1.8

ANALOG FILTER OUTPUT ANALYSIS

3.2.2) Short Voltage Pulse response

Description of the waveform's parameters:

Definition of the short voltage pulse:

$$V4(t, \tau_3, \tau_{pw}, V_{in}) = V_{in} \cdot \text{rect1}(t, \tau_3, \tau_{pw}), \quad V_{in} = \frac{V_{pp}}{V} \quad (3.2.2.1)$$

Parameters description:

$$V4(t, \tau_3, \tau_{pw}, V_{in}) = V4(\text{time}, \text{Rising Edge}, \text{Pulse Width}, \text{Dimensionless Amplitude}).$$

Pulse amplitude: $V_{pp} = 5 \cdot \text{mV}$ Pulse width: $\tau_{pw} := T3_s \cdot 20$ $\tau_{pw} = 27777.78 \cdot \text{ns}$

Pulse displacement from the origin: $\xi_{sl} := 0.8, \quad \tau_{pw} = \tau_{pw} \cdot (1 - \xi_{sl}) + \xi_{sl} \cdot \tau_{pw}$

Time delay from the origin: $\tau_{3svp} := -\tau_{pw} \cdot (1 - \xi_{sl}), \quad \text{risingedge} = \tau_{3svp}, \quad \text{width} = \tau_{pw}$.

Generic pulse definition defined in "Fourier Series.xmcd":

$$\text{Input signal defined in Test Signal.xmcd:} \quad V_w(t) := V4(t, \tau_{3svp}, \tau_{pw}, V_{pp}) \quad (3.2.2.3)$$

Consider a Short Voltage Pulse delayed τ_{3svp} seconds: $\tau_{3svp} = -5.56 \cdot \mu\text{s}$

$$t := -2 \cdot \tau_{pw}, -2 \cdot \tau_{pw} + \frac{4 \cdot \tau_{pw}}{5000}, \dots, 2 \cdot \tau_{pw}$$

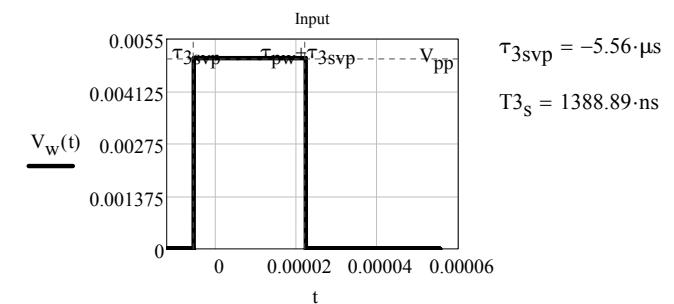


Fig.: (3.2.2.1)

Analog filter Input sampling.

Consider now the same signal repeated periodically, with period $T_{vp} := 4 \cdot (\tau_{pw} + \tau_{3svp})$, in such a way that it is possible to calculate the bandwidth using the program BCSA defined in "Fourier Analysis.xmcd":

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)

BCSA stands for Bandwidth Calculation and Signal Analysis"

$$Sb_{vp0} := \text{BCSA}[V_w, r_{gd}, N_{gd}, 0.0, 2 \cdot (\tau_{pw} + \tau_{3svp})] \quad r_{gd} = 10\% \quad (3.2.2.4)$$

Bandwidth Calculation

$$\text{Signal bandwidth: } B_{vp0} = 0 \cdot \text{GHz}$$

$$f_{test} = 0 \cdot \text{GHz}$$

$$\text{Parseval}_{vp0} = 0 \text{ V}^2$$

$$\text{Average}_{vp0} = 0 \text{ V}$$

$$\text{RMS}_{vp0} = 0 \text{ V}$$

Sampling frequency:

$$f_{samp} = \frac{1}{T_{samp}} \geq 2 \cdot f_1$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{samp} := 2 \cdot B_{vp0} \quad f_{samp} = 0 \cdot \text{GHz} \quad (3.2.2.5)$$

$$\text{sampling period: } T_{3svp} := \frac{1}{f_{samp}} \quad (3.2.2.6)$$

Samples are taken at the instants: $n_{3svp_k} := k \cdot T_{3svp} + \tau_{3svp}$ assuming that it is periodic.

$$V_{pp} = 0.01 \text{ V}$$

$$\text{Pulse sampling: } u_{44k} := V_w(n_{3svp_k}) \quad \frac{N_0 g_d}{f_{samp}} \cdot \frac{1}{T_{test}} = 8.89 \quad (3.2.2.7)$$

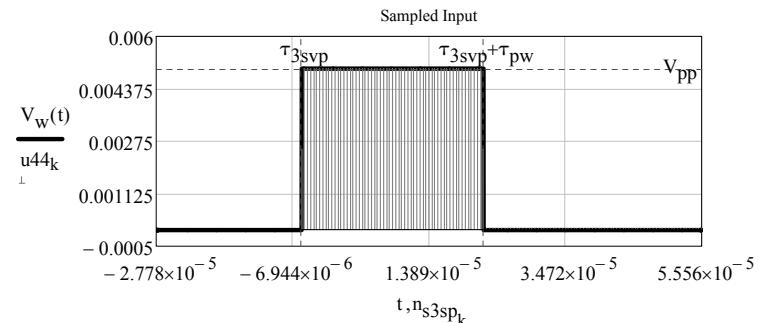


Fig.:3.2.2.2

Input signal approximate reconstruction according to the Shannon sampling theorem:

$$\omega_{sp} := 2 \cdot \pi \cdot B_{vp0}$$

$$shpulse(t) := \sum_{n=0}^{N_0 g_d - 1} [u_{44n} \cdot \text{sinc}[\omega_{sp} \cdot (t - \tau_{3svp}) - n \cdot \pi]]$$

$$N_0 g_d - 1 = 255$$

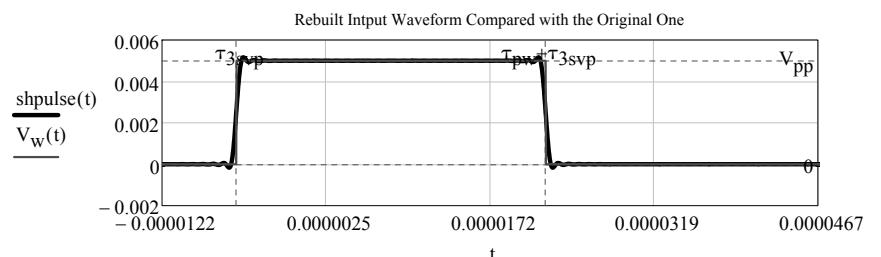


Fig.: (3.2.2.3)

Fourier Transform of the output signal:

$$Oywnd := \text{fft}(u44) \quad f_{test} = 0.08 \cdot \text{MHz} \quad \frac{f_s}{f_{test}} = 9.6 \quad \omega_{sp} = 6.79 \cdot \frac{\text{Mrads}}{\text{sec}}$$

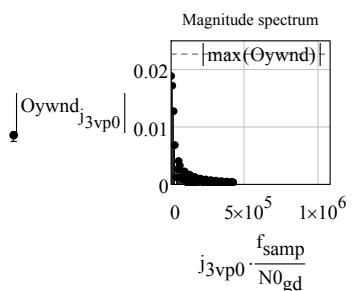


Fig.: (3.2.2.4)

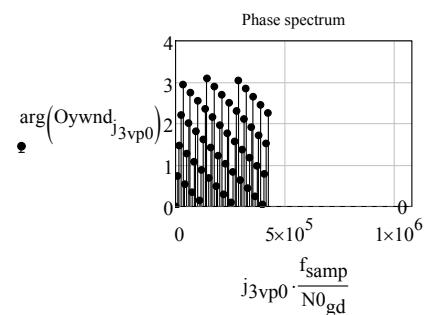


Fig.: (3.2.2.5)

Filter's response calculation

Description of the waveform's parameters:

$$\text{Definition: } V_4(t, \tau_{3svp}, \tau_{pw}, V_{in}) = V_{in} \cdot \text{rect1}(t, \tau_{3svp}, \tau_{pw}), \quad V_{in} = \frac{V_{pp}}{V} \quad (3.2.2.1)$$

$$V4(t, \tau_{3svp}, \tau_{pw}, V_{in}) = V4(\text{time}, \text{Rising Edge}, \text{Pulse Width}, \text{Dimensionless Amplitude}).$$

$$\text{Input signal: } V_w(t) = V_i \cdot (\Phi(t - \tau_{3svp}) - \Phi(t - \tau_{pw} - \tau_{3svp})) \quad V_i = 0.01 \text{ V}$$

$$\text{Laplace transform of the input signal: } V_w(s) = \frac{V_i}{s} \cdot e^{-\tau_{3svp} \cdot s} \cdot (1 - e^{-\tau_{pw} \cdot s})$$

$$\text{Laplace transform of the output signal: } y_{wndw}(s) = \frac{V_i}{s} \cdot e^{-\tau_{3svp} \cdot s} \cdot (1 - e^{-\tau_{pw} \cdot s}) \cdot \frac{A_3 \cdot \omega_3}{(s + \omega_3)}$$

Hence, Laplace anti-transforming I get the time step response :

$$y_{wndw}(t) := A_3 \cdot V_i \cdot [e^{-(t - \tau_{3svp}) \cdot \omega_3} - 1] \cdot \Phi(t - \tau_{3svp}) + [e^{-(t - \tau_{3svp} - \tau_{pw}) \cdot \omega_3} - 1] \cdot \Phi(t - \tau_{3svp} - \tau_{pw})$$

$$y_{wndwst}(t) := \text{if}(-V_{sat} \leq y_{wndw}(t) \leq V_{sat}, y_{wndw}(t), \text{if}(y_{wndw}(t) \leq 0.0 \cdot \text{volt}, -V_{sat}, V_{sat}))$$

$$N_{gd} = 256 \quad Q_3 = 10$$

$$A_3 = -10$$

$$\tau_{pw} + \tau_{3svp} = 22.22 \cdot \mu s$$

$$y_{wndw}(\tau_{pw} + \tau_{3svp}) = -0.05 V$$

$$T_s \ll (2 \cdot \pi \cdot \tau_{3svp}) \quad \tau_{3svp} = -5.56 \cdot \mu s$$

$$N_{gd} = 256$$

Graph of the response

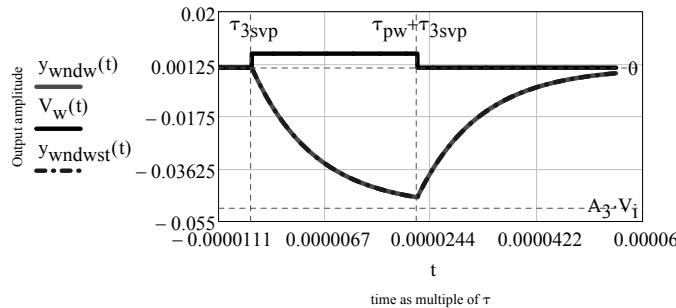


Fig.: (3.2.2.6)

$$\text{Dimensionless output: } y_{wnd}(t) := \frac{y_{wndw}(t)}{V}$$

Analog filter Output sampling.

Consider now the same signal repeated periodically, with period $T_{svp} := 2 \cdot (\tau_{pw} + \tau_{3svp})$, in such a way that it is possible to calculate the bandwidth using BCSA:

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for Bandwidth Calculation and Signal Analysis"

$$N_{gd} = 50 \quad r_{tgd} = 10\% \quad S_{vpo} := \text{BCSA}(y_{wnd}, r_{tgd}, N_{gd}, 0.0 \cdot \text{sec}, T_{svp}) \quad (3.2.2.24)$$

Bandwidth Calculation

$$\text{Signal bandwidth: } B_{vpo} = 0 \cdot \text{GHz}$$

$$f_{test} = 0 \cdot \text{GHz}$$

$$\text{Parseval}_{vpo} = 0 \text{ V}^2$$

$$\text{Average}_{vpo} = -0.03 \text{ V}$$

$$\text{RMS}_{vpo} = 0.03 \text{ V}$$

Sampling frequency:

$$f_{3o_{ssp}} = \frac{1}{T_{samp}} \geq 2 \cdot B_{vpo}$$

$$\text{Chosen sampling frequency: } f_{3o_{ssp}} := 2 \cdot B_{vpo} \quad f_{3o_{ssp}} = 0 \cdot \text{GHz}$$

$$\frac{N_{gd}}{f_{3o_{ssp}}} \cdot \frac{1}{T_{test}} = 8.89 \quad T_{3o_{ssp}} := \frac{1}{f_{3o_{ssp}}} \quad n^{3o_{ssp}}_k := \frac{k}{f_{3o_{ssp}}} + \tau_{3svp} \quad (3.2.2.25)$$

Output sampling considering Op Amp saturation:

$$V_{pk} := \frac{y_{wndw}(n^{3o_{ssp}}_k)}{V} \quad (3.2.2.26)$$

Output sampling without considering Op Amp saturation:

$$y_{wndk} := \frac{y_{wndw}(n^{3o_{ssp}}_k)}{\text{volt}} \quad (3.2.2.27)$$

$$N_{gd} = 256 \quad Q_3 = 10$$

$$A_3 = -10$$

Dimensionless output sampling considering Op Amp saturation:

$$A_3 \cdot V_{pp} = -0.05 \text{ V} \quad y_{wndsk} := \frac{y_{wndw}(n^{3o_{ssp}}_k)}{\text{volt}} \quad (3.2.2.28)$$

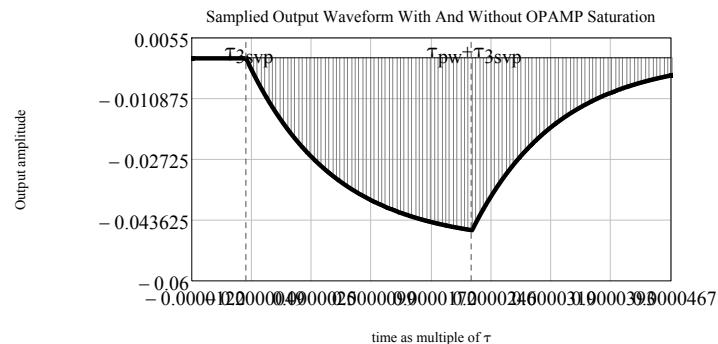


Fig.:3.2.2.
7

Approximate output signal reconstruction according to the Shannon sampling theorem:

$$\omega_{spo} := 2 \cdot \pi \cdot B_{vpo} \quad shpo(t) := \sum_{n=0}^{N_{gd}-1} [y_{wndn} \cdot \text{sinc}[\omega_{spo} \cdot (t - \tau_{3svp}) - n \cdot \pi]]$$

$$N_{gd} - 1 = 255$$

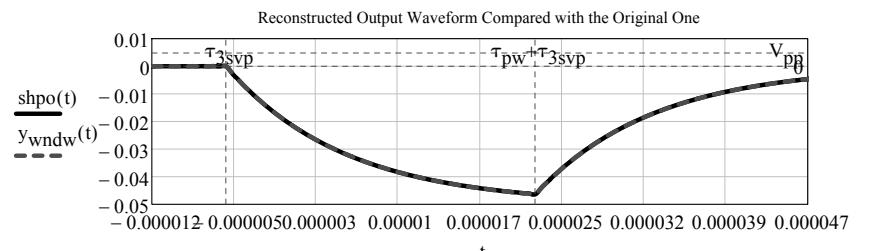


Fig.: (3.2.2.8)

Convolution:

The second method to obtain the filter response, is to calculate **the time domain convolution product** between the signal and the impulse response.

$$\text{Dimensionless Input: } v_p(t) := \frac{V_w(t)}{V}$$

$$\text{Exact output (convolution): } v_{ow}(t) = A_3 \cdot \omega_3 \cdot e^{-\omega_3 \cdot t} \cdot \int_0^t V_w(\tau) \cdot e^{\omega_3 \cdot \tau} d\tau \quad (3.2.2.29)$$

To obtain the output I replace to $V_w(\tau)$, as argument of the integral operator, with its definition, that is:

$$v_{ow}(t) = A_3 \cdot V_i \cdot \omega_3 \cdot e^{-\omega_3 \cdot t} \cdot \int_0^t (\Phi(\tau - \tau_{3svp}) - \Phi(\tau - \tau_{pw} - \tau_{3svp})) \cdot e^{\omega_3 \cdot \tau} d\tau$$

the integration gives:

$$v_{ow}(t) := A_3 \cdot e^{-\omega_3 \cdot t} \cdot [\Phi(t - \tau_{3svp}) \cdot (e^{t \cdot \omega_3} - e^{\tau_{3svp} \cdot \omega_3}) - \Phi(t - \tau_{pw} - \tau_{3svp}) \cdot (e^{t \cdot \omega_3} - e^{\omega_3 \cdot (\tau_{3svp} + \tau_{pw})})] \cdot V_i \quad (3.2.2.30)$$

$$\text{Approximated output: } v_{ow}(t) := -\omega_3 dB \cdot \int_0^t v_p(\sigma) d\sigma \quad T_{test} = 13.33 \mu s$$

$$\tau_{3svp} = -5.56 \mu s \quad t_w := \tau_{3svp} \cdot 2.2, \tau_{3svp} \cdot 2.2 + \frac{4.4 \cdot (\tau_{pw} + \tau_{3svp}) - \tau_{3svp} \cdot 2.2}{100} \dots 4.4 \cdot (\tau_{pw} + \tau_{3svp})$$

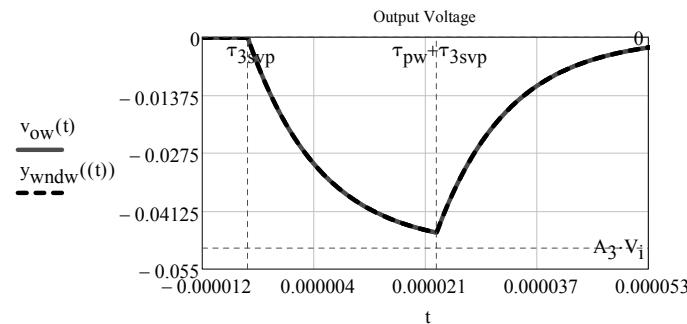


Fig.: (3.2.2.9)

$$v_{ow}(n_3 o_{ssp_k}) \\ v_{ow_k} := \frac{v_{ow}(n_3 o_{ssp_k})}{volt} \quad (3.2.2.31)$$

$$v_{ow}^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \dots \\ 0 & 0 & -0 & -0 & -0.01 & -0.01 & -0.01 & \dots \end{bmatrix}$$

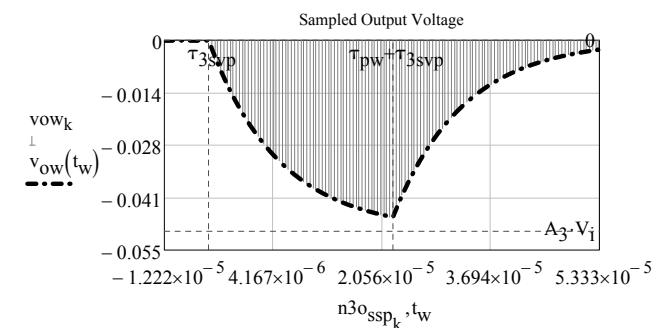


Fig.: (3.2.2.10)

Fourier Transform of the test signal

$$f_{test} = 0.08 \text{ MHz}$$

$$\frac{f_s}{f_{test}} = 9.6$$

$$Ow := fft(vow)$$

$$(3.2.2.32)$$

$$Ow^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & \dots \\ 0 & -0.19 & -0.05 - 0.14j & 0.06 - 0.06j & \dots \end{bmatrix}$$

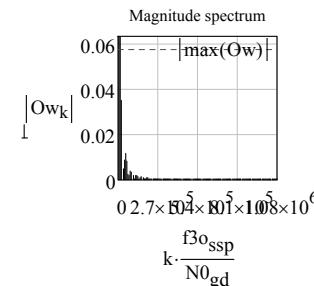


Fig.: (3.2.2.11)

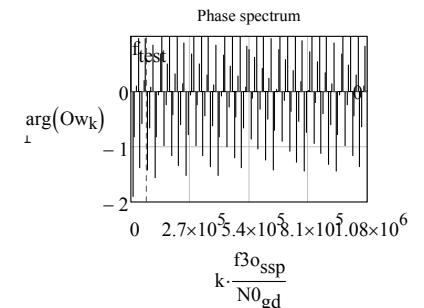


Fig.: (3.2.2.12)

ANALOG FILTER OUTPUT ANALYSIS

3.2.3) Bipolar pulse train response

Calculations.

$$N_{gd} = 50$$

$$\tau_3 = 10.61 \cdot \mu s \quad T_{test} = 13.33 \cdot \mu s \quad T_{test} \ll (2 \cdot \pi \cdot \tau_3)$$

$$V_{bpt}(t) := v_{sqw}(t, T_{test}, V_{pp}, N_{gd}) \quad \text{Defined in Signals.xcd}$$

$$t := 0 \cdot T_{test}, 0 \cdot T_{test} + \frac{4 \cdot T_{test} - 0 \cdot T_{test}}{1000} \dots 4 \cdot T_{test}$$

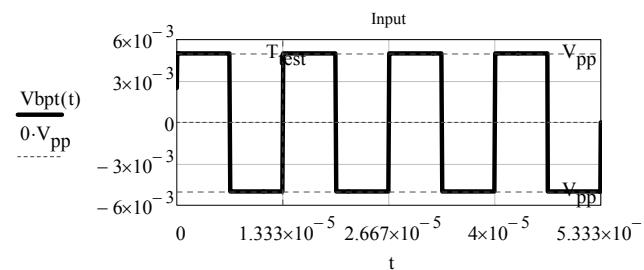


Fig.: (3.2.3.1)

Signal Bandwidth calculation using the Fourier series' harmonics. Are excluded all harmonics with amplitude less than $r_{gd} = 10\%$ of the fundamental.

$$V_{pp} = 5 \cdot mV \quad v_{sqwb}(t) := \frac{v_{sqw}(t, T_{test}, V_{pp}, N_{gd})}{volt}$$

$$T_{test} = 0 \text{ s}$$

Signal bandwidth:

Description of the program's parameters:

BCSA (Dimensionless signal name, relative error, polynomial degree, start time, signal period)

BCSA stands for Bandwidth Calculation and Signal Analysis"

$$r_{gd} = 10\%$$

$$Sb_{sqw} := BCSA(V_{sqwb}, r_{gd}, 50, 0.0 \cdot sec, T_{test}) \quad (3.2.3.2)$$

	0	1	2	3
0	0.1	0	0	0
1	$3.6 \cdot 10^6$	0	0.01	0
2	49	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0.01	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0

23

11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
...				

$$cffaSb_{sqw} := Sb_{sqw}^{(1)} \quad cffbSb_{sqw} := Sb_{sqw}^{(2)}$$

$$cffaSb_{sqw}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ ... \end{bmatrix}$$

$$cffbSb_{sqw}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ ... \end{bmatrix}$$

Bandwidth Calculation

The function returns a three columns matrix.

The first column contains:

- pos. 0: relative error,
- pos. 1: bandwidth (Dimensionless),
- pos. 2: the nth. harmonic number corresponding to the give relative error,
- pos. 3: temporary variable,
- pos. 4: Parseval,
- pos. 5: signal average,
- pos. 6: signal rms.

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

$$\text{Signal bandwidth: } B_{sqw} = 3.6 \cdot \text{MHz}$$

$$\text{Parseval}_{sqwb} = 0 \text{ V}^2$$

$$\text{Average2} = 0 \text{ V}$$

$$\text{RMS2} = 0.01 \text{ V}$$

$$\text{sampling frequency (Nyquist rate): } f_{ssqw} := 2 \cdot B_{sqw}$$

$$f_{ssqw} = 0.01 \cdot \frac{\text{Grads}}{\text{sec}}$$

$$n_{sqw_k} := \frac{k}{f_{ssqw}} \quad T_{ssqw} := \frac{1}{f_{ssqw}}$$

$$sqw_k := V_{sqwb}(n_{sqw_k}) \quad \frac{N_{gd}}{f_{ssqw}} \cdot \frac{1}{T_{test}} = 2.67$$

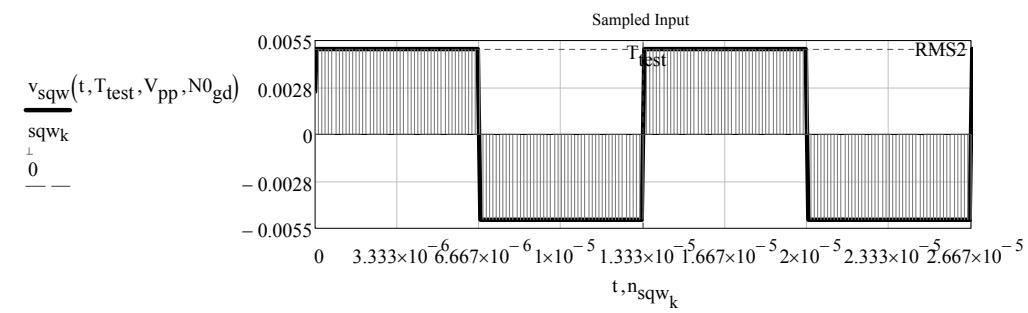


Fig.:3.2.3.2

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{s4} := 2 \cdot \pi \cdot B_{sqw} \quad sh4(t) := \sum_{n=0}^{N_0 gd-1} (sqw_n \cdot \text{sinc}(\omega_{s4} \cdot t - n \cdot \pi)) \quad (3.2.3.6)$$

$$t := 0 \cdot T_{\text{test}}, 0 \cdot T_{\text{test}} + \frac{5 \cdot T_{\text{test}} - 0 \cdot T_{\text{test}}}{1000} \dots 5 \cdot T_{\text{test}} \quad rt_{gd} = 10\%$$

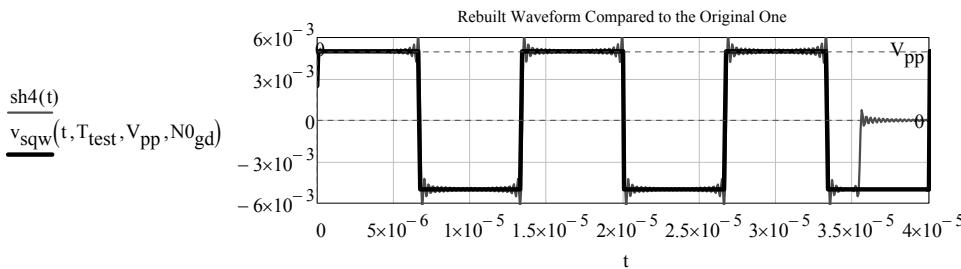


Fig.:3.2.3.3

Output calculation using the Laplace transform of the input and the filter's transfer function.

Output calculation

$$\text{input signal: } v_i(t, T_{\text{test}}, V_p) = V_{\text{pp}} \cdot \sum_{k=0}^{\infty} \left[\Phi(t - k \cdot T_{\text{test}}) - 2 \cdot \Phi \left[t - \left(\frac{2 \cdot k + 1}{2} \right) \cdot T_{\text{test}} \right] \dots \right]$$

Laplace transform of the output signal:

$$V_{\text{opt}} = \frac{A_3 \cdot \omega_3}{s + \omega_3} \cdot \frac{V_i}{s} \cdot \tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right)$$

Laplace transform of the input signal:

$$\mathcal{L}(v_i(t, T_{\text{test}}, V_p)) = \frac{V_p}{s} \cdot \tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right)$$

$$\text{output signal: } V_{\text{opt}}(t) = \mathcal{L}^{-1} \left(\frac{V_p}{s} \cdot \tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right) \cdot \frac{-\omega_3 \text{dB}}{s + \omega_3} \right) \quad V_{\text{pp}} \cdot \omega_3 \text{dB} = 0 \cdot V \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$V_p := V_p \quad T_{\text{test}} := T_{\text{test}} \quad \omega_3 \text{dB} := \omega_3 \text{dB} \quad s := s$$

$$\text{Output final value: } \lim_{s \rightarrow 0} \left(\frac{V_p \cdot s}{s} \cdot \tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right) \cdot \frac{-\omega_3 \text{dB}}{s + \omega_3} \right) \rightarrow 0$$

$$V_p := V_p \quad T_{\text{test}} := T_{\text{test}} \quad \omega_3 \text{dB} := \omega_3 \text{dB} \quad s := s$$

$$\text{Output initial value: } \lim_{s \rightarrow \infty} \left(\frac{V_p \cdot s}{s} \cdot \tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right) \cdot \frac{-\omega_3 \text{dB}}{s + \omega_3} \right) \rightarrow -V_p \cdot \omega_3 \text{dB} \cdot \lim_{s \rightarrow \infty} \frac{\tanh \left(\frac{T_{\text{test}} \cdot s}{4} \right)}{s + \omega_3}$$

Output Laplace anti-transform calculation:

$$\text{Shift theorem } \mathcal{L}(f(t-a)) = e^{-a \cdot s} \cdot F(s)$$

$$V_i(s) = \frac{V_p}{s} \cdot \sum_{k=0}^{N_0 gd} \left[e^{-k \cdot T_{\text{test}} \cdot s} - 2 \cdot e^{-\frac{2 \cdot k + 1}{2} \cdot T_{\text{test}} \cdot s} + e^{-(k+1) \cdot T_{\text{test}} \cdot s} \right]$$

$$\mathcal{L} \left(\sum_{k=0}^{\infty} f(t - k \cdot T) \right) = \frac{F(s)}{1 - e^{-T \cdot s}}$$

$$V_o(s) = V_i(s) \cdot \frac{-\omega_3 \text{dB}}{s + \omega_3}$$

$$V_o(s) = \frac{V_p}{s} \cdot \frac{-\omega_3 \text{dB}}{s + \omega_3} \cdot \sum_{k=0}^{N_0 gd} \left[e^{-k \cdot T_{\text{test}} \cdot s} - 2 \cdot e^{-\frac{2 \cdot k + 1}{2} \cdot T_{\text{test}} \cdot s} + e^{-(k+1) \cdot T_{\text{test}} \cdot s} \right]$$

$$A_3 := A_3 \quad s := s \quad \omega_3 \text{dB} := \omega_3 \text{dB}$$

$$\frac{-\omega_3 \text{dB}}{s + \omega_3} \cdot \frac{1}{s} \quad \begin{array}{l} | \text{invlaplace}, s \\ | \text{simplify}, \max \end{array} \rightarrow \frac{\omega_3 \text{dB} \cdot (e^{-t \cdot \omega_3} - 1)}{\omega_3}$$

Output calculation

$$y_o(t) = \mathcal{L}^{-1} \left(\frac{-\omega_3 \text{dB}}{s + \omega_3} \cdot \frac{1}{s} \right) \quad (3.2.3.7)$$

$$\text{Define the function: } y_o(t) := \frac{\omega_3 \text{dB} \cdot (e^{-t \cdot \omega_3} - 1)}{\omega_3} \cdot \Phi(t) \quad (3.2.3.8)$$

Filter output:

$$\omega_3 \text{dB} = 0 \cdot \frac{\text{Grads}}{\text{sec}} \quad \frac{1}{T_{\text{test}}} = 0 \cdot \text{GHz}$$

$$N_{gd} = 50$$

$$V_{\text{opt}}(t) := V_{\text{pp}} \cdot \sum_{k=0}^{N_0 gd} \left[y_o(t - k \cdot T_{\text{test}}) - 2 \cdot y_o \left(t - \frac{2 \cdot k + 1}{2} \cdot T_{\text{test}} \right) + y_o \left[t - (k+1) \cdot T_{\text{test}} \right] \right] \quad (3.2.3.9)$$

$$N_{gd} = 50 \quad V_{\text{opt}} \left(\frac{T_{\text{test}}}{2} \right) = -0.02 \cdot V \quad \omega_3 \text{dB} = 942477.8 \frac{1}{s}$$

Graph of the bipolar pulse train response considering the Op Amp saturation:

$$V_{\text{obp}}(t) := \begin{cases} V_{\text{opt}}(t), & \text{if } -V_{\text{sat}} \leq V_{\text{opt}}(t) \leq V_{\text{sat}} \\ 0.0 \text{ volt}, & \text{if } (V_{\text{opt}}(t) \leq 0.0 \text{ volt}) \text{ or } (V_{\text{opt}}(t) \geq V_{\text{sat}}) \end{cases} \quad (3.2.3.10)$$

$$A_3 = -10 \quad T_{\text{test}} = 13333.33 \text{ ns} \quad V_{\text{pp}} = 0.01 \text{ V}$$

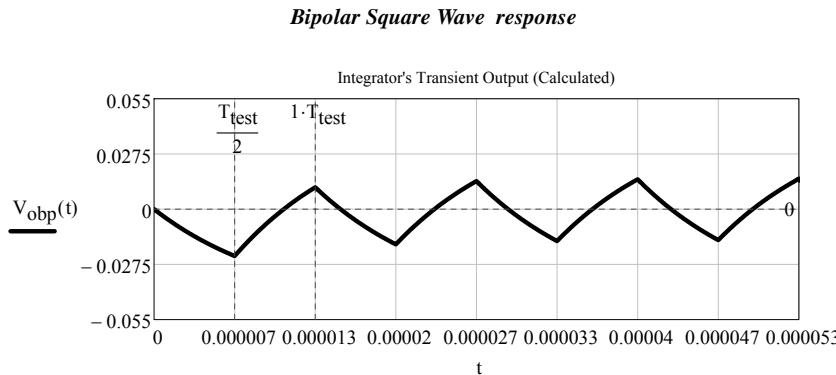


Fig.: (3.2.3.4)

Convolution

$$\text{Exact output (convolution): } v_{\text{opt}}(t) := A_3 \cdot \omega_3 \cdot e^{-t \cdot \omega_3} \cdot \int_0^t V_{\text{bpt}}(\tau) \cdot e^{\tau \cdot \omega_3} d\tau$$

$$\text{Approximated output: } v_{\text{opta}}(t) := A_3 \cdot \omega_3 \cdot \int_0^t V_{\text{bpt}}(\sigma) d\sigma \quad T_{\text{test}} = 13.33 \mu\text{s}$$

$$t := 0 \cdot T_{\text{test}}, 0 \cdot T_{\text{test}} + \frac{2 \cdot T_{\text{test}}}{200}, \dots, 2 \cdot T_{\text{test}}$$

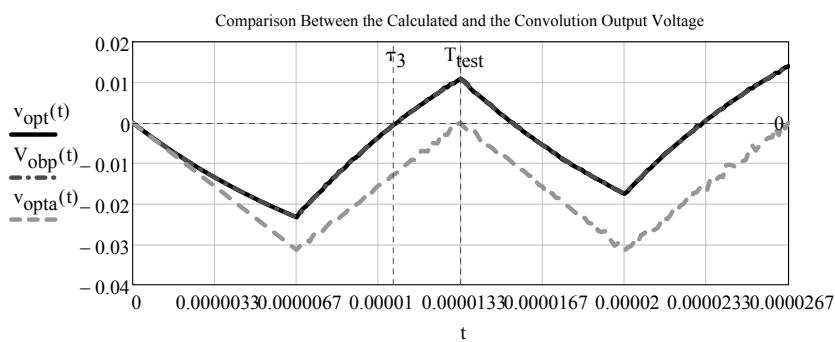


Fig.: (3.2.3.5)

Approximate reconstruction of the output signal (with Op Amp saturation) according to the Shannon sampling theorem:

$$v_{\text{optk}} := \frac{V_{\text{obp}}(n_{\text{sqw}_k})}{\text{volt}}$$

$$\omega_{S3} := 2 \cdot \pi \cdot B_{\text{sqw}} \quad sh3(t) := \left[\sum_{n=0}^{N_0 g_d - 1} (v_{\text{optn}} \cdot \text{sinc}(\omega_{S3} \cdot t - n \cdot \pi)) \right] \quad (3.2.4.21)$$

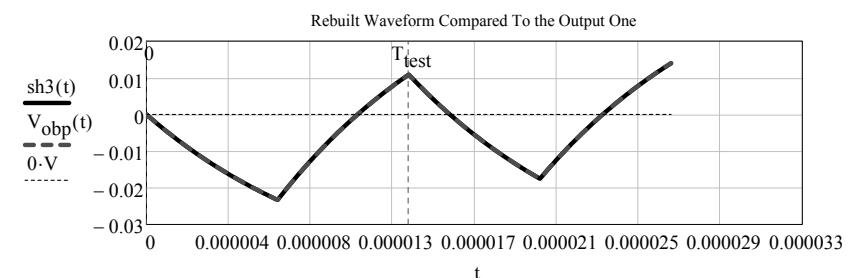


Fig.: 3.2.3.6

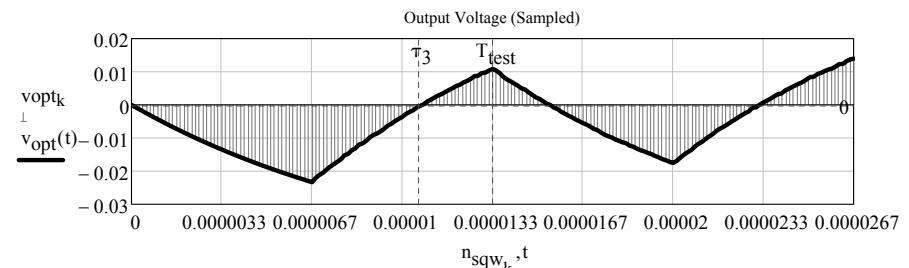


Fig.: (3.2.3.7)

$$f_{\text{test}} = 0.08 \text{ MHz} \quad \frac{f_{\text{ssqw}}}{f_{\text{test}}} = 96 \quad \omega_{S3} = 22.62 \cdot \frac{\text{Mrads}}{\text{sec}}$$

Fourier Transform of the test signal | Opt := fft(vopt) |

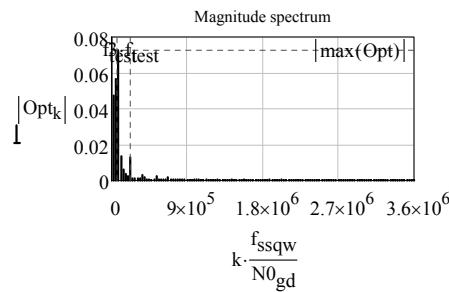


Fig.: (3.2.3.8)

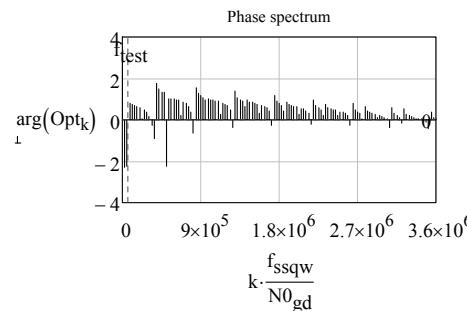


Fig.: (3.2.3.9)

ANALOG FILTER OUTPUT ANALYSIS

3.2.4) Periodic Triangular Cusp's signal, filter response

$$\text{Signal amplitude: } V_{pp} := 4.5 \cdot V \quad V_{pp} = 4.5 \cdot V$$

Duty cycle

$$\text{Duty cycle: } \frac{2 \cdot |V_{sat}|}{V_{pp} \cdot |A_3|} > \delta_{cy} \geq \frac{|V_{sat}|}{V_{pp} \cdot |A_3|} \quad \frac{|V_{sat}|}{V_{pp} \cdot |A_3|} = 0.33 \quad \frac{2 \cdot |V_{sat}|}{V_{pp} \cdot |A_3|} = 0.67$$

$$\text{Duty cycle chosen: } \delta_{cy} := 0.4600$$

$$\text{Max pulse and cusp height ratio: } a := \frac{2 \cdot |V_{sat}| - V_{pp} \cdot \delta_{cy} \cdot |A_3|}{V_{pp} \cdot \delta_{cy} \cdot |A_3|} \quad a = 0.45 \quad a < 1$$

$$\text{Pulse width: } p_{wtc} := \delta_{cy} \cdot T_{test}$$

$$\text{Period: } T_{test} = 13.33 \cdot \mu s \quad f_{cst} := \frac{1}{T_{test}} \quad \omega_{csp} := 2 \cdot \pi \cdot f_{cst}$$

$$\text{Output's final value: } V_{fin} := \frac{|A_3| \cdot V_{pp} \cdot \delta_{cy} \cdot (a + 1)}{2}$$

$$\text{One side Cusp's slope: } c_{cs} := V_{pp} \cdot \frac{2 \cdot (1 - a)}{p_{wtc}} \quad c_{cs} = 0.81 \cdot \frac{V}{\mu s}$$

$$\text{Frequency} \quad f_{test} = 0.08 \cdot \text{MHz} \quad \omega_{csp} = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$N_{gd} = 50$$

The test signal be the following defined in Signals.xmcd:

$$\text{cusp0}(t, p_w, a, V_{pp}) = V_{pp} \cdot \left[\begin{aligned} & \left[1 + t \cdot \frac{2 \cdot (1 - a)}{p_w} \right] \cdot \left(\Phi\left(t + \frac{p_w}{2}\right) - \Phi(t) \right) \dots \\ & + \left[1 - t \cdot \frac{2 \cdot (1 - a)}{p_w} \right] \cdot \left(\Phi(t) - \Phi\left(t - \frac{p_w}{2}\right) \right) \end{aligned} \right] \quad (3.2.4.1)$$

whose graph is below depicted:

$$t_{wtc} := -4 \cdot p_{wtc}, -4 \cdot p_{wtc} + \frac{(8 \cdot p_{wtc})}{1000} \dots 4 \cdot p_{wtc}$$

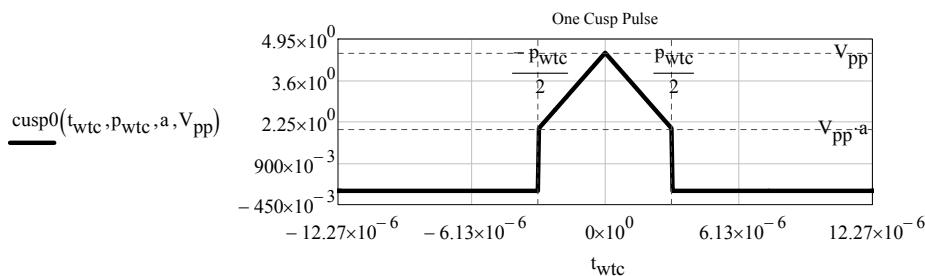


Fig.: (3.2.4.1)

With it I create the following periodic signal defined in Signals.xmcd:

$$csp01(t, p_{w\text{tc}}, a, T_0, V_{pp}, N_{gd}) = \sum_{k=0}^{N_{gd}} \left(cusp0\left(t - k \cdot T_0 - \frac{p_{w\text{tc}}}{2}, p_{w\text{tc}}, a, V_{pp}\right) \right) \quad (3.2.4.2)$$

$$p_{w\text{tc}} = 6.13 \cdot \mu\text{s}$$

$$a = 0.45$$

$$fx1(t) := csp01(t, p_{w\text{tc}}, a, T_{\text{test}}, V_{pp}, N_{gd}) \quad (3.2.4.3)$$

$$T_{\text{test}} = 13.33 \cdot \mu\text{s} \quad t_w := 0 \cdot T_{\text{test}}, 0 \cdot T_{\text{test}} + \frac{30 \cdot T_{\text{test}}}{1000} \dots 30 \cdot T_{\text{test}}$$

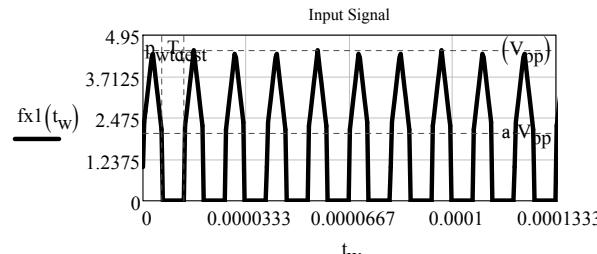


Fig.: (3.2.4.2)

Analog filter input sampling

$$v_{\text{incsp}01}(t) := \frac{fx1(t)}{V}$$

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for Bandwidth Calculation and Signal Analysis"

$$rt_{\text{gd}} = 10\% \quad S_{\text{b,csp}} := \text{BCSA}(v_{\text{incsp}01}, rt_{\text{gd}}, 50, 0.0 \cdot \text{sec}, T_{\text{test}}) \quad (3.2.4.3)$$

Calculation results:

$$\text{Parseval}_{\text{csp}} = 10.24 \text{ V}^2$$

$$\text{Average}_{\text{csp}} = 1.5 \text{ V}$$

$$\text{RMS}_{\text{csp}} = 2.26 \text{ V}$$

$$(3.2.4.4)$$

$$\text{Input signal bandwidth: } B_{\text{csp}} = 3.6 \cdot \text{MHz}$$

$$\text{Chosen sampling frequency: } f_{\text{in,csp}} := 2 \cdot B_{\text{csp}} \quad f_{\text{in,csp}} = 7.2 \cdot \text{MHz}$$

$$\omega_{\text{csp}} := \pi \cdot f_{\text{in,csp}}$$

$k = k \dagger$ defined in global data

$$\frac{N_{0\text{gd}}}{f_{\text{in,csp}}} \cdot \frac{1}{T_{\text{test}}} = 2.67 \quad T_{\text{in,csp}} := \frac{1}{f_{\text{in,csp}}} \quad n_{\text{in,csp},k} := \frac{k}{f_{\text{in,csp}}} \quad (3.2.4.5)$$

$$N_{0\text{gd}} = 256 \quad Q_3 = 10 \quad \text{Voltage gain: } A_3 = -10 \quad f_{\text{test}} = 0.08 \cdot \text{MHz} \quad (3.2.4.6)$$

$$\text{Dimensionless input sampling: } v_{\text{in,csp},k} := \frac{fx1(n_{\text{in,csp},k})}{\text{volt}} \quad (3.2.4.7)$$

$$A_3 \cdot V_{pp} = -45 \text{ V}$$

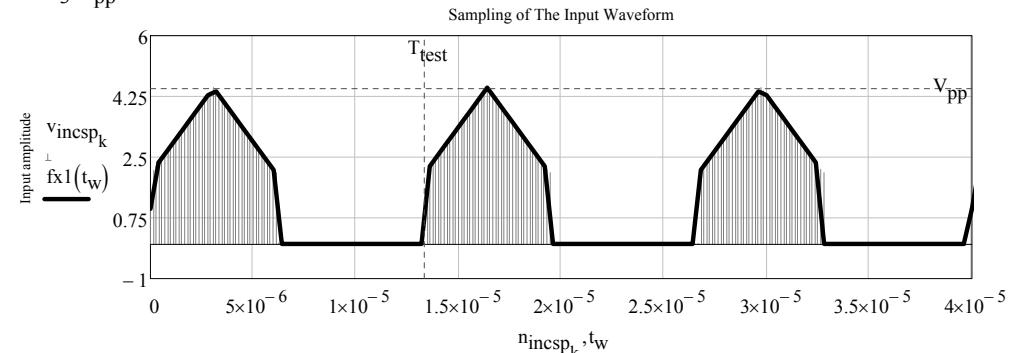


Fig.: (3.2.4.2)

$$f_{\text{test}} = 0.08 \cdot \text{MHz} \quad \frac{f_{\text{in,csp}}}{f_{\text{test}}} = 96 \quad \omega_{\text{csp}} = 22.62 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\text{Fourier Transform of the output signal } V_{\text{incsp}} := \text{fft}(v_{\text{in,csp}}) \quad (3.2.4.8)$$

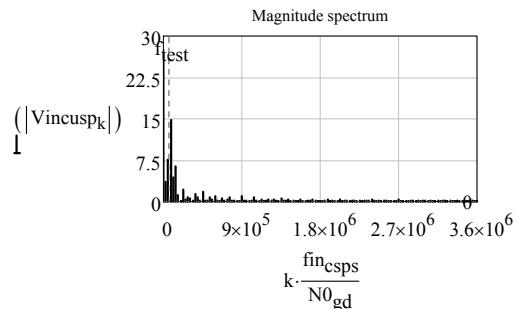


Fig.: (3.2.4.2)

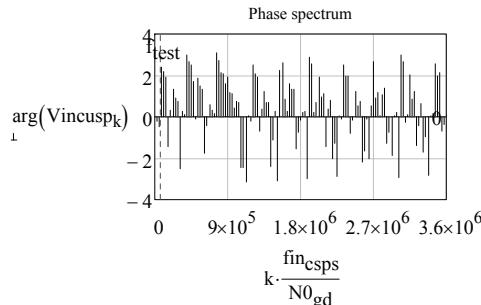


Fig.: (3.2.4.3)

Input signal rebuilt according to the Shannon sampling theorem:

$$shcsp0(t) := \sum_{k=0}^{N0gd-1} \left(v_{incsp_k} \cdot \text{sinc}(\omega_{csp} \cdot t - k \cdot \pi) \right) \quad (3.2.4.9)$$

$$N0gd - 1 = 255$$

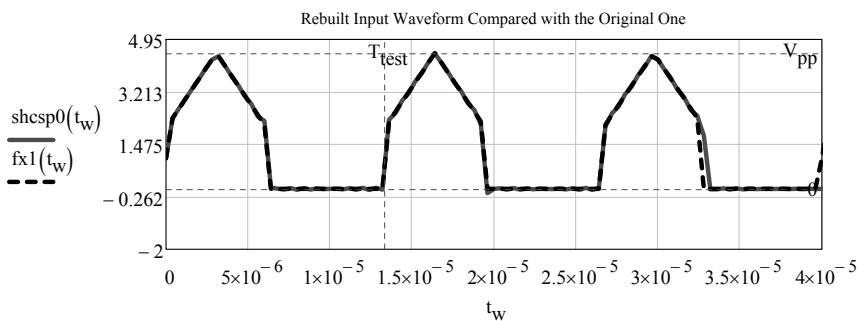


Fig.: (3.2.4.4)

$$\max(|v_{incsp}|) = 38.41$$

Laplace transform of the signal (see Laplace Transform.xmed)

$$\mathcal{L}(csp01(t, p_{wtc}, a, T_{test}, V_{pp})) = \frac{2 \cdot V_{pp}}{p_{wtc} \cdot s^2 \cdot (1 - e^{-T_{test} \cdot s})} \cdot \begin{aligned} & \left[a \cdot \left(\frac{p_{wtc}}{2} \cdot s - 1 \right) + 1 \right] \dots \\ & + \left[1 - a \cdot \left(\frac{p_{wtc}}{2} \cdot s + 1 \right) \right] \cdot e^{-p_{wtc} \cdot s} \dots \\ & + 2 \cdot (a - 1) \cdot e^{-\frac{p_{wtc}}{2} \cdot s} \end{aligned} \quad (3.2.4.10)$$

Filter Output calculation

$$\text{Transfer function: } W_{lp}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3} \quad (3.2.4.11)$$

Laplace transform of the filter's output signal:

$$V_{ocsp}(t) = \frac{2 \cdot V_{pp} \cdot A_3 \cdot \omega_3}{p_{wtc}} \cdot \mathcal{L}^{-1} \left[\frac{1}{s^2 \cdot (s + \omega_3) \cdot (1 - e^{-T_{test} \cdot s})} \cdot \begin{aligned} & \left[a \cdot \left(\frac{p_{wtc}}{2} \cdot s - 1 \right) + 1 \right] \dots \\ & + \left[1 - a \cdot \left(\frac{p_{wtc}}{2} \cdot s + 1 \right) \right] \cdot e^{-p_{wtc} \cdot s} \dots \\ & + 2 \cdot (a - 1) \cdot e^{-\frac{p_{wtc}}{2} \cdot s} \end{aligned} \right] \quad (3.2.4.11)$$

The Laplace transform is composed by the sum of three terms. The inverse Laplace transform of each term are the following:

▪ Inverse Laplace transform calculations

Define the following functions:

$$1) \quad y_{00}(t) := \frac{2 \cdot \omega_3 \cdot (1 - a) \cdot t + [2 - a \cdot (p_{wtc} \cdot \omega_3 + 2)] \cdot (e^{-t \cdot \omega_3} - 1)}{2 \cdot \omega_3^2} \cdot \Phi(t) \quad (3.2.4.12)$$

$$2) \quad y_{01}(t) := \frac{a \cdot (p_{wtc} \cdot \omega_3 - 2) + 2}{2 \cdot \omega_3^2} \cdot e^{-t \cdot \omega_3} + \frac{t \cdot [-2 \cdot \omega_3 \cdot (a - 1)]}{2 \cdot \omega_3^2} - \frac{a \cdot (p_{wtc} \cdot \omega_3 - 2) + 2}{2 \cdot \omega_3^2} \cdot \Phi(t) \quad (3.2.4.13)$$

$$3) \quad y_{02}(t) := \frac{2 \cdot (a - 1) \cdot (e^{-t \cdot \omega_3} + t \cdot \omega_3 - 1)}{\omega_3^2} \cdot \Phi(t) \quad (3.2.4.14)$$

The filter response is the following:

$$V_{ocsp}(t) := \frac{2 \cdot V_{pp} \cdot A_3 \cdot \omega_3}{p_{wtc}} \cdot \sum_{k=0}^{100} \left(y_{00}(t - T_{test} \cdot k) + y_{01}(t - p_{wtc} - T_{test} \cdot k) + y_{02}\left(t - \frac{p_{wtc}}{2} - T_{test} \cdot k\right) \right) \quad (3.2.4.15)$$

The filter response considering the opamp saturation:

$$v_{ocuspst}(t) := \text{if}(-V_{sat} \leq V_{ocsp}(t) \leq V_{sat}, V_{ocsp}(t), \text{if}(V_{ocsp}(t) \leq 0.0 \cdot \text{volt}, -V_{sat}, V_{sat})) \quad (3.2.4.16)$$

$$\text{Output max value} \quad V_{ocspmx} := V_{ocsp}\left[\left(k + \frac{1}{2}\right) \cdot T_{test}\right]$$

$$\text{Output min value} \quad V_{ocspmin} := \min(V_{ocspmx}) \quad V_{ocspmin} = -18.98 \text{ V}$$

$$A_3 = -10 \quad f_{\text{test}} = 75 \cdot \text{kHz}$$

$$V_{\text{fin}} = 15 \text{ V}$$

$$\omega_3 = 94.25 \cdot \frac{\text{krad}}{\text{sec}}$$

Graph of the Op Amp response considering **saturation**:

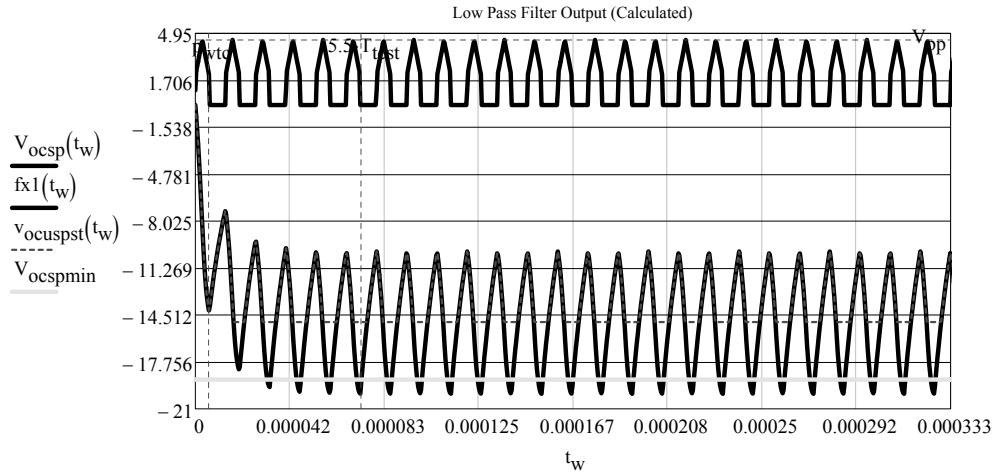
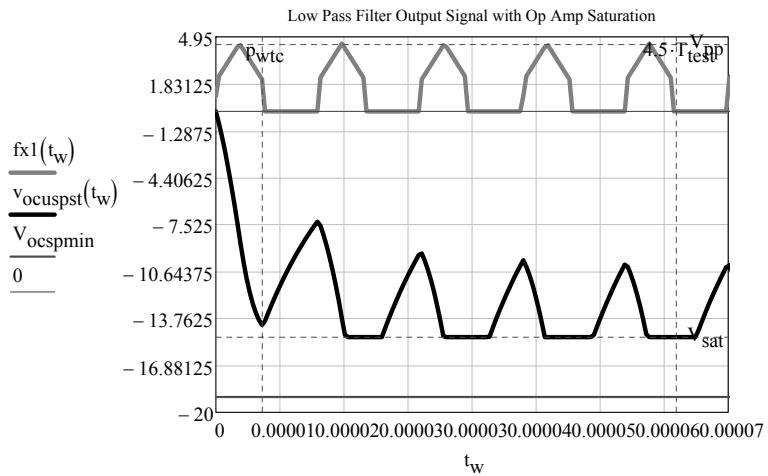


Fig.: (3.2.4.7)



Convolution

$$\text{Exact output: } v_{\text{ocusp}}(t) := A_3 \cdot \omega_3 \cdot \int_0^t e^{-\omega_3 \cdot (t-\tau)} \cdot \text{csp01}(\tau, p_{\text{wtc}}, a, T_{\text{test}}, V_{\text{pp}}, N_{\text{gd}}) d\tau \quad V_i = 5 \cdot \text{mV}$$

$$\text{Approximated output: } v_{\text{ocuspax}}(t) := -\omega_3 \cdot \int_0^t f_{\text{x1}}(\sigma) d\sigma \quad T_{\text{test}} = 13.33 \cdot \mu\text{s}$$

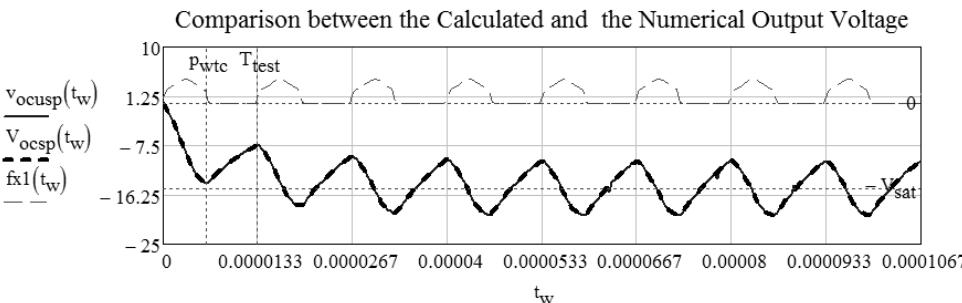


Fig.: (3.2.4.8)

$$\text{Dimensionless output } v_{\text{ocusp}}(t) := \frac{V_{\text{ocusp}}(t)}{V}$$

Analog filter Output sampling

Description of the program's parameters:

BCSA(*Dimensionless signal name, relative error, polynomial degree, start time, signal period*)
BCSA stands for *Bandwidth Calculation and Signal Analysis*"

$$N_{\text{gd}} = 50 \quad r_{\text{tg}} = 10\% \quad S_{\text{bo}}_{\text{csp}} := \text{BCSA}(v_{\text{ocusp}}, r_{\text{tg}}, N_{\text{gd}}, 0.0 \cdot \text{sec}, T_{\text{test}}) \quad (3.2.4.24)$$

Bandwidth Calculation

Signal bandwidth: $B_{\text{csp}} = 3.6 \cdot \text{MHz}$

$f_{\text{test}} = 75 \cdot \text{kHz}$

Parseval_{ocsp} = 197.73 V²

Average_{ocsp} = -9.24 V

RMS_{ocsp} = 9.95 V

Chosen sampling frequency: $f_{\text{ocsp}} := 2 \cdot B_{\text{csp}}$ $f_{\text{ocsp}} = 7.2 \cdot \text{MHz}$ $\omega_{\text{ocsp}} := 2 \cdot \pi \cdot f_{\text{ocsp}}$

$$\frac{N_{\text{gd}}}{f_{\text{ocsp}}} \cdot \frac{1}{T_{\text{test}}} = 2.67 \quad T_{\text{ocsp}} := \frac{1}{f_{\text{ocsp}}} \quad n_{\text{ocsp}_k} := \frac{k}{f_{\text{ocsp}}} \quad (3.2.4.25)$$

Output sampling $V_{\text{ocsp}_k} := v_{\text{ocusp}}(n_{\text{ocsp}_k})$

(3.2.4.26)

$N_{\text{gd}} = 256 \quad Q_3 = 10 \quad A_3 = -10$

Dimensionless output sampling considering Op Amp saturation:

$$v_{\text{ocsp}}(t) := \text{if}(-V_{\text{sat}} \leq v_{\text{ocusp}}(t) \leq V_{\text{sat}}, v_{\text{ocusp}}(t), \text{if}(v_{\text{ocusp}}(t) \leq 0.0 \cdot \text{volt}, -V_{\text{sat}}, V_{\text{sat}})) \quad (3.2.4.27)$$

$$A_3 \cdot V_{pp} = -45 \text{ V} \quad V2op_{ocsp_k} := \frac{v_{ocusp}(n_{ocsp_k})}{\text{volt}} \quad (3.2.4.28)$$

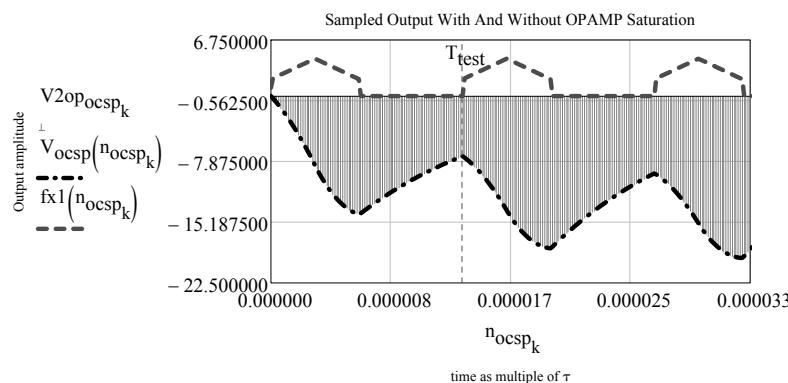


Fig.: (3.2.4.9)

$$f_{test} = 0.08 \cdot \text{MHz} \quad \frac{f_{ocsp}}{f_{test}} = 96 \quad \omega_{csp} = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}}$$

Fourier Transform of the output signal: $V_{ocusp} := \text{fft}(V2op_{ocsp})$ $\max(|V_{ocusp}|) = \blacksquare$

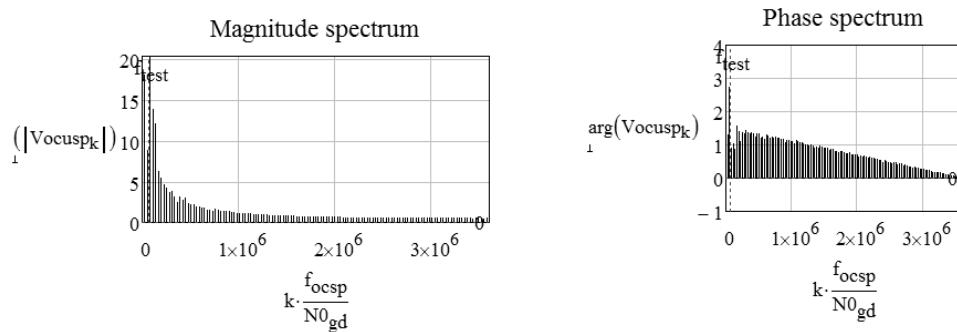


Fig.: (3.2.4.10)

Fig.: (3.2.4.11)

Approximate output signal reconstruction according to the Shannon sampling theorem:

$$\omega_{scsp} := 2 \cdot \pi \cdot B_{ocsp} \quad shcsp(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(V2op_{ocsp_n} \cdot \text{sinc}(\omega_{scsp} \cdot t - n \cdot \pi) \right) \right]$$

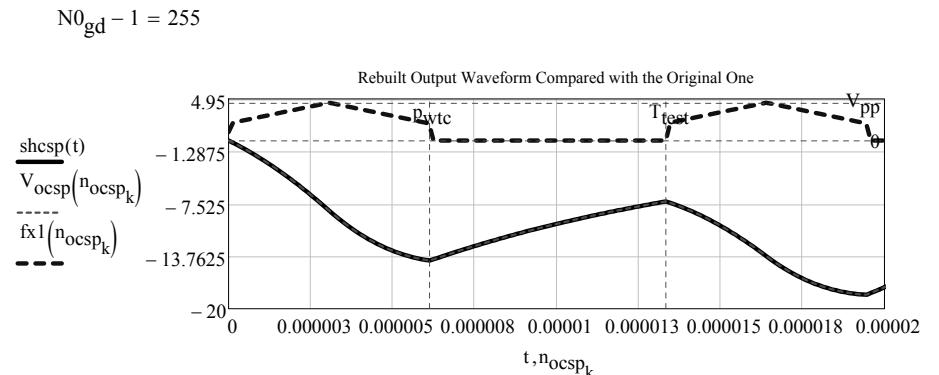


Fig.: (3.2.4.12)

ANALOG FILTER OUTPUT ANALYSIS

3.2.5 Sawtooth response

Input signal defined in Test Signal.xmeds:

$$V_{sw}(t) := \frac{v1_{sw}(t, T_{test}, V_{pp}, N0_{gd})}{volt} \quad (3.2.5.1)$$

$V_{pp} = 4.5\text{ V}$

$$t_{saw} := -T_{sawth_0}, T_{sawth_0} + \frac{5 \cdot T_{sawth_0} + T_{sawth_0}}{1000} .. 5 \cdot T_{sawth_0}$$

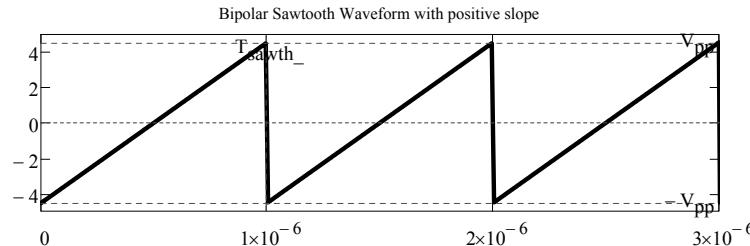


fig.:3.2.5.1

For a correct sampling one must know the signal bandwidth.

Numerical search of the signal bandwidth. All harmonics with amplitude less than $r_{tg_d} = 10\%$ of the fundamental one, are neglected. To do that it is used the function BCSA(...) defined in "Fourier Series.xmcd".

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for Bandwidth Calculation and Signal Analysis"

$$N_{gd} = 50 \quad Sb_{sw} := BCSA(V_{sw}, r_{tg_d}, N_{gd}, 0.0\text{-sec}, T_{test}) \quad (3.2.5.2)$$

Bandwidth Calculation

Signal bandwidth: $B_{sw} = 3.6\text{-MHz}$

Parseval_{sw} = 13.34

Average_{1-volt} = 0 V

RMS1-volt = 2.6 V

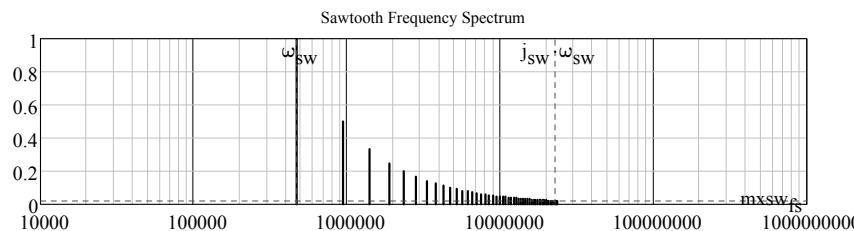


fig.:3.2.5.2

(Min) sampling frequency (Nyquist rate): $f_{ssw} := 2 \cdot B_{sw}$

$$f_{ssw} = 0.01\text{-GHz}$$

$$T_{ssw} := \frac{1}{f_{ssw}}$$

$$\text{sampling time step: } n_{sw_k} := \frac{k}{f_{ssw}} \quad (3.2.5.3)$$

$$\text{rows}(n_{sw}) = 256 \quad \frac{1}{T_{test}} = 0.08\text{-MHz} \quad \frac{N0_{gd}}{f_{ssw}} \cdot \frac{1}{T_{test}} = 2.67$$

$$\text{RMS1} = 2.6 \quad \text{Sampled signal: } u10_k := v1_{sw}(n_{sw_k}, T_{test}, V_{pp}, N0_{gd}) \quad (3.2.5.4)$$

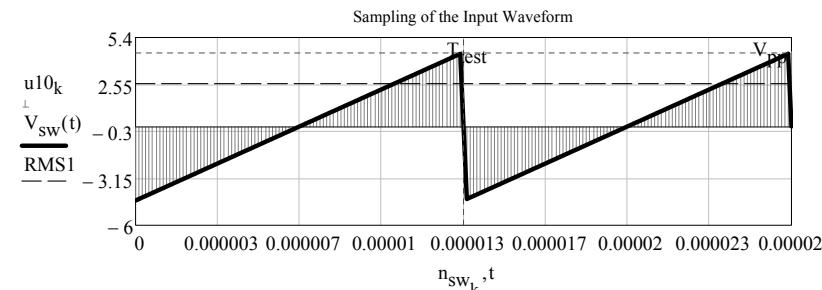


fig.:3.2.5.3

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{s2} := 2 \cdot \pi \cdot B_{sw} \quad sh5(t) := \left[\sum_{n=0}^{N0_{gd}-1} (u10_n \cdot \text{sinc}(\omega_{s2} \cdot t - n \cdot \pi)) \right] \quad (3.2.5.5)$$

$$N0_{gd} - 1 = 255$$

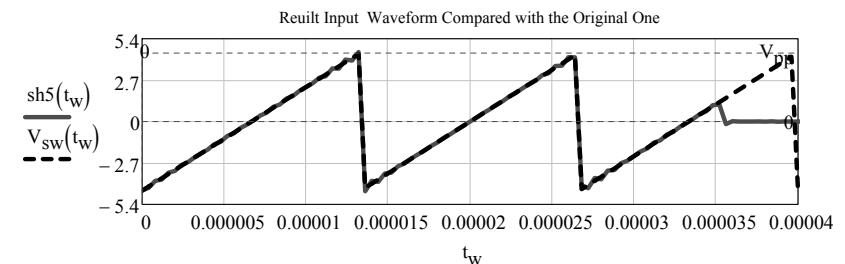


fig.:3.2.5.4

$$r_{tg_d} = 10\%$$

Search of the filter's transient response

Given the signal:

$$v1_{sw}(t, T_{test}, V_{pp}) := 2 \cdot \frac{V_{pp}}{T_{test}} \cdot \sum_{k=0}^{\infty} [(t - k \cdot T_{test}) \cdot \text{rect1}(t - k \cdot T_{test}, 0.0 \cdot T_{test}, T_{test})] - V_{pp} \quad (3.2.5.6)$$

or:

$$v1_{sw0}(t, T_{test}, V_{pp}) := 2 \cdot \frac{V_{pp}}{T_{test}} \cdot \sum_{k=0}^{20} \left[(t - k \cdot T_{test}) \cdot \Phi(t - k \cdot T_{test}) \dots + (-1) \cdot (t - k \cdot T_{test}) \cdot \Phi[t - T_{test} \cdot (k+1)] \right] - V_{pp}$$

RMS1 = 2.6 (3.2.5.7)

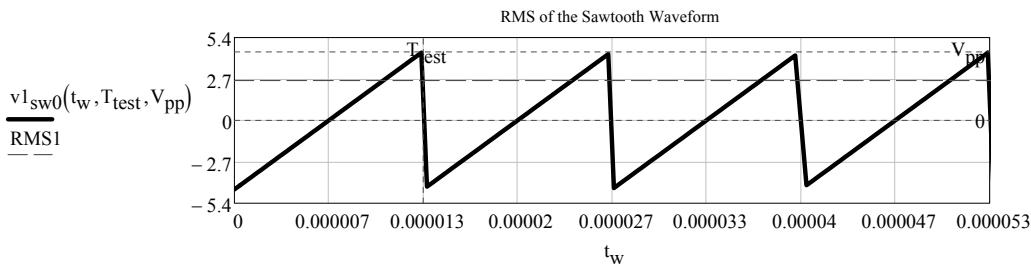


fig.:3.2.5.5

Laplace Transform calculation of the Output Signal .

Hence, the compact laplace transform of the (3.2.5.6) is:

$$\mathcal{L}(v1_{sw}(t, T_{test}, V_{pp})) = 2 \cdot \frac{V_{pp}}{T_{test}} \cdot \frac{1}{s} \left[\frac{1}{s} - \left(T_{test} + \frac{1}{s} \right) e^{-s \cdot T_{test}} \right] \cdot \sum_{k=0}^{\infty} \left(e^{-T_{test} \cdot k \cdot s} \right) - \frac{V_{pp}}{s},$$

or

$$\boxed{\mathcal{L}(v1_{sw}(t, T_{test}, V_{pp})) = \frac{V_{pp}}{s} \left(\frac{2}{T_{test} \cdot s} - \coth\left(\frac{T_{test} \cdot s}{2}\right) \right)} \quad (3.2.5.10)$$

Search of the corresponding output waveform of the filter.

$$\text{Given the transfer function: } W_{lp}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3} \quad (3.2.5.11)$$

the Laplace transform of the filter's output is: $V_{osw}(s) = W_{lp}(s) \cdot \mathcal{L}(v1_{sw}(t, T_{test}, V_{pp}))$

$$\text{that is: } V_{osw}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3} \cdot \frac{V_{pp}}{s} \left(\frac{2}{T_{test} \cdot s} - \coth\left(\frac{T_{test} \cdot s}{2}\right) \right) \quad (3.2.5.12)$$

Now apply the following theorems:

Initial value theorem: $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} (s \cdot F(s))$,

Final value theorem: $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} (s \cdot F(s))$

Final value of the output voltage:

$$T_{test} := T_{test} \quad \omega_3 := \omega_3 \quad A_3 := A_3 \quad V_{pp} := V_{pp} \quad s := s$$

$$V_{ofin} := \lim_{s \rightarrow 0} \left[s \cdot \left[\frac{A_3 \cdot \omega_3}{s + \omega_3} \cdot \frac{V_{pp}}{s} \left(\frac{2}{T_{test} \cdot s} - \coth\left(\frac{T_{test} \cdot s}{2}\right) \right) \right] \right] \quad \begin{array}{l} \text{assume, } (T_{test} > 0) \rightarrow 0 \\ \text{simplify} \end{array} \quad V_{ofin} = 0 \cdot V$$

Initial value of the output voltage:

$$T_{test} := T_{test} \quad \omega_3 := \omega_3 \quad A_3 := A_3 \quad V_{pp} := V_{pp} \quad s := s$$

$$\lim_{s \rightarrow \infty} \left[s \cdot \left[\frac{A_3 \cdot \omega_3}{s + \omega_3} \cdot \frac{V_{pp}}{s} \left(\frac{2}{T_{test} \cdot s} - \coth\left(\frac{T_{test} \cdot s}{2}\right) \right) \right] \right] \quad \begin{array}{l} \text{assume, } A_3 = \text{real} \\ \text{assume, } T_{test} = \text{real} \rightarrow \\ \text{assume, } \omega_3 = \text{real} \\ \text{simplify} \end{array}$$

Transient response calculation:

As seen, the Laplace transform of the filter output is:

$$\boxed{v_{osw}(t) = V_{pp} \cdot A_3 \cdot \omega_3 \cdot \mathcal{L}^{-1} \left[\frac{1}{s + \omega_3} \cdot \frac{1}{s} \left(\frac{2}{T_{test} \cdot s} - \coth\left(\frac{T_{test} \cdot s}{2}\right) \right) \right]} \quad (3.2.5.14)$$

$$V_{pp} \cdot A_3 \cdot \omega_3 = -4.24 \cdot \frac{V \cdot \text{rad}}{\mu\text{s}}$$

Calculations of the Inverse Laplace transform of the Output

The inverse Laplace transform of the output is composed by the two following functions:

$$\text{Given the two functions: } g1_{sw}(t) := \left(\frac{t}{\omega_3} + \frac{e^{-t \cdot \omega_3} - 1}{\omega_3^2} \right) \cdot \Phi(t)$$

$$g2_{sw}(t) := \frac{1 - e^{-t \cdot \omega_3}}{\omega_3} \cdot \Phi(t)$$

The resulting output is:

$$\boxed{v_{osw}(t) = V_{pp} \cdot A_3 \cdot \omega_3 \cdot \left[\frac{2}{T_{test}} \cdot \sum_{k=0}^{\infty} \left[g1_{sw}(t - k \cdot T_{test}) \dots + (-1) \cdot T_{test} \cdot g2_{sw}[t - (k+1) \cdot T_{test}] \dots + (-1) \cdot g1_{sw}[t - (k+1) \cdot T_{test}] \right] \dots \right] + -1 \cdot g2_{sw}(t)}$$

$$V_{pp} \cdot A_3 \cdot \omega_3 = -4.24 \cdot \frac{mV}{ns}$$

For the first 20 terms, gives:

$$v_{oswc}(t) := V_{pp} \cdot A_3 \cdot \omega_3 \cdot \left[\frac{2}{T_{test}} \cdot \sum_{k=0}^{20} \left[g1sw(t-k \cdot T_{test}) \dots + (-1) \cdot T_{test} \cdot g2sw[t-(k+1) \cdot T_{test}] \dots + (-g2sw(t)) \right] \dots \right]$$

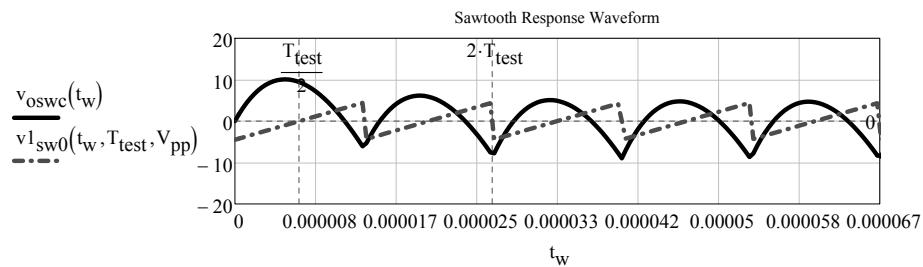


fig.:3.2.5.7

Convolution

$$\text{Exact output: } v_{oswcc}(t) := A_3 \cdot \omega_3 \cdot \int_0^t e^{-\omega_3 \cdot (t-\tau)} \cdot v1_{sw}(\tau, T_{test}, V_{pp}, N_{gd}) d\tau \quad V_i = 0.01 V$$

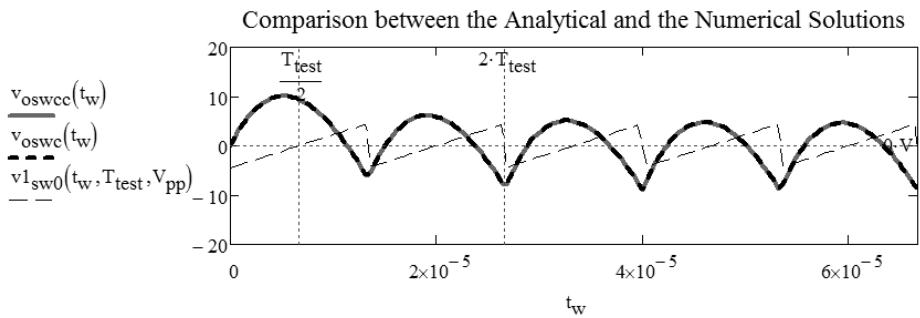


fig.:3.2.5.8

Approximated output signal reconstruction, according to the Shannon sampling theorem:

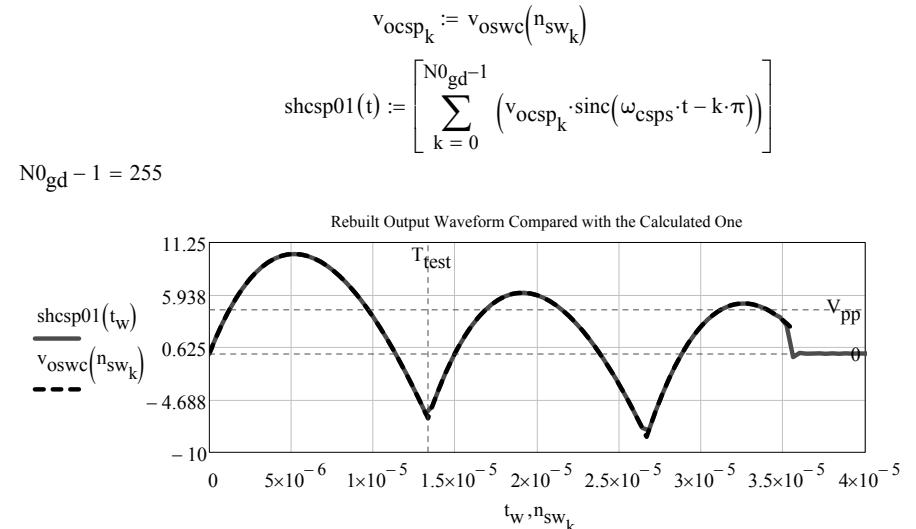


Fig.: (3.2.4.4)

3.2 ANALOG FILTER OUTPUT ANALYSIS

3.2.6 (single tone) AM Signal response

☒ Data

$$\text{Carrier Amplitude: } A_{1\text{sl}} := 10 \cdot \text{volt}$$

$$\text{Modulating signal's amplitude: } B_{1\text{sl}} := 5.5 \cdot \text{volt}$$

$$\omega_{1\text{csl}} := \frac{\omega_{0\text{gd}}}{2} \quad T_{1\text{csl}} := \frac{2 \cdot \pi}{\omega_{1\text{csl}}} \quad \omega_{1\text{msl}} := \frac{\omega_{0\text{gd}}}{10} \quad T_{1\text{msl}} := \frac{2 \cdot \pi}{\omega_{1\text{msl}}} \quad f_{1\text{msl}} := \frac{\omega_{1\text{msl}}}{2 \cdot \pi} \quad f_{15\text{sl}} := \frac{\omega_{1\text{csl}}}{2 \cdot \pi}$$

$$v_{\text{ammaxsl}} := A_{1\text{sl}} + B_{1\text{sl}} \quad v_{\text{amminsl}} := A_{1\text{sl}} - B_{1\text{sl}} \quad A_{1\text{sl}} = v_{\text{ammaxsl}} + v_{\text{amminsl}} \quad B_{1\text{sl}} = v_{\text{ammaxsl}} - v_{\text{amminsl}}$$

$$v_{\text{ammaxsl}} = 15.5 \cdot \text{volt}$$

$$v_{\text{amminsl}} = 4.5 \cdot \text{volt} \quad m_{\text{am}} := \frac{v_{\text{ammaxsl}} - v_{\text{amminsl}}}{v_{\text{ammaxsl}} + v_{\text{amminsl}}} \quad m_{\text{am}} = 0.55$$

☒ Data

$$\text{Carrier max amplitude: } A_{1\text{sl}} = 10 \text{ V}$$

$$\text{Modulating single tone max amplitude: } B_{1\text{sl}} = 5.5 \text{ V}$$

$$\text{Carrier pulsation: } \omega_3 c := 1 \cdot \omega_{\text{test}}$$

$$\text{Carrier frequency: } f_3 c := \frac{\omega_3 c}{2 \cdot \pi} \quad f_3 c = 0.08 \cdot \text{MHz}$$

$$\text{Carrier period: } T_3 c := \frac{1}{f_3 c} \quad T_3 c = 0 \text{ s}$$

$$\text{Modulating single tone pulsation: } \omega_{\text{mam}} := \frac{1}{5} \cdot \omega_3 c$$

$$\text{Modulating single tone frequency: } f_{\text{mam}} := \frac{\omega_{\text{mam}}}{2 \cdot \pi} \quad f_{\text{mam}} = 0.02 \cdot \text{MHz}$$

$$\text{Modulating single tone period: } T_{\text{mam}} := \frac{1}{f_{\text{mam}}}$$

$$\text{modulation index: } m_{\text{am}} = 55\%$$

$$\omega_{\text{mam}} = 0.09 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \omega_3 c = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\text{Modulated signal bandwidth: } Bw_{3\text{am}} := f_3 c + 2 \cdot f_{\text{mam}}$$

$$\text{sampling frequency (Nyquist rate): } f_{\text{sam}} := 2 \cdot Bw_{3\text{am}}, \quad f_{\text{sam}} = 0.21 \cdot \text{MHz}$$

$$\text{sampling angular frequency: } \omega_{\text{sam}} := 2 \cdot \pi \cdot f_{\text{sam}}, \quad \omega_{\text{sam}} = 0 \cdot \frac{\text{Grads}}{\text{sec}},$$

$$\text{sampling period: } T_{\text{sam}} := \frac{1}{f_{\text{sam}}}, \quad T_{\text{sam}} = 4.76 \cdot \mu\text{s},$$

$$\text{sampling time step: } \text{nam}_k := \frac{k}{f_{\text{sam}}},$$

$$N_0 \cdot \frac{T_{\text{sam}}}{T_{\text{test}}} = 91.43$$

$$\text{nam}^T = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 0 & 4.76 & 9.52 & 14.29 & 19.05 & 23.81 & \dots \\ \hline \end{array} \cdot \mu\text{s}$$

$$\text{Modulated carrier: } V2_i(t) := v2_i(t, \omega_{\text{mam}}, \omega_3 c, A_{1\text{sl}}, B_{1\text{sl}}) \quad (3.2.5.1)$$

$$\frac{\omega_3 c}{\omega_{\text{mam}}} = 5 \quad \text{Sampling: } u_{-7k} := \frac{V2_i(\text{nam}_k)}{\text{volt}} \quad (3.2.5.2)$$

$$\frac{N_0 \cdot \frac{1}{f_{\text{sam}}}}{T_{\text{test}}} = 91.43 \quad t := 0 \cdot T_3 c, 0 \cdot T_3 c + \frac{20 \cdot T_3 c}{1000} \dots 20 \cdot T_3 c$$

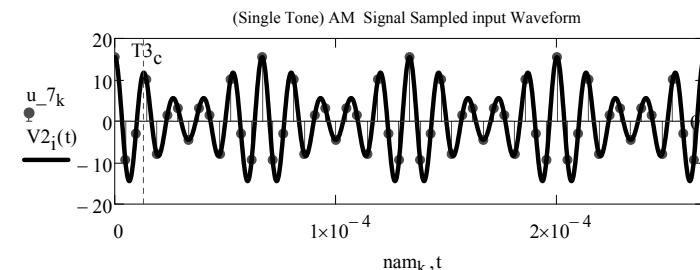


Fig.: (3.2.6.1)

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{\text{sh6}} := 2 \cdot \pi \cdot Bw_{3\text{am}} \quad \text{sh6}(t) := \left[\sum_{n=0}^{N_0 \cdot \frac{1}{f_{\text{sam}}} - 1} (u_{-7n} \cdot \text{sinc}(\omega_{\text{sh6}} \cdot t - n \cdot \pi)) \right] \quad (3.2.5.3)$$

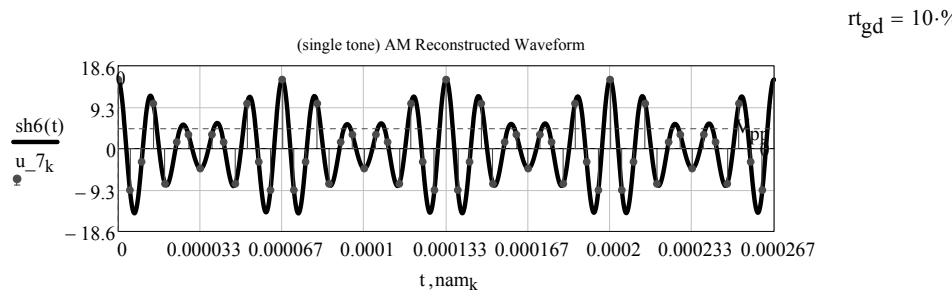


Fig.: (3.2.6.2)

$Spec_7 := fft(u_7)$

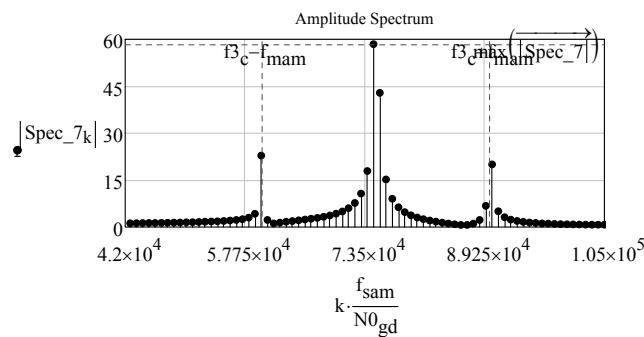


Fig.: (3.2.6.3)

$V_{pp} = 4.5V$

$$\text{Exact output: } v_{oam}(t) := \int_0^t w(t-\sigma) \cdot V2_i(\sigma) d\sigma \quad (3.2.5.4)$$

$$m_{am} = 55\% \quad t_m := 0 \cdot T3_c, 0 \cdot T3_c + \frac{40 \cdot T3_c}{1000} \dots 40 \cdot T3_c \quad T3_c = 13.33 \cdot \mu s$$

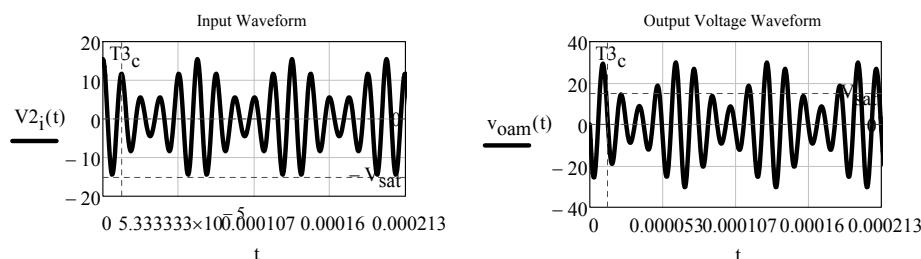


Fig.: (3.2.6.4)

Fig.: (3.2.6.5)

$Vam_k := v_{oam}(nam_k)$

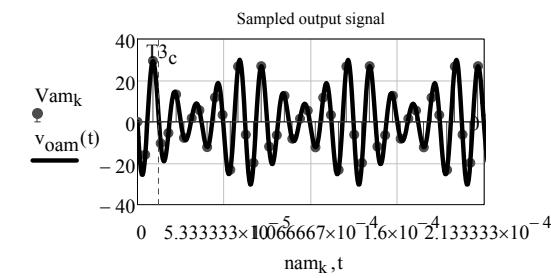


Fig.: (3.2.6.6)

Output sampling:

Fourier Transform of the output signal $Spec_{am} := fft(Vam)$

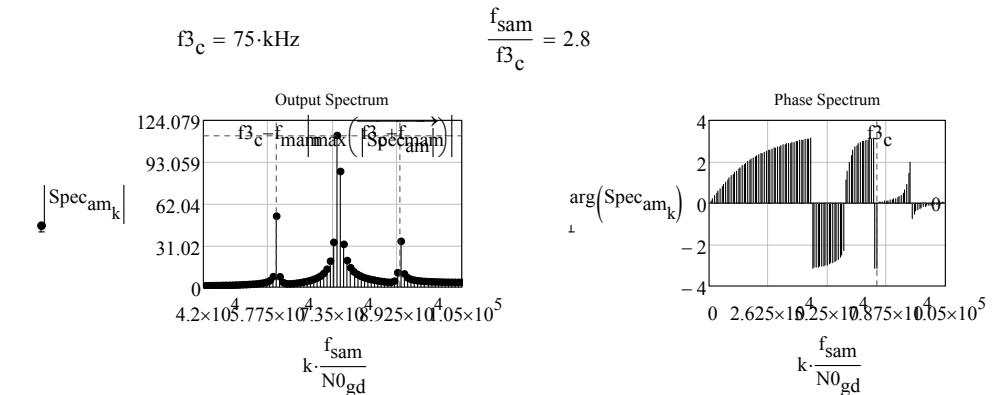


Fig.: (3.2.6.7)

Fig.: (3.2.6.8)

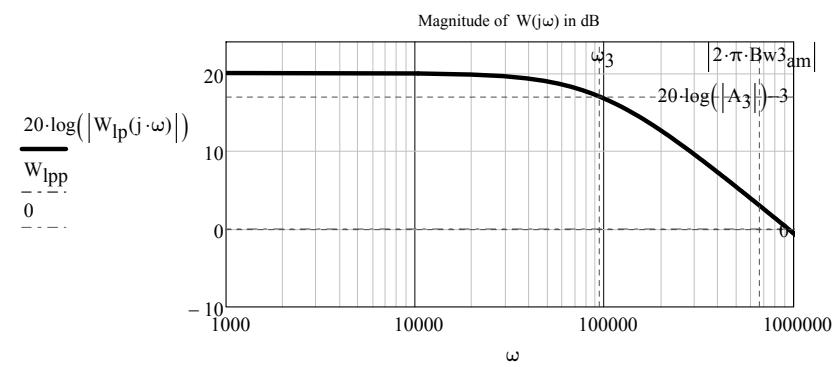


Fig.: (3.2.6.9)

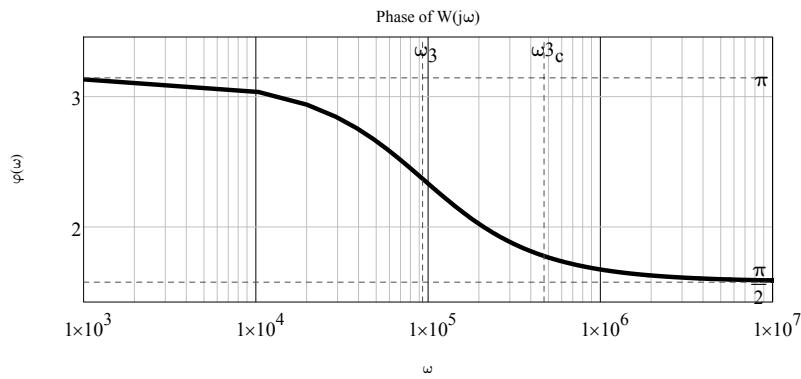


Fig.: (3.2.6.10)

3.2 ANALOG FILTER OUTPUT ANALYSIS

3.2.7 (single tone) Frequency Modulated carrier response

$$\text{Carrier max amplitude: } A_{fm} = 0.2 \text{ V}$$

$$\text{Modulating single tone max amplitude: } B_{fmm} = 15 \text{ V}$$

$$\text{Carrier pulsation: } \omega_{cfm} = 0.02 \cdot \frac{\text{Grads}}{\text{s}}$$

$$\text{Carrier frequency: } f_{cfm} = 0 \cdot \text{GHz}$$

$$\text{Carrier period: } T_{cfm} = 333.33 \cdot \text{ns}$$

$$\text{Modulating single tone pulsation: } \omega_{fmm} = 0.94 \cdot \frac{\text{Mrads}}{\text{s}}$$

$$\text{Modulating single tone frequency: } f_{fmm} = 0.15 \cdot \text{MHz}$$

$$\text{Modulating single tone period: } T_{fmm} = 6666.67 \cdot \text{ns}$$

$$\text{frequency modulation index: } m_{fm} := \frac{2 \cdot Kst \cdot \pi \cdot B}{\omega_m} \quad m_{fm} = 8 \quad (3.2.7.1)$$

$$\text{Carson bandwidth: } Cars1 := 2 \cdot \omega_{cfm} \cdot (m_{fm} + 1) \quad Cars1 = 0.34 \cdot \frac{\text{Grads}}{\text{sec}} \quad (3.2.7.2)$$

$$\text{Sampling frequency (Nyquist rate): } f_{sfm} := 10 \cdot f_{cfm}, \quad f_{sfm} = 0.03 \cdot \text{GHz}, \quad (3.2.7.3)$$

$$\text{sampling angular frequency: } \omega_{sfm} := 2 \cdot \pi \cdot f_{sfm} \quad \omega_{sfm} = 0.19 \cdot \frac{\text{Grads}}{\text{sec}},$$

$$\text{sampling period: } T_{sfm} := \frac{1}{f_{sfm}}, \quad T_{sfm} = 0.03 \cdot \mu\text{s},$$

$$\text{sampling time step: } nfm_k := \frac{k}{f_{sfm}}, \quad (3.2.7.4)$$

$$\frac{N_0 gd}{f_{sfm}} \cdot \frac{1}{T_{test}} = 6.4 \times 10^{-1}$$

$$nfm^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0.03 & 0.07 & 0.1 & 0.13 & 0.17 & 0.2 & 0.23 & 0.27 \end{bmatrix} \cdot \mu\text{s}$$

$$\frac{\omega_{cfm}}{\omega_{fmm}} = 20 \quad V_{fm}(t) := v_{fm,sl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd}) \quad (3.2.7.5)$$

$$A_{fm} = 0.2 \text{ V} \quad u_{8k} := \frac{V_{fm}(nfm_k)}{\text{volt}} \quad V_{pp} = 4.5 \text{ V} \quad (3.2.7.6)$$

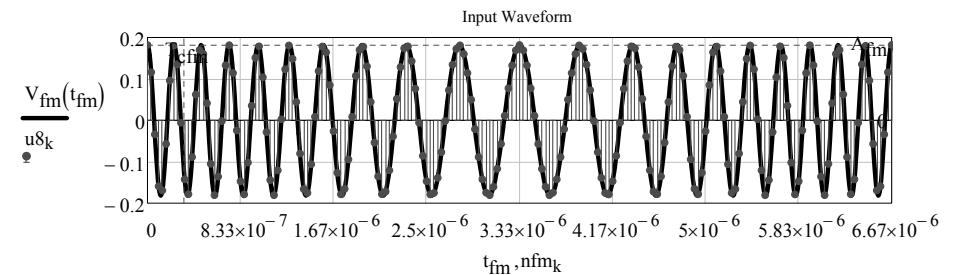


Fig.: (3.2.7.1)

Input signal's reconstruction according to the Shannon sampling theorem:

$$rt_{gd} = 10\% \quad \omega_{sh7} := \omega_{sfm} \quad sh8(t) := \sum_{n=0}^{N_0 gd-1} (u_{8n} \cdot \text{sinc}(\omega_{sh7} \cdot t - n \cdot \pi)) \quad (3.2.7.7)$$

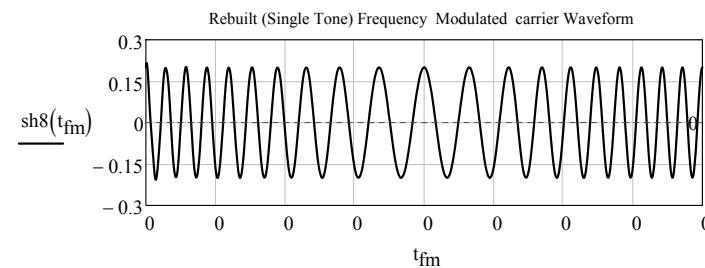


Fig.: (3.2.7.2)

Exact output

$$f2fm(t, \sigma, \omega_3, \zeta_5, \omega_{3cfm}, \omega_{fmm}, m_f) := \sin[(t - \sigma) \cdot \sqrt{\omega_3^2 - \zeta_5^2}] \cdot e^{\zeta_5 \cdot \sigma} \cdot \cos(\omega_{3cfm} \cdot \sigma + m_f \cdot \sin(\omega_{fmm} \cdot \sigma))$$

$$v_{ofm}(t) := A_{fm} \cdot A_5 \cdot \omega_3^2 \cdot \begin{cases} \left(\frac{e^{-\zeta_5 \cdot t}}{\sqrt{\omega_3^2 - \zeta_5^2}} \cdot \int_0^t f2fm(t, \sigma, \omega_3, \zeta_5, \omega_3^{cfm}, \omega_{mfm}, m_f) d\sigma \right) & \text{if } \zeta_5 \neq \omega_3 \\ \int_0^t (t - \sigma) \cdot e^{-(t-\sigma) \cdot \omega_3} \cdot \cos(\omega_3^{cfm} \cdot \sigma + m_f \cdot \sin(\omega_{mfm} \cdot \sigma)) d\sigma & \text{otherwise} \end{cases} \quad (3.2.7.8)$$

$$v_{ofm}(t) := \int_0^t w(t - \sigma) \cdot V_{fm}(\sigma) d\sigma \quad (3.2.7.9)$$

$$T_{test} = 13.33 \cdot \mu s \quad t_{fm} := \frac{T_{fmm} \cdot 0}{100}, \frac{T_{fmm} \cdot 0}{100} + \frac{2 \cdot T_{fmm} - \frac{T_{fmm} \cdot 0}{100}}{200} \dots 2 \cdot T_{fmm} \quad m_{fm} = 800\% \quad$$

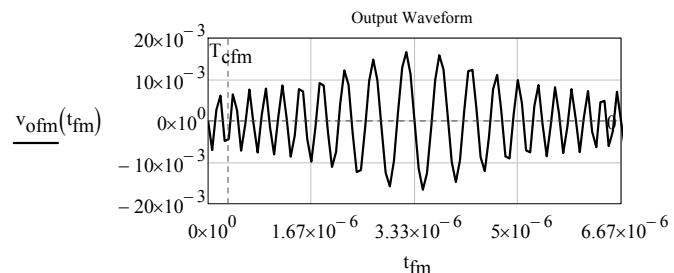


Fig.: (3.2.7.3)

$$v_{ofm}(nfm_{100}) = -0 V$$

Output sampling: Ofmk := v_ofm(nfmk)

(3.2.7.10)

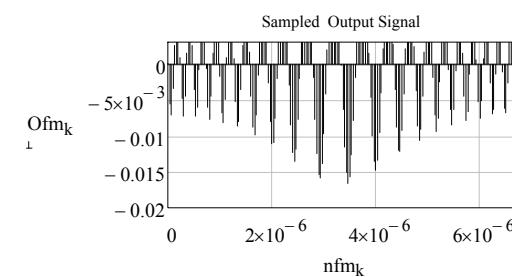


Fig.: (3.2.7.4)

$$m_{fm} = 800\% \quad f3_c = 75 \cdot \text{kHz} \quad \omega_{fmm} = 0 \cdot \frac{\text{Grads}}{\text{sec}} \quad \frac{f_{sfm}}{f3_c} = 400 \quad \frac{N0_{gd}}{f_{sfm}} \cdot \frac{1}{T_{test}} = 0.64$$

Fourier Transform of the Test signal OSpecfm := fft(Ofmk)

(3.2.7.11)

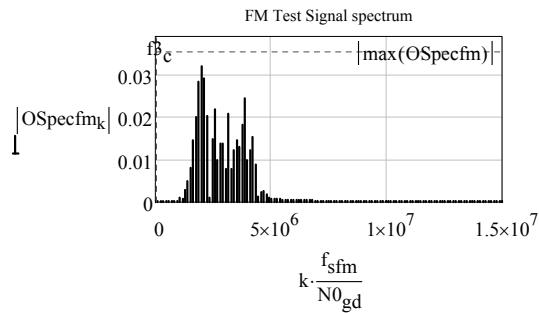


Fig.:3.2.7.5

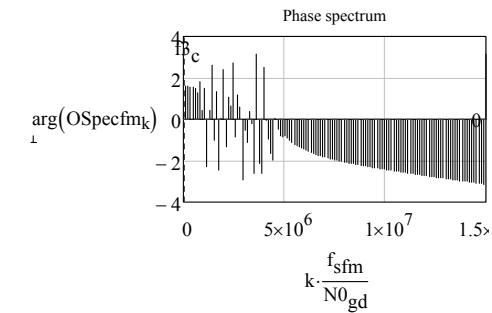


Fig.:3.2.7.6

On the other hand if the carrier frequency is located in the filter's passing band, its response will change. In the following I show what happens.

$$\omega_{31cfm} := \frac{\omega_3 c}{100} \quad \omega_{3m} := \frac{\omega_{fmm}}{100} \quad T31_{cfm} := \frac{2 \cdot \pi}{\omega_{3c}} \quad T3m := \frac{2 \cdot \pi}{\omega_{3m}}$$

Carson bandwidth: $Cars3 := 2 \cdot \omega_{3m} \cdot (m_{fm} + 1)$ $Cars3 = 0.17 \cdot \frac{\text{Mrads}}{\text{sec}}$ (3.2.7.12)

Carrier amplitude $A_{fm} = 0.2 \text{ V}$

Carrier frequency: $f31_c := \frac{\omega_{31cfm}}{2 \cdot \pi}$, $f31_c = 750 \cdot \text{Hz}$

sampling frequency (Nyquist rate): $f3_{sfm} := 10 \cdot f_{cfm}$, $f3_{sfm} = 30 \cdot \text{MHz}$,

sampling angular frequency: $\omega_{3sfm} := 2 \cdot \pi \cdot f_{sfm}$, $\omega_{3sfm} = 188.5 \cdot \frac{\text{Mrads}}{\text{sec}}$,

$$\text{sampling period: } T_{\text{sfm}} := \frac{1}{f_{\text{sfm}}}, \quad T_{\text{sfm}} = 0.03 \cdot \mu\text{s},$$

$$\text{sampling time step: } n_3 f_{\text{fm}} := \frac{k}{f_{\text{sfm}}}$$

$$\frac{N_0 g_d}{f_{\text{sfm}}} \cdot \frac{1}{T_{31} c_{\text{fm}}} = 0.64$$

the filter response is: $V1_{\text{fm}}(t) := v_{\text{fm}} s(t, \omega_3, \omega_3, A_{\text{fm}}, m_{\text{fm}}, N_{\text{gd}})$ (3.2.7.13)

$$u_8 b_k := \frac{V1_{\text{fm}}(n_3 f_{\text{fm}})}{\text{volt}} \quad (3.2.7.14)$$

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{s7} := \omega_{\text{sfm}} \quad sh7b(t) := \left[\sum_{n=0}^{N_0 g_d - 1} (u_8 b_n \cdot \text{sinc}(\omega_{s7} \cdot t - n \cdot \pi)) \right] \quad (3.2.7.15)$$

$$r_{\text{gd}} = 10\%$$

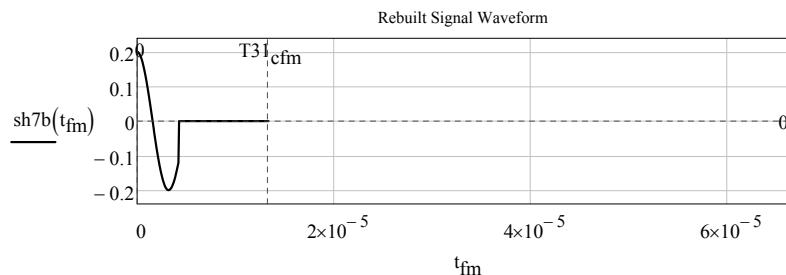


Fig.:3.2.7.7

Exact output:

$$v2_{\text{ofm}}(t) := A_{\text{fm}} \cdot A_5 \cdot \omega_3^2 \cdot \begin{cases} \frac{e^{-\zeta_5 \cdot t}}{\sqrt{\omega_3^2 - \zeta_5^2}} \cdot \int_0^t f2_{\text{fm}}(t, \sigma, \omega_3, \zeta_5, \omega_3, \omega_3, m_f) d\sigma & \text{if } \zeta_5 \neq \omega_3 \\ \int_0^t (t - \sigma) \cdot e^{-(t-\sigma) \cdot \omega_3} \cdot \cos(\omega_3 \cdot \sigma + m_f \cdot \sin(\omega_3 \cdot \sigma)) d\sigma & \text{otherwise} \end{cases} \quad (3.2.7.16)$$

$$v2_{\text{ofm}}(t) := \int_0^t w(t - \sigma) \cdot V1_{\text{fm}}(\sigma) d\sigma \quad (3.2.7.17)$$

$$T_{\text{test}} = 13.33 \cdot \mu\text{s}$$

$$T_{31} c_{\text{fm}} = 13.33 \cdot \mu\text{s}$$

$$\frac{1}{f_{\text{sfm}}} = 33.33 \cdot \text{ns}$$

$$V_{\text{pp}} = 4.5 \text{ V}$$

$$t1_{\text{fm}} := T_{\text{fmm}} \cdot 0, T_{\text{fmm}} \cdot 0 + \frac{100 \cdot T_{\text{fmm}} - T_{\text{fmm}} \cdot 0}{400} \dots 100 \cdot T_{\text{fmm}}$$

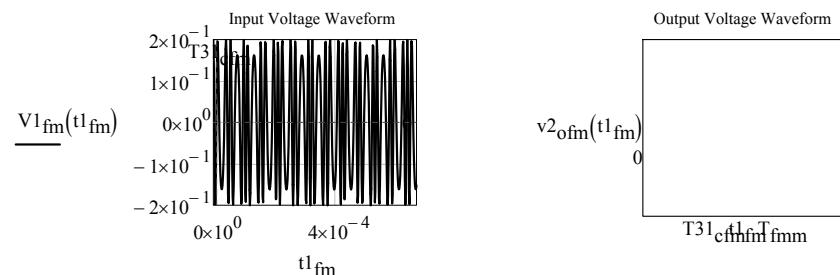


Fig.:3.2.7.8

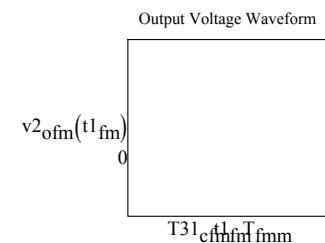


Fig.:3.2.7.9

$$n_3 f_{\text{fm}} := \frac{k}{N_0 g_d} \cdot T_{31} c_{\text{fm}} \quad \text{Output sampling: } O_{\text{fm}} := v2_{\text{ofm}}(n_3 f_{\text{fm}}) \quad (3.2.7.18)$$

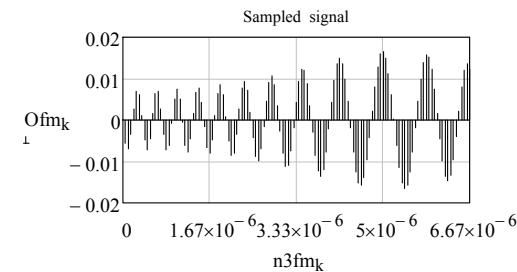


Fig.:3.2.7.10

$$\omega_{\text{fmm}} = 0 \cdot \frac{\text{Grads}}{\text{sec}} \quad f_3 c = 0 \cdot \text{GHz} \quad \frac{f_3 s_{\text{fm}}}{f_3 c} = 400 \quad m_{\text{fm}} = 8$$

Fourier Transform of the Test signal

$$O_{\text{Specfm3}} := \text{fft}(O_{\text{fm}}) \quad (3.2.7.19)$$

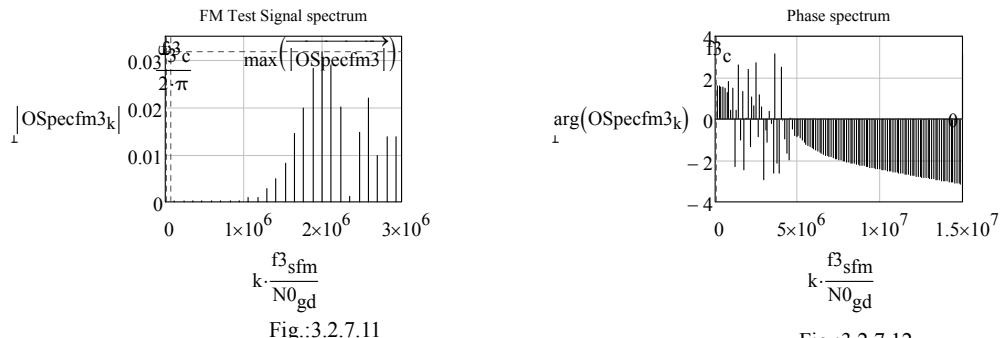


Fig.:3.2.7.11

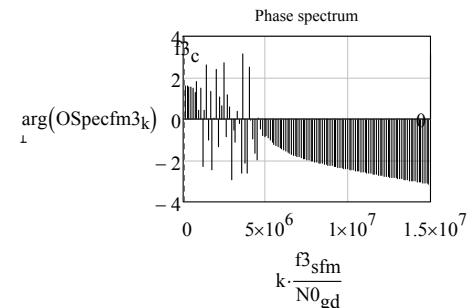


Fig.:3.2.7.12

$$\omega := \frac{\omega_3}{20 \cdot U_0}, \frac{\omega_3}{20 \cdot U_0} + \frac{\omega_3 \cdot U_0 - \frac{\omega_3}{20 \cdot U_0}}{U_0^2} \dots 10 \cdot U_0 \cdot \omega_3$$

$$W_{db\omega_1c} := 20 \cdot \log(|W_{lp}(j \cdot \omega_1 c)|) \quad (3.2.7.20)$$

$W_{db\omega_1c} = 19.99 \text{ dB}$

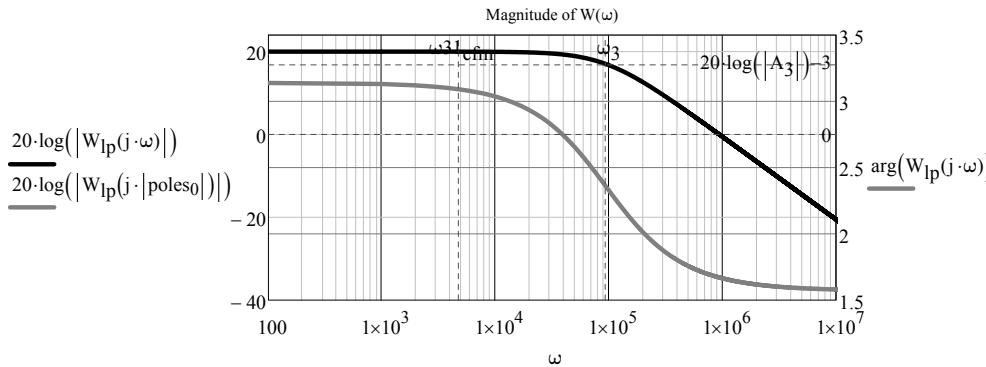


Fig.:3.2.7.13

3.2 ANALOG FILTER OUTPUT ANALYSIS

3.2.8 (single tone) Phase Modulated carrier response

Carrier max amplitude: $A_{pm} = 20 \text{ V}$

Modulating single tone signal max amplitude:

$$\text{Carrier pulsation: } \omega_{cpm} = 4 \cdot \omega_{test} \quad \omega_{cpm} = 3.77 \frac{\text{Grads}}{\text{sec}}$$

$$\text{Carrier frequency: } f_{cpm} = \frac{\omega_{cpm}}{2 \cdot \pi} \quad f_{cpm} = 0.6 \cdot \text{GHz}$$

$$\text{Carrier period: } T_{cpm} = \frac{1}{f_{cpm}} \quad T_{cpm} = 1.67 \cdot \text{ns}$$

$$\text{Modulating single tone pulsation: } \omega_{pmm} = \frac{\omega_{cpm}}{20} \quad \omega_{pmm} = 0.09 \frac{\text{Grads}}{\text{sec}}$$

$$\text{Modulating single tone frequency: } f_{pmm} = \frac{\omega_{pmm}}{2 \cdot \pi} \quad f_{pmm} = 15 \cdot \text{MHz} \quad (3.2.7.1)$$

$$\text{Modulating single tone period: } T_{pmm} = \frac{1}{f_{pmm}} \quad T_{pmm} = 0.07 \cdot \mu\text{s} \quad (3.2.7.2)$$

$$\text{Phase modulation index: } m_{pm} = 6$$

$$\text{Carson bandwidth: } \text{Cars4} := 2 \cdot \omega_{pmm} \cdot (m_{pm} + 1) \quad (3.2.7.3)$$

$$\text{Sampling frequency (Nyquist rate): } f_{spm} := 2 \cdot \text{Cars4} \quad , f_{spm} := 100 \cdot f_{cpm} \\ f_{spm} = 60 \cdot \text{GHz} \quad (3.2.7.4)$$

$$\text{Sampling angular frequency: } \omega_{spm} := 2 \cdot \pi \cdot f_{spm} \cdot \omega_{spm} = 376.99 \frac{\text{Grads}}{\text{sec}},$$

$$\text{Sampling period: } T_{spm} := \frac{1}{f_{spm}}, \quad T_{spm} = 0 \cdot \mu\text{s},$$

$$\text{Sampling time step: } npmk := \frac{k}{f_{spm}}, \quad (3.2.7.5)$$

$$\frac{N0gd}{f_{spm}} \cdot \frac{1}{T_{test}} = 0 \quad (3.2.7.6)$$

$$npmt^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \dots \end{bmatrix} \cdot \mu\text{s}$$

$$A_{pm} = 20 \text{ V} \quad V_{pm}(t) := V9_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, 40) \quad (3.2.7.7)$$

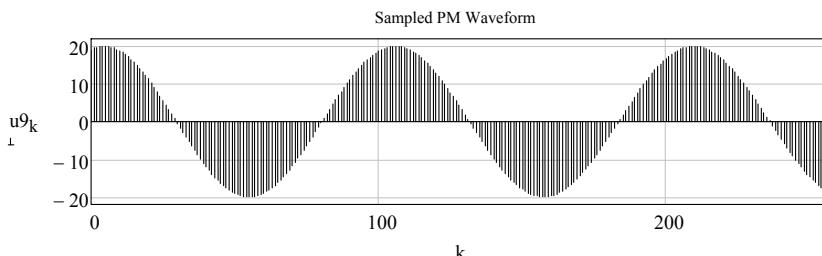
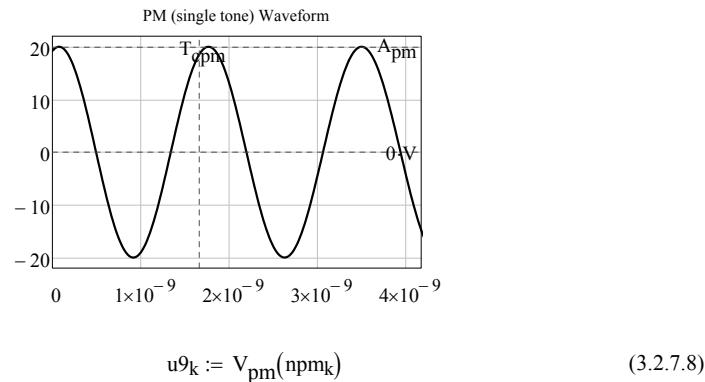


Fig.:3.2.7.1

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{ts8} := \omega_{spm} \quad sh8(t) := \left[\sum_{n=0}^{N_0 gd-1} (u_9 n \cdot \text{sinc}(\omega_{ts8} \cdot t - n \cdot \pi)) \right] \quad (3.2.7.9)$$

$$t_{pm} := 0 \cdot T_{cpm}, 0 \cdot T_{cpm} + \frac{40 \cdot T_{cpm}}{10000} \dots 40 \cdot T_{cpm} \quad rt_{gd} = 10\%$$

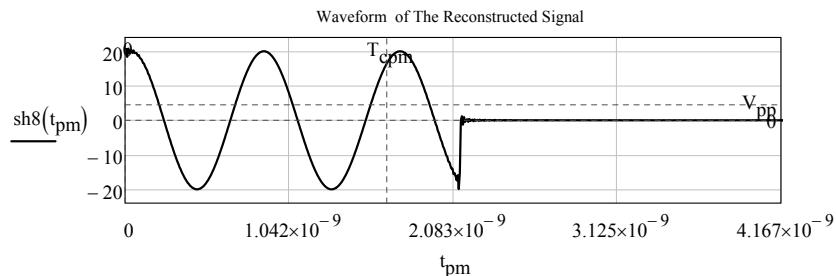
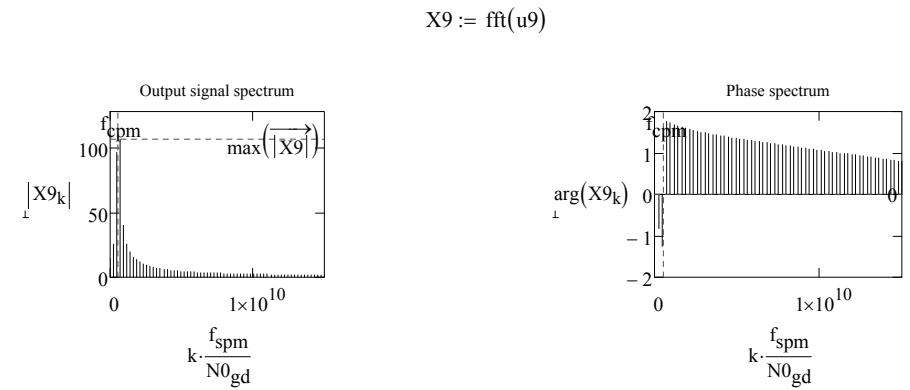


Fig.:3.2.7.2



$$V_{pp} = 4.5 \text{ V} \quad \text{Exact output: } v_{opm}(t) := \int_0^t w(t-\sigma) \cdot V_{pm}(\sigma) d\sigma \quad (3.2.7.10)$$

$$T_{cpm} = 0 \cdot \mu\text{s} \quad T_{pmm} = 0.07 \cdot \mu\text{s}$$

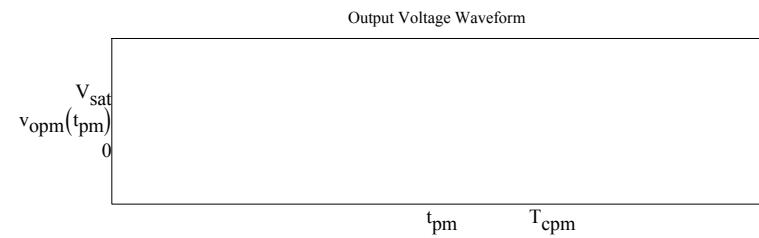


Fig.:3.2.7.3

$$Op_{mk} := \frac{v_{opm}(n\mu m_k)}{\text{volt}} \quad (3.2.7.11)$$

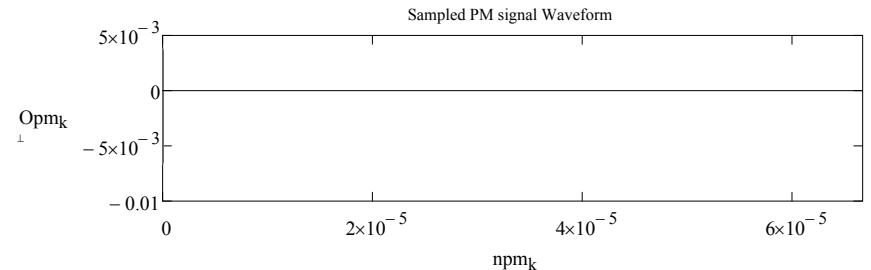


Fig.:3.2.7.4

Fourier Transform of the Test signal

$$f_{\text{cpm}} = 0.6 \cdot \text{GHz}$$

$$\frac{f_{\text{spm}}}{f_{\text{cpm}}} = 100$$

$$m_{\text{pm}} = 6 \quad \omega_{\text{pmm}} = 0.09 \cdot \frac{\text{Grads}}{\text{sec}}$$

$$OSpecpm := \text{fft}(Opm)$$

(3.2.7.12)

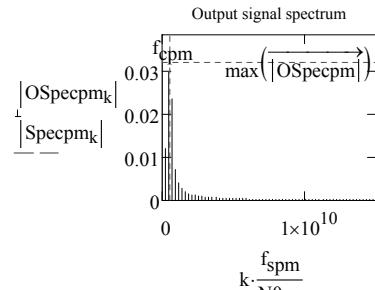


Fig.:3.2.7.5

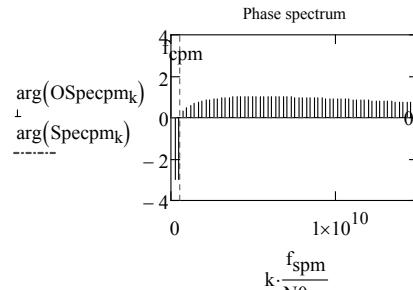


Fig.:3.2.7.6

$$W_{\text{db}}\omega_3 = 20 \cdot \log(|W_{\text{lp}}(j \cdot \omega_3)|)$$

$$W_{\text{db}}\omega_3 = 5.85 \cdot \text{dB}$$

ω_c is the angular frequency of the carrier

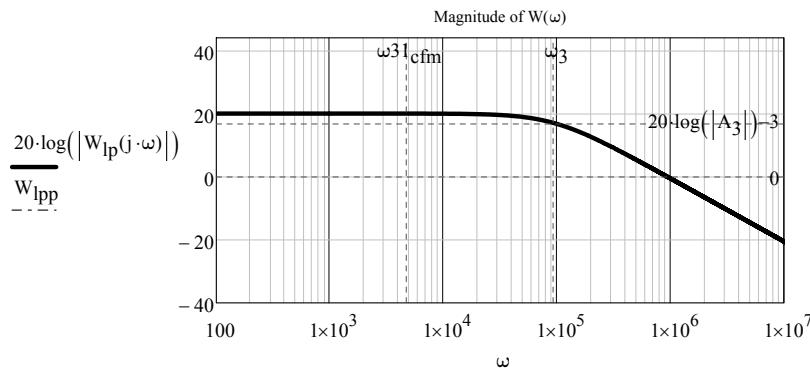


Fig.:3.2.7.7

3.3

Equivalent Digital Low Pass Filter (I^o order)

3.3.1) Z-transfer function of the I^o Order Low Pass Digital Filter

Chosen sampling period: $T_{\text{s}} = 1388.89 \cdot \text{ns}$ place: $T_{\text{s}} := T_{\text{s}}$

Given the transfer function: $W_{\text{lp}}(s) = \frac{A_3 \cdot \omega_3}{s + \omega_3}$, I can find its z-transform in this way:

with the change of variable: $s = \frac{1 - z^{-1}}{T_{\text{s}}}$, we can place: $\frac{1}{T_{\text{s}}} = \frac{\omega_s}{2 \cdot \pi}$, $s = \frac{\omega_s}{2 \cdot \pi} \cdot (1 - z^{-1})$,

$A_3 := A_3 \quad T_{\text{s}} := T_{\text{s}} \quad \omega_3 := \omega_3 \quad s := s \quad z := z$

Transfer function z-transform:

$$H_{\text{lp}}(z) := \frac{A_3 \cdot \omega_3}{s + \omega_3} \begin{cases} \text{substitute, } s = \frac{1 - z^{-1}}{T_{\text{s}}} \\ \text{collect, } z \end{cases} \rightarrow \frac{A_3 \cdot T_{\text{s}} \cdot \omega_3 \cdot z}{z \cdot (T_{\text{s}} \cdot \omega_3 + 1) - 1}$$

and after some algebraic manipulation and the definition of the following parameters:

$$\alpha_0 := \frac{A_3 \cdot \omega_3 \cdot T_{\text{s}}}{(T_{\text{s}} \cdot \omega_3 + 1)} \quad \beta_0 := (1 + \omega_3 \cdot T_{\text{s}})^{-1} \quad A_3 = -10$$

$$\alpha_0 = -1.157482795, \quad \beta_0 = 0.88425172,$$

you get the following result for the t. f. as a function of z:

$$H_{\text{lp}}(z) = \alpha_0 \cdot \frac{1}{1 - \beta_0 \cdot z^{-1}} \quad T_{\text{s}} = 1388.89 \cdot \text{ns}$$

3.3 Equivalent Digital Low Pass Filter (I^oorder)

3.3.2) Difference equations (Low Pass filter(I^oorder)). Canonical form

Given the z transfer function $H1_0(z) = \alpha_0 \cdot \frac{1}{1 - \beta_0 \cdot z^{-1}}$, I can split it so that:

$$H1_0(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W_{-}(z)} \cdot \frac{W_{-}(z)}{X(z)}$$

and place:

$$1) \quad \frac{Y(z)}{W_{-}(z)} = \alpha_0 \quad Y(z) = \alpha_0 \cdot W_{-}(z)$$

$$2) \quad \frac{W_{-}(z)}{X(z)} = \frac{1}{(1 - \beta_0 \cdot z^{-1})}$$

$$X(z) = (1 - \beta_0 \cdot z^{-1}) \cdot W_{-}(z) = W_{-}(z) - \beta_0 \cdot z^{-1} \cdot W_{-}(z)$$

which, inverting the z transform, gives :

$$x(n) = w_{-}(n) - \beta_0 \cdot w_{-}(n-1)$$

The corresponding set of difference equations is:

$$1) \quad w(n) = x(n) + \beta_0 \cdot w(n-1)$$

$$2) \quad y(n) = \alpha_0 \cdot w(n)$$

$$\alpha_0 := \alpha_0 \quad \beta_0 := \beta_0 \quad \lim_{z \rightarrow \infty} \left(\alpha_0 \cdot \frac{1}{1 - \beta_0 \cdot z^{-1}} \right) \rightarrow \alpha_0$$

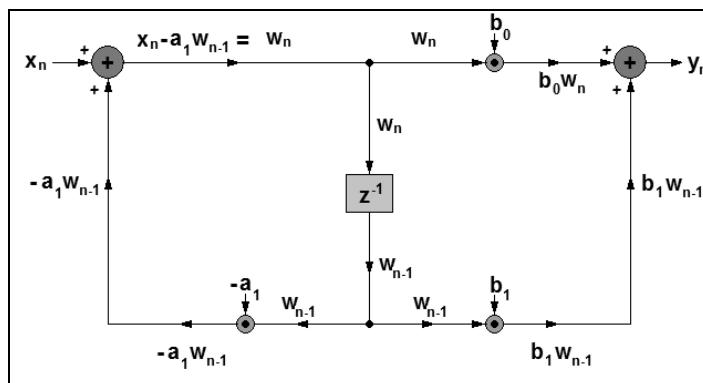


Fig.:3.3.2.1

3.3 Equivalent Digital Low Pass Filter (I^oorder)

For each test signal there would be shown the following results:

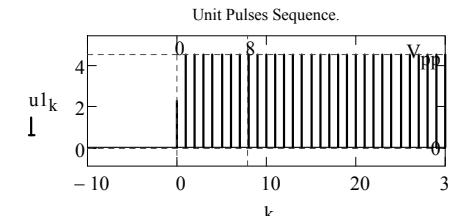
- 1) Sequence of the periodic response,
- 2) Digital first order low pass filter difference equations,
- 3) Schematic,
- 4) Graphics,
- 5) Comparison of the Bode plots of the z and s transfer functions

3.3.3) Sequence of the Voltage step response

Digital first order low pass filter difference equations:

$$\text{dimensionless input signal: } u1_k := V_{\text{stpls}}(n3k, V_{\text{pp}}) \quad v_{i1}(k) := \frac{u1_k}{\text{volt}}$$

$$V_{\text{pp}} = 4500 \cdot \text{mV}$$



$$\text{rows}(u1) = 256$$

fig.:3.3.3.1)

$$w1y1 := \text{DELPF1OCF}\left(v_{i1}, A_3, \frac{T_3}{\text{sec}}, \frac{\omega_3}{\text{rad}}, N_0 \text{gd}\right)$$

$$w1 := w1y1^{(0)} \quad y1 := w1y1^{(1)} \quad \alpha1 := (w1y1^{(2)})_0 \quad \beta1 := (w1y1^{(3)})_0$$

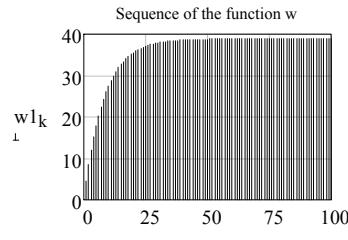


fig.:3.3.3.2

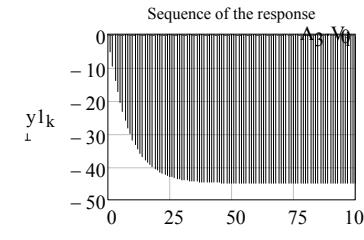


fig.:3.3.3.3

$$t1 := 0 \cdot \tau_3, 0 \cdot \tau_3 + \frac{100 \cdot \tau_3}{10000} \dots 100 \cdot \tau_3$$

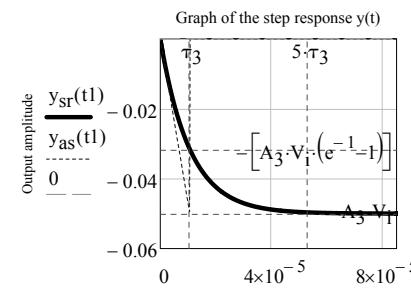


fig.:3.3.3.4

3.3 Equivalent Digital Low Pass Filter (I^{\bullet} order)

3.3.4) Sequence of the short Voltage pulse response.

$$\tau_{sv} := T_{s3sp}$$

$$t := -2 \cdot \tau_{pw}, -2 \cdot \tau_{pw} + \frac{4 \cdot \tau_{pw}}{5000} \dots 2 \cdot \tau_{pw}$$

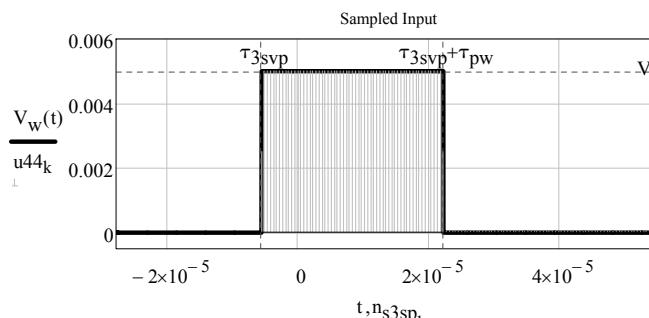


fig.:3.3.4.1

Digital first order low pass filter difference equations:

$$\text{dimensionless input signal: } v_{i2}(k) := u_{44k}$$

$$w2y2 := \text{DELPF1OCF}\left(v_{i2}, A_3, \frac{T_{s3sp}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N0 \cdot \text{gd}\right)$$

$$w2 := w2y2^{(0)} \quad y2 := w2y2^{(1)} \quad \alpha2 := (w2y2^{(2)})_0 \quad \beta2 := (w2y2^{(3)})_0$$

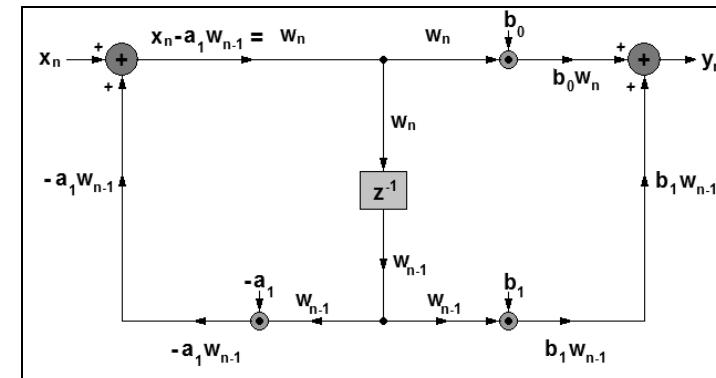


Fig.:3.3.4.2

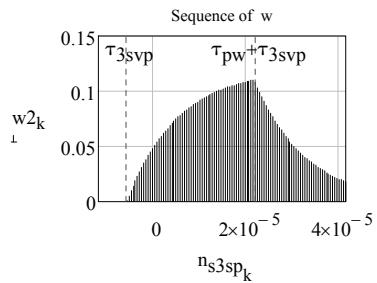


fig.:3.3.4.3

$$t_m := -2 \cdot \tau_{pw}, -2 \cdot \tau_{pw} + \frac{4 \cdot \tau_{pw}}{5000} .. 2 \cdot \tau_{pw}$$

Graph of the Short Pulse Response

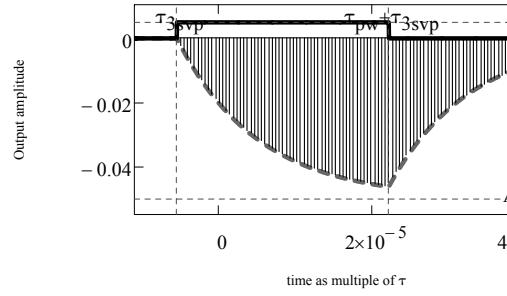


fig.:3.3.4.5

$$\begin{aligned} V_i &= 0.01 \text{ V} \\ V_{pp} &= 4.5 \text{ V} \end{aligned}$$

Sequence of the response

fig.:3.3.4.4

3.3 Equivalent Digital Low Pass Filter (I^* order)

3.3.5) Sequence of the Bipolar Pulse train response

$$\begin{aligned} T_{test} &= 13.33 \cdot \mu\text{s} & T_S &= 138.89 \cdot \text{ns} & \tau_3 &= 10.61 \cdot \mu\text{s} & f_S &:= \frac{1}{T_S} & \omega_S &:= 2 \cdot \pi \cdot f_S \\ \text{Chosen test signal period, } T_{test} &= 13333.33 \cdot \text{ns} & \frac{1}{T_{test}} &= 0.08 \cdot \text{MHz} \end{aligned}$$

$$\text{Laplace transform of the test signal: } V_{ip}(s) := \frac{V_i}{s} \cdot \tanh\left(\frac{T_{test} \cdot s}{4}\right)$$

$$u3_k := V_{sqwb}(n_{sqw_k})$$

Pulse amplitude $\pm V_i$:

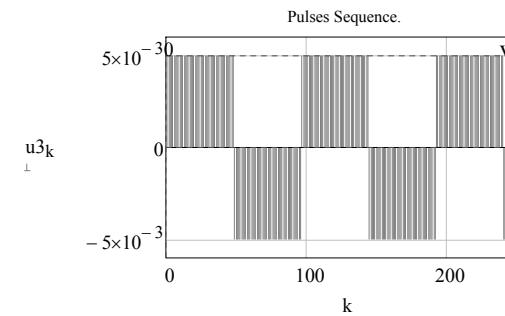


fig.:3.3.5.1

Digital first order low pass filter difference equations:

$$\text{dimensionless input signal: } v_{i3}(k) := u3_k$$

$$w3y3 := \text{DELPF1OCF}\left(v_{i3}, A_3, \frac{T_{ssqw}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N0_{gd}\right)$$

$$w3 := w3y3^{(0)} \quad y3 := w3y3^{(1)} \quad \alpha3 := (w3y3^{(2)})_0 \quad \beta3 := (w3y3^{(3)})_0$$

$$\alpha3 = -0.12920836, \quad \beta3 = 0.987079164,$$

you get the following result for the t. f. as a function of z :

$$H1_{\emptyset}(z) := \alpha3 \cdot \frac{1}{1 - \beta3 \cdot z^{-1}} \quad T_S = 138.89 \cdot \text{ns}$$

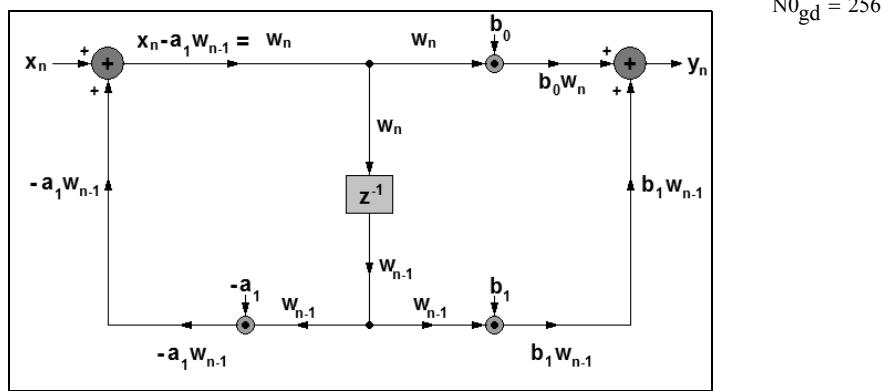


Fig.:3.3.5.2

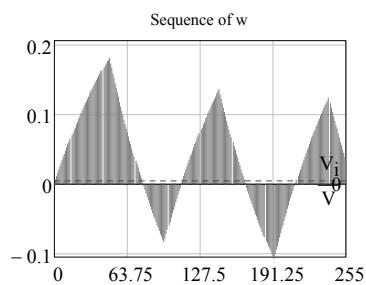


Fig.:3.3.5.3

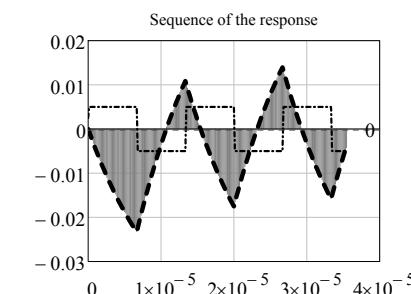


Fig.:3.3.5.4

$$N0_{gd} = 256$$

$$20 \cdot \log \left(\left| H1_o \left(e^{j \cdot \omega_{\text{test}} \cdot T_s} \right) \right| \right) = 5.8 \cdot \text{dB}$$

$$HdB1(\omega) := 20 \cdot \log \left(\left| H1_o \left(e^{j \cdot \omega \cdot T_s} \right) \right| \right)$$

$$\varphi 1(\omega) := \arg \left(H1_o \left(e^{j \cdot \omega \cdot T_s} \right) \right)$$

$$HdBc := 20 \cdot \log \left(\left| H1_o \left(e^{j \cdot \omega_{\text{test}} \cdot T_s} \right) \right| \right)$$

$$\varphi 1c := \arg \left(H1_o \left(e^{j \cdot \omega_3 \cdot T_s} \right) \right)$$

$$\underline{\omega} := \frac{\omega_3}{U_0}, \quad \omega_3 + \frac{\omega_{\text{test}} \cdot U_0 - \frac{\omega_3}{U_0}}{4 \cdot U_0^2} \dots U_0 \cdot \omega_{\text{test}} \quad \frac{\omega_s}{\omega_{\text{test}}} = 96$$

$$\underline{\omega} := \frac{\omega_3}{U_0}, \quad \omega_3 + \frac{\omega_{\text{test}} \cdot U_0 - \frac{\omega_3}{U_0}}{4 \cdot U_0^2} \dots U_0 \cdot \omega_{\text{test}}$$

Magnitude of $H(z)$

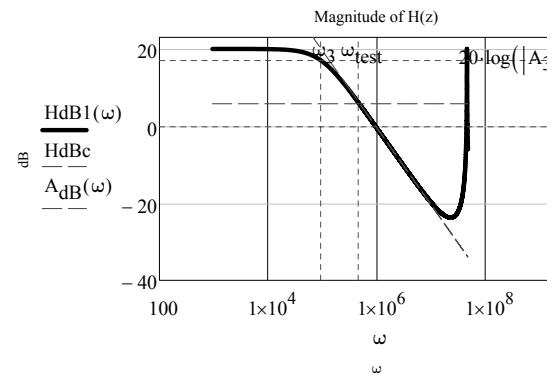


Fig.:3.3.5.7

Phase of $H(z)$

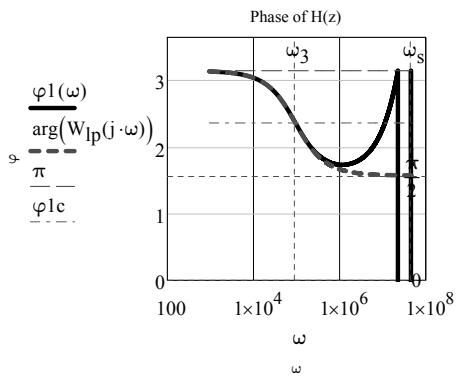


Fig.:3.3.5.8

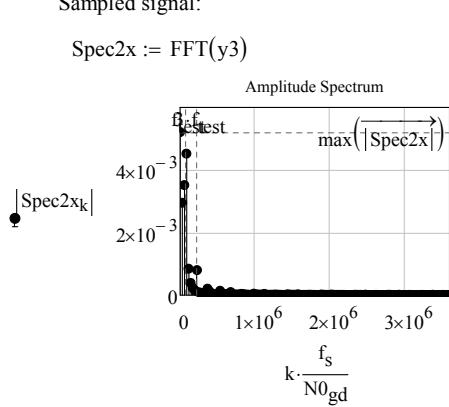


Fig.:3.3.5.5

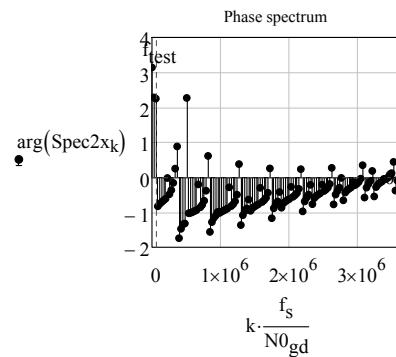


Fig.:3.3.5.6

$$\omega_3 = 0.09 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\omega_{\text{test}} = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}}$$

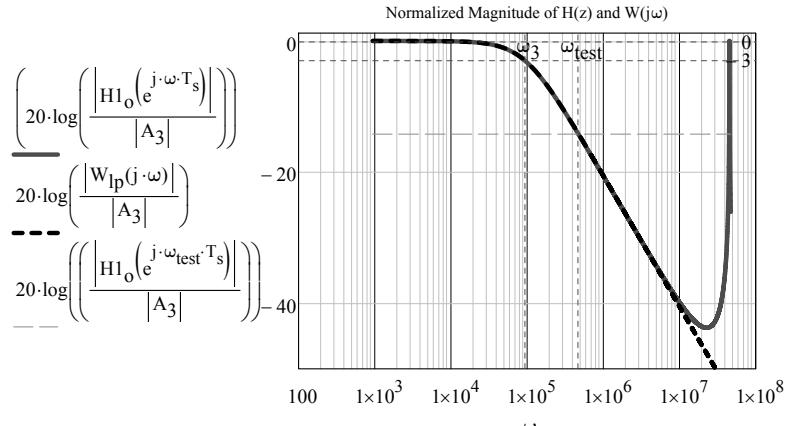


Fig.:3.3.5.9

3.3 Equivalent Digital Low Pass Filter (Ith order)

3.3.6) Sequence of the Cusp test signal response. Definition:

$$u_{4k} := v_{\text{incsp}_k}$$

$$T_{\text{ocsp}} := T_{\text{test}}$$

$$A_3 = -10$$

$$t_3 := -4 \cdot T_{\text{test}}, -4 \cdot T_{\text{test}} + \frac{8 \cdot T_{\text{test}}}{1000} \dots 4 \cdot T_{\text{test}}$$

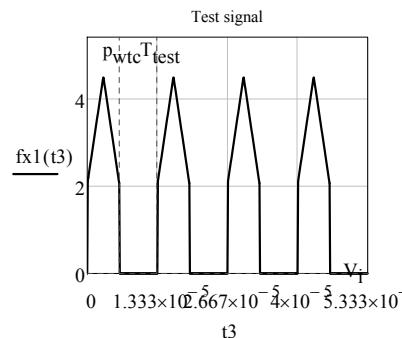


Fig.:3.3.6.1

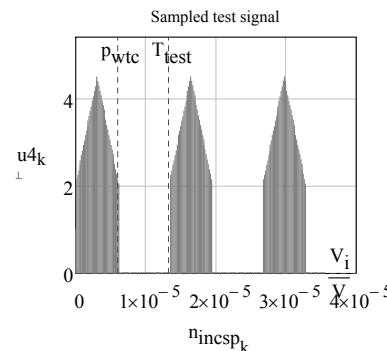


Fig.:3.3.6.2

Cusps sequence of amplitude V_i :

Digital first order low pass filter recurrence relations:

dimensionless input signal: $v_{i4}(k) := u_{4k}$ $\frac{f_s}{f_{3c}} = 96$

$$w4y4 := \text{DELPF1OCF}\left(v_{i4}, A_3, \frac{T_{\text{ocsp}}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N_0 \text{gd}\right)$$

$$w4 := w4y4^{(0)} \quad y4 := w4y4^{(1)} \quad \alpha4 := (w4y4^{(2)})_0 \quad \beta4 := (w4y4^{(3)})_0$$

$$\alpha4 = -0.12920836, \quad \beta4 = 0.987079164,$$

you get the following result for the t. f. as a function of z:

$$H1_0(z) := \alpha4 \cdot \frac{1}{1 - \beta4 \cdot z^{-1}} \quad T_s = 138.89 \cdot \text{ns}$$

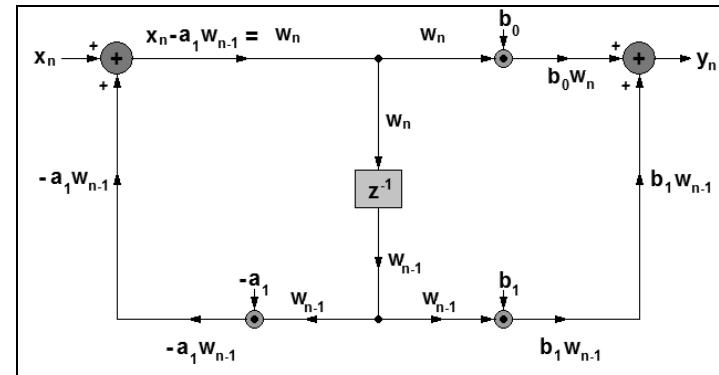


Fig.:3.3.6.3

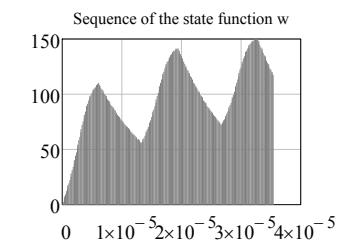


Fig.:3.3.6.4

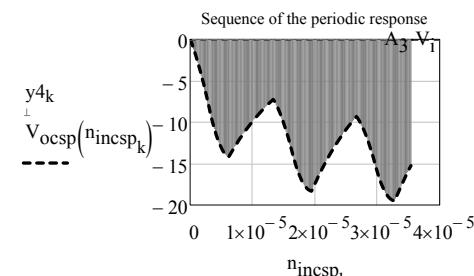


Fig.:3.3.6.5

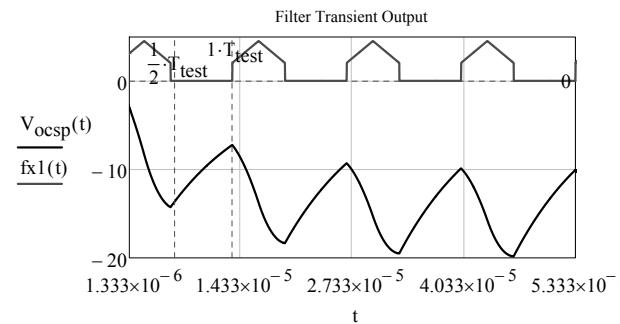


Fig.:3.3.6.6

$$\text{Spec4x} := \text{FFT}(y4)$$

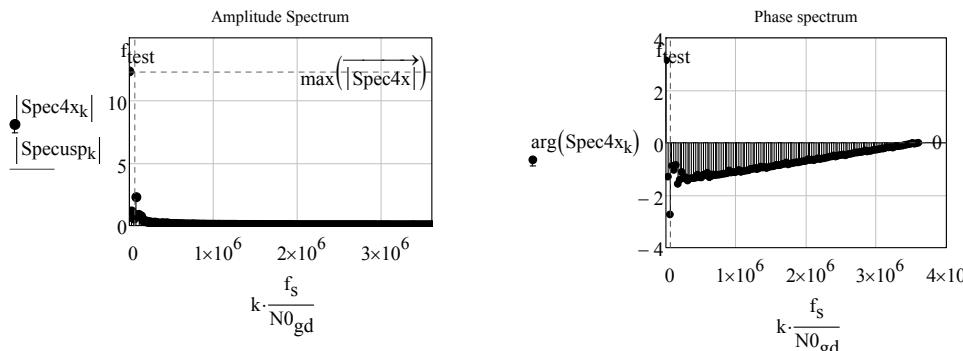


Fig.:3.3.6.7

Fig.:3.3.6.8

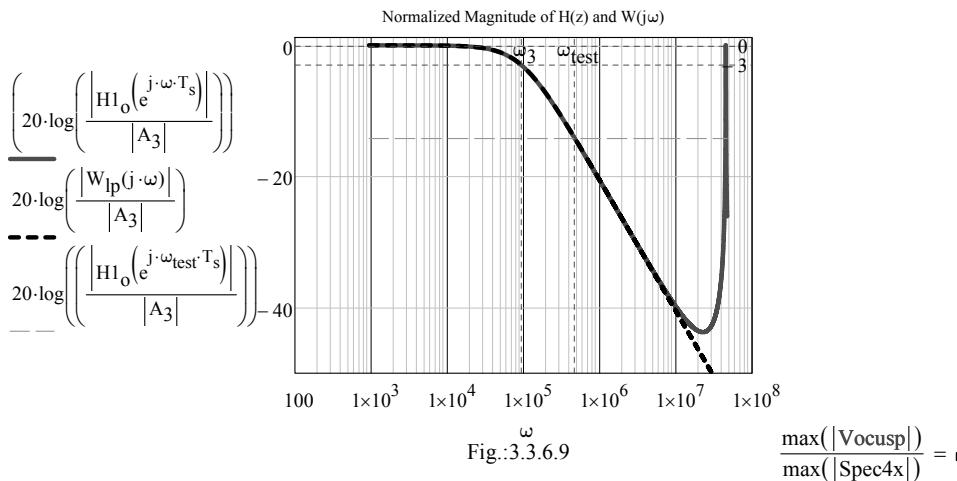


Fig.:3.3.6.9

$$\frac{\max(|V_{ocsp}|)}{\max(|\text{Spec4x}|)} = ■$$

3.3 Equivalent Digital Low Pass Filter (I[•]order)

3.3.7) Sequence of the Sawtooth response

$$T_{sw} := T_{ssw}$$

Sawtooth sequence of amplitude V_i :

Digital first order low pass filter recurrence relations:

$$\text{dimensionless input signal: } v_{i5}(k) := \frac{u10_k}{V} \quad \frac{f_s}{f_3 c} = 96$$

$$w5y5 := \text{DELPF1OCF}\left(v_{i5}, A_3, \frac{T_{ssw}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N0_{gd}\right)$$

$$w5 := w5y5^{(0)} \quad y5 := w5y5^{(1)} \quad \alpha5 := (w5y5^{(2)})_0 \quad \beta5 := (w5y5^{(3)})_0$$

$$\alpha5 = -0.12920836, \quad \beta5 = 0.987079164,$$

you get the following result for the t. f. as a function of z:

$$H1_0(z) := \alpha5 \cdot \frac{1}{1 - \beta5 \cdot z^{-1}} \quad T_s = 138.89 \cdot \text{ns}$$

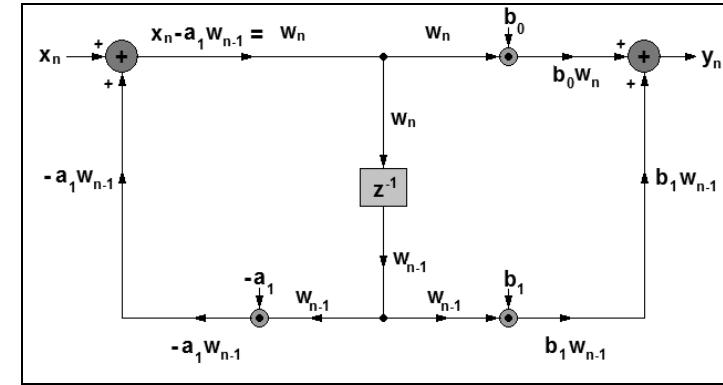


Fig.:3.3.7.1

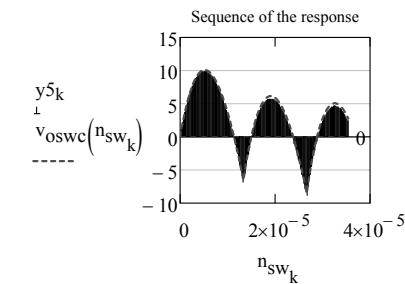
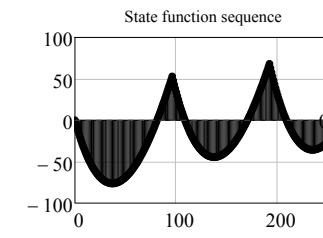


Fig.:3.3.7.2

$\text{Spec5o} := \text{FFT}(y5)$

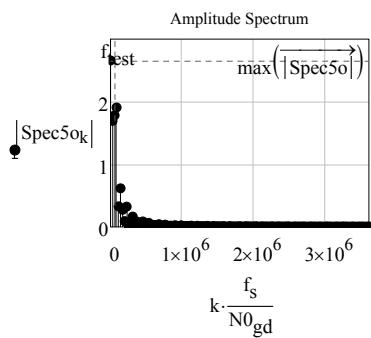
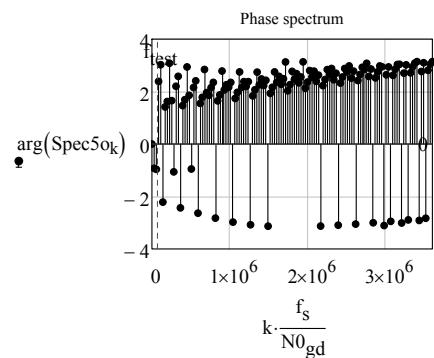


Fig.:3.3.7.3



$$\frac{\max(|\text{Spec5o}|)}{\max(|\text{Spec5o}|)} = 1 \quad T_s = 0.14 \cdot \mu\text{s}$$

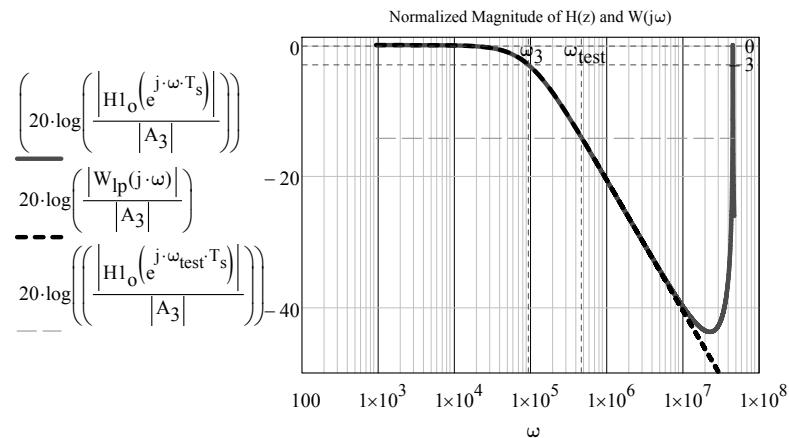


Fig.:3.3.7.6

Fig.:3.3.7.5

3.3 Equivalent Digital Low Pass Filter (I[•]order)

3.3.8) Sequence of the (single tone) Frequency Modulated carrier response

$m_{\text{fm}} = 8$

dimensionless input signal: $v_{i6}(k) := u8_k$

$T_s := T_{\text{sfm}}$

$$\begin{aligned} w6y6 &:= \text{DELPF1OCF}\left(v_{i6}, A_3, \frac{T_{\text{sfm}}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N0_{\text{gd}}\right) \\ w6 &:= w6y6^{(0)} \quad y6 := w6y6^{(1)} \quad \alpha6 := (w6y6^{(2)})_0 \quad \beta6 := (w6y6^{(3)})_0 \\ \alpha6 &= -0.03131754, \quad \beta6 = 0.996868246, \end{aligned}$$

Digital first order low pass filter difference relations:

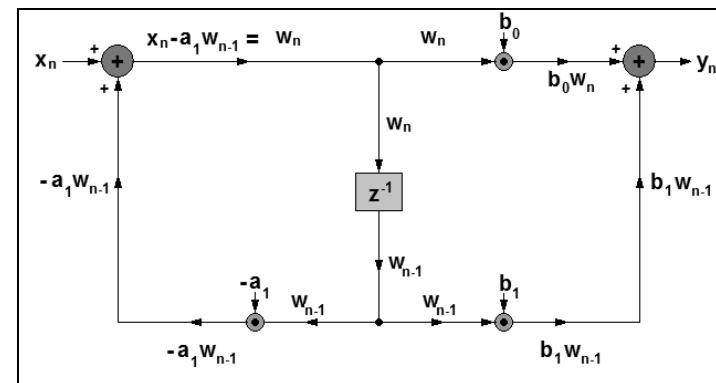


Fig.:3.3.8.1

you get the following result for the t. f. as a function of z:

$$H1_o(z) := \alpha6 \cdot \frac{1}{1 - \beta6 \cdot z^{-1}} \quad T_s = 33.33 \cdot \text{ns}$$

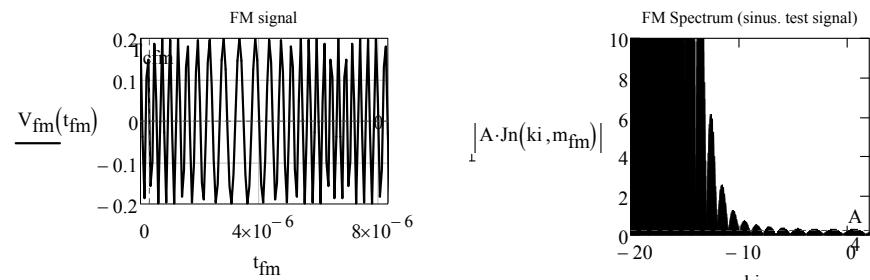


Fig.:3.3.8.2

Fig.:3.3.8.3

$X8 := \text{fft}(u8)$

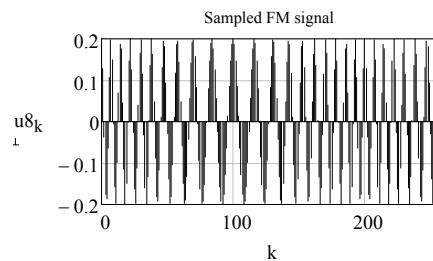


Fig.:3.3.8.4

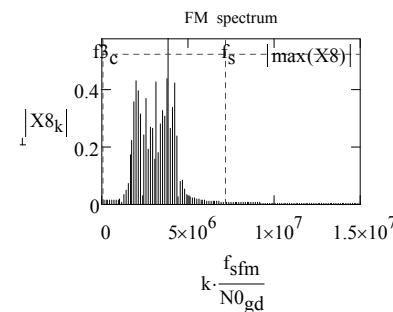


Fig.:3.3.8.5

$$\frac{f_{\text{sfm}}}{f_{3c}} = 400$$

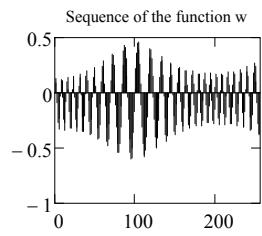


Fig.:3.3.8.6

$$f_{3c} = 0.08 \cdot \text{MHz}$$

$$\frac{f_s}{f_{3c}} = 96$$

$$\text{Spec6} := \text{fft}(y6)$$

$$m_{\text{fm}} = 8$$

$$\omega_{\text{fmm}} = 942477.8 \frac{1}{\text{s}}$$

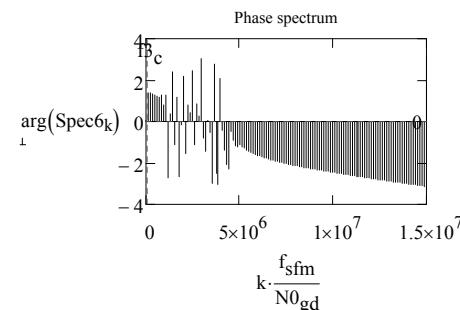


Fig.:3.3.8.8

Fig.:3.3.8.9

$\max(|\text{Spec6}|) = 0.09$

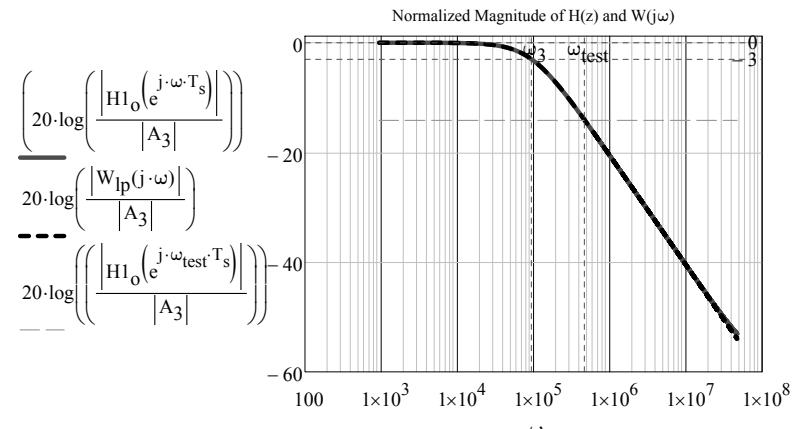


Fig.:3.3.8.10

3.3 Equivalent Digital Low Pass Filter (I^oorder)

3.3.9) Sequence of the Phase Modulated carrier response.

$$m_{\text{pm}} = 6$$

dimensionless input signal: $v_{i7}(k) := u9_k \quad T_{\text{spm}} := T_{\text{spm}}$

$$w7y7 := \text{DELPF1OCF}\left(v_{i7}, A_3, \frac{T_{\text{spm}}}{s}, \omega_3 \cdot \frac{s}{\text{rad}}, N_0 \cdot g_d\right)$$

$$w7 := w7y7^{(0)} \quad y7 := w7y7^{(1)} \quad \alpha7 := (w7y7^{(2)})_0 \quad \beta7 := (w7y7^{(3)})_0$$

$$\alpha7 = -1.570793859 \times 10^{-5}, \quad \beta7 = 0.999998429,$$

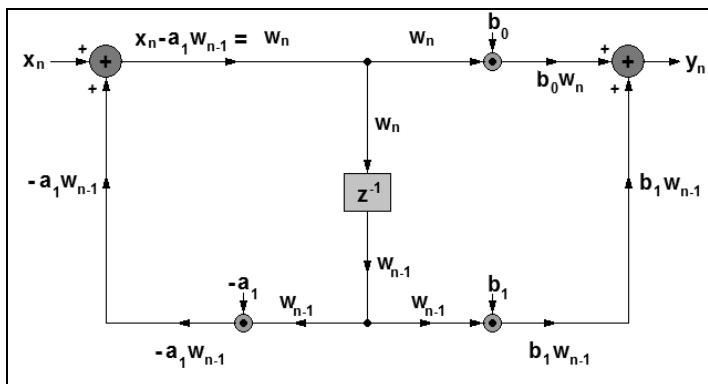
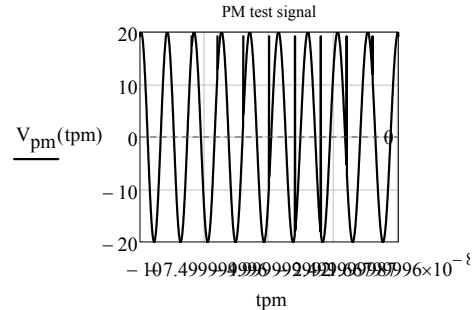


Fig.:3.3.9.1

you get the following result for the t. f. as a function of z:

$$H_{\text{pm}}(z) := \alpha7 \cdot \frac{1}{1 - \beta7 \cdot z^{-1}} \quad T_s = 0.02 \cdot \text{ns}$$



$$m_{\text{pm}} = 6$$

Fig.:3.3.9.2

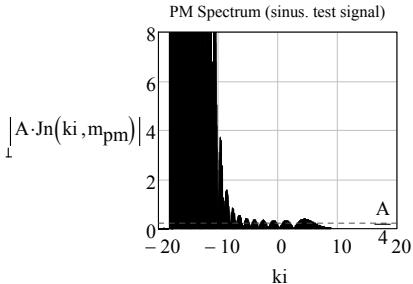


Fig.:3.3.9.3

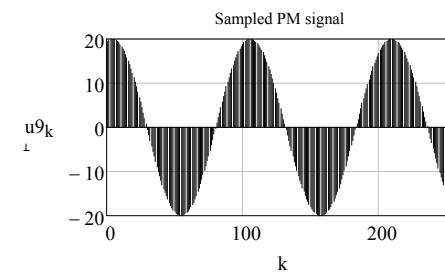


Fig.:3.3.9.4

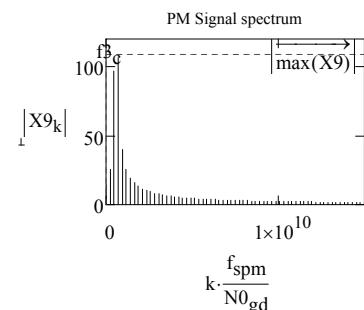


Fig.:3.3.9.5

$$\frac{f_{\text{spm}}}{f_{3c}} = 800000$$

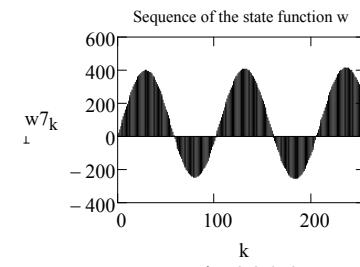


Fig.:3.3.9.6

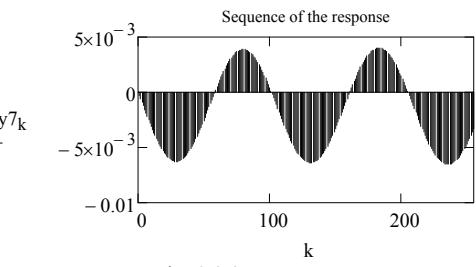


Fig.:3.3.9.7

$$f_{3c} = 0.08 \cdot \text{MHz} \quad \frac{f_{\text{spm}}}{f_{3c}} = 800000$$

$$\omega_{\text{pmm}} = 94.25 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\text{Spec7} := \text{FFT}(y7) \quad m_{\text{pm}} = 6$$

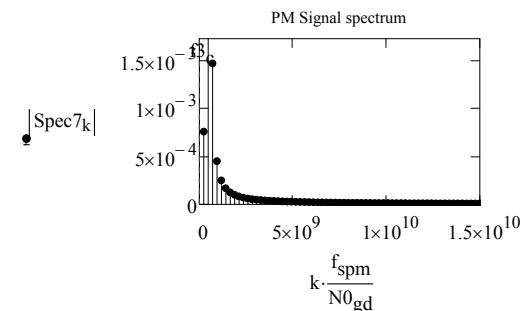


Fig.:3.3.9.8

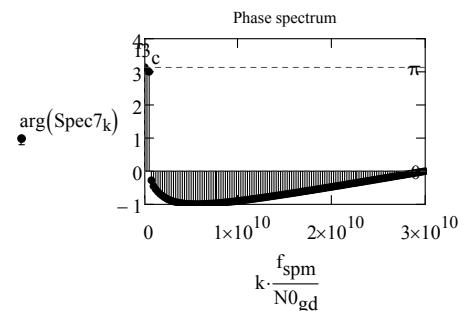


Fig.:3.3.9.9

3.3 Equivalent Digital Low Pass Filter (I^oorder)

3.3.10) BODE plot (Low Pass Analog v. s. Digital filter(I^oorder))

$$20 \cdot \log(|\alpha_7|) = -96.08 \quad T_s = 0.02 \cdot \text{ns}$$

$$W_{1dB}(\omega) := 20 \cdot \log(|W_{lp}(j\omega)|) \quad \varphi_{w1}(\omega) := \arg(W_{lp}(j\omega))$$

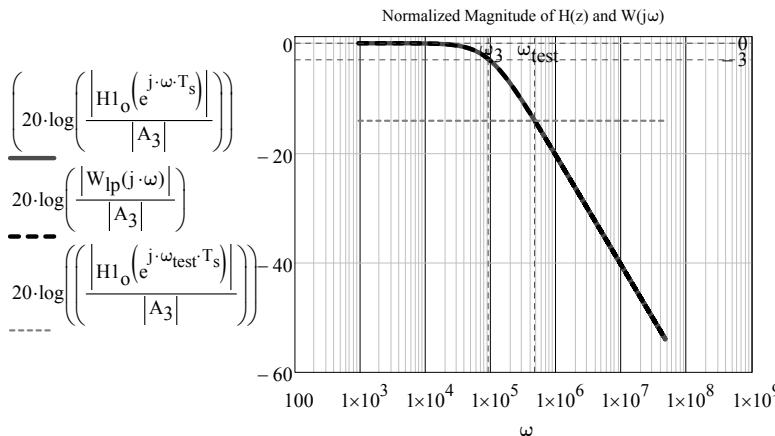


fig.:3.3.10.1

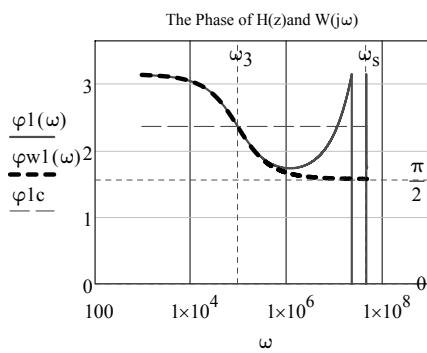


fig.:3.3.10.2

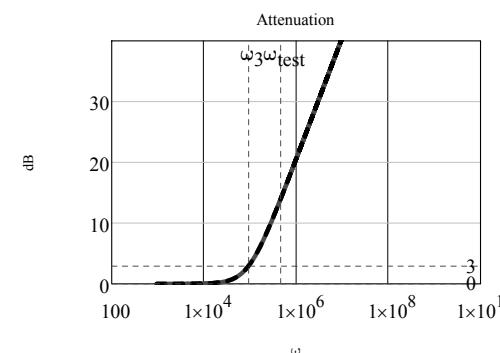


fig.:3.3.10.3

3.4

Transfer Function Sequence Obtained by the Synthetic Division Algorithm.

For each test signal would be shown the following results:

- 1) Sequence of the periodic response,
- 2) Digital first order low pass filter difference equations,
- 3) Schematic,
- 4) Graphics,
- 5) Comparison of the Bode plots of the z and s transfer functions

α_0 and β_0 are functions of the sampling period which in turn it depends from the kind of signal used.

$$\bar{T}_s := T_3 s$$

$$\alpha_0 = \frac{A_3 \cdot \omega_3 \cdot T_s}{(T_s \cdot \omega_3 + 1)} \quad \beta_0 = (1 + \omega_3 \cdot T_s)^{-1} \quad A_3 = -10$$

$$H1_o(z) = \alpha_0 \cdot \frac{1}{1 - \beta_0 \cdot z^{-1}}$$

$$\text{Numerator degree } N_n := 1 \quad \text{Denominator degree } M_d := 1$$

$$N1 := N_n + M_d \quad N0_{gd} = 256 \quad h1_k := 0$$

A generic first order transfer function in the z domain takes this form:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1}}{a_0 + a_1 \cdot z^{-1}}$$

The coefficients of the numerator and denominator can be defined as the elements of two vectors, namely a and b, hence:

Numerator coeffs	Denominator coeffs
$n1 := 1 .. N0_{gd} - 1$	$b_{n1} = 0.0$
$b_0 = \alpha_0$	$a_{n1} = 0.0$
$b_1 = 0$	$a_0 = 1$
	$a_1 = -\beta_0$

and divide the two polynomial by means of the following algorithm:

$$N1 = 2 \quad h1_0 = \frac{b_0}{a_0} \quad h1_{n1} = \frac{1}{a_0} \left[b_{n1} - \sum_{i=1}^{n1} (h1_{n1-i} \cdot a_i) \right]$$

$$y8v := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h1k \cdot u1_{\nu-k}, 0))$$

IACAN

$$tfs1 := \text{IACAN}\left(\frac{u1}{V}, A_3, T_{3s}, \omega_3, N_{0gd}\right)$$

$$\alpha_0 := (tfs1^{(0)})_0 \quad \beta_0 := (tfs1^{(0)})_1 \quad S1 := (tfs1^{(0)})_2 \quad E11 := (tfs1^{(0)})_3 \quad h11 := tfs1^{(1)}$$

$$a := (tfs1^{(2)}) \quad b := (tfs1^{(3)})$$

T. F. Numerator coefficients:

$$a^T = \begin{bmatrix} 1 & -0.88 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -1.16 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

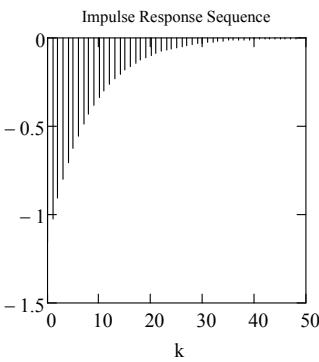
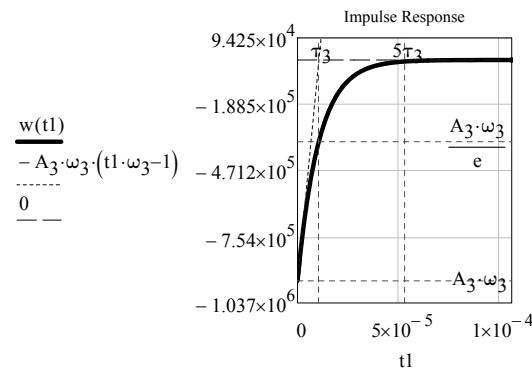
Sequence of the Impulse response:

$$h1^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Stability ($S1 < \infty$):

$$S1 = \sum_{k=0}^{\text{rows}(h1)-1} |h1_k| \quad S1 = 10$$

Energy of the sequence $h1$: $E11 = \sum_{k=0}^{\text{rows}(h1)-1} (|h1_k|)^2 \quad E11 = 6.14$



3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.1) Sequence of the voltage step response.

$$T_{8s} := T_{s3sp}$$

$$tfs2 := \text{IACAN}\left(\frac{u1}{V}, A_3, T_{s3sp}, \omega_3, N_{0gd}\right)$$

$$\alpha_8 := (tfs2^{(0)})_0 \quad \beta_8 := (tfs2^{(0)})_1 \quad S1 := (tfs2^{(0)})_2 \quad E11 := (tfs2^{(0)})_3 \quad h11 := tfs2^{(1)}$$

$$a := (tfs1^{(2)}) \quad b := (tfs1^{(3)}) \quad y8 := (tfs1^{(4)})$$

T. F. Numerator coefficients:

$$a^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.88 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -1.16 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h1^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Pulses Sequence.

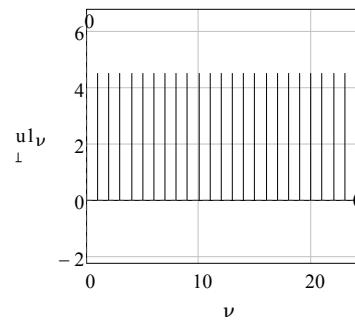


fig.:3.4.1.1

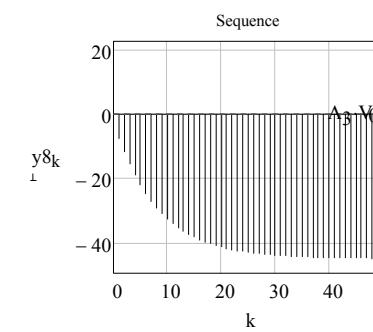


fig.:3.4.1.2

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.2) Sequence of the Short Voltage Pulse response.

$$T_{sv} := T_{svp}$$

$$tfs3 := IACAN(u44, A_3, T_{svp}, \omega_3, N0_{gd})$$

$$\alpha_9 := (tfs3^{(0)})_0 \quad \beta_9 := (tfs3^{(0)})_1 \quad S1 := (tfs3^{(0)})_2 \quad E11 := (tfs3^{(0)})_3 \quad h11 := tfs3^{(1)}$$

$$a9 := (tfs3^{(2)}) \quad b9 := (tfs3^{(3)}) \quad y9 := (tfs3^{(4)})$$

T. F. Numerator coefficients:

$$a9^T = \begin{bmatrix} 0 & 1 & -0.19 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b9^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h11^T = \begin{bmatrix} 0 & 1 & 2 & 3 & \dots \end{bmatrix}$$

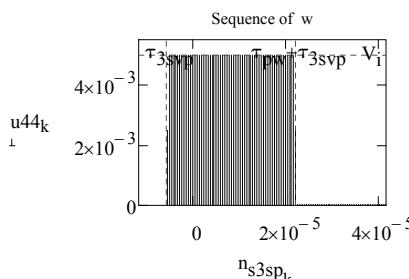


fig.:3.4.2.1

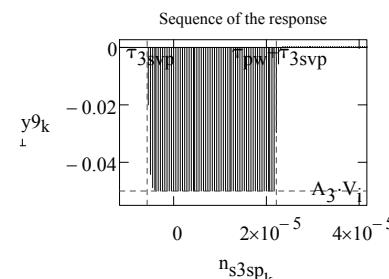


fig.:3.4.2.2

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

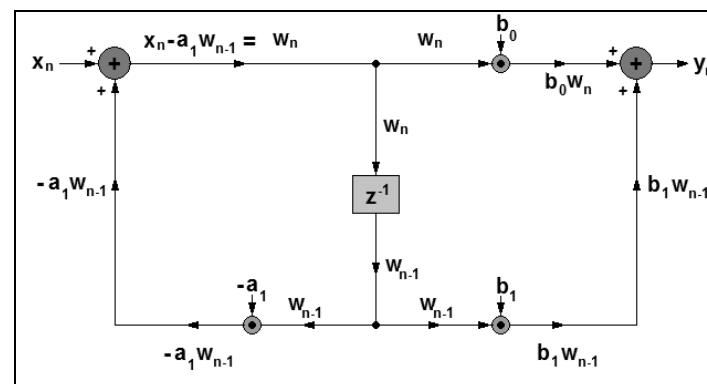
3.4.3) Sequence of the Bipolar Pulse train response:

$$T_{sv} := T_{ssqw}$$

$$tfs4 := IACAN(u3, A_3, T_{ssqw}, \omega_3, N0_{gd})$$

$$\alpha_{10} := (tfs4^{(0)})_0 \quad \beta_{10} := (tfs4^{(0)})_1 \quad S1 := (tfs4^{(0)})_2 \quad E11 := (tfs4^{(0)})_3 \quad h11 := tfs4^{(1)}$$

$$a11 := (tfs4^{(2)}) \quad b11 := (tfs4^{(3)}) \quad y10 := (tfs4^{(4)})$$



T. F. Numerator coefficients:

$$a11^T = \begin{bmatrix} 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b11^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h11^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & \dots \end{bmatrix}$$

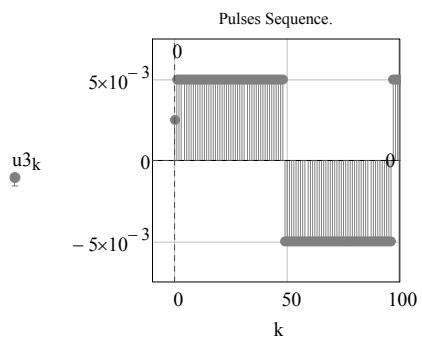


fig.:3.4.3.1

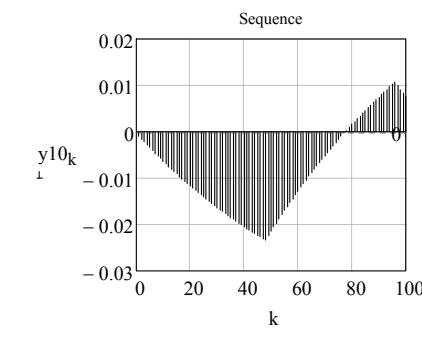


fig.:3.4.3.2

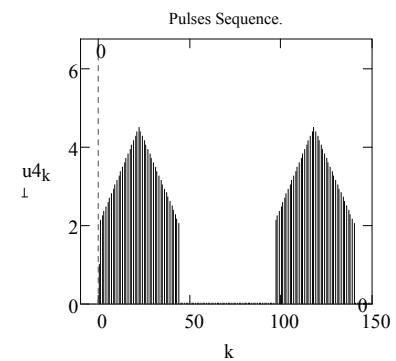


fig.:3.4.4.1

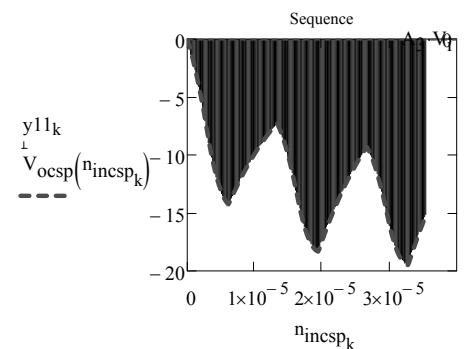


fig.:3.4.4.2

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.4) Sequence of the Cusp wave response.

$$T_{\text{ocsp}} := T_{\text{ocsp}}$$

$$\text{tfs5} := \text{IACAN}(u4, A3, T_{\text{ocsp}}, \omega_3, N0_{\text{gd}})$$

$$\alpha_{11} := (\text{tfs5}^{(0)})_0 \quad \beta_{11} := (\text{tfs5}^{(0)})_1 \quad S1 := (\text{tfs5}^{(0)})_2 \quad E11 := (\text{tfs5}^{(0)})_3 \quad h12 := \text{tfs5}^{(1)}$$

$$a12 := (\text{tfs5}^{(2)}) \quad b12 := (\text{tfs5}^{(3)}) \quad y11 := (\text{tfs5}^{(4)})$$

T. F. Numerator coefficients:

$$a12^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b12^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.13 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h12^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.1292 & -0.1275 & -0.1259 & -0.1243 & \dots \end{bmatrix}$$

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.5) Sequence of the Sawtooth response.

$$T_{ssv} := T_{ssw}$$

$$tfs6 := IACAN\left(\frac{u10}{V}, A_3, T_{ocsp}, \omega_3, N0_{gd}\right)$$

$$\alpha_{12} := (tfs6^{(0)})_0 \quad \beta_{12} := (tfs6^{(0)})_1 \quad S1 := (tfs6^{(0)})_2$$

$$E11 := (tfs6^{(0)})_3 \quad h13 := tfs6^{(1)}$$

$$a13 := (tfs6^{(2)}) \quad b13 := (tfs6^{(3)}) \quad y12 := (tfs6^{(4)})$$

T. F. Numerator coefficients:

$$a13^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

T. F. Denominator coefficients:

$$b13^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & -0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

Sequence of the Impulse response:

$$h13^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & -0.1292 & -0.1275 & -0.1259 & -0.1243 & \dots \\ \hline \end{array}$$

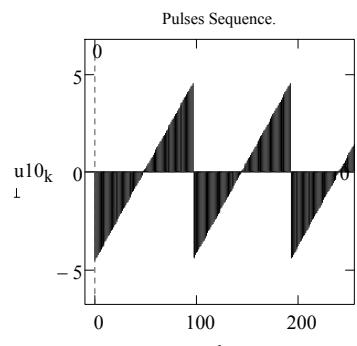


fig.:3.4.5.1

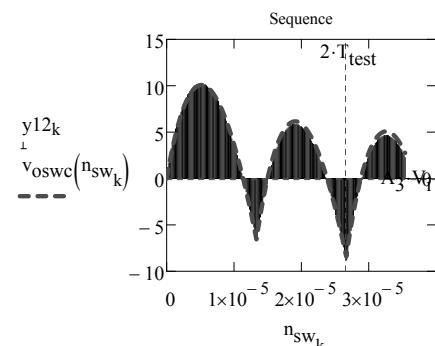


fig.:3.4.5.2

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.6) Sequence of the Frequency Modulated carrier response.

$$T_{sv} := T_{sfm}$$

$$tfs7 := IACAN(u8, A_3, T_{ocsp}, \omega_3, N0_{gd})$$

$$\alpha_{13} := (tfs7^{(0)})_0 \quad \beta_{13} := (tfs7^{(0)})_1 \quad S1 := (tfs7^{(0)})_2 \quad E11 := (tfs7^{(0)})_3 \quad h14 := tfs7^{(1)}$$

$$a14 := (tfs7^{(2)}) \quad b14 := (tfs7^{(3)}) \quad y13 := (tfs7^{(4)})$$

T. F. Numerator coefficients:

$$a14^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

T. F. Denominator coefficients:

$$b14^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & -0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

Sequence of the Impulse response:

$$h14^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & -0.1292 & -0.1275 & -0.1259 & -0.1243 & \dots \\ \hline \end{array}$$

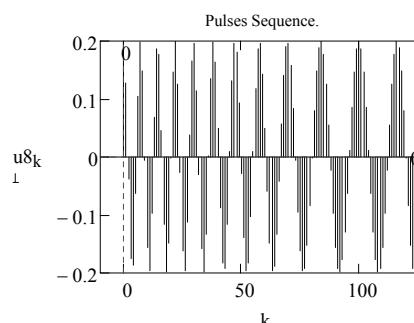


fig.:3.4.6.1

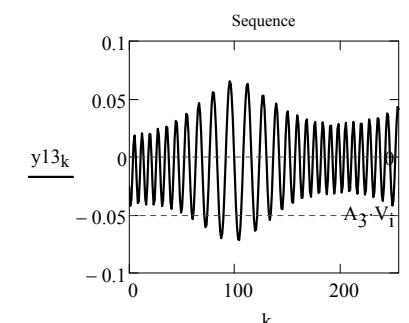


fig.:3.4.6.2

3.4 Transfer Function Sequence obtained by an Algorithm. Convoluting Output.

3.4.7) Sequence of the Phase Modulated carrier response.

$$T_{\text{sp}} := T_{\text{spm}}$$

$$\text{tfs8} := \text{IACAN}(u_9, A_3, T_{\text{ocsp}}, \omega_3, N_0_{\text{gd}})$$

$$a_{14} := (\text{tfs8}^{(0)})_0 \quad b_{14} := (\text{tfs8}^{(0)})_1 \quad S_1 := (\text{tfs8}^{(0)})_2 \quad E_{11} := (\text{tfs8}^{(0)})_3 \quad h_{15} := \text{tfs8}^{(1)}$$

$$a_{14} := (\text{tfs8}^{(2)}) \quad b_{14} := (\text{tfs8}^{(3)}) \quad y_{14} := (\text{tfs8}^{(4)})$$

T. F. Numerator coefficients:

$$a_{14}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b_{14}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.13 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h_{15}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.1292 & -0.1275 & -0.1259 & -0.1243 & \dots \end{bmatrix}$$

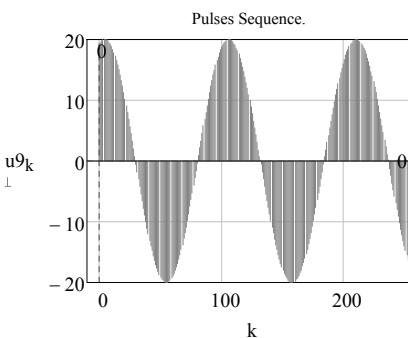


fig.:3.4.7.1

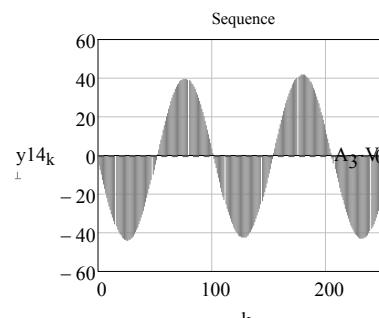


fig.:3.4.7.2

3.5

Search of the output discrete time sequence by a discrete convolution

For each test signal would be shown the following results:

- 1) Sequence of the periodic response,
- 2) Digital first order low pass filter difference equations,
- 3) Schematic,
- 4) Graphics,
- 5) Comparison of the Bode plots of the z and s transfer functions

The sequence corresponding to the transfer function, can be found using the "invztrans" MATHCAD's operator as follows:

$$\alpha_0 := \alpha_0 \quad \beta_0 := \beta_0 \quad k := k$$

Transfer function parameters definition:

$$\alpha_0 = \frac{A_3 \cdot \omega_3 \cdot T_3 s}{(T_3 s \cdot \omega_3 + 1)} \quad \beta_0 = (1 + \omega_3 \cdot T_3 s)^{-1} \quad A_3 = -10$$

$$\alpha_0 = -1.157482795, \quad \beta_0 = 0.88425172,$$

$$h_{14k} := \alpha_0 \cdot \frac{1}{1 - \beta_0 \cdot z^{-1}} \text{ invztrans}, z, k \rightarrow \alpha_0 \cdot \beta_0^k$$

The result is the sequence of the impulse response, here depicted:

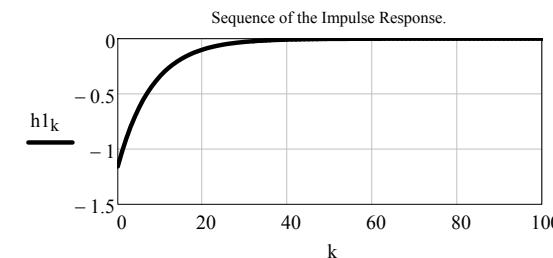
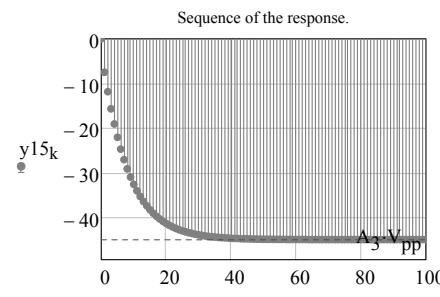
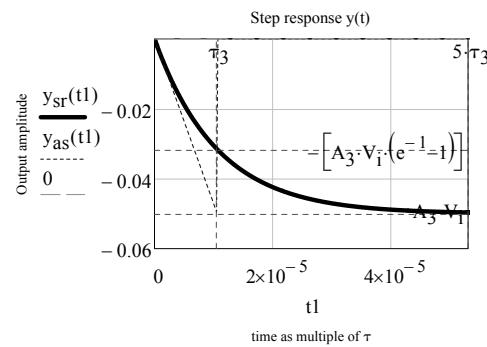


fig.:3.5.1

The Output of the Digital System is given by the **discrete convolution** between the input signal (the discrete time sequence) and the discrete impulse response of the System:

$$y15n1 := \sum_{k=0}^{n1} (\text{if}(n1 - k \geq 0, h1k \cdot u1_{n1-k}, 0))$$



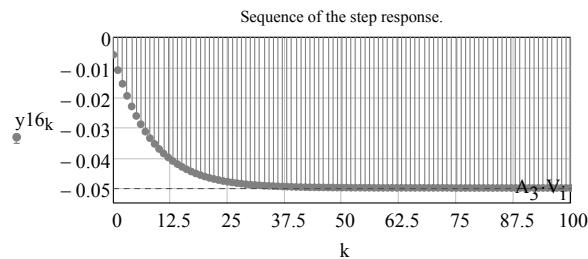
Knowing the sequences of any input, that of the impulse response and the relative Z transforms, the **z-inverse transform of the product** of the two z functions can be determined, it corresponds to the convolution of the two sequences, as follows:

$$X(z) := \sum_{n=0}^{N1-1} (u1_n \cdot z^{-n}) \quad H3(z) := \sum_{n=0}^{N0gd-1} (h1_n \cdot z^{-n}) \quad Y9(z) := H3(z) \cdot X(z)$$

$$\text{input signal: } V_i \text{ ztrans } \rightarrow \frac{V_i \cdot z}{z - 1}$$

System output corresponding to the z-anti transform of the product:

$$y16k := \alpha0 \cdot \frac{1}{1 - \beta0 \cdot z^{-1}} \cdot \frac{V_i \cdot z}{z - 1} \Big| \text{invztrans, } z, k \xrightarrow{\text{simplify}} \frac{V_i \alpha0 \cdot (\beta0^{k+1} - 1)}{\beta0 - 1}$$



$$T_{test} = 13.33 \cdot \mu\text{s}$$

Example: sinusoidal input:

$$\begin{aligned} \text{Signal amplitude: } & V_m := V_{pp}, \\ \text{Signal frequency: } & f_{test} = 0.08 \cdot \text{MHz}, \\ \text{arbitrary sampling frequency: } & f_s := 10 \cdot f_{test}, \quad f_s = 0.75 \cdot \text{MHz}, \\ \text{sampling angular frequency: } & \omega_s := 2 \cdot \pi \cdot f_s, \quad \omega_s = 0 \cdot \frac{\text{Grads}}{\text{sec}}, \end{aligned} \quad (5.8.)$$

$$\begin{aligned} \text{sampling period: } & T_s := \frac{1}{f_s}, \quad T_s = 1333.33 \cdot \text{ns}, \\ \text{sampling time step: } & n_k := \frac{k}{f_s}, \quad N0_{gd} = 256 \end{aligned} \quad (5.8.)$$

$$\frac{N0_{gd}}{f_s} \cdot f_3 = 5.12 \quad N0_{gd} = 256. \quad (3.8.)$$

$$\text{L. p. filter Input: } x2_k := V_m \cdot \sin(\omega_{test} \cdot n_k) \quad (3.8.39)$$

$$\Delta T := n_{10} - n_9 \quad \Delta T = 1.33 \cdot \mu\text{s}$$

$$\text{Z transform of the input signal: } \omega := \omega_{test} \quad \omega = 0.47 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \Delta T = 1333.33 \cdot \text{ns}$$

$$\Delta T := \Delta T \quad \omega := \omega \quad V_i = 0.01 \text{ V}$$

$$n := n \quad V_i := V_i \quad V_i \cdot \sin(\omega \cdot n \cdot \Delta T) \text{ ztrans } \rightarrow \frac{V_i \cdot z \cdot \sin(\omega \cdot \Delta T)}{z^2 - 2 \cdot \cos(\omega \cdot \Delta T) \cdot z + 1}$$

$$\text{I place: } K2 := \sin(\Delta T \cdot \omega) \quad \cos(\Delta T \cdot \omega) = \sqrt{1 - K2^2} \quad \sqrt{1 - K2^2} = 0.81$$

Computing the corresponding sequence *the result returned for the symbolic operation is too large to be displayed, but can be used for other calculations if assigned to a function.*

$$A_3 = -10 \quad 20 \cdot \log(|W_{lp}(j \cdot \sqrt{\omega_3 \cdot \omega_s})|) = 2.92$$

$$\omega := \frac{\omega_3}{U_0}, \frac{\omega_3}{U_0} + \frac{\omega_{test} \cdot U_0 - \frac{\omega_3}{U_0}}{4 \cdot U_0^2} \cdot U_0 \cdot \omega_{test}$$

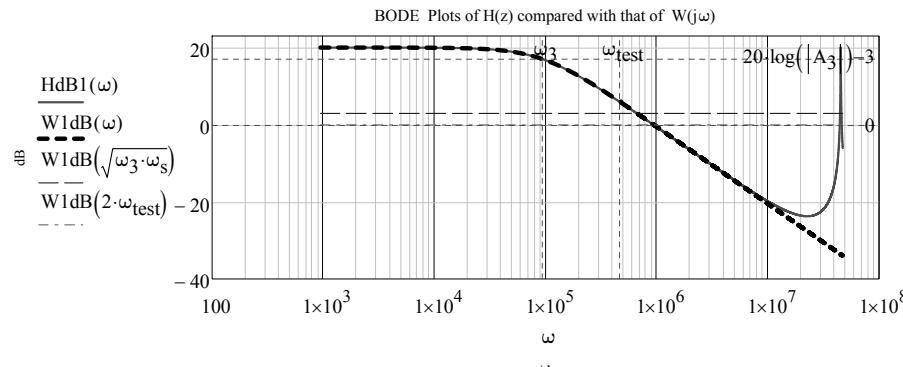


fig.:3.5.5

3.6

The bilinear transformation

3.6.1) Z-transfer function of the I^o Order Low Pass Digital Filter

For each test signal would be shown the following results:

- 1) Sequence of the periodic response,
- 2) Digital first order low pass filter difference equations,
- 3) Schematic,
- 4) Graphics,
- 5) Comparison of the Bode plots of the z and s transfer functions

$$\text{Basic assumption: Bilinear transformation: } s = \frac{2}{T_s} \cdot \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right),$$

the amplitude response of the analog function is preserved.

$$A_3 := A_3 \quad \omega_3 := \omega_3 \quad \omega_s := \omega_s \quad \omega_s = \frac{2 \cdot \pi}{T_s} \quad \frac{2}{T_s} = \frac{\omega_s}{\pi}$$

$$H11_t(z) := \frac{A_3 \cdot \omega_3}{s + \omega_3} \quad \begin{cases} \text{substitute, } s = \frac{\omega_s}{\pi} \cdot \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right) \\ \text{collect, } z \\ \text{collect, } A_3 \cdot \pi \cdot \omega_3 \end{cases} \rightarrow \frac{z + 1}{z \cdot (\omega_s + \pi \cdot \omega_3) - \omega_s + \pi \cdot \omega_3} \cdot (\pi \cdot A_3 \cdot \omega_3)$$

Hence the Transfer function's z-transform is:

$$H11(z) := \frac{[\pi \cdot A_3 \cdot \omega_3 \cdot (1 + z^{-1})]}{(\omega_s + \pi \cdot \omega_3) \cdot \left[1 - \frac{\omega_s - \pi \cdot \omega_3}{(\omega_s + \pi \cdot \omega_3)} \cdot z^{-1} \right]}$$

The following new parameters are necessary for the design of the digital filter:

$$\delta_0 := \frac{\omega_s - \pi \cdot \omega_3}{\omega_s + \pi \cdot \omega_3}, \quad \chi_0 := \frac{\pi \cdot A_3 \cdot \omega_3}{\omega_s + \pi \cdot \omega_3},$$

$$\omega_3 = 94.25 \cdot \frac{\text{krad/s}}{\text{s}}, \quad \delta_0 = 0.881765205, \quad \chi_0 = -0.5911739744$$

the new t. f. is:

$$H11(z) := \chi_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}}$$

$$\delta_0 := \delta_0 \quad \chi_0 := \chi_0$$

$$\text{Z T. Initial value theorem: } \lim_{z \rightarrow \infty} \left(\chi_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}} \right) \rightarrow \chi_0 \quad \chi_0 = -0.59$$

Z T. Final value theorem:

$$\lim_{z \rightarrow 0} \left(\chi_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}} \right) \rightarrow \begin{cases} -\frac{\chi_0}{\delta_0} & \text{if } \delta_0 \neq 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$H11dB(\omega) := 20 \cdot \log \left(\frac{|H11(e^{j\omega T_s})|}{|A_3|} \right)$$

$$W1dB(\omega) := 20 \cdot \log \left(\frac{|W_{lp}(j\omega)|}{|A_3|} \right)$$

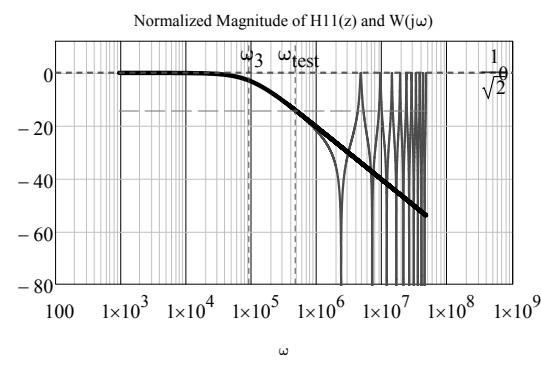


fig.:3.6.1

3.6 Equivalent Digital Low Pass Filter (I^oorder) - The bilinear transformation

3.6.2) Difference equations (Low Pass filter(I^oorder)). Canonical form

$$H11(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = \chi_0 \cdot (1 + z^{-1})$$

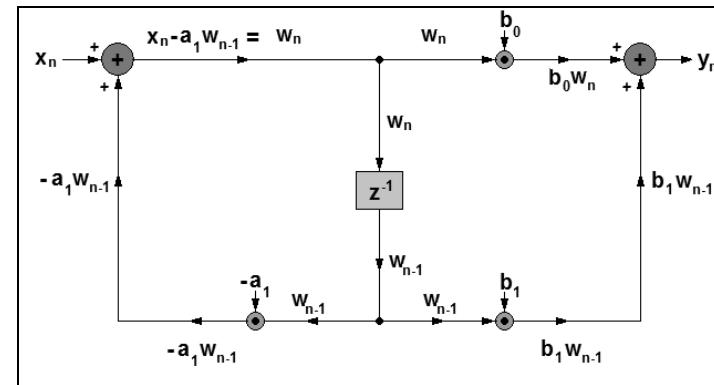
$$Y(z) = \chi_0 \cdot W(z) + \chi_0 \cdot z^{-1} \cdot W(z)$$

$$1) \quad y(k) = \chi_0 \cdot (w(k) + w(k-1))$$

$$\frac{W(z)}{X(z)} = \frac{1}{(1 - \delta_0 \cdot z^{-1})}$$

$$X(z) = (1 - \delta_0 \cdot z^{-1}) \cdot W(z) = W(z) - \delta_0 \cdot z^{-1} \cdot W(z)$$

$$2) \quad x(n) = w(n) - \delta_0 \cdot w(n-1)$$



DIFFEBILIN

3.6.3) Sequence of the voltage step response.

$$\begin{aligned} T_{\text{ss}} &:= T_3 s \quad \omega_s := \frac{2 \cdot \pi}{T_s} \\ \text{sqout0} &:= \text{DIFFEBILIN}\left(\frac{u1}{V}, A_3, T_3 s, \omega_3, N_0 \text{gd}\right) \\ w &:= (\text{sqout0}^{(0)}) \quad y := (\text{sqout0}^{(1)}) \quad \delta_0 := (\text{sqout0}^{(2)})_0 \quad x_0 := (\text{sqout0}^{(2)})_1 \end{aligned}$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_0 = 0.877141384, \quad x_0 = -0.6142930813$$

the new t. f. is:

$$H11(z) := x_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}}$$

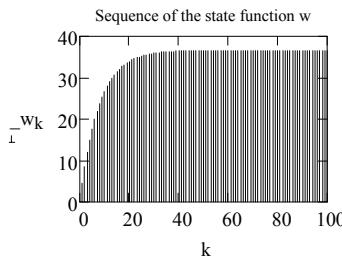


fig.:3.6.3.1

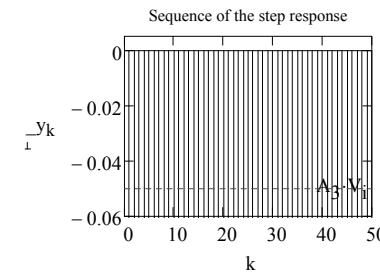


fig.:3.6.3.2

$$t1 := 0 \cdot \tau_3, 0 \cdot \tau_3 + \frac{10 \cdot \tau_3}{1000} \dots 10 \cdot \tau_3$$

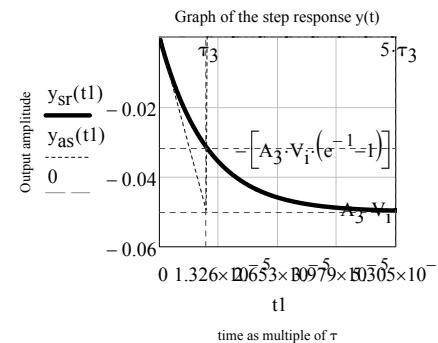


fig.:3.6.3.3

3.6.4) Sequence of the Voltage Short Pulse response.

$$\begin{aligned} T_{\text{ss}} &:= T_{s3sp} \quad \omega_s := \frac{2 \cdot \pi}{T_s} \\ \text{sqout1} &:= \text{DIFFEBILIN}(u44, A_3, T_{s3sp}, \omega_3, N_0 \text{gd}) \\ w18 &:= (\text{sqout1}^{(0)}) \quad y18 := (\text{sqout1}^{(1)}) \quad \delta_1 := (\text{sqout1}^{(2)})_0 \quad x_1 := (\text{sqout1}^{(2)})_1 \end{aligned}$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_1 = 0.957298374, \quad x_1 = -0.2135081317$$

the new t. f. is:

$$H11(z) := x_1 \cdot \frac{1 + z^{-1}}{1 - \delta_1 \cdot z^{-1}}$$

$$\begin{aligned} T_{\text{test}} &= 13333.33 \cdot \text{ns} \quad T_s = 462.96 \cdot \text{ns} \quad \tau_3 = 10.61 \cdot \mu\text{s} \\ \text{Chosen test signal period, } T_{\text{test}} &= 13333.33 \cdot \text{ns} \quad \frac{1}{T_{\text{test}}} = 0.08 \cdot \text{MHz} \end{aligned}$$

Short pulse sequence of amplitude V_i :

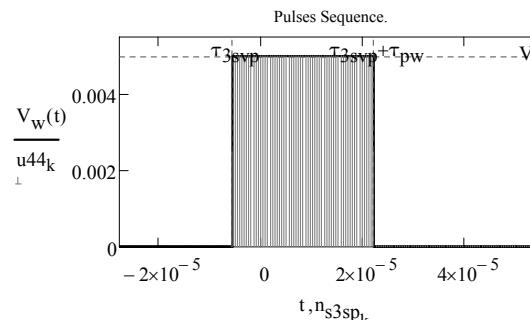


fig.:3.6.4.1

Digital first order low pass filter difference equations:

$$\text{dimensionless input signal: } v_{i18}(k) = \frac{u44_k}{\text{volt}}$$

$$1) \quad w18(k) = \begin{cases} v_{i18}(k) + \delta_0 \cdot w18(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y18(k) = \begin{cases} x_0 \cdot (w18(k) + w18(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

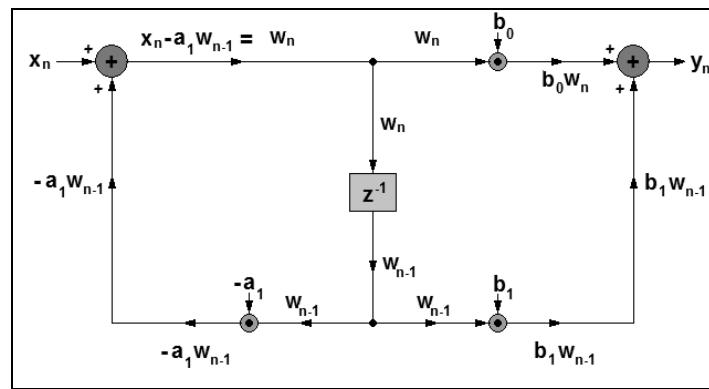


fig.:3.6.4.2

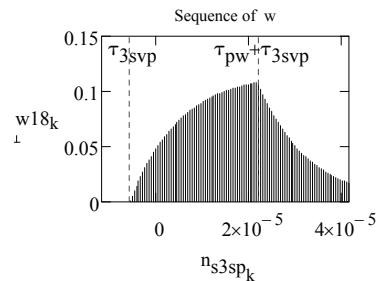
 $N0_{gd} = 256$ 

fig.:3.6.4.2

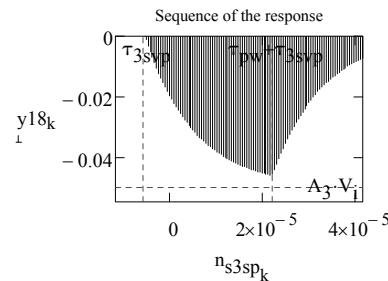


fig.:3.6.4.3

$$t2 := 0 \cdot T_{test}, 0 \cdot T_{test} + \frac{2 \cdot \delta 1 \cdot T_{test}}{100} \dots 2 \cdot \delta 1 \cdot T_{test}$$

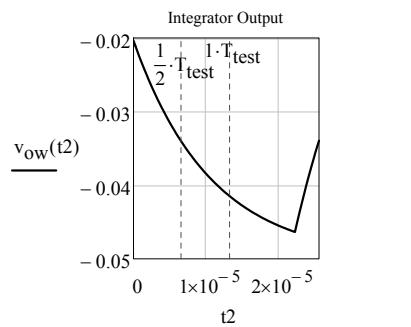
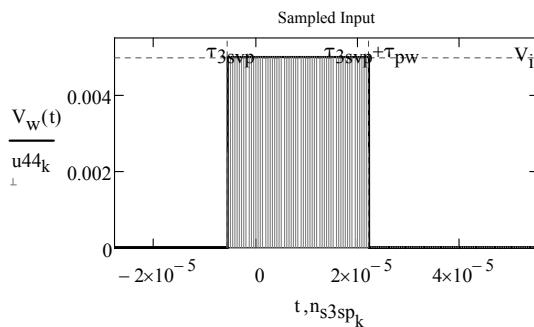
fig.:3.6.4.
4

fig.:3.6.4.5

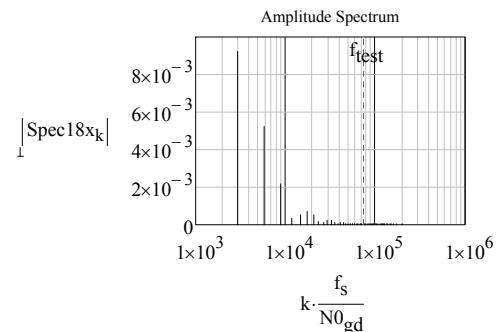


fig.:3.6.4.6

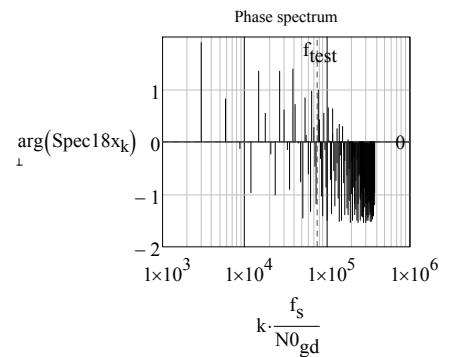


fig.:3.6.4.7

Spec18x := FFT(y18)

3.6 Equivalent Digital Low Pass Filter (Ith order) - The bilinear transformation

3.6.5) Sequence of the Bipolar Pulse train response

$$T_{ssq} := T_{ssqw}$$

$$\omega_s := \frac{2\pi}{T_s}$$

sqout2 := DIFFEBILIN(u3, A3, T_{ssqw}, ω₃, N_{0gd})

$$w19 := (\text{sqout2}^{(0)}) \quad y19 := (\text{sqout2}^{(1)}) \quad \delta2 := (\text{sqout2}^{(2)})_0 \quad \chi2 := (\text{sqout2}^{(2)})_1$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta2 = 0.986995147, \quad \chi2 = -0.0650242641$$

the new t. f. is:

$$H11(z) := \chi2 \cdot \frac{1 + z^{-1}}{1 - \delta2 \cdot z^{-1}}$$

$$T_{\text{test}} = 13333.33 \cdot \text{ns}$$

$$T_s = 138.89 \cdot \text{ns}$$

$$\tau_3 = 10.61 \cdot \mu\text{s}$$

Chosen test signal period, $T_{\text{test}} = 13333.33 \cdot \text{ns}$

$$\frac{1}{T_{\text{test}}} = 0.08 \cdot \text{MHz}$$

$$\text{Laplace transform of the test signal: } V_{ip}(s) := \frac{V_i}{s} \cdot \tanh\left(\frac{T_{\text{test}} \cdot s}{4}\right)$$

Square wave sequence of amplitude $\pm V_i$:

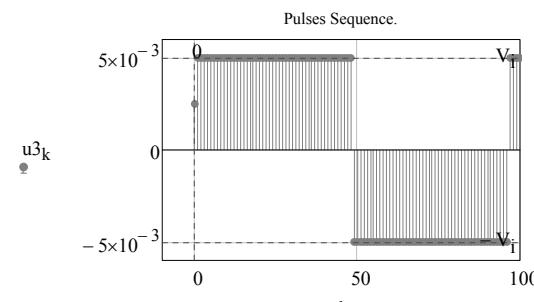


fig.:3.6.5.1

Digital first order low pass filter recurrence relations:

dimensionless input signal: $v19_i(k) = u3_k$

$$\frac{f_s}{f_3 c} = 10$$

$$1) \quad w19(k) = \begin{cases} v19_1(k) + \delta0 \cdot w19(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y19(k) = \begin{cases} \chi0 \cdot (w19(k) + w19(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

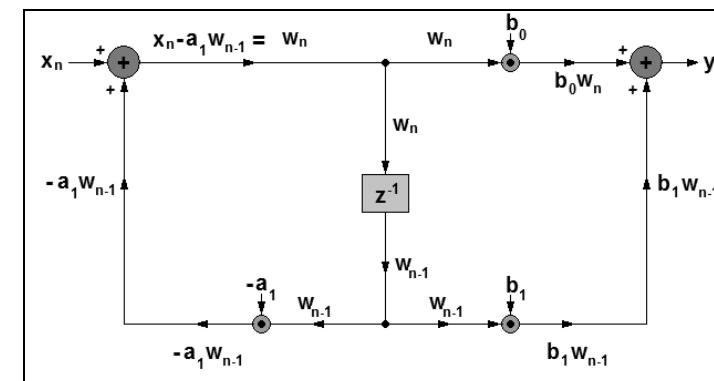


fig.:3.6.5.2

$$N0_{gd} = 256$$

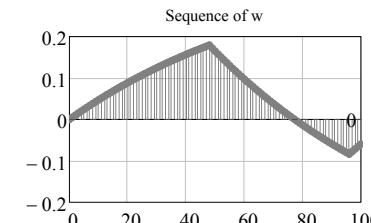


fig.:3.6.5.3

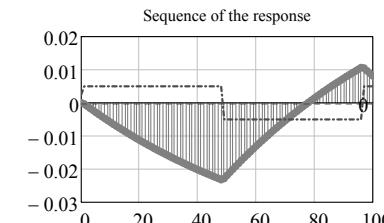
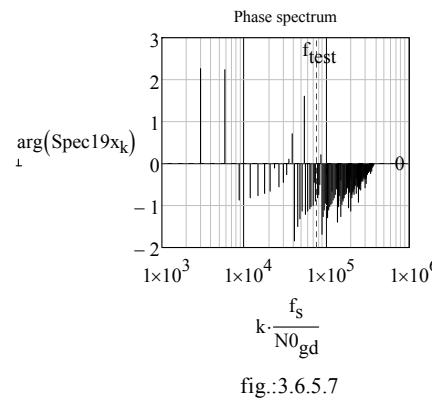
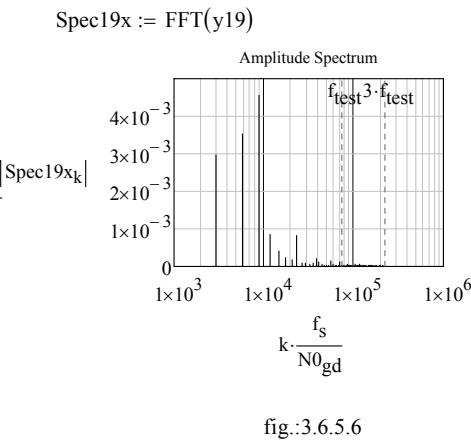
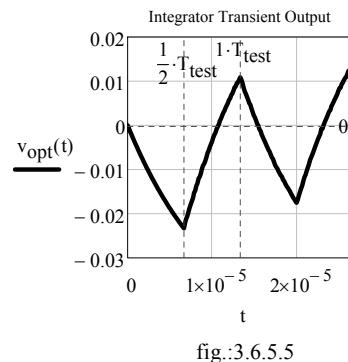


fig.:3.6.5.4

$$t := 0 \cdot T_{\text{test}}, 0 \cdot T_{\text{test}} + \frac{2 \cdot T_{\text{test}}}{200} \dots 2 \cdot T_{\text{test}}$$



3.6 Equivalent Digital Low Pass Filter (I^{order}) - The bilinear transformation

3.6.6) Sequence of the Cusp wave response.

$$T_{ocsp} := T_{ocsp} \quad \omega_{s*} := \frac{2 \cdot \pi}{T_s}$$

$$sqout3 := DIFFEBILIN(u4, A_3, T_{ssqw}, \omega_3, N0_{gd})$$

$$w20 := (sqout3^{(0)}) \quad y20 := (sqout3^{(1)}) \quad \delta3 := (sqout3^{(2)})_0 \quad \chi3 := (sqout3^{(2)})_1$$

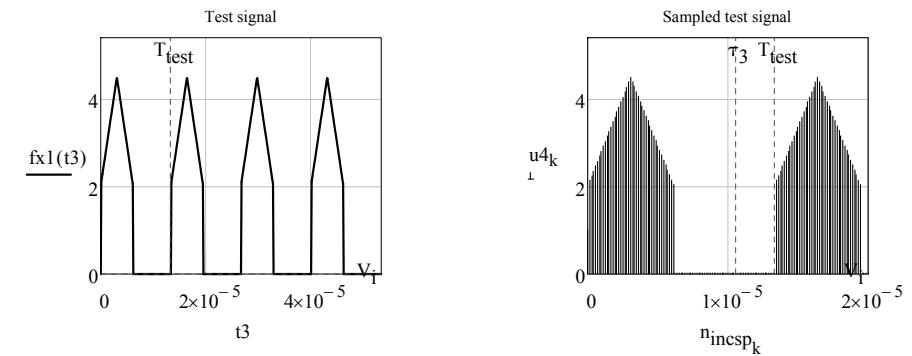
The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta3 = 0.986995147, \quad \chi3 = -0.0650242641$$

the new t. f. is:

$$H11(z) := \chi3 \cdot \frac{1 + z^{-1}}{1 - \delta3 \cdot z^{-1}}$$

$$t3 := -4 \cdot T_{test}, -4 \cdot T_{test} + \frac{8 \cdot T_{test}}{1000} \dots 4 \cdot T_{test}$$



Step sequence of amplitude V_i :

Digital first order low pass filter recurrence relations:

dimensionless input signal: $v20_i(k) = u4_k$

$$\frac{f_s}{f_{c*}} = 10$$

$$1) \quad w20(k) = \begin{cases} v20_i(k) + \delta0 \cdot w20(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y20(k) = \begin{cases} \chi0 \cdot (w20(k) + w20(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

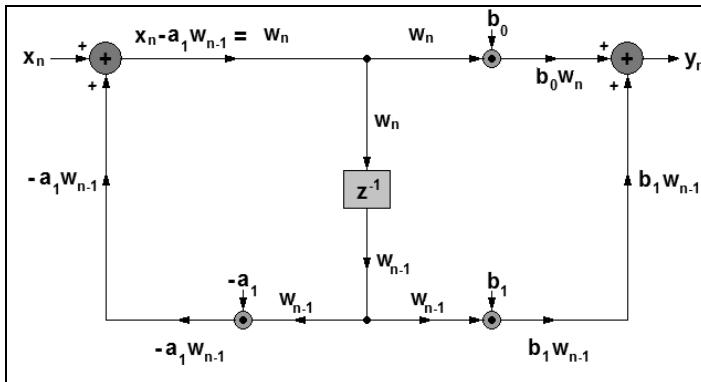


fig.:3.6.6.1

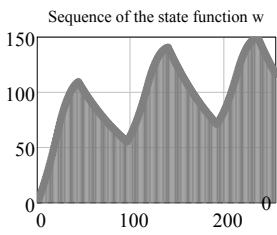


fig.:3.6.6.2

Sampled signal: y_{20k}

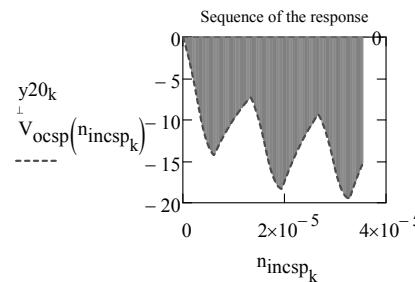


fig.:3.6.6.3

$\text{Spec20x} := \text{FFT}(y_{20})$

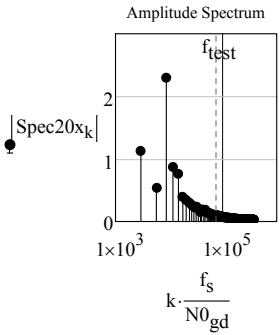


fig.:3.6.6.4

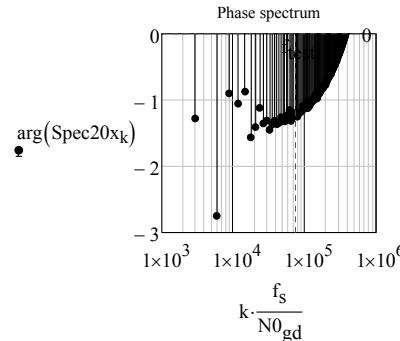


fig.:3.6.6.5

3.6 Equivalent Digital Low Pass Filter (I^{order}) - The bilinear transformation

3.6.7) Sequence of the Sawtooth response

$$T_{ssw} := T_{ssw} \quad \omega_{ssw} := \frac{2 \cdot \pi}{T_{ssw}}$$

$$\text{sqout4} := \text{DIFFEBILIN}\left(\frac{u10}{V}, A_3, T_{ssw}, \omega_3, N0_{gd}\right)$$

$$w21 := (\text{sqout4}^{(0)}) \quad y21 := (\text{sqout4}^{(1)}) \quad \delta4 := (\text{sqout4}^{(2)})_0 \quad \chi4 := (\text{sqout4}^{(2)})_1$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta4 = 0.986995147, \quad \chi4 = -0.0650242641$$

the new t. f. is:

$$H111(z) := \chi4 \cdot \frac{1 + z^{-1}}{1 - \delta4 \cdot z^{-1}}$$

Step sequence of amplitude V_i :

Digital first order low pass filter recurrence relations:

dimensionless input signal: $v21_i(k) = u10_k$

$$\frac{f_s}{f_3 c} = 10$$

$$1) \quad w21(k) = \begin{cases} v21_i(k) + \delta0 \cdot w21(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y21(k) = \begin{cases} \chi0 \cdot (w21(k) + w21(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

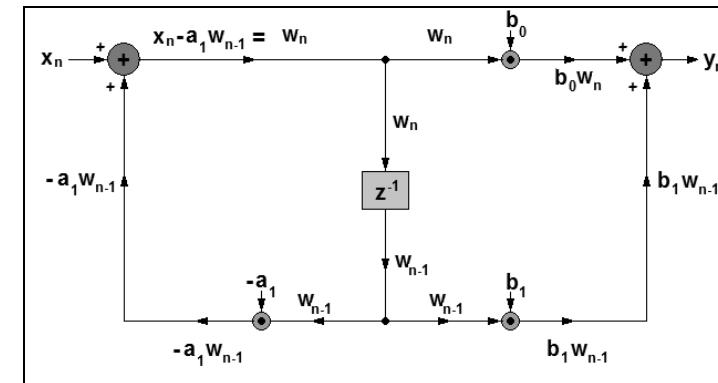


fig.:3.6.7.1

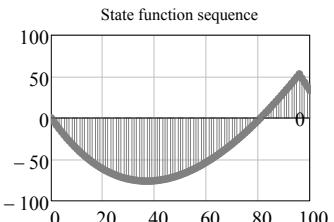


fig.:3.6.7.2
Spec21o := FFT(y21)

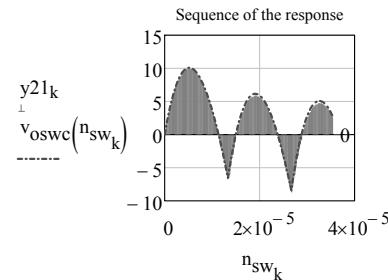


fig.:3.6.7.3

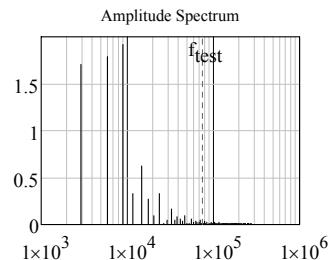


fig.:3.6.7.4

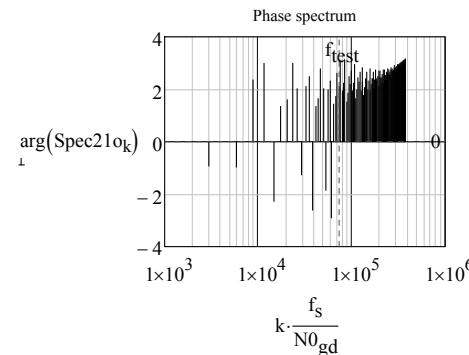


fig.:3.6.7.5

3.6 Equivalent Digital Low Pass Filter (Istorder) - The bilinear transformation

3.6.8) Sequence of the Frequency Modulated carrier response.

$$T_{\text{sfm}} := T_{\text{sfm}} \quad \omega_{\text{sfm}} := \frac{2 \cdot \pi}{T_{\text{sfm}}} \quad m_f = 1$$

$$\text{sqout5} := \text{DIFFEBILIN}(u8, A_3, T_{\text{sfm}}, \omega_3, N0_{\text{gd}})$$

$$w22 := (\text{sqout5}^{(0)}) \quad y22 := (\text{sqout5}^{(1)}) \quad \delta5 := (\text{sqout5}^{(2)})_0 \quad \chi5 := (\text{sqout5}^{(2)})_1$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta5 = 0.996863334, \quad \chi5 = -0.015683328$$

the new t. f. is:

$$H11(z) := \chi5 \cdot \frac{1 + z^{-1}}{1 - \delta5 \cdot z^{-1}}$$

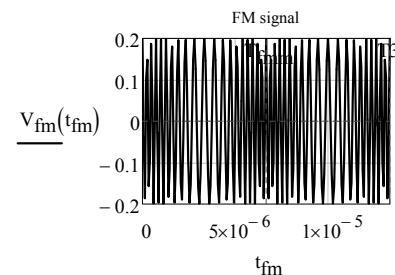


fig.:3.6.8.1

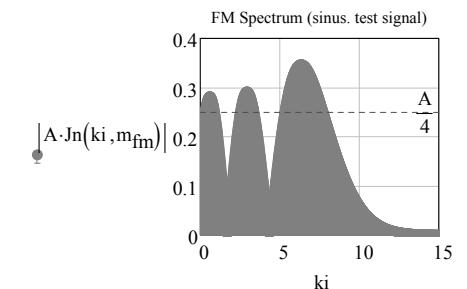


fig.:3.6.8.2

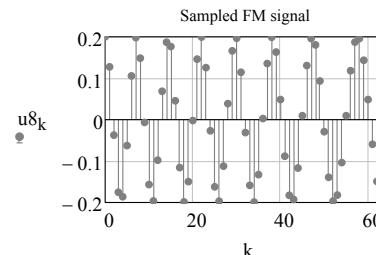


fig.:3.6.8.3

$$\frac{f_{\text{sfm}}}{f_3 c} = 400$$

Digital first order low pass filter difference relations:

dimensionless input signal: $v22_1(k) = u8_k$

$$1) \quad w22(k) = \begin{cases} v22_1(k) + 80 \cdot w22(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y22(k) = \begin{cases} x0 \cdot (w22(k) + w22(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases}$$

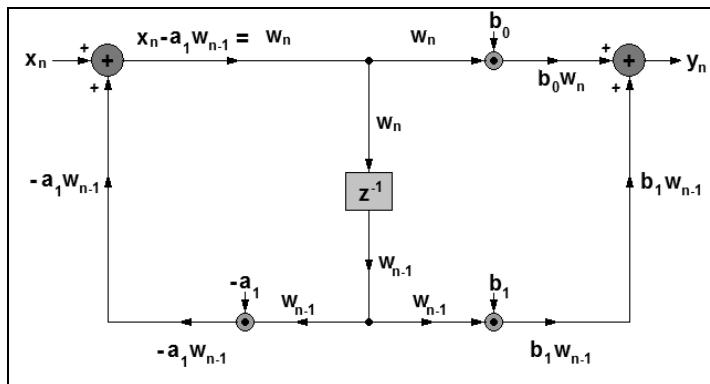


fig.:3.6.8.4

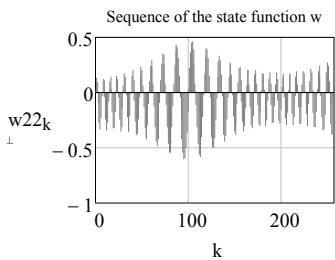


fig.:3.6.8.5

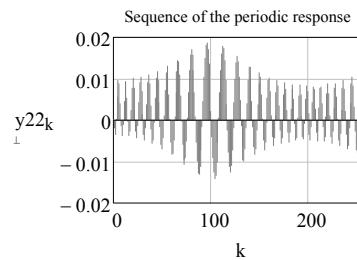


fig.:3.6.8.6

$$\text{Spec22} := \text{fft}(y22) \quad m_{fm} = 8 \quad \omega_{fmm} = 0.94 \cdot \frac{\text{Mrads}}{\text{sec}} \quad f3_c = 0.08 \cdot \text{MHz} \quad \frac{f_s}{f3_c} =$$

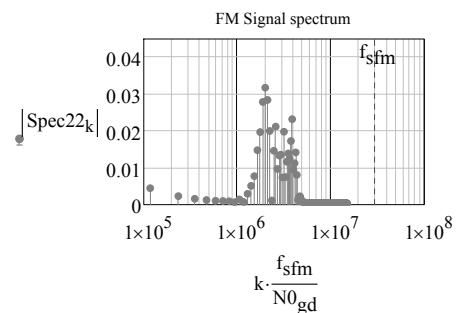


fig.:3.6.8.7

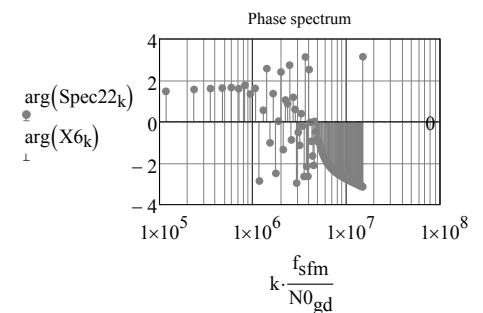


fig.:3.6.8.8

3.6 Equivalent Digital Low Pass Filter (Ist order) - The bilinear transformation

3.6.9) Sequence of the Phase Modulated carrier response.

(sinusoidal test signal)

$$T_{\text{spm}} := T_{\text{spm}}$$

$$\omega_3 := \frac{2 \cdot \pi}{T_s}$$

$$\text{sqout6} := \text{DIFFEBILIN}(u9, A_3, T_{\text{spm}}, \omega_3, N0_{\text{gd}})$$

$$w23 := (\text{sqout6}^{(0)}) \quad y23 := (\text{sqout6}^{(1)}) \quad \delta6 := (\text{sqout6}^{(2)})_0 \quad \chi6 := (\text{sqout6}^{(2)})_1$$

The following new parameters are necessary for the design of the digital filter:

$$\omega_3 = 94.25 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta6 = 0.999998429, \quad \chi6 = -7.8539754655 \times 10^{-6}$$

the new t. f. is:

$$H11(z) := \chi6 \cdot \frac{1 + z^{-1}}{1 - \delta6 \cdot z^{-1}}$$

$$A_3 = -10 \quad m_{\text{pm}} = 6 \quad |A_3 \cdot J_n(0, m_{\text{pm}})| = 1.51$$

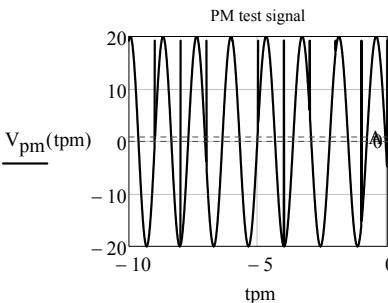


fig.:3.6.9.1

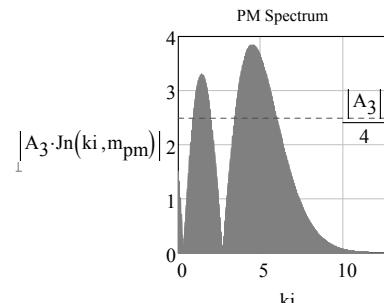


fig.:3.6.9.2

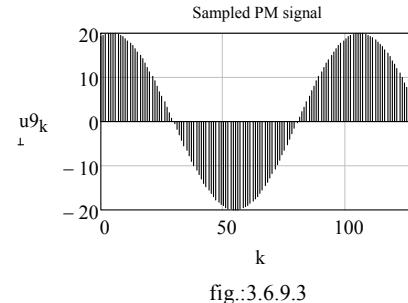


fig.:3.6.9.3

dimensionless input signal:

$$v23_i(k) = u9_k$$

$$1) \quad w23(k) = \begin{cases} v23_i(k) + \delta0 \cdot w23(k-1) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases} \quad \delta0 = 0.877141384$$

$$2) \quad y23(k) = \begin{cases} \chi0 \cdot (w23(k) + w23(k-1)) & \text{if } k > 0 \\ 0 & \text{otherwise} \end{cases} \quad \chi0 = -0.614293$$

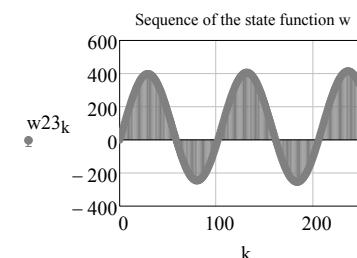


fig.:3.6.9.4

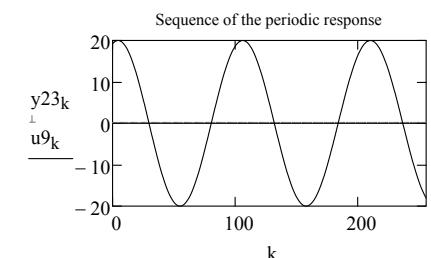


fig.:3.6.9.5

$$\text{Spec23} := \text{FFT}(y23) \quad m_{\text{pm}} = 6$$

$$\omega_{\text{pmm}} = 94.25 \cdot \frac{\text{Mrads}}{\text{sec}}$$

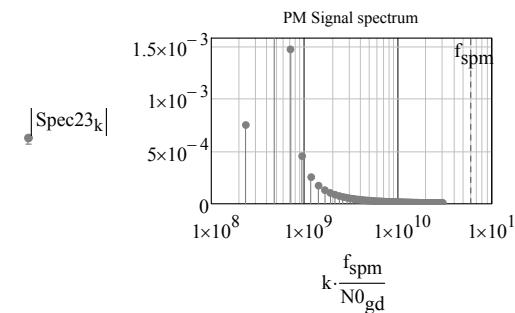


fig.:3.6.9.6

$$\max(|\text{Spec23}|) = 0$$

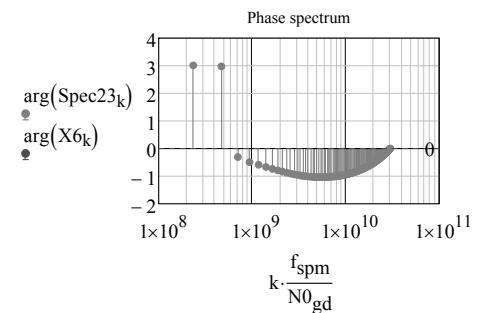


fig.:3.6.9.7

3.7

Synthetic Division algorithm (considering the bilinear transformation)

$$T_{\text{sfm}} := T_{\text{sfm}}$$

$$\omega_s := \frac{2 \cdot \pi}{T_s}$$

The following new parameters are necessary for the design of the digital filter:

$$\delta_0 := \frac{\omega_s - \pi \cdot \omega_3}{\omega_s + \pi \cdot \omega_3}, \quad \chi_0 := \frac{\pi \cdot A_3 \cdot \omega_3}{\omega_s + \pi \cdot \omega_3},$$

$$\omega_3 = 94.25 \cdot \frac{\text{krad/s}}{\text{s}}, \quad \delta_0 = 0.996863334, \quad \chi_0 = -0.015683328$$

The sequence corresponding to the following t. f. :

$$m_{\text{pm}} = 6 \quad H11(z) = \chi_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}} \quad z_0 := -1 \quad p_0 := \beta_0 \quad p_0 = 0.88$$

is realized using an method.

$$\text{Numerator degree } N_{\text{n}} := 1 \quad \text{Denominator degree } M_{\text{d}} := 1$$

$$N1 := N_{\text{n}} + M_{\text{d}} \quad N0_{\text{gd}} = 256 \quad h11_k := 0$$

$$N1 = 2 \quad H11(z) = \frac{b_0 + b_1 \cdot z^{-1}}{a_0 + a_1 \cdot z^{-1}}$$

I can define the coefficients of the numerator and denominator as elements of two vectors, namely a and b:

Numerator coeffs. Denominator coeffs.

$$b_k := 0.0 \quad a_k := 0.0$$

$$b_0 := \chi_0 \quad a_0 := 1$$

$$b_1 := \chi_0 \quad a_1 := -\delta_0$$

and divide the two polynomial by means of the following algorithm:

$$k := n1 \quad h11_0 := \frac{b_0}{a_0} \quad h11_k := \frac{1}{a_0} \left[b_k - \sum_{i=1}^k (h11_{k-i} \cdot a_i) \right]$$

T. F. Numerator coefficients:

$$a^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & -0.02 & -0.02 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence Impulse Response:

$$h11^T = \begin{bmatrix} -0.0157 & -0.0313 & -0.0312 & -0.0311 & -0.031 & \dots \end{bmatrix}$$

Stability (S1< ∞): $S2 := \sum_{k=0}^{\text{rows}(h11)-1} |h11_k| \quad S2 = 5.52$

Energy of the sequence h11: $E11 := \sum_{k=0}^{\text{rows}(h11)-1} (|h11_k|)^2 \quad E11 = 0.13$

Iterative algorithm (considering the bilinear transformation)

$$\tau_3 = 10.61 \cdot \mu\text{s} \quad 100 \cdot T_{\text{test}} = 1333.33 \cdot \mu\text{s}$$

$$t3 := 0 \cdot T_{\text{test}}, \frac{T_{\text{test}}}{100} \dots 1000 \cdot T_{\text{test}} \quad T_{\text{test}} = 13333.33 \cdot \text{ns}$$

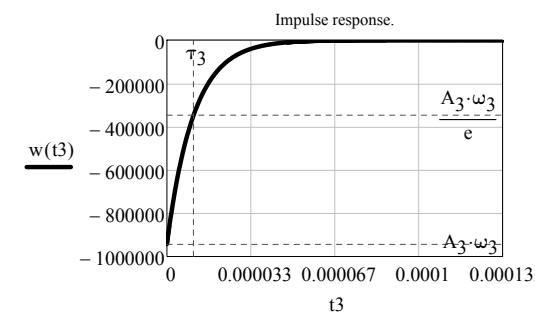


fig.:3.7.0.1

$$f_3 = 0.08 \cdot \text{MHz}$$

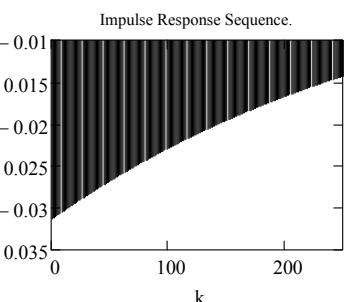


fig.:3.7.0.2

$$\frac{f_s}{f_3} = 10$$

3.7 algorithm (considering the bilinear transformation)

3.7.1) Sequence of the voltage step response.

$$T_{sv} := T_{s3sp}$$

$$\omega_s := \frac{2\pi}{T_s}$$

$$iabil1 := IABILIN\left(\frac{u_1}{V}, A_3, T_{s3sp}, \omega_3, N0_{gd}\right)$$

$$S1 := (iabil1^{(0)})_0 \quad E := (iabil1^{(0)})_1 \quad a1 := (iabil1^{(1)}) \quad b1 := iabil1^{(2)} \quad h11 := iabil1^{(3)} \quad y24 := iabil1^{(4)}$$

T. F. Numerator coefficients:

$$a1^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & 1 & -0.96 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

T. F. Denominator coefficients:

$$b1^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & -0.21 & -0.21 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

Sequence Impulse Response:

$$h11^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & -0.2135 & -0.4179 & -0.4001 & -0.383 & \dots \\ \hline \end{array}$$

$$y24^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & -1.9 & -3.74 & -5.5 & -7.19 & -8.8 & -10.35 & \dots \\ \hline \end{array}$$

$\nu = \bullet$

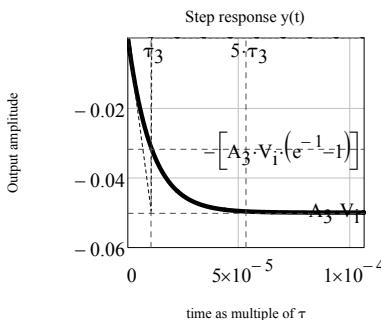


fig.:3.7.1.1

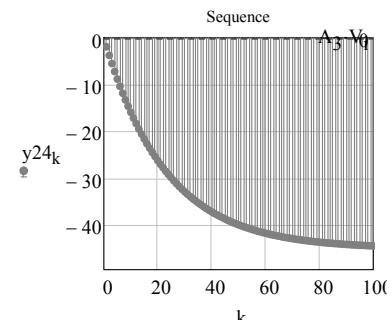


fig.:3.7.1.2

3.7 algorithm (considering the bilinear transformation)

3.7.2) Sequence of the Short Voltage Pulse response.

$$T_{sv} := T_{svp}$$

$$\omega_s := \frac{2\pi}{T_s}$$

$$iabil2 := IABILIN(u44, A_3, T_{svp}, \omega_3, N0_{gd})$$

$$S2 := (iabil2^{(0)})_0 \quad E2 := (iabil2^{(0)})_1 \quad a2 := (iabil2^{(1)}) \quad b2 := iabil2^{(2)} \quad h2 := iabil2^{(3)} \quad y25 := iabil2^{(4)}$$

T. F. Numerator coefficients:

$$a2^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & 1 & 0.35 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

T. F. Denominator coefficients:

$$b2^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 0 & -6.77 & -6.77 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline \end{array}$$

Sequence Impulse Response:

$$h2^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & -0.2135 & -0.4179 & -0.4001 & -0.383 & \dots \\ \hline \end{array}$$

$$y25^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & -0.04 & -0.05 & -0.05 & \dots \\ \hline \end{array}$$

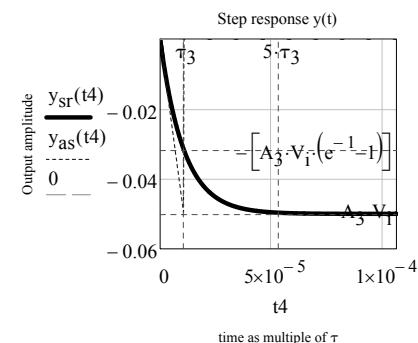


fig.:3.7.2.1

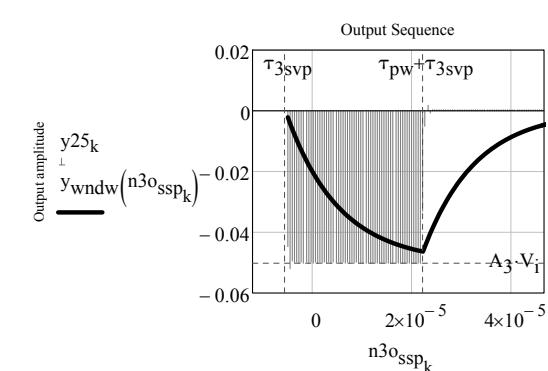


fig.:3.7.2.2

3.7 algorithm (considering the bilinear transformation)

3.7.3) Sequence of the Bipolar Pulse train response:

$$T_{ssw} := T_{ssqw}$$

$$\omega_s := \frac{2 \cdot \pi}{T_s}$$

iabil3 := IABILIN(u3, A3, Tssqw, omega3, N0gd)

$$S3 := (iabil3^{(0)})_0 \quad E3 := (iabil3^{(0)})_1 \quad a3 := (iabil3^{(1)}) \quad b3 := iabil3^{(2)} \quad h3 := iabil3^{(3)} \quad y26 := iabil3^{(4)}$$

T. F. Numerator coefficients:

$$a3^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b3^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.07 & -0.07 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{bmatrix}$$

Sequence Impulse Response:

$$h3^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.065 & -0.1292 & -0.1275 & -0.1259 \dots \end{bmatrix}$$

Output amplitude

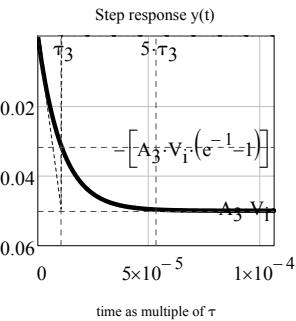


fig.:3.7.3.1

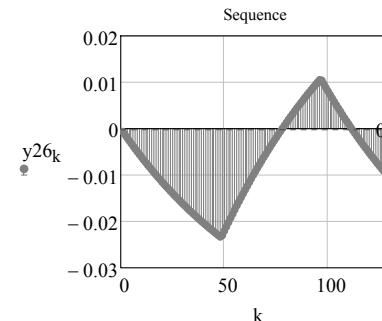


fig.:3.7.3.2

3.7 algorithm (considering the bilinear transformation)

3.7.4) Sequence of the Cusp wave response.

$$T_{ocsp} := T_{ocsp}$$

$$\omega_s := \frac{2 \cdot \pi}{T_s}$$

iabil4 := IABILIN(u4, A3, Tocsp, omega3, N0gd)

$N0_{gd} = 256$

$$S4 := (iabil4^{(0)})_0 \quad E4 := (iabil4^{(0)})_1 \quad a4 := (iabil4^{(1)}) \quad b4 := iabil4^{(2)} \quad h4 := iabil4^{(3)} \quad y27 := iabil4^{(4)}$$

T. F. Numerator coefficients:

$$a4^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b4^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.07 & -0.07 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{bmatrix}$$

Sequence Impulse Response:

$$h4^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.065 & -0.1292 & -0.1275 & -0.1259 \dots \end{bmatrix}$$

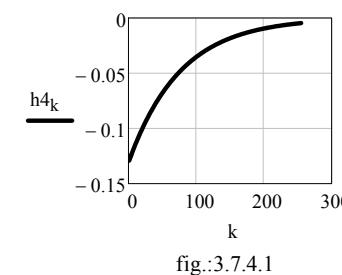


fig.:3.7.4.1

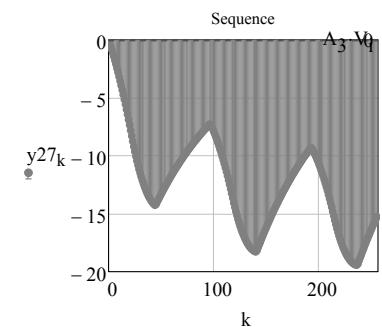


fig.:3.7.4.2

3.7 algorithm (considering the bilinear transformation)

3.7.5) Sequence of the Sawtooth response.

$$T_{ssw} := T_{ssw} \quad \omega_s := \frac{2\pi}{T_s}$$

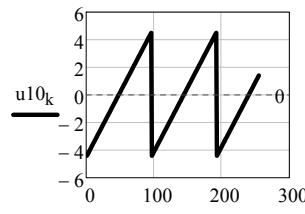


fig.:3.7.5.1

$$iabil5 := IABILIN\left(\frac{u10}{V}, A_3, T_{ssw}, \omega_3, N0_{gd}\right) \quad N0_{gd} = 256$$

$$S5 := (iabil5^{(0)})_0 \quad E5 := (iabil5^{(0)})_1 \quad a5 := (iabil5^{(1)}) \quad b5 := iabil5^{(2)} \quad h5 := iabil5^{(3)} \quad y28 := iabil5^{(4)}$$

T. F. Numerator coefficients:

$$a5^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

T. F. Denominator coefficients:

$$b5^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.07 & -0.07 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

Sequence Impulse Response:

$$h5^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.065 & -0.1292 & -0.1275 & -0.1259 \end{bmatrix} \dots$$

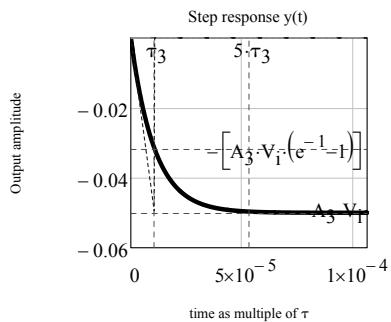


fig.:3.7.5.2

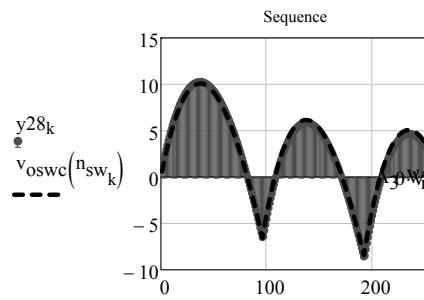


fig.:3.7.5.3

3.7 algorithm (considering the bilinear transformation)

3.7.6) Sequence of the Frequency Modulated carrier response.

$$T_{sfm} := T_{sfm} \quad \omega_s := \frac{2\pi}{T_s}$$

$$iabil6 := IABILIN(u8, A_3, T_{ssw}, \omega_3, N0_{gd}) \quad N0_{gd} = 256$$

$$S6 := (iabil6^{(0)})_0 \quad E6 := (iabil6^{(0)})_1 \quad a6 := (iabil6^{(1)}) \quad b6 := iabil6^{(2)} \quad h6 := iabil6^{(3)} \quad y29 := iabil6^{(4)}$$

T. F. Numerator coefficients:

$$a6^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1 & -0.99 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

$$b6^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.07 & -0.07 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots$$

Impulse Response Sequence:

$$h6^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.065 & -0.1292 & -0.1275 & -0.1259 \end{bmatrix} \dots$$

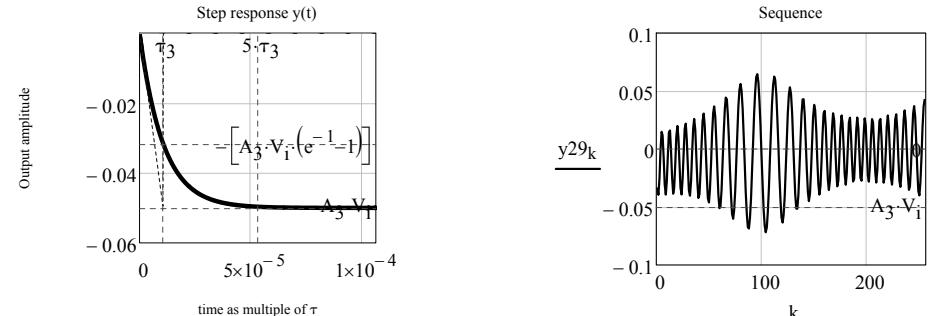


fig.:3.7.6.1

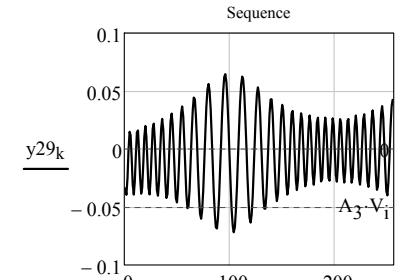


fig.:3.7.6.2

3.7 algorithm (considering the bilinear transformation)

3.7.7) Sequence of the Phase Modulated carrier response.

$$T_{\text{spm}} := T_{\text{spm}}$$

$$\omega_s := \frac{2\pi}{T_s}$$

$$\text{iabil7} := \text{IABILIN}(u9, A_3, T_{\text{ssw}}, \omega_3, N0_{\text{gd}})$$

$$N0_{\text{gd}} = 256$$

$$S7 := (\text{iabil7}^{(0)})_0 \quad E7 := (\text{iabil7}^{(0)})_1 \quad a7 := (\text{iabil7}^{(1)}) \quad b7 := \text{iabil7}^{(2)} \quad h7 := \text{iabil7}^{(3)} \quad y30 := \text{iabil7}^{(4)}$$

Numerator coefficients of the T. F.:

$$a^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & ... \end{bmatrix}$$

Denominator coefficients of the T. F.:

$$b^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & -0.02 & -0.02 & 0 & 0 & 0 & 0 & 0 & ... \end{bmatrix}$$

Sequence of the Impulse Response:

$$h11^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & -0.2135 & -0.4179 & -0.4001 & -0.383 & ... \end{bmatrix}$$

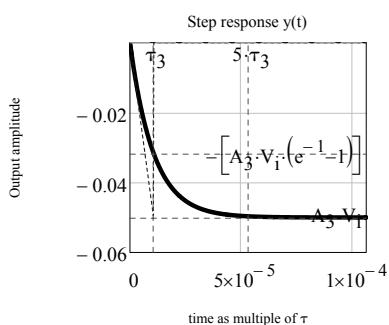


fig.:3.7.7.1

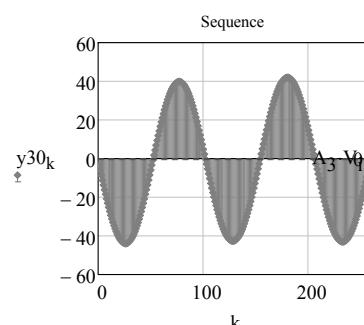


fig.:3.7.7.2

3.8

Analytical search of the output sequence by means of the residues method (considering the bilinear transformation)

$$\beta_0 := \beta_0 \quad \chi_0 := \chi_0 \quad \delta_0 := \delta_0$$

$$\text{Poles and zeroes of } H101(z) := \chi_0 \cdot \frac{1+z^{-1}}{1-\delta_0 \cdot z^{-1}} :$$

$$\delta_0 = 0.996863334 , \quad \chi_0 = -0.015683328$$

$$v := \text{numer}(H101(z)) \text{ coeffs}, z \rightarrow \begin{pmatrix} 0 \\ -\chi_0 \\ -\chi_0 \end{pmatrix}$$

$$\text{zeros} := \text{polyroots}(v) \quad \text{zeros} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$y := \text{denom}(H101(z)) \text{ coeffs}, z \rightarrow \begin{pmatrix} 0 \\ \delta_0 \\ -1 \end{pmatrix}$$

$$\text{poles} := \text{polyroots}(v) \quad \text{poles} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\delta_0 = 1 , \quad \chi_0 = -0.02$$

The calculation gives: $k := 0 .. N0_{\text{gd}} - 1$

$$H101(z) = \frac{\chi_0 \cdot (z+1)}{z - \delta_0}$$

$$F(z, n) = z^{n-1} H(z) = \chi_0 \frac{z^n \cdot (z+1)}{z \cdot (z - \delta_0)}$$

$$h_n = \frac{1}{2 \cdot \pi \cdot j} \oint_{\Gamma} F(z, n) dz = \sum_{k=0}^{P-1} (\text{Res}(F(z, n)))$$

$$\sum_{k=0}^{P-1} (\text{Res}(F(z, n))) = \sum_{k=0}^{P-1} \left[\lim_{z \rightarrow p_k} [(z - p_k) \cdot (F(z, n))] \right]$$

$$\text{If } F(z, n) \text{ is a rational function } F(z, n) = \frac{A(z)}{B(z)} = \frac{A(z)}{(z - z_1) \cdot (z - z_2) \cdot \dots \cdot (z - z_n)} \quad (1.2.1.5)$$

with all simple poles, then:

$$h_n = \sum_{k=0}^{P-1} \lim_{z \rightarrow p_k} \frac{\partial}{\partial z} \frac{A(z)}{B(z)}$$

$$\chi_0 := \chi_0 \quad z := z \quad \omega_3 := \omega_3 \quad A_3 := A_3 \quad \omega_s := \omega_s \quad T_s := T_s \quad v := v \quad u := u$$

$$F(z, n) = \chi_0 \frac{z^n \cdot (z+1)}{z \cdot (z-\delta_0)} \quad \frac{\partial}{\partial z} [z \cdot (z-\delta_0)] \rightarrow 2 \cdot z - \delta_0$$

$$\sum_{k=0}^{P-1} \lim_{z \rightarrow p_k} \frac{\partial}{\partial z} \frac{A(z)}{B(z)} = \lim_{z \rightarrow 0} \frac{\chi_0 \cdot z^n \cdot (z+1)}{\frac{\partial}{\partial z} [z \cdot (z-\delta_0)]} + \lim_{z \rightarrow \delta_0} \frac{\chi_0 \cdot z^n \cdot (z+1)}{\frac{\partial}{\partial z} [z \cdot (z-\delta_0)]}$$

$$n = 0 \quad \chi_0 \left[\lim_{z \rightarrow 0} \frac{(z+1)}{2 \cdot z - \delta_0} + \lim_{z \rightarrow \delta_0} \frac{(z+1)}{2 \cdot z - \delta_0} \right] \left| \begin{array}{l} \text{collect, } \chi_0 \\ \text{simplify} \end{array} \right. \rightarrow \begin{cases} \chi_0 & \text{if } \delta_0 \neq 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$n \neq 0 \quad \chi_0 \cdot \lim_{z \rightarrow 0} \frac{z^n \cdot (z+1)}{2 \cdot z - \delta_0} \quad \text{simplify} \rightarrow \text{undefined} \quad k := k$$

$$\chi_0 \cdot \lim_{z \rightarrow \delta_0} \frac{z^n \cdot (z+1)}{2 \cdot z - \delta_0} \quad \text{simplify} \rightarrow \chi_0 \cdot \delta_0^{n-1} \cdot (\delta_0 + 1)$$

$$h_k := \chi_0 \cdot [\delta(k, 0) + \delta_0^{k-1} \cdot (\delta_0 + 1)]$$

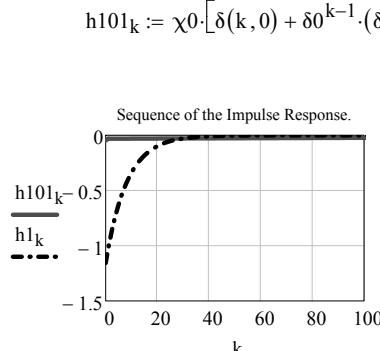


fig.:3.8.0.1

1.6 iii) By using the "invztrans" operator:

$$\chi_0 := \chi_0 \quad \delta_0 := \delta_0 \quad k := k$$

$$h101_k := \chi_0 \cdot \frac{1 + z^{-1}}{1 - \delta_0 \cdot z^{-1}} \quad \left| \begin{array}{l} \text{invztrans, } z, k \\ \text{collect, } \chi_0 \end{array} \right. \rightarrow \frac{\delta_0^{k+1} - \delta(k, 0) + \delta_0^k}{\delta_0} \cdot \chi_0 \quad \frac{\chi_0 \cdot (z+1)}{z - \delta_0} \quad \left| \begin{array}{l} \text{invztran} \\ \text{collect, } \chi_0 \end{array} \right.$$

$$t4 := 0 \cdot \tau_3, \frac{\tau_3}{1000} .. 50 \cdot \tau_3 \quad h101_1 = -0.03$$

Impulse response w(t).

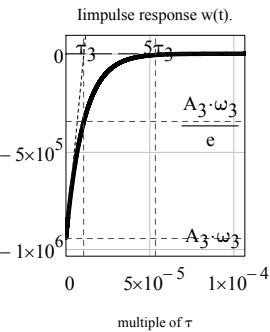


fig.:3.8.0.2

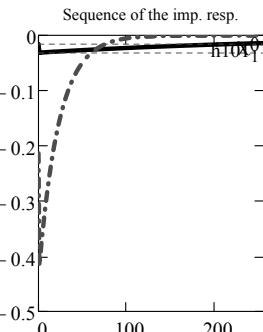


fig.:3.8.0.3

Stability ($S_{11} < \infty$):

$$S_{11} := \sum_{j=0}^{\text{rows}(h101)-1} |h101_j| \quad S_{11} = 5.52$$

$$\text{Energy of the sequence } h11: \quad E101 := \sum_{j=0}^{\text{rows}(h101)-1} (|h101_j|)^2 \quad E101 = 0.13$$

The Output of the Digital System is given by the discrete convolution between the input signal (in this case the sequence of a step function) and the impulse response of the System:

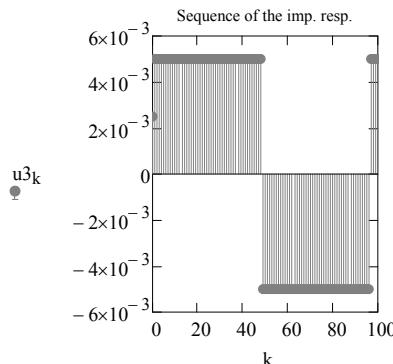


fig.:3.8.0.4

$$\nu := k$$

$$y_{31\nu} := \sum_{j=0}^{\nu} (\text{if}(\nu-j \geq 0, h_{11j} \cdot u_{3\nu-j}, 0))$$

$$y_{32\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu-k \geq 0, h_{101k} \cdot u_{3\nu-k}, 0))$$

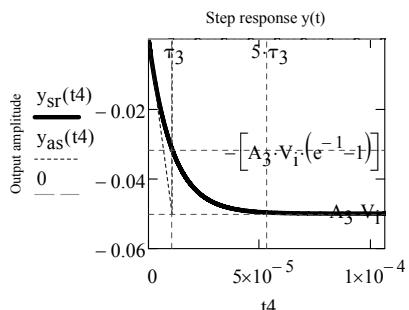


fig.:3.8.0.5

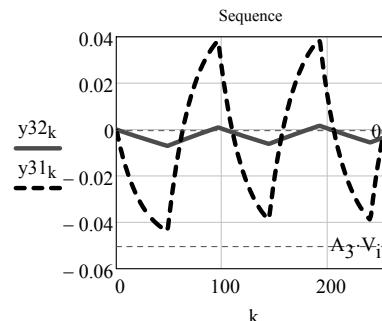


fig.:3.8.0.6

Example 1) Bipolar pulse train:

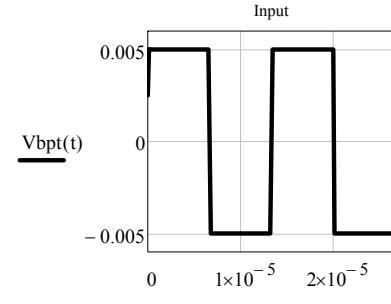


fig.:3.8.0.7

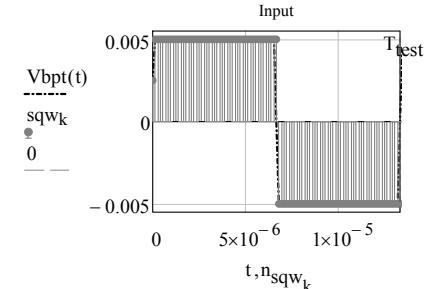


fig.:3.8.0.8

$$\nu := k \quad y_{101\nu} := \sum_{j=0}^{\nu} (\text{if}(\nu-j \geq 0, h_{101j} \cdot \text{sqw}_{\nu-j}, 0))$$

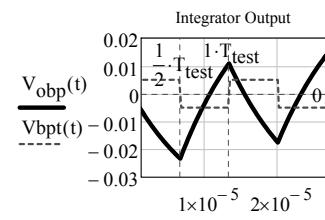


fig.:3.8.0.9

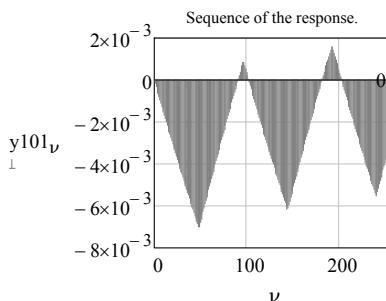


fig.:3.8.0.10