

Equivalent Digital High Pass Filter (I^oorder)

Introduction.

This worksheet, developed using a common operational amplifier in an inverting configuration, begins with a brief summary of the main results of the circuit analysis, enriched with graphics and examples. Seven signals from an external file (Signals List.xmcd), among the most common, are generated and used as input of the amplifier or, once sampled, as input to the digital filter. The many algorithms to implement the corresponding digital filter are derived applying the z-transform. Two approximations are applied to each result to derive the corresponding difference equations. Thus one will see that, as the analog filter is effective, just is the digital one with the used approximations. After reviewing this worksheet, the reader just has to choose the algorithm and implement the firmware for the DSP.#

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File Reference

Definitions and a few necessary constants:

$$\text{Amplifier Gain: } A_0 = -6, \quad (4.1)$$

$$\text{Cut off frequency: } f_0 = 0.250 \cdot \text{MHz}, \quad (4.2)$$

$$\text{Cut off period: } T_0 = \frac{1}{f_0}, \quad T_0 = 4 \cdot \mu\text{s} \quad (4.3)$$

$$\text{Pass Band edge: } \omega_0 = 2 \cdot \pi \cdot f_0, \quad (4.4)$$

$$\text{time constant: } \tau_0 = \frac{1}{\omega_0} \quad (4.5)$$

$$\text{Quality factor: } Q_0 = 10.0, \quad (4.6)$$

$$\text{damping factor: } \zeta_0 = \frac{\omega_0}{2 \cdot Q_0}, \quad \zeta_0 = 78.54 \cdot \frac{\text{krads}}{\text{sec}}, \quad (\omega_0 = \frac{1}{\tau_0} = 2 \cdot \zeta_0 \cdot Q_0) \quad (4.7)$$

Defined in "global data.xmcd":

$$\text{Op. Amp. saturation voltage: } V_{\text{sat}} = 15 \cdot \text{V} \quad (4.8)$$

$$\text{Number of samples for the FFT: } N_{\text{gd}} = 256, \quad (4.9)$$

$$\text{Number of elements of a series: } N_{\text{gd}} = 50 \quad (4.10)$$

$$\text{An integer constant: } U_0 := U^\dagger, \quad (4.11)$$

$$k := k^\dagger \text{ (defined once for all in "global data")} \quad (4.12)$$

The Bode diagrams will have an extension defined by a multiple $U_0 = 100$ of ω , freely chosen.

$$T_{\text{test}} := 1 \cdot 2 \cdot \pi \cdot \tau_0 \cdot 8 \quad (4.13)$$

$$\omega_{\text{test}} := \frac{2 \cdot \pi}{T_{\text{test}}} \quad f_{\text{test}} := \frac{1}{T_{\text{test}}} \quad \omega_{\text{test}} = 0.196 \cdot \frac{\text{Mrads}}{\text{sec}} \quad (4.14)$$

$$V_i := \frac{V_{\text{pp}}}{2}$$

4.1 Analog HIGH PASS Filter (I° order)

Consider the simple analog high pass active filter (derivative) below depicted:

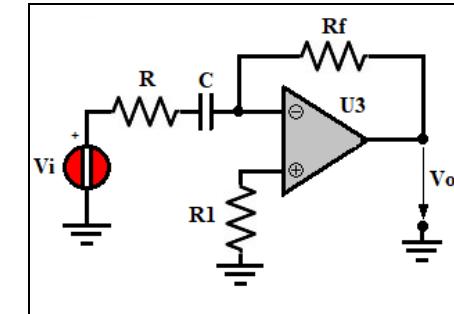


Fig.: (4.1.1)

Hereafter are reported the main results of the analysis that is: the transfer function, the graph of the impulse response, the Bode plots.

$$\text{The transfer function (ideal Op. Amp.) is: } W_{\text{hp}}(s) = \frac{-R_f}{R + \frac{1}{s \cdot C}} = -\frac{R_f}{R} \cdot \frac{s}{s + \frac{1}{R \cdot C}}. \quad (4.1.1)$$

Placing the low cut off filter pulsation at $\omega_0 = 2 \cdot \zeta \cdot Q = \frac{1}{R \cdot C} = \frac{1}{\tau_0}$ and the high frequency amplitude $A_0 = -\frac{R_f}{R}$, ($\tau_0 = R \cdot C$), the filter transfer function takes the form:

$$W_{\text{hp}}(s) = A_0 \cdot \frac{s}{s + \omega_0} \quad (4.1.2)$$

The angular frequency by which the voltage gain is 0dB, is:

$$\omega_{0\text{dB}} = \frac{1}{C \cdot \sqrt{R_f^2 - R^2}} = \frac{\omega_0}{\sqrt{(A_0)^2 - 1}}, \quad (4.1.3)$$

so the 0dB pulsation is:

$$\omega_{0\text{dB}} := \frac{\omega_0}{\sqrt{(A_0)^2 - 1}} \quad (4.1.4)$$

Finally, since $\omega_0 = \frac{1}{\tau_0}$, the transfer function can be rewritten

$$W_{\text{hp}}(s) := \frac{A_0 \cdot s}{s + \omega_0}, \quad A_0 = -6, \quad (4.1.5)$$

or

$$W_{hp}(s) = \frac{A_0 \cdot s}{\omega_0 \cdot \left(\frac{s}{\omega_0} + 1 \right)} = \frac{A_0 \cdot \tau_0 \cdot s}{(\tau_0 \cdot s + 1)}. \quad (4.1.6)$$

$s_-(\sigma, \omega) := \sigma + j\omega$

$\sigma := 0$

$$\omega := -40 \cdot \omega_0, -40 \cdot \omega_0 + \frac{40 \cdot \omega_0}{10000} \dots 40 \cdot \omega_0$$

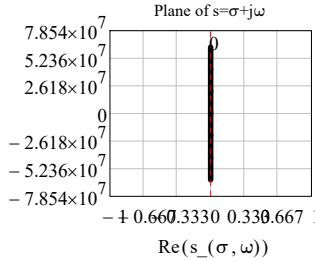


Fig.: (4.1.2)

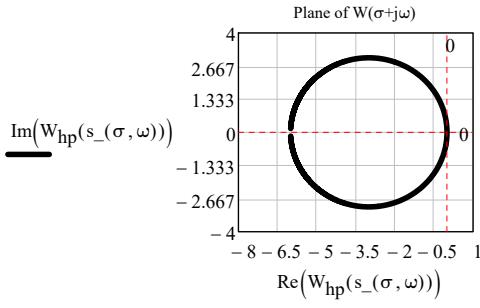


Fig.: (4.1.3)

Calculation of the pulse response that is the inverse Laplace transform of the transfer function:

$$\begin{aligned} A_0 &:= A_0 & \tau_0 &:= \tau_0 \\ \frac{A_0 \cdot \tau_0 \cdot s}{(\tau_0 \cdot s + 1)} &\xrightarrow[\text{collect, } A_0]{\text{inlaplace, simplify}} \left(\Delta(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \right) \cdot A_0 & \Delta(t) &\text{ is the Dirac Pulse.} \end{aligned}$$

$$\text{The pulse response is: } w(t) = A_0 \cdot \left(\Delta(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \right) \quad \text{where } \tau_0 = \frac{1}{\omega_0} \quad (4.1.7)$$

From the definition of the transfer function it is simple to derive that filter output in fact it is given by:

$$V_o(s) = W_{hp}(s) \cdot V_i(s),$$

Anti Laplace transforming, in the time domain corresponds a convolution product:

$$v_o(t) = \int_0^t v_i(\tau) \cdot w(t - \tau) d\tau. \quad (4.1.8)$$

So that the exact system output is (assuming that for $t < 0$, $v_i(t) = 0$ and $\int_{-\infty}^{\infty} v_i(\tau) \cdot \Delta(t - \tau) d\tau = v_i(t)$):

$$v_o(t) = A_0 \cdot \int_0^t v_i(\tau) \cdot \left(\Delta(t - \tau) - \frac{e^{-\frac{t-\tau}{\tau_0}}}{\tau_0} \right) d\tau = A_0 \cdot v_i(t) - \frac{A_0}{\tau_0} \cdot \int_0^t v_i(\tau) \cdot e^{-\frac{t-\tau}{\tau_0}} d\tau, \quad (4.1.9)$$

Hence the time domain filter's output is:

$$v_o(t) = A_0 \cdot \left[v_i(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_{-\infty}^t v_i(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right]. \quad (4.1.10)$$

Example: $v_i(t) = V_i \Delta(t)$,

$$v_o(t) = A_0 \cdot V_i \cdot \left(\Delta(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_{-\infty}^t \Delta(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) = A_0 \cdot V_i \cdot \left(\Delta(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \Phi(t) \right) \quad (4.1.11)$$

Transfer function approximation

$$\text{Given the transfer function: } W_{hp}(s) = \frac{A_0 \cdot s}{\omega_0 \cdot \left(\frac{s}{\omega_0} + 1 \right)}, \quad (4.1.12)$$

if $\left| \frac{s}{\omega_0} \right| \ll 1$, namely for time harmonic signals since the imaginary pulsation is $s = j\omega$, hence $\omega \ll \omega_0$, or

$T \gg (2 \cdot \pi \cdot \tau_0)$, the fraction can be developed in a Maclaurin series after having placed $x = \frac{s}{\omega_0}$, so that:

$$\left(\frac{A_0 \cdot x}{1 + x} \right) (\approx) \left[A_0 \cdot x \cdot (1 - x + x^2 - x^3 + x^5) \right] \text{ and, in a first approximation:} \quad (4.1.13)$$

$$W_{hp}(s) = \frac{A_0 \cdot s}{\omega_0}, \quad \frac{A_0}{\omega_0} = A_0 \cdot \tau_0 = -\frac{R_f}{R} \cdot \tau_0 = -R_f \cdot C = -\tau_f \quad (4.1.14)$$

$$\tau_f := -\frac{A_0}{\omega_0}, \quad \tau_f = -A_0 \cdot \tau_0 = |A_0| \cdot \tau_0$$

from which the approximated input-output bond is:

$$V_o(s) \approx (-\tau_f \cdot s \cdot V_i(s)), \quad (4.1.15)$$

where $V_i(s)$ is the Laplace transform of the input signal.

Anti-transforming and considering zero initial conditions, the approximated time-trend of the output signal is:

$$v_o(t) \approx \left(-\tau_f \frac{d}{dt} v_i(t) \right) \text{ for } \omega \ll \omega_0 \text{ or } T \gg (2 \cdot \pi \cdot \tau) \quad (4.1.16)$$

that is, the output is proportional to the input derivative.

Exact pulse response calculations

$$A_0 = -6 \quad A_0 := A_0 \quad s := s \quad a := a \quad \omega_0 := \omega_0 \quad t := t$$

Dirac pulse response: $\frac{A_0 \cdot s}{s + \omega_0}$ invlaplace $\rightarrow A_0 \cdot (\Delta(t) - \omega_0 \cdot e^{-t \cdot \omega_0})$ (4.1.17)

Impulse response: $w_{hp}(t) = A_0 \cdot (\Delta(t) - \omega_0 \cdot e^{-t \cdot \omega_0})$

Dirac pulse definition: $\int_{-\infty}^{\infty} \Delta(t) dt \rightarrow 1$

$$\Delta(t) := \begin{cases} \infty \cdot \text{sec}^{-1} & \text{if } t = 0.0 \cdot \text{sec}^{-1} \\ 0.0 \cdot \text{sec}^{-1} & \text{otherwise} \end{cases}$$

Graph of the filter impulse response

$$\varepsilon_{gd} = 0.1 \cdot \text{ns} \quad t_\delta := -10 \cdot \varepsilon_{gd}, -10 \cdot \varepsilon_{gd} + \frac{\varepsilon_{gd}}{100} \dots 2000 \cdot \varepsilon_{gd}$$

$$\tau_0 := \frac{1}{\omega_0} \quad w_{hp}(t) := A_0 \cdot \left(\Delta(t) - \frac{1}{\tau_0} \cdot e^{-\frac{t}{\tau_0}} \right) \quad \tau_0 = 0.637 \cdot \mu\text{s} \quad (4.1.18)$$

(Consider a negative Dirac pulse of area $|A_0|$ at the origin $A_0 = -6$).

Geometric tangent to the curve at the point $(0, w(0))$:

$$f_a(t) := A_0 \cdot \omega_0 \cdot (\omega_0 \cdot t - 1) \cdot (\Phi(t) - \Phi(t - \tau_0)) \quad (4.1.19)$$

$$\varepsilon_{gd} = 0.1 \cdot \text{ns} \quad \tau_0 = 636.62 \cdot \text{ns} \quad t := 0.1 \cdot \tau_0, 0.1 \cdot \tau_0 + \frac{20 \cdot \tau_0 - 0.1 \cdot \tau_0}{2000} \dots 20 \cdot \tau_0 \quad A_0 = -6$$

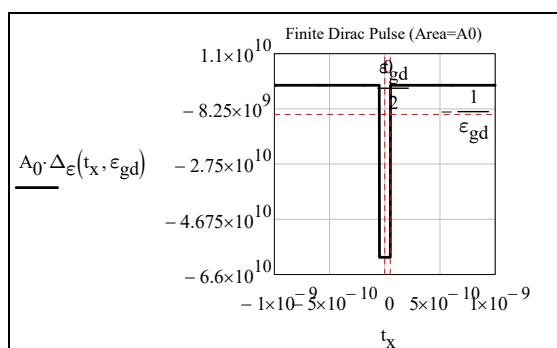


Fig.: (4.1.4)

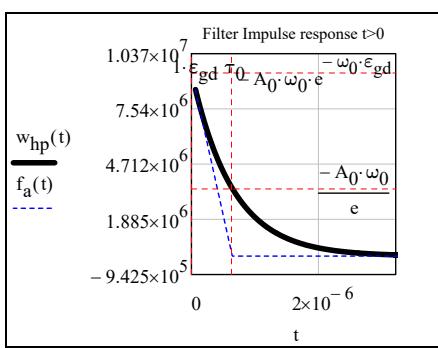


Fig.: (4.1.5)

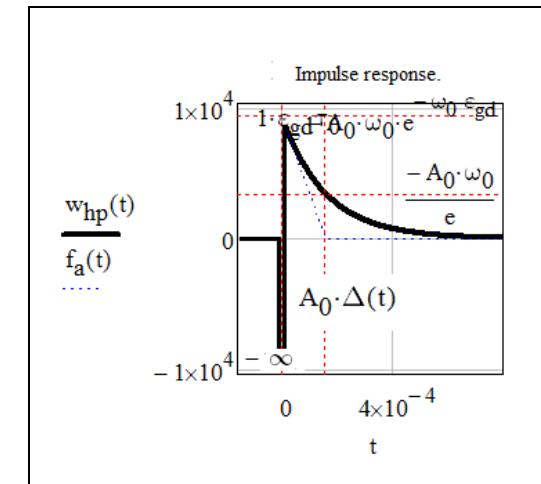


Fig.: (4.1.6)

Fig.: (4.1.5)

Bode plots (FIRST ORDER HIGH PASS FILTER)

For time harmonic input signals the complex variable (or the complex pulsation) is purely imaginary : $s = j \cdot \omega$

$$\omega_0 = 1.571 \cdot \frac{\text{Mrads}}{\text{sec}}$$

The transfer function takes the form (negative voltage gain):

$$A_0 = -6 \quad A_0 j \cdot \omega = -|A_0| \cdot j \cdot \omega = |A_0| \cdot \omega \cdot e^{-j \cdot \frac{\pi}{2}}$$

$$\frac{A_0 j \cdot \omega}{j \cdot \omega + \omega_0} = \frac{|A_0| \cdot e^{-j \cdot \frac{\pi}{2}}}{\left(j + \frac{\omega_0}{\omega}\right)} = \frac{|A_0|}{\sqrt{1 + \left(\frac{\omega_0}{\omega}\right)^2}} \cdot e^{-j \cdot \left(\frac{\pi}{2} + \arctan\left(\frac{\omega}{\omega_0}\right)\right)} \quad \omega := \omega \quad (4.1.20)$$

Its magnitude in dB is: $W_{hpdB}(\omega) := 20 \cdot \log(|A_0|) - 20 \cdot \log \sqrt{1 + \left(\frac{\omega_0}{\omega}\right)^2}$ (4.1.21)

while the phase is: $\varphi_{hp}(\omega) := -\frac{\pi}{2} - \arctan\left(\frac{\omega}{\omega_0}\right)$ (4.1.22)

Limit values: $\lim_{\omega \rightarrow \infty} W_{hpdB}(\omega) = 20 \cdot \log(|A_0|)$ (4.1.23)

$$\lim_{\omega \rightarrow 0} W_{hpdB}(\omega) = -\infty \quad (4.1.24)$$

Asymptote: $\text{Asy}_{dB}(\omega) := 20 \cdot \log(|A_0|) - 20 \cdot \log\left(\frac{\omega_0}{\omega}\right)$ (4.1.25)

$$\omega_{0dB} = 2.655 \times 10^5 \frac{1}{\text{s}} \quad \omega := \frac{\omega_{0dB}}{U_0}, \frac{\omega_{0dB}}{U_0} + \frac{10 \cdot \omega_0 \cdot U_0 - \frac{\omega_{0dB}}{U_0}}{10 \cdot U_0^2} \dots 10 \cdot U_0 \cdot \omega_0 \quad [U_0 = 100]$$

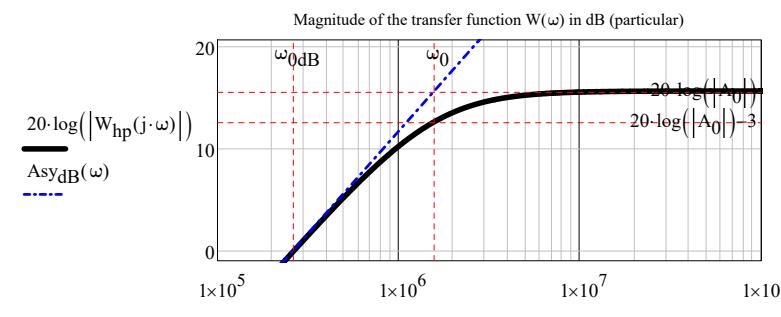
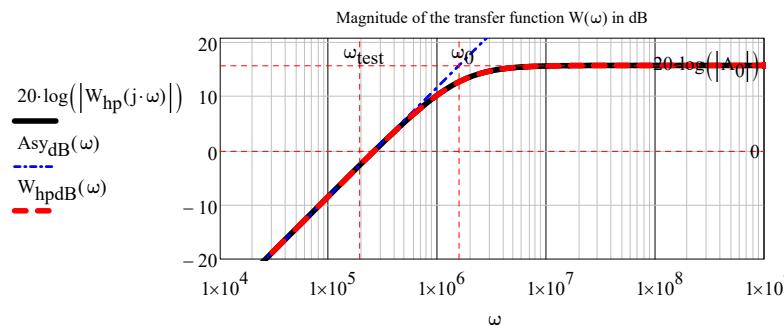


Fig.: (4.1.6)

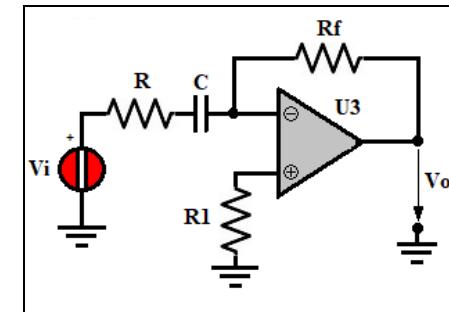
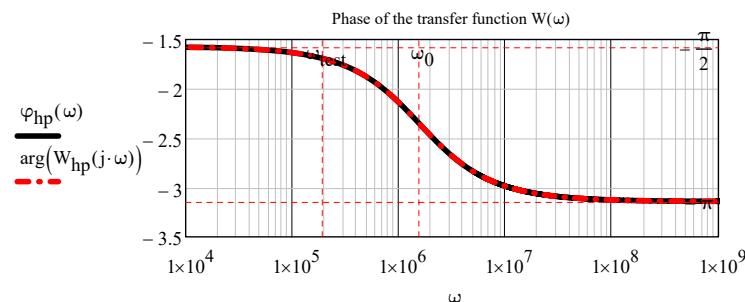


Fig.: (4.1.7)

$$\omega_0 = 1.571 \cdot \frac{\text{Mrads}}{\text{sec}}, \omega_{test} = 0.196 \cdot \frac{\text{Mrads}}{\text{sec}}, \frac{\omega_0}{\omega_{test}} = 8, \omega_{0dB} = 265.513 \cdot \frac{\text{krads}}{\text{s}}, \frac{\omega_{0dB}}{2 \cdot \pi} = 42.258 \cdot \text{kH}\zeta$$

Attenuation:

$$\text{Att}_{\text{hpdB}}(\omega) := -20 \cdot \log(\omega \cdot \text{sec}) + 20 \cdot \log\left(\sqrt{\omega^2 + \omega_0^2} \cdot \text{sec}\right) \quad (4.1.26)$$

$$\text{Attenuation's Phase } \varphi_{\text{atthp}}(\omega) := -\left(\frac{3 \cdot \pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)\right) \quad (4.1.27)$$

$$\text{Final value } \lim_{\omega \rightarrow \infty} \text{Att}_{\text{hpdB}}(\omega) = 0 \quad (4.1.28)$$

$$\text{Initial value } \lim_{\omega \rightarrow 0} \text{Att}_{\text{hpdB}}(\omega) = -20 \cdot \log(\omega) + 20 \cdot \log(\omega_0) \quad (4.1.29)$$

$$\begin{aligned} \text{Attenuation Asymptote} \quad B_{\text{dB}}(\omega) &:= -20 \cdot \log(\omega \cdot \text{sec}) + 20 \cdot \log(\omega_0 \cdot \text{sec}) \\ \text{Att}_{\text{hpdB}}(\omega_{0 \text{dB}}) &= 15.563 \end{aligned} \quad (4.1.30)$$

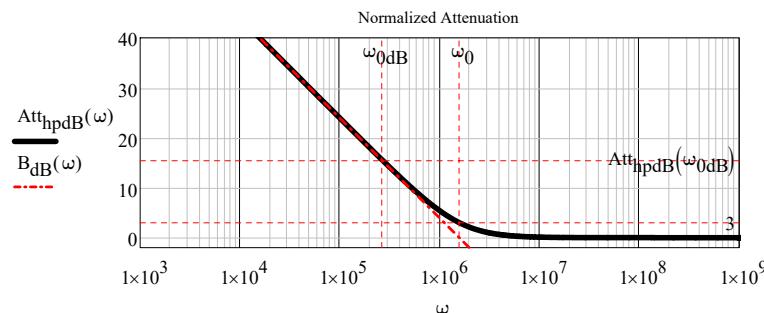


Fig.: (4.1.8)

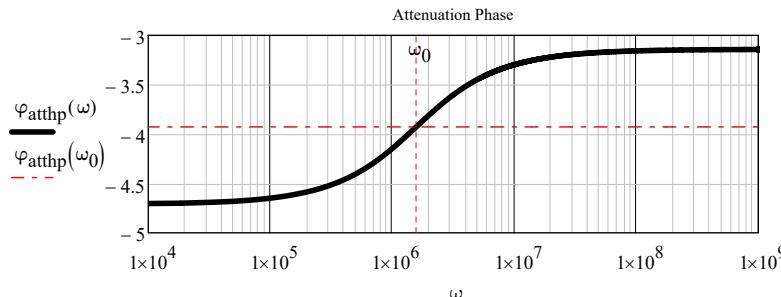


Fig.: (4.1.9)

4.2

ANALOG FILTER OUTPUT ANALYSIS

Chosen period of the test signal: $T_{\text{test}} = 32 \cdot \mu\text{s}$. At the corresponding frequency, the voltage gain of the filter is $20 \cdot \log(|W_{\text{hp}}(j \cdot \omega_{\text{test}})|) = -2.566 \cdot \text{dB}$.

ANALOG FILTER OUTPUT ANALYSIS

4.2.1) The sinus response Sinus amplitude: $V_i = 2.5 \cdot \text{mV}$, period: $T_{\text{test}} = 32 \cdot \mu\text{s}$

$$y_{\sin}(t) := V_i \sin(\omega_{\text{test}} t)$$

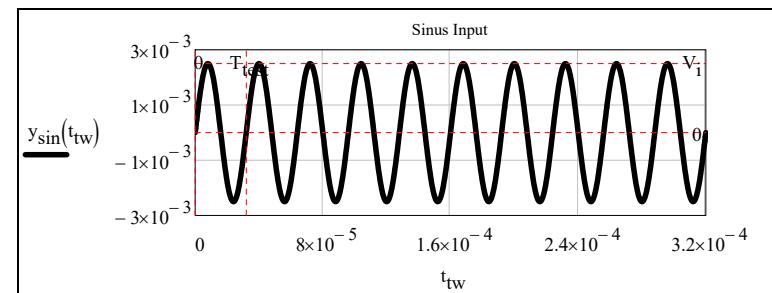


Fig.: (4.2.1)

$$\text{Laplace transform of The sinus response } V_o(s) = \frac{A_0 \cdot s}{s + \omega_0} \cdot \frac{V_i \omega_{\text{test}}}{s^2 + \omega_{\text{test}}^2}, \quad (4.2.1.1)$$

$$A_0 := A_0, \quad V_i := V_i$$

$$\begin{aligned} &\text{invlaplace} s, t \\ &\text{assume, ALL} > 0 \\ &\text{simplify} \\ &\text{collect, sin}\left(t \cdot \sqrt{\omega_{\text{test}}^2}\right) \rightarrow \\ &\text{collect, A}_0 \\ &\text{collect, V}_i \end{aligned} \quad (4.2.1.2)$$

$$Y_{\text{Outsin}}(t) = \frac{A_0 \cdot V_i \omega_{\text{test}} \left[\omega_0 \left(\cos(t \cdot \omega_{\text{test}}) - e^{-t \cdot \omega_0} \right) + \omega_{\text{test}} \cdot \sin(t \cdot \omega_{\text{test}}) \right]}{\omega_0^2 + \omega_{\text{test}}^2} \cdot \Phi(t) \quad (4.2.1.3)$$

► Output calculations

$$Y_{Outsin}(t) := \frac{A_0 \cdot V_i \cdot \omega_{test} \cdot \omega_0}{\omega_0^2 + \omega_{test}^2} \cdot \left[\sqrt{1 + \left(\frac{\omega_{test}}{\omega_0} \right)^2} \cdot \cos \left(\text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) - t \cdot \omega_{test} \right) - e^{-\frac{t}{\tau_0}} \right]$$

$$t \gg \tau_0 \Rightarrow Y_{Outsin}(t) \approx \frac{-|A_0| \cdot V_i \cdot \omega_{test}}{\sqrt{\omega_0^2 + \omega_{test}^2}} \cdot \cos \left(\text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) - t \cdot \omega_{test} \right)$$

$\cos(-\alpha)$ simplify $\rightarrow \cos(\alpha)$

$$\cos \left(\text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) - t \cdot \omega_{test} \right) = \cos \left[- \left(t \cdot \omega_{test} - \text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) \right) \right] = \cos \left(t \cdot \omega_{test} - \text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) \right)$$

$$\cos \left(t \cdot \omega_{test} - \text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) \right) \text{ takes its negative minima for } t_k \cdot \omega_{test} - \text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) = (2k+1) \cdot \pi$$

$$\text{maxima at } t_{0k} := \frac{1}{\omega_{test}} \cdot \left[(2k+1) \cdot \pi + \text{atan} \left(\frac{\omega_{test}}{\omega_0} \right) \right] \quad k = 0, 1, \dots$$

$$\text{Maximum output value: } Y_{Omax} := \frac{|A_0| \cdot V_i \cdot \omega_{test}}{\sqrt{\omega_0^2 + \omega_{test}^2}} = 1.861 \cdot \text{mV} \quad \frac{Y_{Omax}}{\sqrt{2}} = 1.316 \cdot \text{mV} \quad (4.2.1.4)$$

Drawings

$$\frac{A_0 \cdot V_i}{\omega_0} = -9.549 \times 10^{-9} \cdot \text{volt} \cdot \text{sec} \quad t_1 := 0 \cdot \tau_0, \frac{1000 \cdot \tau_0}{10000} \dots 1000 \cdot \tau_0 \quad \frac{A_0 \cdot V_i}{\omega_0} = -\tau_f \cdot V_i$$

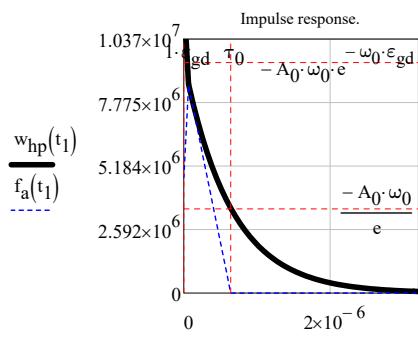
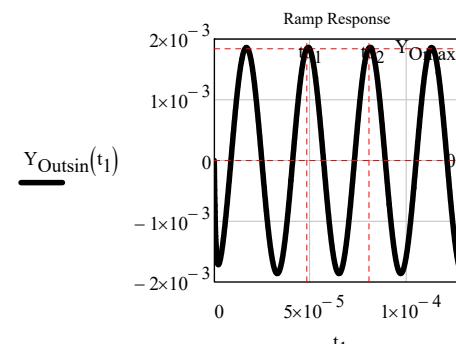


Fig.: (4.2.2)



A₀ V_i = -15 · mV

Analog filter Input sampling.

Calculate the bandwidth of the signal, using the program BCSA defined in "Fourier Analysis.xmcd", to sample it correctly:

$$\text{The dimensionless input signal be: } Isin(t) := \frac{y_{sin}(t)}{V}$$

Description of the program's parameters:

BCSA (Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{sin} := \text{BCSA}(Isin, rt_{gd}, N_{gd}, 0.0, T_{test}) \quad rt_{gd} = 10\% \quad (4.2.2.9)$$

Bandwidth Calculation

$$\text{Signal bandwidth: } B_{sin} = 437.5 \cdot \text{kHz} \quad f_{test} = 31.25 \cdot \text{kHz}$$

$$\text{Parseval}_{sin} = 6.25 \cdot \text{mV}^2 \quad \text{Average}_{sin} = 0 \cdot \text{mV} \quad \text{RMS}_{sin} = 1.768 \cdot \text{mV}$$

$$\text{Sampling frequency: } f_{sin} = \frac{1}{T_{samp}} \geq 2 \cdot f_{test} \quad (4.2.2.10)$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{s3} := 2 \cdot B_{sin} \quad f_{s3} = 0.875 \cdot \text{MHz} \quad (4.2.2.11)$$

$$\text{sampling period: } T_{s3} := \frac{1}{f_{s3}} \quad (4.2.2.12)$$

Samples are taken at the instants: $n_{s3k} := k \cdot T_{s3}$

$$n_{s3}^T = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 0 & 1.143 \cdot 10^{-6} & 2.286 \cdot 10^{-6} & 3.429 \cdot 10^{-6} & 4.571 \cdot 10^{-6} & 5.714 \cdot 10^{-6} & \dots \\ \hline \end{array} \text{s}$$

$$V_i = 2.5 \times 10^{-3} \text{ V} \quad \text{Input sampling: } u3k := y_{sin}(n_{s3k}) \quad \frac{N_{gd}}{f_{s3}} \cdot \frac{1}{T_{test}} = 1.786 \quad (4.2.2.13)$$

$$n_{s3}^T = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 0 & 1.143 \cdot 10^{-6} & 2.286 \cdot 10^{-6} & 3.429 \cdot 10^{-6} & 4.571 \cdot 10^{-6} & 5.714 \cdot 10^{-6} & \dots \\ \hline \end{array} \text{s}$$

$$u3^T = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 0 & 0 & 5.563 \cdot 10^{-4} & 1.085 \cdot 10^{-3} & 1.559 \cdot 10^{-3} & 1.955 \cdot 10^{-3} & 2.252 \cdot 10^{-3} & \dots \\ \hline \end{array} \text{V}$$

$$A_0 = -6$$

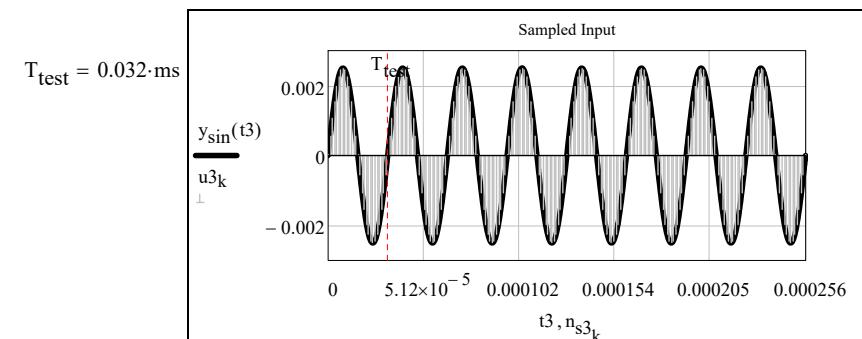


Fig.: (4.2.4)

Input signal reconstruction according to the Shannon sampling theorem:

$$\omega_{sh3o} := 2 \cdot \pi \cdot B_{sin} \quad sh3in(t) := \sum_{n=0}^{N_0 gd^{-1}} (u_{3n} \cdot \text{sinc}(\omega_{sh3o} \cdot t - n \cdot \pi)) \quad (4.2.6.10)$$

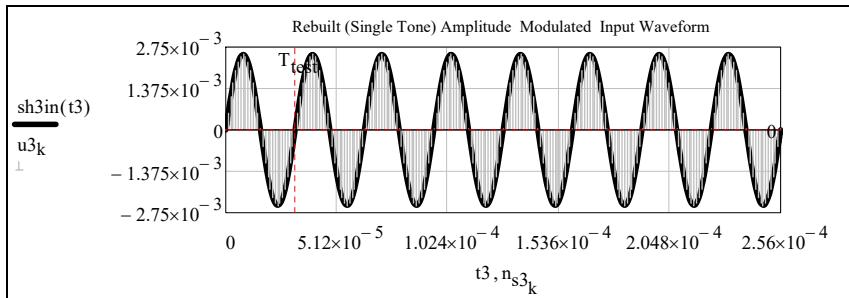


Fig.: (4.2.5)

Output signal reconstruction according to the Shannon sampling theorem:

$$\text{Output sampling: } u_{3ok} := Y_{Outsin}(n_s3_k) \frac{N_0 gd}{f_3} \cdot \frac{1}{T_{test}} = 9.143 \quad (4.2.2.13)$$

$$\omega_{sh3o} := 2 \cdot \pi \cdot B_{sin} \quad sh3o(t) := \sum_{n=0}^{N_0 gd^{-1}} (u_{3n} \cdot \text{sinc}(\omega_{sh3o} \cdot t - n \cdot \pi)) \quad (4.2.6.10)$$

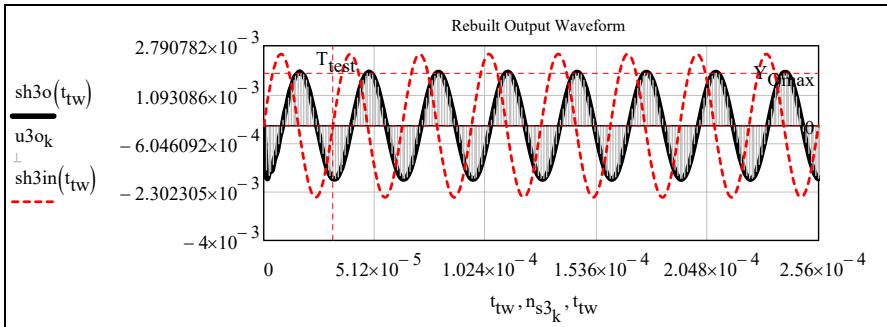


Fig.: (4.2.6)

ANALOG FILTER OUTPUT ANALYSIS

4.2.2) Short-Pulse function filter response

Pulse amplitude: $V_i = 2.5 \cdot mV$

$$\text{Pulse delay: } \tau_\delta := \frac{\tau_{ptd}}{10}$$

$$\text{Pulse duration: } \tau_{ptd} = 250 \cdot \mu s$$

$$\tau_{dp} := (\tau_\delta + \tau_{ptd})$$

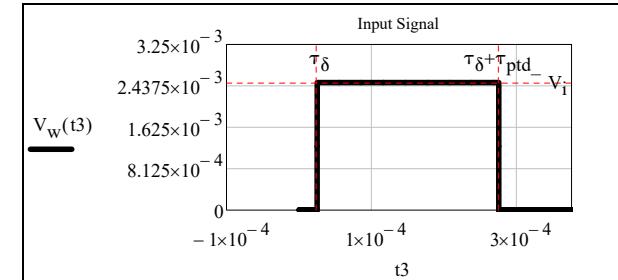
$$\text{The dimensionless input signal be: } V_w(t) := V_4(t, \tau_\delta, \tau_{ptd}, V_i) \quad (4.2.2.1)$$

(For the definition see reference)

Definition of the function rect1 and of the function V4:

Graph of the input signal:

$$t3 := -0 \cdot \tau_{ptd}, 0 \cdot \tau_{ptd} + \frac{2 \cdot \tau_{ptd}}{25000} \dots 2 \cdot \tau_{ptd}$$



$$V_i = 2.5 \cdot mV \\ \tau_\delta = 25 \cdot \mu s \\ \tau_{ptd} = 250 \cdot \mu s$$

Fig.: (4.2.2.3)

For the calculations I will use the Laplace transform.

$$\text{Laplace transform of the filter output: } V_o(s) = \frac{A_0 \cdot s}{s + \omega_0} \cdot \mathcal{L}(V_w(t)) \quad (4.2.2.3)$$

where $\mathcal{L}(V_w(t))$ is the Laplace transform of the input signal, called also window function.

Calculation of the Laplace transform of the input signal $V_w(t)$:

$$\tau_0 := \tau_0 \quad \tau_\delta := \tau_\delta \quad \tau_{ptd} := \tau_{ptd} \quad t := t \quad \tau_{dp} := \tau_{dp}$$

$$\Phi(t - \tau_\delta) - \Phi[t - (\tau_{dp})] \begin{cases} \text{laplace} & \rightarrow \frac{e^{-s \cdot \tau_\delta} - e^{-s \cdot \tau_{dp}}}{s} \\ \text{assume, } \tau_\delta > 0 & \\ \text{assume, } \tau_{dp} > 0 & \end{cases}$$

$$\mathcal{L}(V_w(t)) = V_i \frac{e^{-s \cdot \tau_\delta} - e^{-s \cdot \tau_{dp}}}{s} \quad (4.2.2.4)$$

Now that the Laplace transform of the input signal is known, I can define the filter output:

$$\text{Output: } V_o(s) = V_i \frac{A_0 \cdot s}{s + \omega_0} \cdot \frac{e^{-s \cdot \tau_\delta} - e^{-s \cdot \tau_{dp}}}{s} = V_i \frac{A_0}{s + \omega_0} \cdot (e^{-s \cdot \tau_\delta} - e^{-s \cdot \tau_{dp}})$$

It follows the time domain filter's output. Calculation of the output signal as the inverse Laplace transform of $V_o(s)$:

$$\frac{A_0}{s + \frac{1}{\tau_0}} \cdot \begin{bmatrix} e^{-s \cdot \tau_\delta} & \dots \\ \vdots & \vdots \\ +(-1) \cdot e^{-s \cdot \tau_{\delta p}} & \end{bmatrix} \begin{array}{l} \text{assume, } \tau_\delta > 0 \\ \text{assume, } \tau_0 > 0 \\ \text{assume, } \tau_{\delta p} > 0 \end{array} \rightarrow A_0 \cdot \left(-\frac{t - \tau_\delta}{\tau_0} \cdot \Phi(t - \tau_\delta) - e^{-\frac{t - \tau_{\delta p}}{\tau_0}} \cdot \Phi(t - \tau_{\delta p}) \right)$$

invlaplace

$$\text{Calculated output: } v_{2oc}(t) := V_i \cdot A_0 \cdot \left(-\frac{t - \tau_\delta}{\tau_0} \cdot \Phi(t - \tau_\delta) - e^{-\frac{t - \tau_{\delta p}}{\tau_0}} \cdot \Phi(t - \tau_{\delta p}) \right) \quad (4.2.2.5)$$

The general time domain filter's output as previously seen is:

$$v_o(t) = A_0 \cdot \left(V_i(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_{-\infty}^t v_i(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad \text{where } \boxed{\tau_0 = \frac{1}{\omega_0}}$$

while for the window function I get:

$$\text{Output } v_{2o}(t) = A_0 \cdot \left(V_w(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V_w(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.2.6)$$

Approximated output:

$$v_{2oapp}(t) = -\tau_f \frac{\partial}{\partial t} v_i(t) = -\tau_f \cdot V_i \frac{\partial}{\partial t} (\Phi(t - \tau_\delta) - \Phi(t - \tau_{ptd} - \tau_\delta))$$

$$v_{2oapp}(t) := (\Delta_\epsilon(t - \tau_\delta - \tau_{ptd}, \epsilon_{gd}) - \Delta_\epsilon(t - \tau_\delta, \epsilon_{gd})) \cdot \tau_f \cdot V_i \quad (4.2.2.7)$$

$$\tau_f \cdot V_i = 9.549 \times 10^{-3} \cdot V \cdot \mu s$$

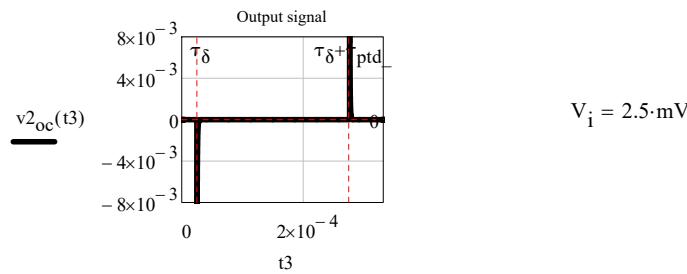


Fig.: (4.2.2.4)

Graph of the Pulse response considering the Op Amp saturation:

$$V2op_o(t) := \text{if}[-V_{sat} \leq v_{2oapp}(t) \leq V_{sat}, v_{2oapp}(t), \text{if}[v_{2oapp}(t) \leq -V_{sat}, -V_{sat}, V_{sat}]] \quad (4.2.2.8)$$

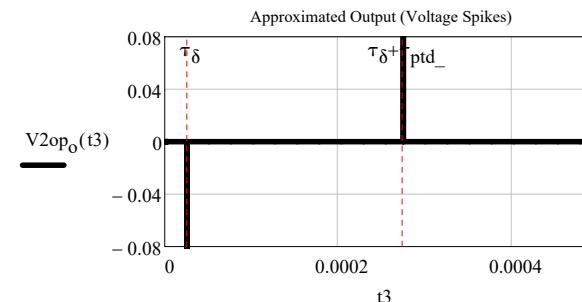


Fig.: (4.2.2.5)

(Always Avoid spikes!)

Analog filter Input sampling.

Consider now the same signal repeated periodically, with period $T_{vp} := 4 \cdot (\tau_\delta + \tau_{ptd})$, in such a way that it is possible to calculate the bandwidth of such a new signal, using the program BCSA defined in "Fourier Analysis.xmcd", to sample it correctly:

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{vp0} := \text{BCSA}[V_w, rt_{gd}, N_{gd}, 0.0, 2 \cdot (\tau_\delta + \tau_{ptd})] \quad rt_{gd} = 10\% \quad (4.2.2.9)$$

Bandwidth Calculation

The function returns a three columns matrix.

The first column contains:

- pos. 0: relative error,
- pos. 1: bandwidth (Dimensionless),
- pos. 2: the nth. harmonic number corresponding to the given relative error,
- pos. 3: temporary variable,
- pos. 4: Parseval,
- pos. 5: signal average,
- pos. 6: signal rms.

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

	0	1	2	3
0	0.1	$2.273 \cdot 10^{-3}$	0	0
1	$8.727 \cdot 10^4$	$-2.24 \cdot 10^{-4}$	$1.559 \cdot 10^{-3}$	0
2	49	$-2.149 \cdot 10^{-4}$	$-6.304 \cdot 10^{-5}$	0
3	$2.487 \cdot 10^{-5}$	$-2.003 \cdot 10^{-4}$	$4.391 \cdot 10^{-4}$	0
4	$5.659 \cdot 10^{-6}$	$-1.809 \cdot 10^{-4}$	$-1.161 \cdot 10^{-4}$	0
5	$1.137 \cdot 10^{-3}$	$-1.576 \cdot 10^{-4}$	$1.82 \cdot 10^{-4}$	0
6	$1.689 \cdot 10^{-3}$	$-1.359 \cdot 10^{-4}$	$-1.513 \cdot 10^{-4}$	0
7	0	$-1.035 \cdot 10^{-4}$	$6.666 \cdot 10^{-5}$	0
8	0	$-7.532 \cdot 10^{-5}$	$-1.644 \cdot 10^{-4}$	0
9	0	$-4.799 \cdot 10^{-5}$	$1.416 \cdot 10^{-5}$	0

10	0	$-2.263 \cdot 10^{-5}$	$-1.56 \cdot 10^{-4}$	0
11	0	$-3.644 \cdot 10^{-7}$	$-5.611 \cdot 10^{-9}$	0
12	0	$1.847 \cdot 10^{-5}$	$-1.301 \cdot 10^{-4}$	0
13	0	$3.291 \cdot 10^{-5}$	$9.598 \cdot 10^{-6}$	0
14	0	$4.281 \cdot 10^{-5}$	$-9.423 \cdot 10^{-5}$	0
15	0	$4.805 \cdot 10^{-5}$	$3.081 \cdot 10^{-5}$...

Fourier series coefficients: $r_{gd} = 10\%$

$$cffaSb_{vp0} := Sb_{vp0}^{(1)} \quad cffbSb_{vp0} := Sb_{vp0}^{(2)}$$

$$cffaSb_{vp0}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2.273 \cdot 10^{-3} & -2.24 \cdot 10^{-4} & -2.149 \cdot 10^{-4} & -2.003 \cdot 10^{-4} & -1.809 \cdot 10^{-4} \\ \dots & & & & & \end{bmatrix}$$

$$cffbSb_{vp0}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1.559 \cdot 10^{-3} & -6.304 \cdot 10^{-5} & 4.391 \cdot 10^{-4} & -1.161 \cdot 10^{-4} \\ \dots & & & & & \end{bmatrix}$$

$$\text{Signal Bandwidth: } B_{vp0} := \frac{\left(Sb_{vp0}^{(0)}\right)_1}{\text{sec}}$$

This is the number of the harmonic for which $r_{gd} = 10\%$

$$j_{vp0} := \left(Sb_{vp0}^{(0)}\right)_2 \quad j_{vp0} = 49$$

$$\text{Frequency of the pulse train: } T_{vp0} := \left(Sb_{vp0}^{(0)}\right)_3 \quad T_{vp0} = 2.487 \times 10^{-5}$$

$$\text{Parseval}_{vp0} := \left(Sb_{vp0}^{(0)}\right)_4 \cdot V^2 \quad \text{Average}_{vp0} := \left(Sb_{vp0}^{(0)}\right)_5 \cdot \text{volt} \quad \text{RMS}_{vp0} := \left(Sb_{vp0}^{(0)}\right)_6 \cdot \text{volt}$$

$$j_{3vp0} := 0.. \text{rows}(cffaSb_{vp0}) - 1$$

Bandwidth Calculation

$$\text{Signal bandwidth: } B_{vp0} = 87.273 \cdot \text{kHz} \quad f_{\text{test}} = 0.031 \cdot \text{MHz}$$

$$\text{Parseval}_{vp0} = 5.659 \times 10^{-6} \text{V}^2 \quad \text{Average}_{vp0} = 1.137 \times 10^{-3} \text{V} \quad \text{RMS}_{vp0} = 1.689 \times 10^{-3} \text{V}$$

$$\text{Sampling frequency: } f_{s0} = \frac{1}{T_{\text{samp}}} \geq 2 \cdot f_1 \quad (4.2.2.10)$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{s0} := 2 \cdot B_{vp0} \quad f_{s0} = 174.545 \cdot \text{kHz} \quad (4.2.2.11)$$

$$\text{sampling period: } T_{s4} := \frac{1}{f_{s0}} \quad (4.2.2.12)$$

Samples are taken at the instants: $n_{s4_k} := k \cdot T_{s4} + \tau_\delta$ assuming that it is periodic.

$$V_i = 2.5 \times 10^{-3} \text{V} \quad \text{Pulse sampling: } u_{-4k} := V_w(n_{s4_k}) \quad \frac{N_0}{f_{s0}} \cdot \frac{1}{T_{\text{test}}} = 45.833 \quad (4.2.2.13)$$

$$n_{s4}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 2.5 \cdot 10^{-5} & 3.073 \cdot 10^{-5} & 3.646 \cdot 10^{-5} & 4.219 \cdot 10^{-5} & 4.792 \cdot 10^{-5} & 5.365 \cdot 10^{-5} \\ \dots & & & & & & \end{bmatrix} \text{s}$$

$$u_{-4}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 1.25 \cdot 10^{-3} & 2.5 \cdot 10^{-3} \\ \dots & & & & & & & & \end{bmatrix}$$

$$A_0 = -6$$

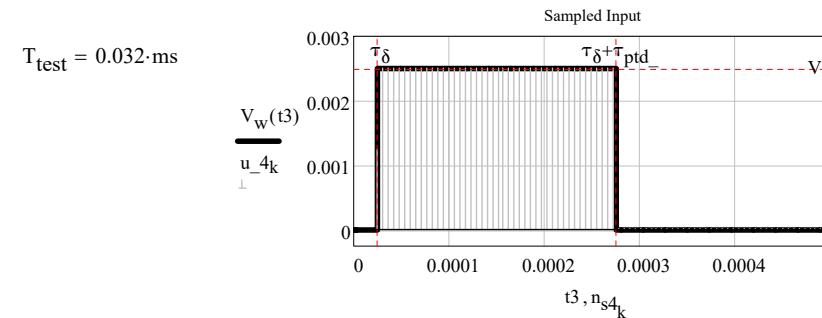


Fig.: (4.2.2.6)

Input signal reconstruction according to the Shannon sampling theorem:

$$\omega_{sh40} := 2 \cdot \pi \cdot B_{vp0} \quad sh40(t) := \sum_{n=0}^{N_0 gd^{-1}} (u_{-4k} \cdot \text{sinc}(\omega_{sh40} \cdot t - n \cdot \pi)) \quad (4.2.2.14)$$

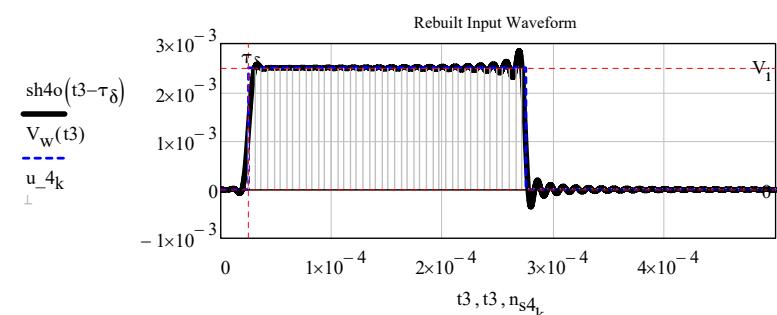


Fig.: (4.2.2.7)

Analog filter Output sampling.

Consider now the same signal repeated periodically, with period $T_{wp} := 2 \cdot (\tau_\delta + \tau_{ptd})$, in such a way that it is possible to calculate the bandwidth of such a new signal, using the program BCSA defined in "Fourier Analysis.xmdc", to sample it correctly:

$$\text{the dimensionless output signal is } V_o(t) := \frac{v^2_{oc}(t)}{V} \quad (4.2.2.15)$$

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{vpo} := \text{BCSA}[V_o, rt_{gd}, N0_{gd}, 0.0, 2 \cdot (\tau_\delta + \tau_{ptd})] \quad rt_{gd} = 10\% \quad (4.2.2.16)$$

Bandwidth Calculation

$$\text{Signal bandwidth: } B_{vpo} = 461.818 \text{-kHz} \quad f_{test} = 0.031 \text{-MHz}$$

$$\text{Parseval}_{vpo} = 0.178 \cdot \text{mV}^2 \quad \text{Average}_{vpo} = 0.017 \cdot \text{mV} \quad \text{RMS}_{vpo} = 0.318 \cdot \text{mV}$$

$$\text{Sampling frequency: } f_{so} = \frac{1}{T_{samp}} \geq 2 \cdot f_l \quad (4.2.2.17)$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{so} := 2 \cdot B_{vpo} \quad f_{so} = 923.636 \text{-kHz} \quad (4.2.2.18)$$

$$\text{sampling period: } T_{so} := \frac{1}{f_{so}} \quad (4.2.2.19)$$

Samples are taken at the instants: $n_{so_k} := k \cdot T_{so} + \tau_\delta$ assuming that it is periodic.

$$V_i = 2.5 \times 10^{-3} \text{ V} \quad \text{Pulse sampling: } u_{4ok} := V_o(n_{so_k}) \quad \frac{N0_{gd}}{f_{so}} \cdot \frac{1}{T_{test}} = 8.661 \quad (4.2.2.13)$$

$$n_{so}^T = \begin{array}{|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 \\ \hline 0 & 2.5 \cdot 10^{-5} & 2.608 \cdot 10^{-5} & 2.717 \cdot 10^{-5} & \dots \\ \hline \end{array} \text{ s}$$

$$u_{4o}^T = \begin{array}{|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 \\ \hline 0 & -7.5 \cdot 10^{-3} & -2.738 \cdot 10^{-3} & -4.999 \cdot 10^{-4} & \dots \\ \hline \end{array}$$

$$A_0 = -6$$

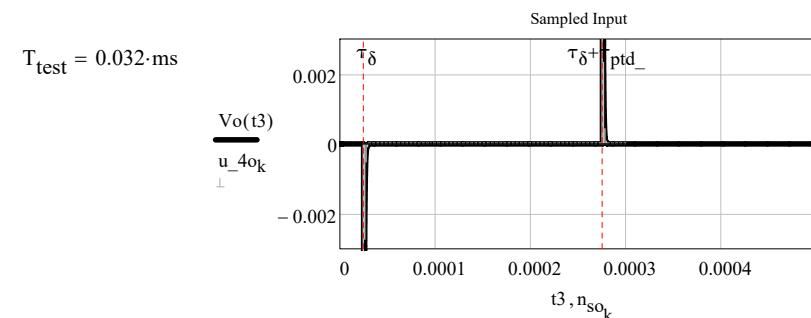


Fig.: (4.2.2.8)

Output signal reconstruction according to the Shannon sampling theorem:

$$\omega_{sh4o} := 2 \cdot \pi \cdot B_{vpo} \quad sh4o0(t) := \sum_{n=0}^{N0_{gd}-1} (u_{4on} \cdot \text{sinc}(\omega_{sh4o} \cdot t - n \cdot \pi)) \quad (4.2.2.20)$$

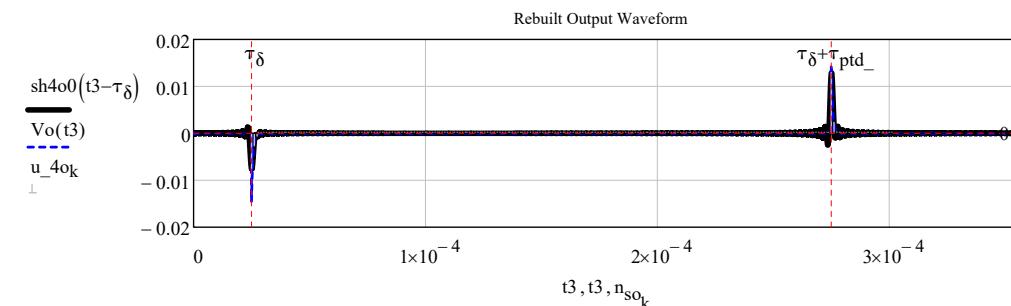


Fig.: (4.2.2.9)

ANALOG FILTER OUTPUT ANALYSIS

4.2.3) Triangular wave response

$$\text{the dimensionless input signal is } V_{tri}(t) := \frac{v_{tri0}(t, T_{test}, V_i, N0_{gd})}{V} \quad N0_{gd} = 256 \quad (4.2.3.1)$$

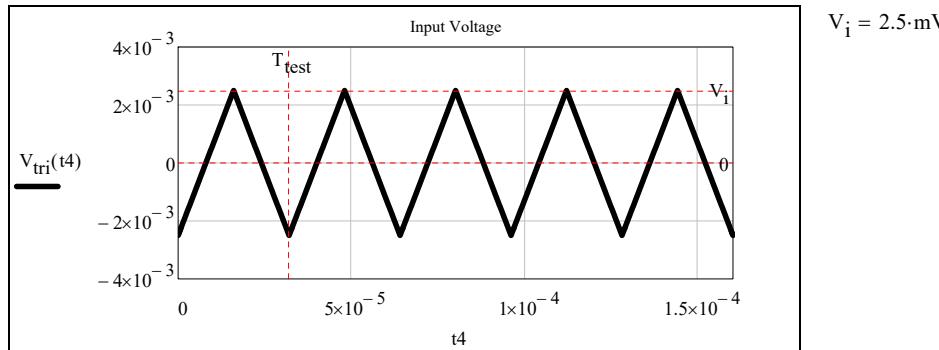


Fig.: (4.2.3.1)

Analog filter Input sampling

At first I calculate the signal bandwidth for a correct sampling.

$$T_{\text{test}} = 32 \cdot \mu\text{s}$$

$$\tau_0 = 0.637 \cdot \mu\text{s}$$

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$S_{\text{tri}} := \text{BCSA}(V_{\text{tri}}, r_{\text{tg}}, N_{\text{gd}}, 0.0, 1 \cdot T_{\text{test}}) \quad r_{\text{tg}} = 10\% \quad (4.2.3.2)$$

Bandwidth Calculation —

$$\text{Signal bandwidth: } B_{\text{tri}} = 0.625 \cdot \text{MHz} \quad f_{\text{test}} = 3.125 \times 10^{-5} \cdot \text{GHz}$$

$$\text{Parseval}_{\text{tri}} = 4.167 \times 10^{-6} \text{ V}^2$$

$$\text{Average}_{\text{tri}} = 0 \text{ V} \quad \text{RMS}_{\text{tri}} = 1.443 \times 10^{-3} \text{ V}$$

Sampling frequency:

$$f_{\text{stri}} = \frac{1}{T_{\text{stri}}} \geq 2 \cdot f_1$$

Chosen sampling frequency (Nyquist rate):

$$f_{\text{stri}} := 2 \cdot B_{\text{tri}} \quad f_{\text{stri}} = 1.25 \cdot \text{MHz} \quad (4.2.3.3)$$

$$\text{sampling period: } T_{\text{stri}} := \frac{1}{f_{\text{stri}}} \quad (4.2.3.4)$$

Samples are taken at the instants: $n_{\text{stri},k} := k \cdot T_{\text{stri}}$

$$\text{Input sampling: } u_{-5k} := V_{\text{tri}}(n_{\text{stri},k}) \quad (4.2.3.5)$$

$$\omega_0 := 0 \cdot T_{\text{test}}, \frac{20 \cdot T_{\text{test}}}{1000} \dots 20 \cdot T_{\text{test}}$$

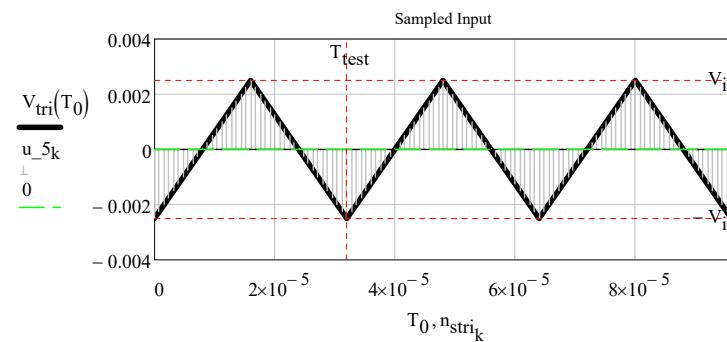


Fig.: (4.2.3.2)

$$\text{Exact output: } v_{\text{otri}}(t) = A_0 \cdot \left(V_{\text{tri}}(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V_{\text{tri}}(\tau) e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.3.6)$$

$$\text{Laplace transform of the filter output: } V_o(s) = \frac{A_0 \cdot s}{s + \omega_0} \cdot \mathcal{L}(V_{\text{tri}}(t)) \quad (4.2.3.7)$$

Laplace transform of the filter input:

$$\mathcal{L}(V_{\text{tri}}(t)) = \mathcal{L}(v_{\text{tri}0}(t, T_{\text{test}}, V_i, N_0 \cdot g_d))$$

Laplace transform calculations —

$$\mathcal{L}(v_{\text{tri}0}(t, T_{\text{test}}, V_i, N_0 \cdot g_d)) = \frac{V_i}{s} \cdot \left(\frac{4}{T_{\text{test}} \cdot s} \cdot \frac{2 \cdot \sinh\left(\frac{T_{\text{test}} \cdot s}{4}\right)^2}{\sinh\left(\frac{T_{\text{test}} \cdot s}{2}\right)} - 1 \right) \quad (4.2.3.8)$$

$$V_o(s) = \frac{A_0 \cdot s}{s + \omega_0} \cdot \mathcal{L}(V_{\text{tri}}(t)) \quad (4.2.3.9)$$

Calculations of the filter response to the triangular signal —

Output signal:

$$v_{\text{otri}c}(t) = V_i \cdot A_0 \cdot \left[\frac{4}{T_{\text{test}} \cdot \omega_0} \cdot \sum_{k=0}^{10} \left[\begin{array}{l} \left[1 - e^{-(t-k \cdot T_{\text{test}}) \cdot \omega_0} \right] \cdot \Phi(t - k \cdot T_{\text{test}}) \dots \\ + (-2) \cdot \left[1 - e^{-\left[t - \left(k + \frac{1}{2}\right) \cdot T_{\text{test}}\right] \cdot \omega_0} \right] \cdot \Phi\left[t - \left(k + \frac{1}{2}\right) \cdot T_{\text{test}}\right] \dots \\ + \left[\Phi\left[(k+1) \cdot T_{\text{test}} - t\right] - 1 \right] \cdot \left[e^{\omega_0 \left[(k+1) \cdot T_{\text{test}} - t\right]} - 1 \right] \cdot \Phi(t - k \cdot T_{\text{test}}) \end{array} \right] - e^{-t \cdot \omega_0} \cdot \Phi(t) \right] \quad (4.2.3.10)$$

Approximated output:
see (4.1.16).

$$v_{\text{otrid}}(t) = \frac{A_0}{\omega_0} \cdot \frac{d}{dt} V_{\text{tri}}(t) \quad \text{holds for } \omega \ll \omega_0 \text{ or } T_{\text{test}} \gg (2 \cdot \pi \cdot \tau_0) \quad (4.2.3.11)$$

$$2 \cdot \pi \cdot \tau_0 = 4 \cdot \mu\text{s}$$

$$v_{otrid}(t) := \frac{4 \cdot V_i}{T_{test}} \cdot \sum_{k=0}^{N_{gd}} \left[\Phi(t - T_{test} \cdot k) + \Phi(t - T_{test} \cdot (k+1)) - 2 \cdot \Phi\left(t - T_{test} \cdot \left(k + \frac{1}{2}\right)\right) \dots \right. \\ \left. + \Delta[t - T_{test} \cdot (k+1)] \cdot [t - T_{test} \cdot (k+1)] \dots \right. \\ \left. + 2 \cdot \Delta\left[t - T_{test} \cdot \left(k + \frac{1}{2}\right)\right] \cdot \left[T_{test} \cdot \left(k + \frac{1}{2}\right) - t\right] \dots \right. \\ \left. + \Delta(t - T_{test} \cdot k) \cdot (t - T_{test} \cdot k) \right] \quad (4.2.3.12)$$

$$A1_t := 1.1 \cdot \left| \frac{A_0}{\omega_0} \cdot V_i \right| \quad A1_t = 10.504 \cdot V \cdot ns$$

$$T_{test} = 32 \cdot \mu s \quad T_{test}^{-1} = 31.25 \cdot kHz \quad f_{stri} = 1.25 \cdot MHz \quad \boxed{\frac{|A_0| \cdot 4 \cdot V_i}{\omega_0 \cdot T_{test}} = 1.194 \cdot mV} \quad A_0 = -6$$

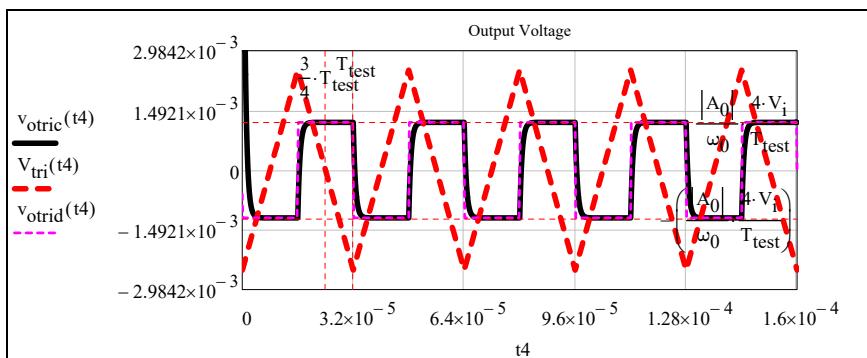


Fig.: (4.2.3.3)

$$\text{Output sampling: } u_{50k} := v_{otrid}(n_{stri_k})$$

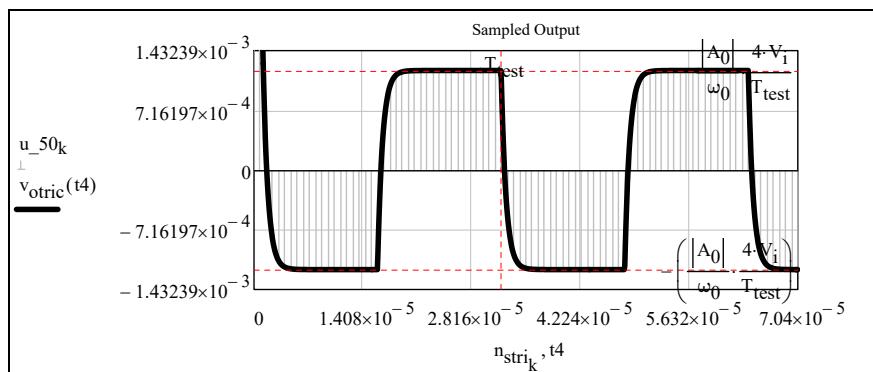


Fig.: (4.2.3.4)

$$\text{Spec50} := \text{fft}(u_{50})$$

(4.2.3.14)

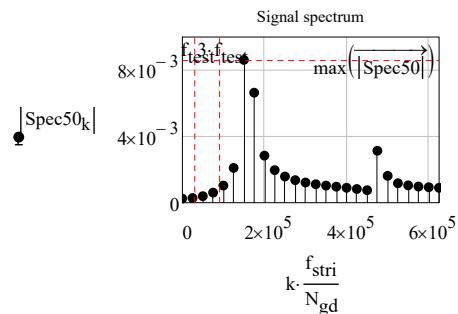


Fig.: (4.2.3.5)

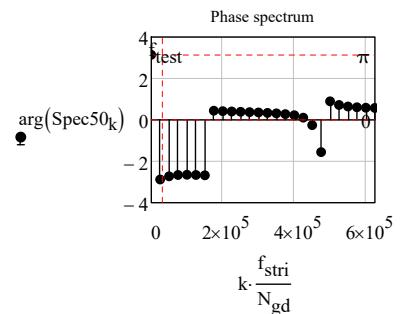


Fig.: (4.2.3.6)

Ouput signal reconstruction according to the Shannon sampling theorem:

$$\omega_{sh50} := 2 \cdot \pi \cdot B_{tri} \quad sh50(t) := \sum_{n=0}^{N_{gd}-1} (u_{50n} \cdot \text{sinc}(\omega_{sh50} \cdot t - n \cdot \pi)) \quad (4.2.6.10)$$

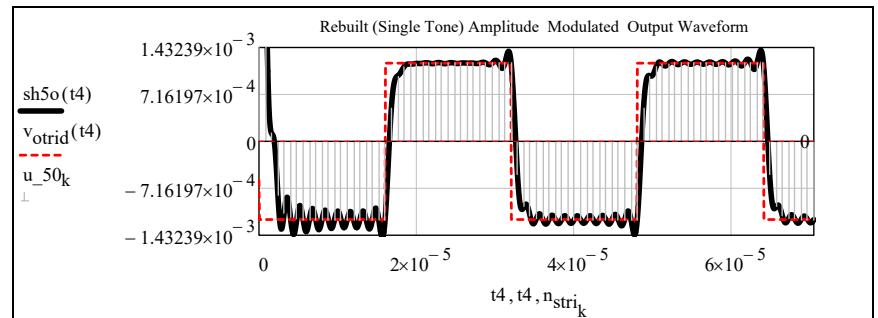


Fig.: (4.2.3.7)

4.2.4) Sawtooth wave response calculation

Amplitude:

$$V_{\text{sawth}} = 50 \text{ V}$$

Sawtooth length:

$$\delta_{\text{sawth}} = 1 \mu\text{s}$$

Slope:

$$sp_{\text{sawth}} = 50 \frac{\text{V}}{\mu\text{s}}$$

Period:

$$T_{\text{sawth}} = 1 \mu\text{s} \quad f_{\text{sawth}} = 1 \text{ MHz}$$

the dimensionless input signal is

$$V_{\text{sawth}} = 5 \times 10^4 \text{ mV} \quad v_{\text{sw}}(t) := \frac{v_{\text{sw}}(t, T_{\text{sawth}}, V_{\text{sawth}}, N_0 \text{ gd})}{V} \quad (4.2.4.1)$$

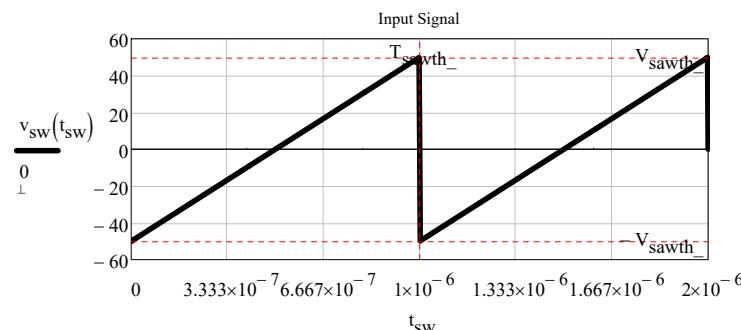


Fig.(4.2.4.1)

Analog filter Input sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$S_{\text{bw}} := \text{BCSA}(v_{\text{sw}}, rt_{\text{gd}}, N_{\text{gd}}, 0.0, T_{\text{sawth}}) \quad rt_{\text{gd}} = 10\% \quad (4.2.4.2)$$

Bandwidth Calculation

$$f_{\text{test}} = 0.031 \text{ MHz} \quad \text{Signal bandwidth: } B_{\text{sw}} = 48 \text{ MHz} \quad (4.2.4.3)$$

$$\text{Parseval}_{\text{sw}} = 1.647 \times 10^3 \text{ V}^2$$

$$\text{Average}_{\text{sw}} = 0 \text{ V}$$

$$\text{RMS}_{\text{sw}} = 28.868 \text{ V}$$

Sampling frequency:

$$f_{\text{ssw}} = \frac{1}{T_{\text{sw}}} \geq 2 \cdot f_{\text{test}}$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{\text{ssw}} := 2 \cdot B_{\text{sw}} \quad f_{\text{ssw}} = 96 \text{ MHz} \quad (4.2.4.4)$$

$$\text{sampling period: } T_{\text{ssw}} := \frac{1}{f_{\text{ssw}}} \quad (4.2.4.5)$$

Samples are taken at the instants: $n_{\text{ssw}} := k \cdot T_{\text{ssw}}$

$$\text{Input sampling: } u_{-6k} := v_{\text{sw}}(n_{\text{ssw}}) \quad (4.2.4.6)$$

$$t_{\text{sw}} := T_{\text{sawth}} \cdot 0, T_{\text{sawth}} \cdot 0 + \frac{40 \cdot T_{\text{sawth}} - T_{\text{sawth}} \cdot 0}{10000} \dots 40 \cdot T_{\text{sawth}}$$

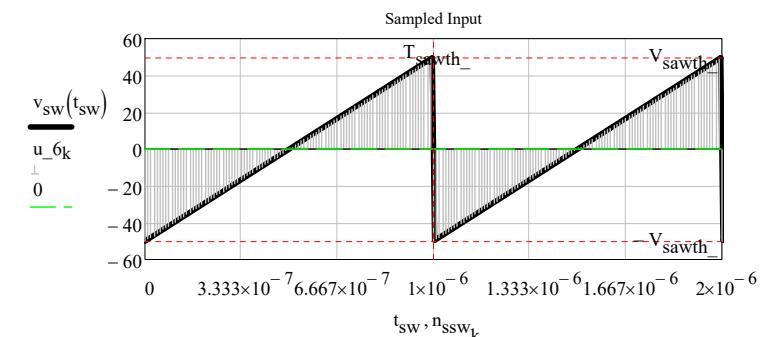


Fig.(4.2.4.2)

$$\text{Exact time domain output: } v_{\text{osw}}(t) = A_0 \left(v_{\text{sw}}(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t v_{\text{sw}}(\tau) e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.4.7)$$

Output calculation

Output signal:

$$V_{\text{saw}} := \frac{V_{\text{sawth}} \cdot A_0}{T_{\text{sawth}}}$$

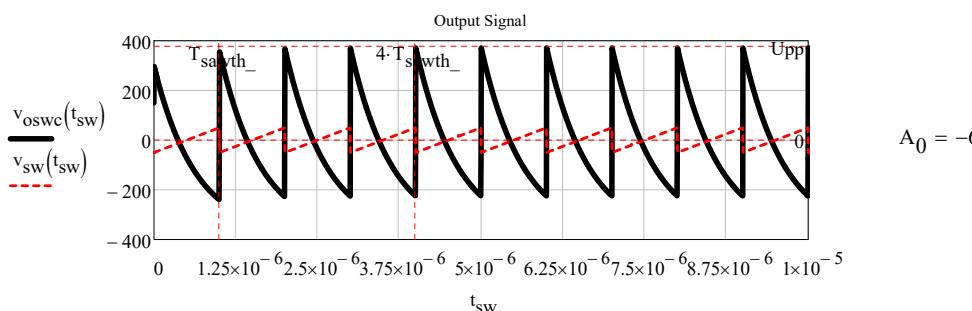
$$v_{\text{oswc}}(t) := V_{\text{saw}} \cdot \left[\frac{2 \cdot (1 - e^{-\frac{t}{\omega_0}})}{\omega_0} \cdot \Phi(t) - T_{\text{sawth}} \cdot \sum_{k=0}^{N_{\text{gd}}} \left[e^{-\frac{(t-k \cdot T_{\text{sawth}}) \cdot \omega_0}{\omega_0}} \cdot \Phi(t - k \cdot T_{\text{sawth}}) \right] \dots \right. \right. \\ \left. \left. + T_{\text{sawth}} \cdot \sum_{k=0}^{N_{\text{gd}}} \left[e^{\frac{\omega_0 \cdot [T_{\text{sawth}} - (t-k \cdot T_{\text{sawth}})]}{\omega_0}} \cdot [\Phi[T_{\text{sawth}} - (t - k \cdot T_{\text{sawth}})] - 1] \right] \right] \right]$$

$$\text{Approximated output: } v_{\text{oswd}}(t) := \frac{A_0}{\omega_0} \cdot \frac{d}{dt} v_{\text{sw}}(t) \quad (4.2.4.8)$$

$$\frac{2 \cdot V_{\text{sawth}} \cdot |A_0|}{T_{\text{sawth}} \cdot \omega_0} = 381.972 \text{ V}_{\text{sawth}} \cdot V_i = 2.5 \cdot \text{volt} \cdot n \cdot \tau_f = -\frac{A_0}{\omega_0} \quad T_{\text{sawth}} = 1 \times 10^{-3} \text{ ms}$$

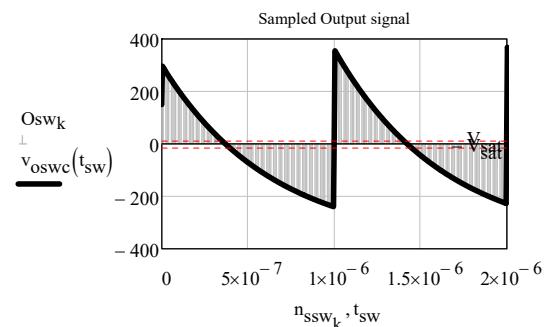
Time average value: $v_{swm} := \frac{1}{\delta_{sawth_}} \int_0^{\delta_{sawth_}} v_{sw}(t) dt$ (4.2.4.9)

$$Upp := \frac{2 \cdot V_{sawth_} \cdot |A_0|}{T_{sawth_} \cdot \omega_0} \quad Upp = 381.972 \text{ V} \quad v_{swm} = -2.704 \times 10^{-15} \quad A_0 = -6$$



Output sampling: $Osw_k := v_{oswc}(n_{ssw_k})$ (4.2.4.10)

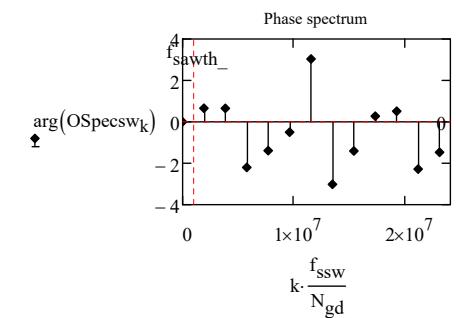
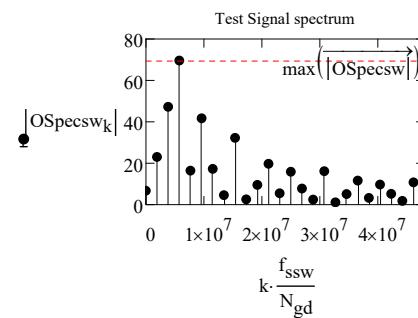
$$T_{sawth_} = 1 \cdot \mu\text{s} \quad T_{ssw} = 0.01 \cdot \mu\text{s} \quad f_{sawth_} = 1 \cdot \text{MHz} \quad f_{ssw} = 96 \cdot \text{MHz}$$



$$f_{sawth_} = 1 \cdot \text{MHz} \quad \frac{f_{ssw}}{f_{sawth_}} = 96$$

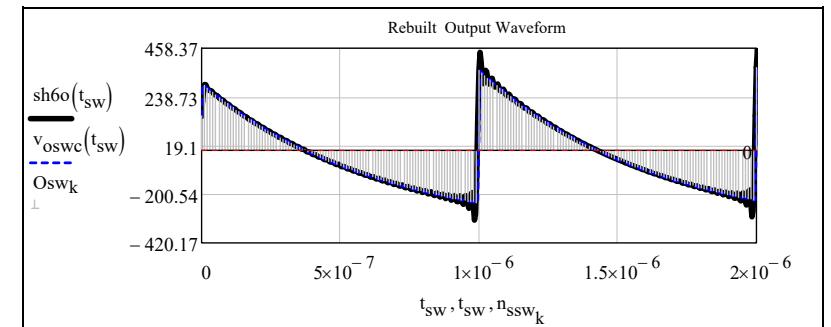
Fourier Transform of the output signal $Ospecsw := FFT(Osw)$ (4.2.4.11)

$$\left| \max(\overrightarrow{Ospecsw}) \right| = 47.275 \text{ V}$$



Output signal reconstruction according to the Shannon sampling theorem:

$$\omega_{sh6o} := 2 \cdot \pi \cdot B_{sw} \quad sh6o(t) := \sum_{n=0}^{N_0 gd^{-1}} (Osw_n \cdot \text{sinc}(\omega_{sh6o} \cdot t - n \cdot \pi)) \quad (4.2.6.10)$$



ANALOG FILTER OUTPUT ANALYSIS

4.2.5) AM Signal response

■ Data

$$\begin{aligned}
 \text{Carrier Amplitude: } A1 &:= 10 \cdot \text{volt} \\
 \text{Modulating signal's amplitude: } B1 &:= 5.5 \cdot \text{volt} \\
 \omega_3_{\text{cam}} &:= 100 \cdot \omega_0_{\text{gd}} \quad T3_{\text{cam}} := \frac{2 \cdot \pi}{\omega_3_{\text{cam}}} \\
 \omega_1_{\text{mam}} &:= \frac{\omega_3_{\text{cam}}}{10} \quad T1_{\text{mam}} := \frac{2 \cdot \pi}{\omega_1_{\text{mam}}} \\
 f1_{\text{msl}} &:= \frac{\omega_1_{\text{mam}}}{2 \cdot \pi} \quad f3_{\text{cam}} := \frac{\omega_3_{\text{cam}}}{2 \cdot \pi} \\
 A1 &= v_{\text{ammax}} + v_{\text{ammin}} \quad B1 = v_{\text{ammax}} - v_{\text{ammin}} \\
 v_{\text{ammax}} &:= A1 + B1 \quad v_{\text{ammin}} := A1 - B1 \\
 v_{\text{ammax}} &= 15.5 \cdot \text{volt} \quad v_{\text{ammin}} = 4.5 \cdot \text{volt} \\
 m_{\text{am}} &:= \frac{v_{\text{ammax}} - v_{\text{ammin}}}{v_{\text{ammax}} + v_{\text{ammin}}} \quad m_{\text{am}} = 0.55
 \end{aligned}$$

■ Data

$$\begin{aligned}
 \text{Carrier max amplitude: } A1 &= 10 \text{ V} \\
 \text{Modulating single tone max amplitude: } B1 &= 5.5 \text{ V} \\
 \text{Carrier pulsation: } \omega_3_{\text{cam}} &= 628.319 \cdot \frac{\text{krads}}{\text{sec}} \\
 \text{Carrier frequency: } f3_{\text{cam}} &= 100 \cdot \text{kHz} \\
 \text{Carrier period: } T3_{\text{cam}} &= 10 \cdot \mu\text{s} \\
 \text{Modulating single tone pulsation: } \omega_1_{\text{mam}} &= 62.832 \cdot \frac{\text{krads}}{\text{sec}} \\
 \text{Modulating single tone frequency: } f_{\text{mam}} &:= \frac{\omega_1_{\text{mam}}}{2 \cdot \pi} \quad f_{\text{mam}} = 0.01 \cdot \text{MHz} \\
 \text{Modulating single tone period: } T_{\text{mam}} &:= \frac{1}{f_{\text{mam}}} \\
 \text{modulation index: } m_{\text{am}} &= 55\%
 \end{aligned}$$

$$\omega_1_{\text{mam}} = 62.832 \cdot \frac{\text{krads}}{\text{sec}} \quad \omega_3_{\text{cam}} = 628.319 \cdot \frac{\text{krads}}{\text{sec}}$$

(Single tone) Modulated signal $V2_1(t) := v2_i(t, \omega_1_{\text{mam}}, \omega_3_{\text{cam}}, A1, B1)$ $m_{\text{am}} = 55\%$

$$A1 = 10 \text{ V} \quad B_{\text{fmm}} = 15 \text{ V}$$

the dimensionless input signal is: $v2_{\text{am}}(t) := \frac{v2_i(t, \omega_1_{\text{mam}}, \omega_3_{\text{cam}}, A1, B1)}{V}$ (4.2.5.1)

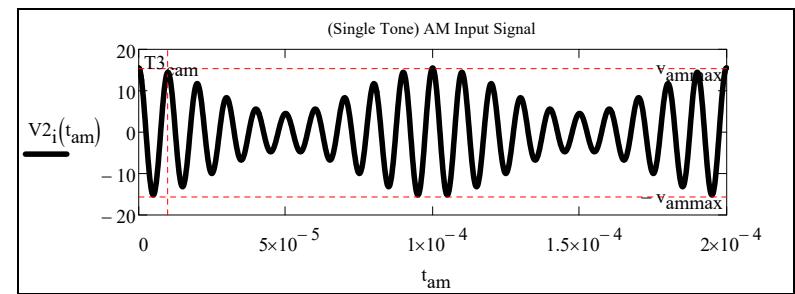


Fig.: (4.2.5.1)

Analog filter Input sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's parameters:

BCSA (Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{\text{am}} := \text{BCSA}(v2_{\text{am}}, rt_{\text{gd}}, N0_{\text{gd}}, 0.0, T_{\text{mam}}) \quad rt_{\text{gd}} = 10\% \quad (4.2.5.2)$$

■ Bandwidth Calculation

$$\begin{aligned}
 N1_{\text{gd}} &:= 2^6 \\
 f_{\text{test}} &= 0.031 \cdot \text{MHz} \quad \boxed{\text{Signal bandwidth: } B_{\text{am}} = 0.77 \cdot \text{MHz}} \\
 \boxed{P_{\text{Parseval}}_{\text{am}} = 115.125 \text{ V}^2} \quad \boxed{A_{\text{am}} = 0 \text{ V}} \quad \boxed{R_{\text{MS}}_{\text{am}} = 7.587 \text{ V}}
 \end{aligned} \quad (4.2.5.3)$$

$$\begin{aligned}
 \text{Sampling frequency: } f_{\text{sam}} &= \frac{1}{T_{\text{am}}} \geq 2 \cdot f_1 \quad \ell := 0..N1_{\text{gd}} - 1 \quad N1_{\text{gd}} = 64 \\
 \text{Chosen sampling frequency (Nyquist rate): } f_{\text{sam}} &:= 2 \cdot B_{\text{am}} \quad f_{\text{sam}} = 1.54 \cdot \text{MHz} \quad (4.2.5.4) \\
 \text{Modulated signal bandwidth: } B_{\text{am}} &= 770 \cdot \text{kHz} \\
 \text{sampling angular frequency: } \omega_{\text{sam}} &:= 2 \cdot \pi \cdot f_{\text{sam}} \cdot \quad \omega_{\text{sam}} = 9.676 \cdot \frac{\text{Mrads}}{\text{sec}}, \\
 \text{sampling period: } T_{\text{sam}} &:= \frac{1}{f_{\text{sam}}}, \quad T_{\text{sam}} = 0.649 \cdot \mu\text{s}, \\
 \text{sampling time step: } namk &:= \frac{k}{f_{\text{sam}}},
 \end{aligned}$$

$$N_0 \cdot \frac{T_{\text{sam}}}{T_{\text{test}}} = 5.195$$

$$\text{nam}^T = \begin{array}{|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & \dots \\ \hline 0 & 0 & 0.649 & 1.299 & 1.948 & \dots & \dots \\ \hline \end{array} \cdot \mu\text{s}$$

Sampling:

$$u_{-7k} := \frac{v_{2\text{am}}(\text{nam}_k)}{\text{volt}} \quad (4.2.5.2)$$

$$t_{\text{am}} := 0 \cdot T_{3\text{cam}}, 0 \cdot T_{3\text{cam}} + \frac{18 \cdot T_{3\text{cam}}}{1000}, \dots, 18 \cdot T_{3\text{cam}}$$

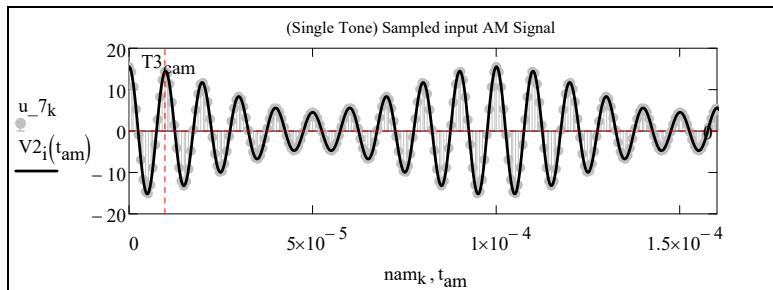


Fig.: (4.2.5.2)

Approximate signal reconstruction according to the Shannon sampling theorem:

$$\omega_{\text{sh}7} := 2 \cdot \pi \cdot B_{\text{am}} \quad \text{sh7}(t) := \sum_{n=0}^{N_0 \cdot g_d - 1} (u_{-7n} \cdot \text{sinc}(\omega_{\text{sh}7} \cdot t - n \cdot \pi)) \quad (4.2.5.3)$$

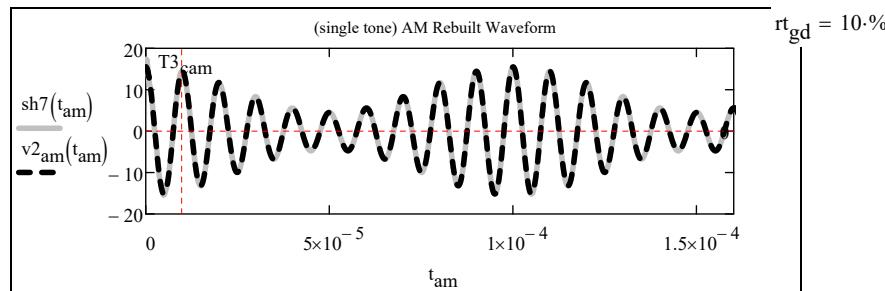


Fig.: (4.2.5.3)

$$0.2 \cdot f_{\text{sam}} = 308000 \frac{1}{\text{s}}$$

$$\text{Spec}_7 := \text{fft}(u_{-7})$$

$$(4.2.5.4)$$

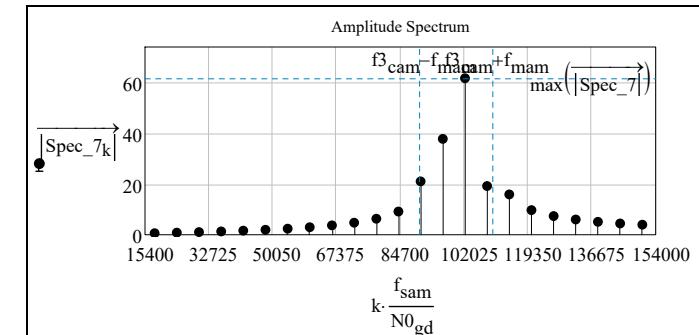


Fig.: (4.2.5.4)

Exact output:

$$v_{\text{osin}}(t) = A_0 \cdot \left(V2_i(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V2_i(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.5.5)$$

Single tone AM filter output calculation

$$\omega_{1\text{cm}} := (\omega_{3\text{cam}} + \omega_{1\text{mam}}) \quad \omega_{2\text{cm}} := (\omega_{3\text{cam}} - \omega_{1\text{mam}})$$

$$C22 := B1 \cdot \frac{1}{2} \left(\frac{1}{\tau_0^2 \cdot \omega_{1\text{cm}}^2 + 1} + \frac{1}{\tau_0^2 \cdot \omega_{2\text{cm}}^2 + 1} \right) + \frac{A1}{\tau_0^2 \cdot \omega_{3\text{cam}}^2 + 1}$$

Filter's output signal:

$$v_{\text{osin}}(t) := A_0 \cdot \left[A1 \cdot \cos(\omega_{3\text{cam}} \cdot t) + \frac{B1}{2} \cdot \cos[(\omega_{1\text{cm}}) \cdot t] + \frac{B1}{2} \cdot \cos[(\omega_{2\text{cm}}) \cdot t] \right. \\ \left. + (-1) \cdot \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \left[B1 \cdot \left[\frac{\tau_0 \cdot (\cos(t \cdot \omega_{1\text{cm}}) + \tau_0 \cdot \omega_{1\text{cm}} \cdot \sin(t \cdot \omega_{1\text{cm}}))}{[2 \cdot (\tau_0^2 \cdot \omega_{1\text{cm}}^2 + 1)]} \dots \right] \dots \right] \cdot e^{\frac{t}{\tau_0}} \dots \right. \\ \left. + (-1) \cdot \frac{\tau_0 \cdot (\cos(t \cdot \omega_{2\text{cm}}) + \tau_0 \cdot \omega_{2\text{cm}} \cdot \sin(t \cdot \omega_{2\text{cm}}))}{[2 \cdot (\tau_0^2 \cdot \omega_{2\text{cm}}^2 + 1)]} \dots \right] \cdot e^{\frac{t}{\tau_0}} \dots \\ \left. + \frac{A1 \cdot (\omega_{3\text{cam}} \cdot \sin(t \cdot \omega_{3\text{cam}}) \cdot \tau_0^2 + \cos(t \cdot \omega_{3\text{cam}}) \cdot \tau_0)}{\tau_0^2 \cdot \omega_{3\text{cam}}^2 + 1} \right. \\ \left. + (-1) \cdot \tau_0 \cdot C22 \right]$$

Approximated output:

$$v_{\text{osind}}(t) := -\tau_f \frac{d}{dt} V2_i(t) \quad \text{holds for } \omega \ll \omega_0 \text{ or } T \gg (2 \cdot \pi \cdot \tau) \quad (4.2.5.6)$$

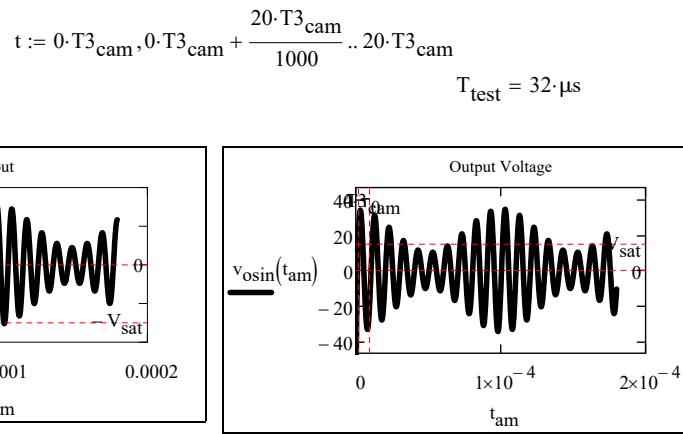


Fig.: (4.2.5.5)

Fig.: (4.2.5.6)

Output sampling: $Osin_k := v_{\text{oscin}}(n \cdot am_k)$ (4.2.5.7)

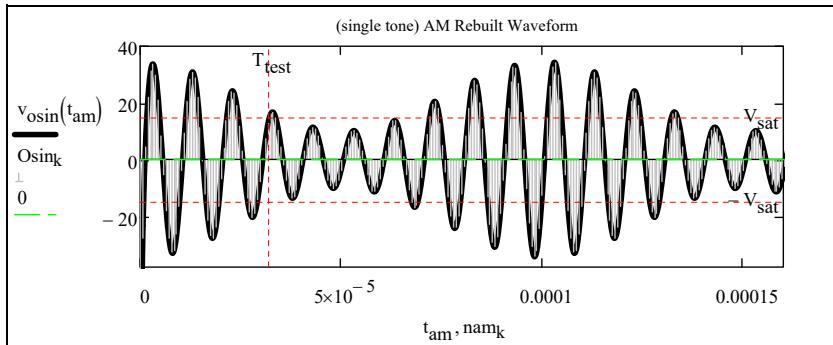


Fig.: (4.2.5.7)

Carrier frequency: $f_{3\text{cam}} = 100 \cdot \text{kHz}$ $\frac{f_{\text{sam}}}{f_{3\text{cam}}} = 15.4$

Fourier Transform of the output signal $O\text{Specsin} := \text{fft}(Osin)$ (4.2.5.8)

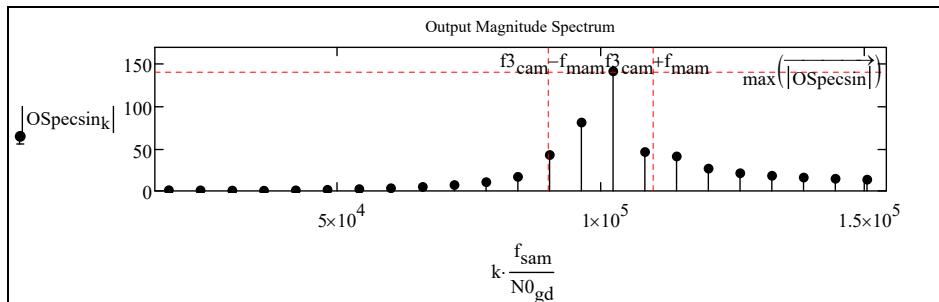


Fig.: (4.2.5.8)

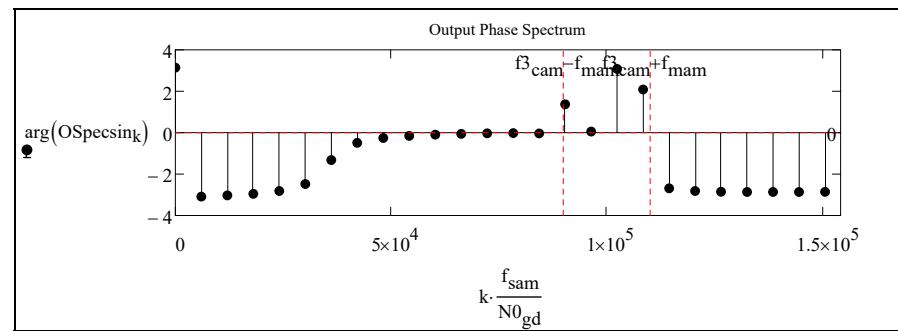


Fig.: (4.2.5.9)

Output signal reconstruction according to the Shannon sampling theorem:

$$\omega_{\text{sh7o}} := 2 \cdot \pi \cdot B_{\text{am}} \quad sh7o(t) := \sum_{n=0}^{\lceil N_0 g_d - 1 \rceil} (Osin_n \cdot \text{sinc}(\omega_{\text{sh7o}} \cdot t - n \cdot \pi)) \quad (4.2.6.10)$$

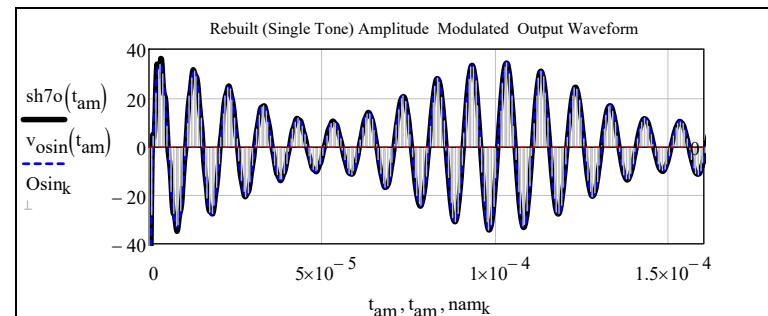


Fig.: (4.2.5.10)

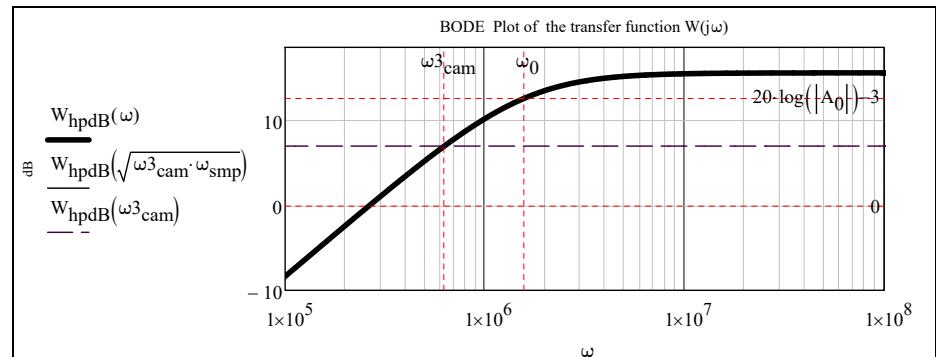


Fig.: (4.2.5.11)

ANALOG FILTER OUTPUT ANALYSIS

4.2.6) (Single tone) Frequency Modulated carrier response

Defined in "FM data.xmcd.xmcd"

Carrier max amplitude:

$$A_{fm} = 0.2 \text{ V}$$

Modulating single tone max amplitude:

$$B_{fmm} = 15 \text{ V}$$

Carrier pulsation:

$$\omega_{cfm} = 18.85 \cdot \frac{\text{Mrads}}{\text{s}}$$

Carrier frequency:

$$f_{cfm} = 3 \cdot \text{MHz}$$

Carrier period:

$$T_{cfm} = 0.333 \cdot \mu\text{s}$$

Modulating single tone pulsation:

$$\omega_{fmm} = 942.478 \cdot \frac{\text{krad}}{\text{s}}$$

Modulating single tone frequency:

$$f_{fmm} = 150 \cdot \text{kHz}$$

Modulating single tone period:

$$T_{fmm} = 6.667 \cdot \mu\text{s}$$

$$\text{frequency modulation index: } m_{fm} = \frac{2 \cdot Kst \cdot \pi \cdot B}{\omega_m} \quad m_{fm} = 8 \quad (4.2.6.1)$$

$$V_{fm}(t) := v_{fmsl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd}) \quad (4.2.6.2)$$

$$v_{fmsl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd}) = \operatorname{Re} \left[A_{fm} \cdot e^{j \cdot 2 \cdot \pi \cdot f_{cfm} \cdot t} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left(J_n(k, m_{fm}) \cdot e^{j \cdot k \cdot 2 \cdot \pi \cdot f_{fmm} \cdot t} \right) \right]$$

$$V_{fm}(t) = \operatorname{Re} \left[A_{fm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left[J_n(k, m_{fm}) \cdot \left[\cos[2 \cdot \pi \cdot t \cdot (f_{cfm} + f_{fmm} \cdot k)] + \sin[2 \cdot \pi \cdot t \cdot (f_{cfm} + f_{fmm} \cdot k)] \cdot i \right] \right] \right]$$

$$V_{fm}(t) = A_{fm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left[J_n(k, m_{fm}) \cdot \cos[2 \cdot \pi \cdot (f_{cfm} + f_{fmm} \cdot k) \cdot t] \right]$$

$$A_{fm} = 0.2 \text{ V}$$

$$V_i = 2.5 \times 10^{-3} \text{ V}$$

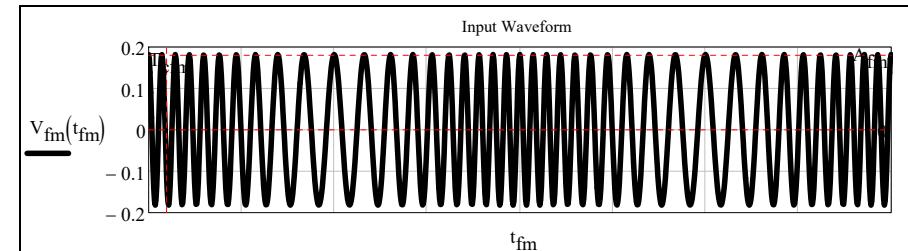


Fig.: (4.2.6.1)

the dimensionless input signal is

$$v2_{fm}(t) := \frac{v_{fmsl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd})}{V} \quad (4.2.6.3)$$

Analog filter Input sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{fm} := \text{BCSA}(v2_{fm}, rt_{gd}, N_{gd}, 0.0, T_{fmm}) \quad rt_{gd} = 10\% \quad (4.2.6.4)$$

Bandwidth Calculation

The function returns a three columns matrix.

The first column contains:

- pos. 0: relative error,
- pos. 1: bandwidth (Dimensionless),
- pos. 2: the nth harmonic number corresponding to the given relative error,
- pos. 3: temporary variable,
- pos. 4: Parseval,
- pos. 5: signal average,
- pos. 6: signal rms.

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

	0	1	2	3
0	0.1	$8.322 \cdot 10^{-8}$	0	0
1	$4.8 \cdot 10^6$	$-2.081 \cdot 10^{-7}$	0	0
2	33	$9.092 \cdot 10^{-7}$	0	0
3	$6.55 \cdot 10^{-4}$	$-3.885 \cdot 10^{-6}$	0	0
4	0.04	$1.56 \cdot 10^{-5}$	0	0
5	$4.161 \cdot 10^{-8}$	$-5.852 \cdot 10^{-5}$	0	0
6	0.141	$2.039 \cdot 10^{-4}$	0	0
7	0	$-6.55 \cdot 10^{-4}$	0	0
8	0	$1.925 \cdot 10^{-3}$	0	0
9	0	$-5.119 \cdot 10^{-3}$	0	0
10	0	0.012	0	0
11	0	-0.025	0	0
12	0	0.045	0	0
13	0	-0.064	0	0

14	0	0.068	0	0
15	0	-0.037	0	...

Fourier series coefficients: $r_{gd} = 10\%$

$$cffaSb_{fm} := Sb_{fm}^{\langle 1 \rangle} \quad cffbSb_{fm} := Sb_{fm}^{\langle 2 \rangle}$$

$$cffaSb_{fm}^T = \begin{array}{|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 8.322 \cdot 10^{-8} & -2.081 \cdot 10^{-7} & 9.092 \cdot 10^{-7} & -3.885 \cdot 10^{-6} & 1.56 \cdot 10^{-5} & ... \\ \hline \end{array}$$

$$cffbSb_{fm}^T = \begin{array}{|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & ... \\ \hline \end{array}$$

$$\text{Signal Bandwidth: } B_{fm} := \frac{(Sb_{fm}^{\langle 0 \rangle})_1}{\text{sec}}$$

$$\text{This is the number of the harmonics for which } r_{gd} = 10\% \quad j_{fm} := (Sb_{fm}^{\langle 0 \rangle})_2 \quad j_{fm} = 33$$

$$\text{Frequency of the pulse train: } T_{fm} := (Sb_{fm}^{\langle 0 \rangle})_3 \quad T_{fm} = 6.55 \times 10^{-4}$$

$$\text{Parseval}_{fm} := (Sb_{fm}^{\langle 0 \rangle})_4 \cdot V^2 \quad \text{Average}_{fm} := (Sb_{fm}^{\langle 0 \rangle})_5 \cdot \text{volt} \quad \text{RMS}_{fm} := (Sb_{fm}^{\langle 0 \rangle})_6 \cdot \text{volt}$$

$$j_{3fm} := 0.. \text{rows}(cffaSb_{fm}) - 1$$

Bandwidth Calculation

$$f_{test} = 0.031 \cdot \text{MHz} \quad \boxed{\text{Signal bandwidth: } B_{fm} = 4.8 \cdot \text{MHz}} \quad (4.2.6.5)$$

$$\boxed{\text{Parseval}_{fm} = 0.04 \text{V}^2}$$

$$\boxed{\text{Average}_{fm} = 0 \text{V}}$$

$$\boxed{\text{RMS}_{fm} = 0.141 \text{V}}$$

$$\text{Sampling frequency: } f_{sfm} = \frac{1}{T_{fm}} \geq 2 \cdot f_1$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{sfm} := 2 \cdot B_{fm} \quad f_{sfm} = 9.6 \times 10^{-3} \cdot \text{GHz} \quad (4.2.6.6)$$

$$\text{Carson bandwidth: } \text{Cars1} := 2 \cdot f_{cfm} \cdot (m_{fm} + 1) \quad \text{Cars1} = 54 \cdot \text{MHz} \quad (4.2.6.7)$$

$$\text{sampling angular frequency: } \omega_{sfm} := 2 \cdot \pi \cdot f_{sfm} \cdot \quad \omega_{sfm} = 0.06 \cdot \frac{\text{Grads}}{\text{sec}}, \quad (4.2.6.8)$$

$$\text{sampling period: } T_{sfm} := \frac{1}{f_{sfm}}, \quad T_{sfm} = 0.104 \cdot \mu\text{s},$$

$$\text{sampling time step: } nfm_k := \frac{k}{f_{sfm}}, \quad (4.2.6.9)$$

$$\frac{N_0 g_d}{f_{sfm}} \cdot \frac{1}{T_{cfm}} = 80$$

$$nfm^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0.104 & 0.208 & 0.313 & 0.417 & 0.521 & 0.625 & ... \\ \hline \end{array} \cdot \mu\text{s} \quad (4.2.6.10)$$

$$\frac{\omega_{cfm}}{\omega_{fmm}} = 20$$

$$A_{fm} = 0.2 \text{V} \quad u_{8k} := \frac{V_{fm}(nfm_k)}{\text{volt}} \quad V_i = 2.5 \times 10^{-3} \text{V} \quad (4.2.6.11)$$

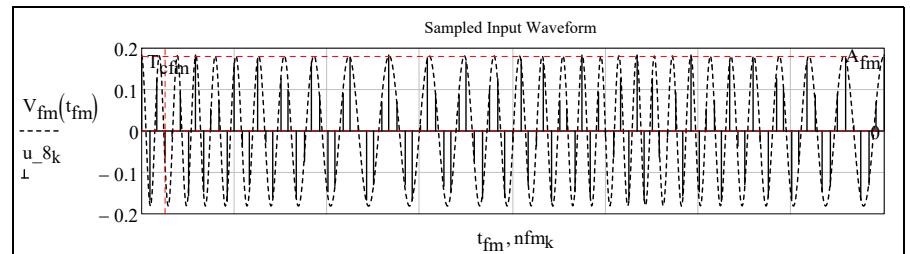


Fig.: (4.2.6.2)

Input signal reconstruction according to the Shannon sampling theorem:

$$r_{gd} = 10\% \quad \boxed{\omega_{sh8} := 2 \cdot \pi \cdot B_{fm}} \quad sh8(t) := \sum_{n=0}^{N_0 g_d - 1} (u_{8n} \cdot \text{sinc}(\omega_{sh8} \cdot t - n \cdot \pi)) \quad (4.2.6.12)$$

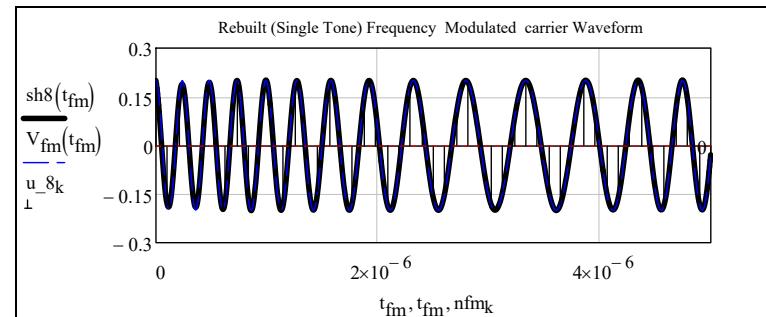


Fig.: (4.2.6.3)

Exact output

$$\text{The filter cutoff frequency is } f_0 = 0.25 \cdot \text{MHz} \quad V_i = 2.5 \times 10^{-3} \text{V}$$

$$\text{Exact output: } v_{ofm}(t) = A_0 \cdot \left(V_{fm}(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V_{fm}(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.6.13)$$

Calculation of the single tone FM filter Output

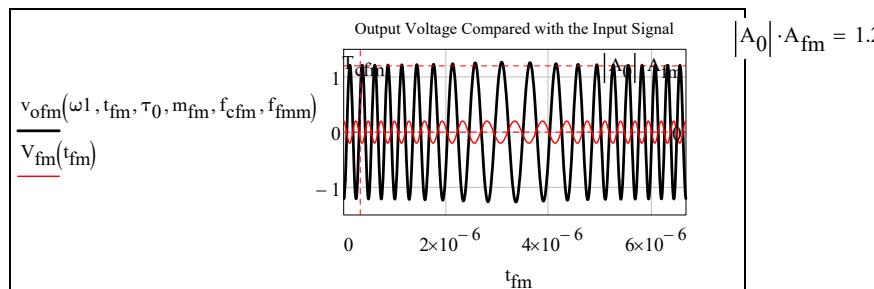
$$\omega_1(k) := 2\pi \cdot (f_{cfm} + f_{fmm} \cdot k)$$

$$v_{ofm}(\omega_1, t, \tau_0, m_{fm}, f_{cfm}, f_{fmm}) := A_0 \cdot A_{fm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left[J_n(k, m_{fm}) \cdot \cos(\omega_1(k) \cdot t) \dots - \frac{t}{\tau_0} \right] \\ + \frac{\left(\cos(\omega_1(k) \cdot t) \dots + 2 \cdot \tau_0 \cdot \omega_1(k) \cdot \sin(t \cdot \omega_1(k)) \right) - e^{-\frac{t}{\tau_0}}}{\tau_0^2 \cdot \omega_1(k)^2 + 1}$$

Approximated output: $v_{ofma}(t) = \frac{A_0}{\omega_0} \cdot \frac{d}{dt} V_{fm}(t)$ $T_{test} = 32 \mu s$ $A_{fm} = 0.2 V$

Calculation of the frequency modulated Approximated Output

$$v_{ofma}(\omega_1, t, m_{fm}, f_{cfm}, f_{fmm}, f_0) := \frac{-A_{fm} \cdot A_0}{\omega_0} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left[J_n(k, m_{fm}) \cdot \sin \left[2\pi \cdot t \cdot (f_{cfm} + f_{fmm} \cdot k) \right] \cdot \omega_1(k) \right]$$



Voltage gain: $A_0 = -6$

Fig.: (4.2.6.4)

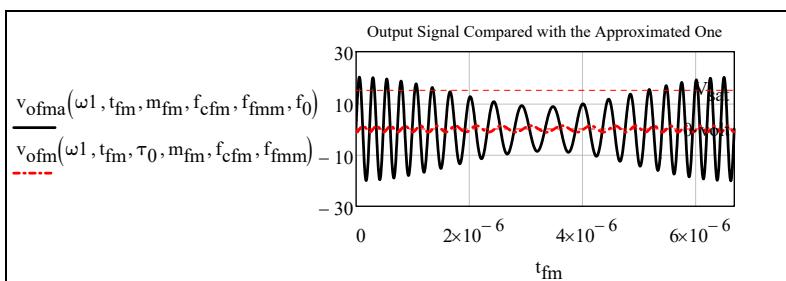


Fig.: (4.2.6.5)

Output sampling: $O_{fmk} := \frac{v_{ofm}(\omega_1, n_{fmk}, \tau_0, m_{fm}, f_{cfm}, f_{fmm})}{volt}$ (4.2.6.14)

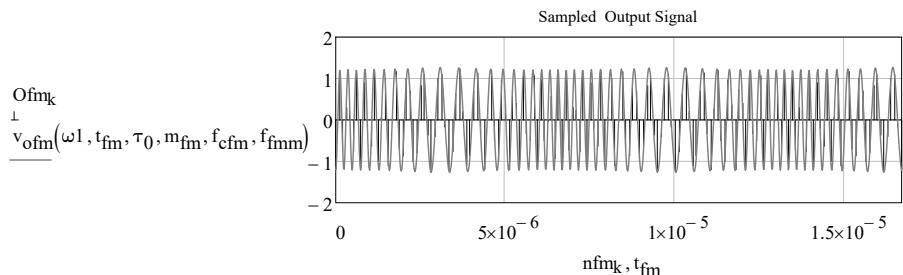


Fig.: (4.2.6.6)

$$\omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}} \quad f_{cfm} = 3 \times 10^{-3} \cdot \text{GHz} \quad \frac{f_{sfm}}{f_{cfm}} = 3.2 \quad m_{fm} = 8$$

Fourier Transform of the output signal $O_{Specfm} := fft(O_{fm})$ (4.2.6.15)

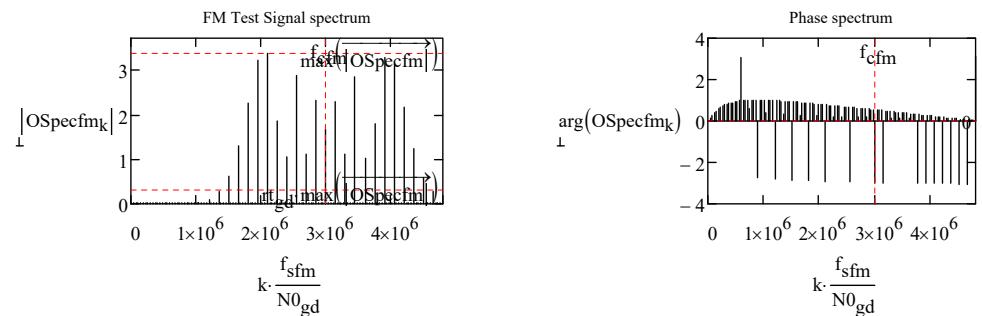


Fig.: (4.2.6.7)

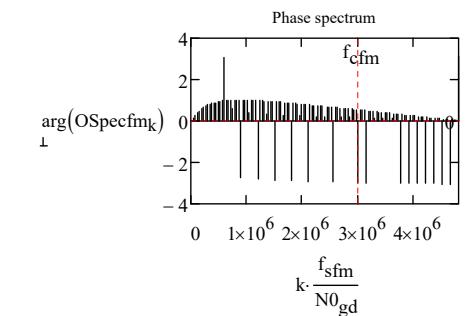


Fig.: (4.2.6.8)

the dimensionless output signal is: $v_{2ofm}(t) := \frac{v_{ofm}(\omega_1, t, \tau_0, m_{fm}, f_{cfm}, f_{fmm})}{V}$ (4.2.6.3)

Analog filter Output sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's parameters:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$S_{bfm} := BCSA(v_{2ofm}, rt_{gd}, N_{gd}, 0.0, T_{fmm}) \quad rt_{gd} = 10\% \quad (4.2.6.4)$$

The function returns a three columns matrix.

The first column contains:

pos. 0: relative err

pos. 1: bandwidth (Dimensionless),

pos. 2: the nth. harmonic number corresponding to the given relative error,

- pos. 3: temporary variable,
 pos. 4: Parseval,
 pos. 5: signal average,
 pos. 6: signal rms.

The second column contains the coefficients a_k of the Fourier series,
 the third column contains the coefficients b_k of the Fourier series.

	0	1	2	3
S _{b_ofm} =	0	0.1	$8.07 \cdot 10^{-4}$	0
	1	$4.8 \cdot 10^6$	$5.963 \cdot 10^{-4}$	$3.576 \cdot 10^{-4}$
	2	33	$3.234 \cdot 10^{-4}$	$3.92 \cdot 10^{-4}$
	3	$3.954 \cdot 10^{-3}$	$2.185 \cdot 10^{-4}$	$3.628 \cdot 10^{-4}$
	4	1.517	$1.207 \cdot 10^{-5}$	$2.204 \cdot 10^{-4}$
	5	$4.035 \cdot 10^{-4}$	$4.67 \cdot 10^{-4}$	$4.531 \cdot 10^{-4}$
	6	0.871	$-1.253 \cdot 10^{-3}$	$-4.225 \cdot 10^{-4}$
	7	0	$4.184 \cdot 10^{-3}$	$1.953 \cdot 10^{-3}$
	8	0	-0.012	$-4.45 \cdot 10^{-3}$
	9	0	0.032	0.011
	10	0	-0.075	-0.024
	11	0	0.155	0.045
	12	0	-0.273	-0.073
	13	0	0.391	0.097
	14	0	-0.411	-0.095
	15	0	0.226	0.049
...				

Bandwidth Calculation

$$f_{\text{test}} = 0.031 \cdot \text{MHz} \quad \text{Signal bandwidth: } B_{\text{ofm}} = 4.8 \cdot \text{MHz} \quad (4.2.6.5)$$

$$\text{Parseval}_{\text{ofm}} = 1.517 \text{ V}^2 \quad \text{Average}_{\text{ofm}} = 0 \text{ V} \quad \text{RMS}_{\text{ofm}} = 0.871 \text{ V}$$

$$\text{Sampling frequency: } f_{\text{sofm}} = \frac{1}{T_{\text{ofm}}} \geq 2 \cdot f_{\text{l}}$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{\text{sofm}} := 2 \cdot B_{\text{ofm}} \quad f_{\text{sofm}} = 9.6 \times 10^{-3} \cdot \text{GHz} \quad (4.2.6.6)$$

$$\text{Carson bandwidth: } \text{Cars1} := 2 \cdot \omega_{\text{cfm}} \cdot (m_{\text{fm}} + 1) \quad \text{Cars1} = 0.339 \cdot \frac{\text{Grads}}{\text{sec}} \quad (4.2.6.7)$$

$$\text{sampling angular frequency: } \omega_{\text{sofm}} := 2 \cdot \pi \cdot f_{\text{sofm}} \cdot \omega_{\text{sofm}} = 0.06 \cdot \frac{\text{Grads}}{\text{sec}}, \quad (4.2.6.8)$$

$$\text{sampling period: } T_{\text{sofm}} := \frac{1}{f_{\text{sofm}}}, \quad T_{\text{sofm}} = 0.104 \cdot \mu\text{s},$$

$$\text{sampling time step: } \text{nofmk} := \frac{k}{f_{\text{sofm}}}, \quad (4.2.6.9)$$

$$\frac{N_0 \text{gd}}{f_{\text{sofm}}} \cdot \frac{1}{T_{\text{cfm}}} = 80$$

$$\text{nofmk}^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 0.104 & 0.208 & 0.313 & 0.417 & 0.521 & 0.625 & 0.729 & \dots \end{bmatrix} \cdot \mu\text{s}$$

$$\frac{\omega_{\text{cfm}}}{\omega_{\text{fmm}}} = 20 \quad (4.2.6.10)$$

$$A_{\text{fm}} = 0.2 \text{ V} \quad u_{\text{8ok}} := \frac{v2o_{\text{fm}}(\text{nofmk})}{\text{volt}} \quad V_i = 2.5 \times 10^{-3} \text{ V} \quad (4.2.6.11)$$

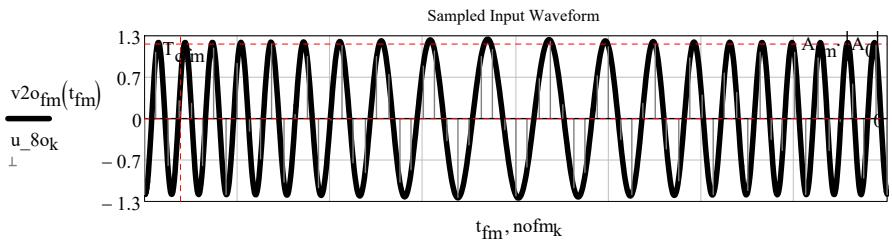


Fig.: (4.2.6.2)

Input signal reconstruction according to the Shannon sampling theorem:

$$rt_{\text{gd}} = 10\% \quad \omega_{\text{sh80}} := 2 \cdot \pi \cdot B_{\text{ofm}} \quad \text{sh80}(t) := \left[\sum_{n=0}^{N_0 \text{gd} - 1} (u_{\text{8ok}} \cdot \text{sinc}(\omega_{\text{sh80}} \cdot t - n \cdot \pi)) \right] \quad (4.2.6.12)$$

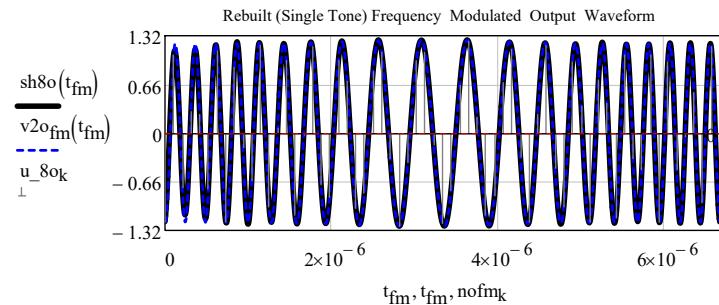


Fig.: (4.2.6.3)



On the other hand if the carrier frequency (or test frequency) is located in the passing band, the filter response is:

$$\begin{aligned} \omega^2_{\text{cfm}} &:= 100 \cdot \omega_{\text{cfm}} & \omega^2_{\text{cfm}} &= 1.885 \cdot \frac{\text{Grads}}{\text{s}} & f^2_{\text{cfm}} &:= \frac{\omega^2_{\text{cfm}}}{2 \cdot \pi} & f^2_{\text{fmm}} &:= 100 \cdot f_{\text{fmm}} \\ T^2_{\text{cfm}} &:= \frac{1}{f^2_{\text{cfm}}} & f^2_{\text{sfm}} &:= f_{\text{sfm}} & T^2_{\text{fmm}} &:= \frac{1}{f^2_{\text{fmm}}} \end{aligned}$$

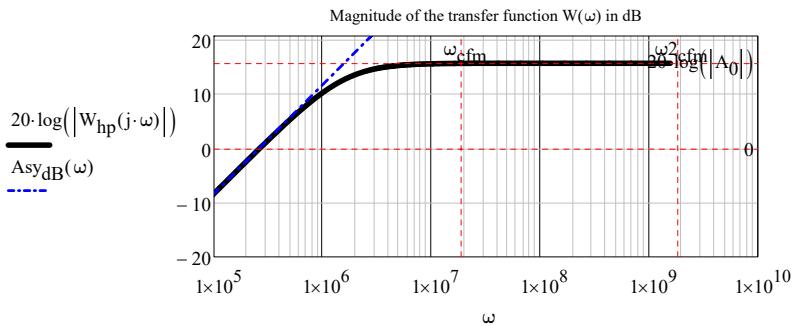


Fig.: (4.2.6.9)

$$\text{Exact output: } v_{1\text{ofm}}(t) = A_0 \left(V1_{\text{fm}}(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V1_{\text{fm}}(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.6.16)$$

$$\text{Approximated output: } v_{1\text{ofma}}(t) = \frac{A_0}{\omega_0} \cdot \frac{d}{dt} V1_{\text{fm}}(t) \quad (4.2.6.17)$$

$$V1_{\text{fm}}(t) := v_{\text{fmssl}}(t, f2_{\text{cfm}}, f2_{\text{fmm}}, A_{\text{fm}}, m_{\text{fm}}, N_{\text{gd}})$$

$$\omega_2(k) := 2 \cdot \pi \cdot (f2_{\text{cfm}} + f2_{\text{fmm}} \cdot k)$$

$$v_{1\text{ofm}}(t) := v_{\text{ofm}}(\omega_2, t, \tau_0, m_{\text{fm}}, f2_{\text{cfm}}, f2_{\text{fmm}})$$

$$t1_{\text{fm}} := T2_{\text{fmm}} \cdot 0, T2_{\text{fmm}} \cdot 0 + \frac{80 \cdot T2_{\text{fmm}} - 0 \cdot T2_{\text{fmm}}}{20000} \dots 80 \cdot T2_{\text{fmm}}$$

$$T1_{\text{cfm}} := \frac{2 \cdot \pi}{100 \cdot \omega_0} \quad \frac{1}{f_{\text{sfm}}} = 0.104 \cdot \mu\text{s} \quad T2_{\text{cfm}} = 3.333 \cdot \text{ns}$$

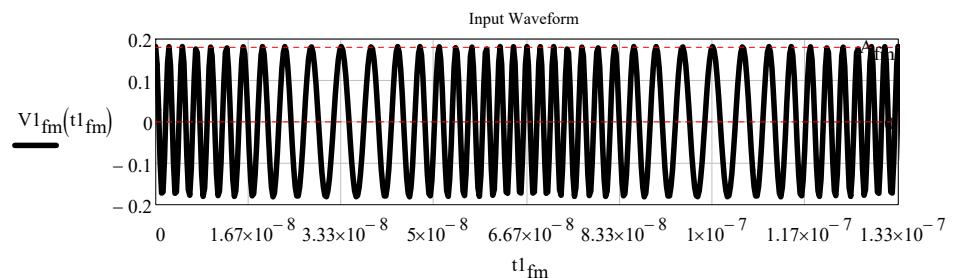


Fig.: (4.2.6.10)

$$A_0 = -6$$

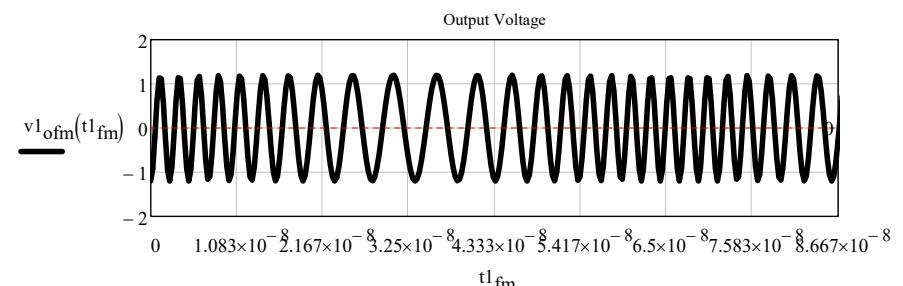


Fig.: (4.2.6.11)

Sampling of the output:

$$n1fm_k := \frac{k}{N0_{\text{gd}}} \cdot T1_{\text{cfm}} \cdot 1 \quad \text{Output sampling: } O1fm_k := \frac{v1_{\text{ofm}}(n1fm_k)}{\text{volt}} \quad (4.2.6.18)$$

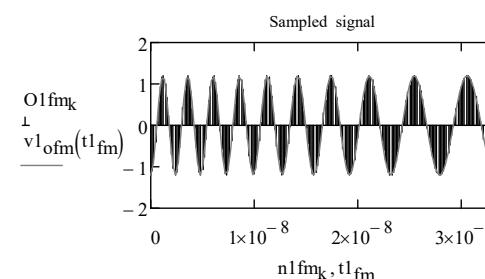


Fig.: (4.2.6.12)

$$\omega_{\text{fmm}} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}} \quad f2_{\text{cfm}} = 300 \cdot \text{MHz} \quad \frac{f_{\text{sfm}}}{f2_{\text{cfm}}} = 0.032 \quad m_{\text{fm}} = 8$$

$$\text{Fourier Transform of the output signal } O1\text{Specfm} := \text{fft}(O1fm) \quad (4.2.6.19)$$

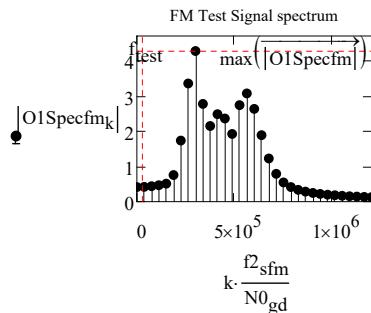


Fig.(4.2.6.13)

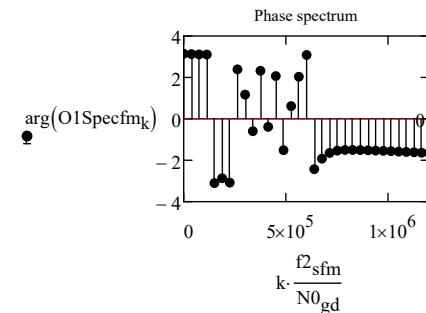


Fig.(4.2.6.14)

$$\omega := \frac{\omega_0}{20 \cdot U_0}, \frac{\omega_0}{20 \cdot U_0} + \frac{\omega_0 \cdot U_0 - \frac{\omega_0}{20 \cdot U_0}}{U_0^2} \dots 10 \cdot U_0 \cdot \omega_0 \quad \omega_{\text{cfm}} = 1.885 \frac{\text{Grads}}{\text{s}}$$

$$W_{\text{db}}\omega_1 = 20 \cdot \log(|W_{\text{hp}}(j \cdot \omega_{\text{cfm}})|) \quad W_{\text{db}}\omega_1 = 15.563 \text{ dB} \quad (4.2.6.20)$$

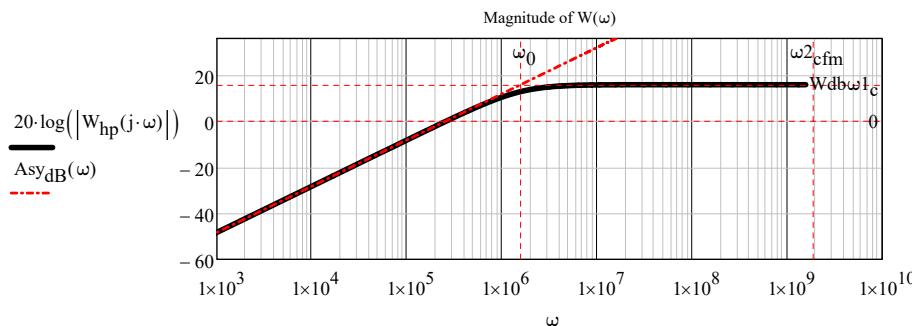


Fig.(4.2.6.15)

ANALOG FILTER OUTPUT ANALYSIS

4.2.7) (Single tone) Phase Modulated carrier response

Defined in "PM data.xmcd.xmcd"

Carrier max amplitude:

$$A_{\text{pm}} = 20 \text{ V}$$

Modulating single tone max amplitude:

$$B_{\text{pm}} = 8 \text{ V}$$

Carrier pulsation:

$$\omega_{\text{epm}} := 4 \cdot \omega_{\text{test}}$$

Carrier frequency:

$$f_{\text{cpm}} := \frac{\omega_{\text{cpm}}}{2 \cdot \pi}$$

Carrier period:

$$T_{\text{epm}} := \frac{1}{f_{\text{cpm}}} \quad T_{\text{cpm}} = 8 \cdot \mu\text{s}$$

Modulating single tone pulsation:

$$\omega_{\text{pmm}} := \frac{\omega_{\text{cpm}}}{20}$$

Modulating single tone frequency:

$$f_{\text{pmm}} := \frac{\omega_{\text{pmm}}}{2 \cdot \pi} \quad f_{\text{pmm}} = 6.25 \times 10^{-3} \text{ MHz}$$

Modulating single tone period:

$$T_{\text{pmm}} := \frac{1}{f_{\text{pmm}}} \quad T_{\text{pmm}} = 160 \cdot \mu\text{s}$$

frequency modulation index:

$$m_{\text{pm}} := 8 \quad m_{\text{pm}} = \frac{2 \cdot K_{\text{st}} \cdot \pi \cdot B_{\text{pm}}}{\omega_{\text{pmm}}}$$

Carson bandwidth:

$$\text{Cars4} := 2 \cdot \omega_{\text{pmm}} \cdot (m_{\text{pm}} + 1)$$

the dimensionless input signal is

$$v_2^{\text{pm}}(t) := V_9^{\text{pm}}(t, f_{\text{cpm}}, f_{\text{pmm}}, A_{\text{pm}}, m_{\text{pm}}, N_{\text{gd}}) \quad (4.2.7.1)$$

$$t_{\text{pm}} := 0 \cdot T_{\text{pmm}}, \frac{8 \cdot T_{\text{pmm}}}{10000} \dots 8 \cdot T_{\text{pmm}}$$

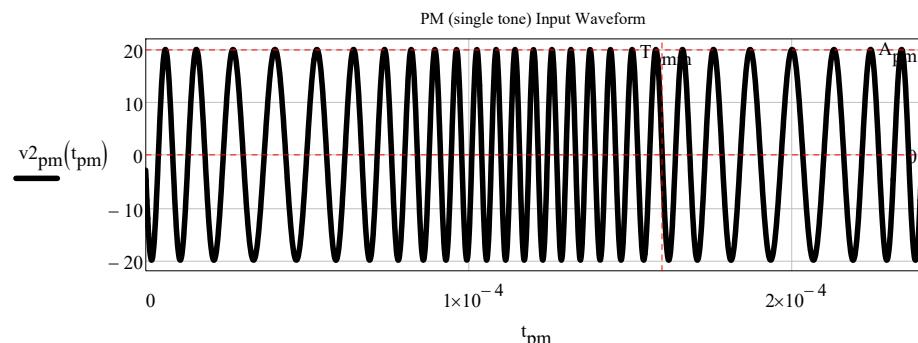


Fig.(4.2.7.1)

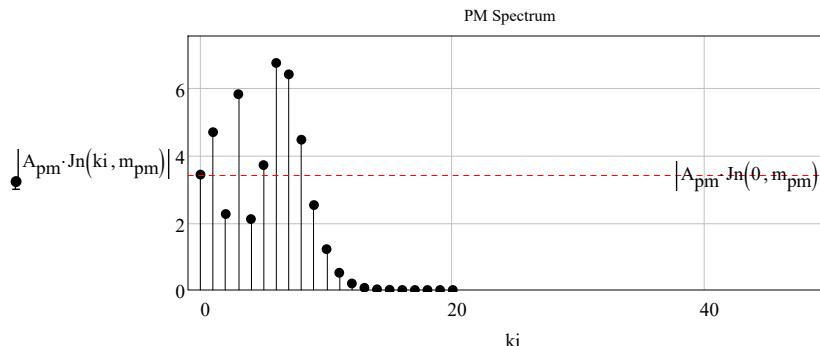


Fig.: (4.2.7.2)

Analog filter Input sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$S_{bpm} := \text{BCSA}(v_{2pm}, rt_{gd}, N_{gd}, 0.0, T_{pmm}) \quad rt_{gd} = 10\% \quad (4.2.7.2)$$

The function returns a three columns matrix.

The first column contains:

- pos. 0: relative error,
- pos. 1: bandwidth (Dimensionless),
- pos. 2: the nth harmonic number corresponding to the given relative error,
- pos. 3: temporary variable,
- pos. 4: Parseval,
- pos. 5: signal average,
- pos. 6: signal rms.

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

	0	1	2	3
0	0.1	$8.322 \cdot 10^{-6}$	0	0
1	$2 \cdot 10^5$	$-1.355 \cdot 10^{-14}$	$2.081 \cdot 10^{-5}$	0
2	33	$-9.092 \cdot 10^{-5}$	$1.253 \cdot 10^{-14}$	0
3	0.065	$9.045 \cdot 10^{-15}$	$-3.885 \cdot 10^{-4}$	0
4	400	$1.56 \cdot 10^{-3}$	$5.42 \cdot 10^{-15}$	0
5	$4.161 \cdot 10^{-6}$	$1.95 \cdot 10^{-14}$	$5.852 \cdot 10^{-3}$	0
6	14.142	-0.02	$-1.636 \cdot 10^{-14}$	0
7	0	$1.942 \cdot 10^{-15}$	-0.065	0
8	0	0.192	$-4.932 \cdot 10^{-15}$	0
9	0	$-2.978 \cdot 10^{-14}$	0.512	0
10	0	-1.215	$3.115 \cdot 10^{-14}$	0
11	0	$2.541 \cdot 10^{-14}$	-2.526	0
12	0	4.469	$-2.474 \cdot 10^{-14}$	0
13	0	$3.634 \cdot 10^{-15}$	6.412	0
14	0	-6.752	$-7.654 \cdot 10^{-15}$	0
15	0	$-1.592 \cdot 10^{-14}$	-3.715	...

Bandwidth Calculation

$$f_{test} = 0.031 \cdot \text{MHz} \quad \boxed{\text{Signal bandwidth: } B_{pm} = 0.2 \cdot \text{MHz}} \quad (4.2.7.3)$$

$$\boxed{\text{Parseval}_{pm} = 400 \text{ V}^2}$$

Sampling frequency:

$$f_{spm} = \frac{1}{T_{pm}} \geq 2 \cdot f_1$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{spm} := 2 \cdot B_{pm} \quad f_{spm} = 4 \times 10^{-4} \cdot \text{GHz} \quad (4.2.7.4)$$

$$\text{Carson bandwidth: } \boxed{\text{Cars1} := 2 \cdot \omega_{cpm} \cdot (m_{pm} + 1)} \quad \text{Cars1} = 14.137 \cdot \frac{\text{Mrads}}{\text{sec}} \quad (4.2.7.5)$$

$$\text{sampling angular frequency: } \omega_{spm} := 2 \cdot \pi \cdot f_{spm} \quad \omega_{spm} = 2.513 \times 10^{-3} \cdot \frac{\text{Grads}}{\text{sec}}$$

(4.2.7.6)

$$\text{sampling period: } T_{spm} := \frac{1}{f_{spm}}, \quad T_{spm} = 2.5 \cdot \mu\text{s},$$

$$\text{sampling time step: } npm_k := \frac{k}{f_{spm}}, \quad (4.2.7.7)$$

$$V_{pm}(\tau) = A_{pm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left(J_n(k, m_{pm}) \cdot \cos(\tau \cdot k \cdot \omega_{pmm}) \cdot \cos\left(\frac{\pi \cdot k}{2} + \tau \cdot \omega_{cpm}\right) \right) \quad (4.2.7.8)$$

$$A_{pm} = 20 \text{ V} \quad u_{9k} := \frac{v_{pm}(npm_k, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})}{volt} \quad (4.2.7.9)$$

$$V_{pm}(t) := v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})$$

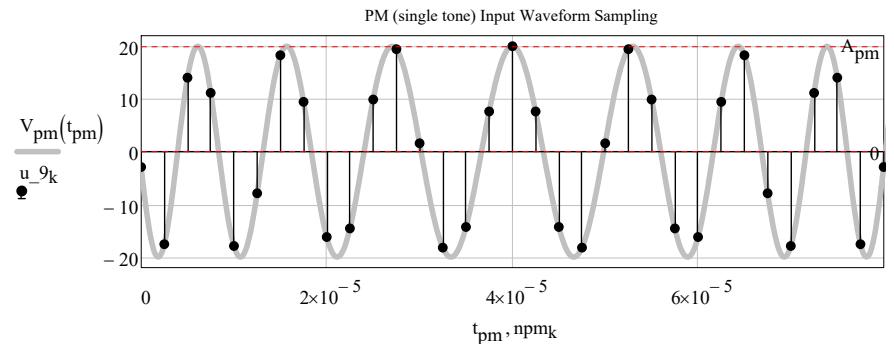


Fig.: (4.2.7.3)

Input signal reconstruction according to the Shannon sampling theorem:

$$r_{gd} = 10\% \quad \omega_{sh9} := 2 \cdot \pi \cdot B_{pm} \quad sh9(t) := \left[\sum_{n=0}^{N_0 gd^{-1}} (u_{-9n} \cdot \text{sinc}(\omega_{sh9} \cdot t - n \cdot \pi)) \right] \quad (4.2.7.10)$$

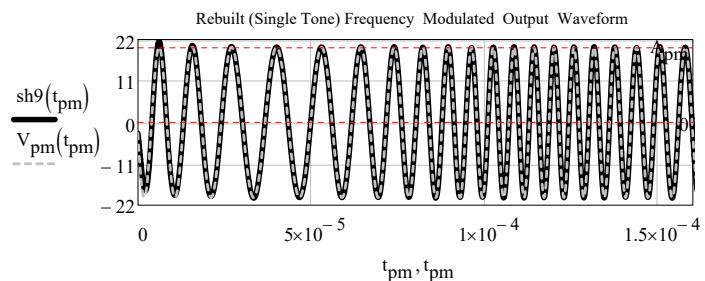


Fig.: (4.2.7.4)

$$\text{Exact output: } v_{opm}(t) = A_0 \cdot \left(V_{pm}(t) - \frac{e^{-\frac{t}{\tau_0}}}{\tau_0} \cdot \int_0^t V_{pm}(\tau) \cdot e^{\frac{\tau}{\tau_0}} d\tau \right) \quad (4.2.7.11)$$

► Output filter PM signal calculations

$$\begin{aligned} \omega_k(k) &= \omega_{cpm} + k \cdot \omega_{pmm} & \omega_\mu(k) &= \omega_{cpm} - k \cdot \omega_{pmm} \\ \theta_k(t, k) &= \frac{\pi \cdot k}{2} + \omega_k(k) \cdot t & \theta_\mu(t, k) &= \frac{\pi \cdot k}{2} + \omega_\mu(k) \cdot t \\ v_{opm}(t) &= A_0 \left[V_{pm}(t) \dots \right. \\ &\quad \left. + \frac{-e^{-\frac{t}{\tau_4}}}{\tau_4} \cdot A_{pm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} J_{n(k, m_{pm})} \cdot \left[\frac{\tau_4}{2} \left[\begin{array}{l} \left[\cos(\theta_k(t, k)) \dots \right. \\ \left. + \omega_k(k) \cdot \tau_4 \cdot \sin(\theta_k(t, k)) \right] \dots \end{array} \right] \dots \right] \cdot e^{\frac{t}{\tau_4}} \dots \right] \end{aligned}$$

Fig.: (4.2.7.5)

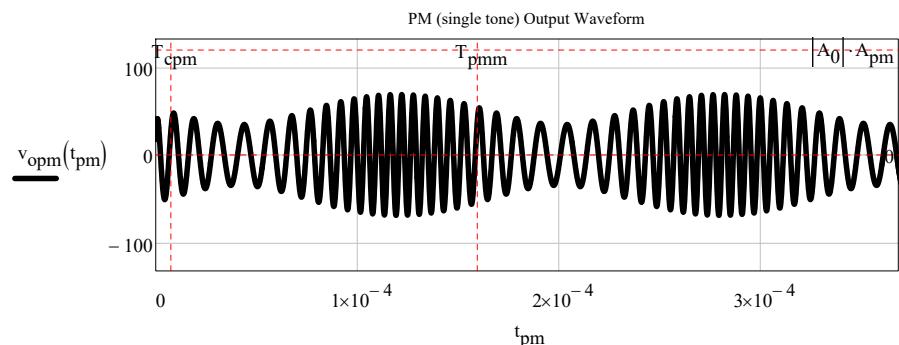


Fig.: (4.2.7.6)

$$v_{opms}(t) := \frac{v_{opm}(t)}{V}$$

Analog filter Output sampling

At first I calculate the signal bandwidth for a correct sampling.

Description of the program's **parameters**:

BCSA(Dimensionless signal name, relative error, polynomial degree, start time, signal period)
BCSA stands for "Bandwidth Calculation and Signal Analysis"

$$Sb_{opm} := \text{BCSA}(v_{opms}, r_{gd}, N_{gd}, 0.0, T_{cpm}) \quad r_{gd} = 10\% \quad (4.2.7.12)$$

► Bandwidth Calculation

$$f_{test} = 0.031 \cdot \text{MHz} \quad \text{Signal bandwidth: } B_{opm} = 6 \cdot \text{MHz} \quad (4.2.7.13)$$

$$\text{Parseval}_{opm} = 2.138 \times 10^3 \text{ V}^2 \quad \text{Average}_{opm} = -5.355 \text{ V} \quad \text{RMS}_{opm} = 32.701 \text{ V}$$

Sampling frequency:

$$f_{sopm} = \frac{1}{T_{pm}} \geq 2 \cdot f_1$$

$$\text{Chosen sampling frequency (Nyquist rate): } f_{sopm} := 2 \cdot B_{opm} \quad f_{sopm} = 0.012 \cdot \text{GHz} \quad (4.2.7.14)$$

$$\text{Carson bandwidth: } \text{Cars1o} := 2 \cdot \omega_{cpm} \cdot (m_{pm} + 1) \quad \text{Cars1} = 0.014 \cdot \frac{\text{Grads}}{\text{sec}} \quad (4.2.7.15)$$

$$\text{sampling angular frequency: } \omega_{sopm} := 2 \cdot \pi \cdot f_{sopm} \cdot \omega_{sopm} = 0.075 \cdot \frac{\text{Grads}}{\text{sec}}, \quad (4.2.7.16)$$

$$\text{sampling period: } T_{sopm} := \frac{1}{f_{sopm}}, \quad T_{sopm} = 0.083 \cdot \mu\text{s},$$

$$\text{sampling time step: } n_{opm} := \frac{k}{f_{sopm}}, \quad (4.2.7.17)$$

$$\frac{N_{gd}}{f_{sopm}} \cdot \frac{1}{T_{test}} = 0.13$$

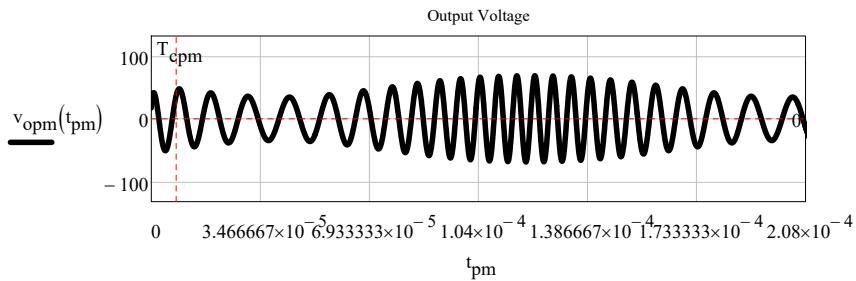


Fig.: (4.2.7.7)

Approximated output: $v_{opmd}(t) = \frac{A_0}{\omega_0} \cdot \frac{d}{dt} V_{pm}(t)$ $T_{test} = 32 \cdot \mu s$

(4.2.7.18)

$$V_{pm}(t) = A_{pm} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left(J_n(k, m_{pm}) \cdot \cos(t \cdot k \cdot \omega_{pmm}) \cdot \cos\left(\frac{\pi \cdot k}{2} + t \cdot \omega_{cpm}\right) \right)$$

$$v_{opmd}(t) := \frac{A_0 \cdot A_{pm}}{\omega_0} \cdot \sum_{k=-N_{gd}}^{N_{gd}} \left[-J_n(k, m_{pm}) \cdot \begin{aligned} & \left(\omega_{cpm} \cdot \sin\left(\frac{\pi \cdot k}{2} + t \cdot \omega_{cpm}\right) \cdot \cos(k \cdot t \cdot \omega_{pmm}) \dots \right. \\ & \left. + k \cdot \omega_{pmm} \cdot \cos\left(\frac{\pi \cdot k}{2} + t \cdot \omega_{cpm}\right) \cdot \sin(k \cdot t \cdot \omega_{pmm}) \right) \end{aligned} \right]$$

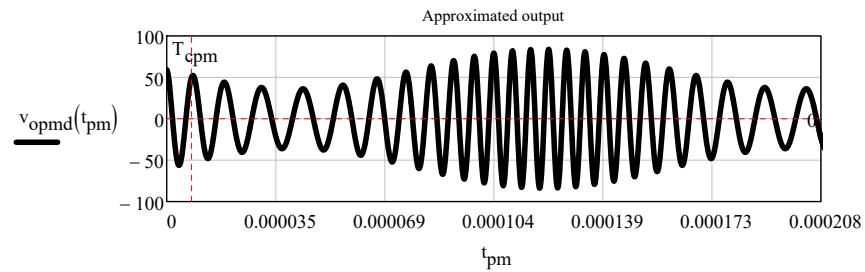


Fig.: (4.2.7.8)

$Opm_k := \frac{v_{opm}(n_{opm_k})}{volt}$

(4.2.7.19)

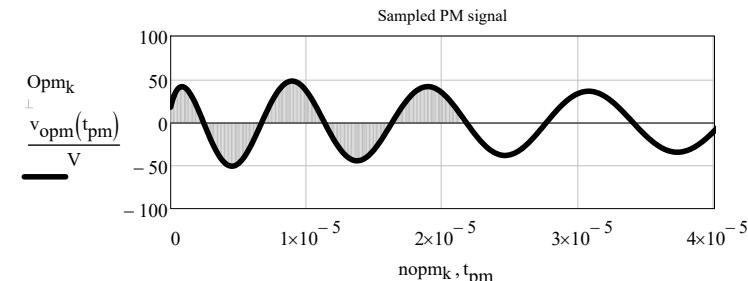


Fig.: (4.2.7.9)

Fourier Transform of the test signal

$$f_{cpm} = 0.125 \cdot MHz \quad \frac{f_{sopm}}{f_{cpm}} = 96$$

$$OSpecpm := fft(Opm) \quad m_{pm} = 8 \quad \omega_{pmm} = 0.039 \cdot \frac{Mrads}{sec}$$

(4.2.7.20)

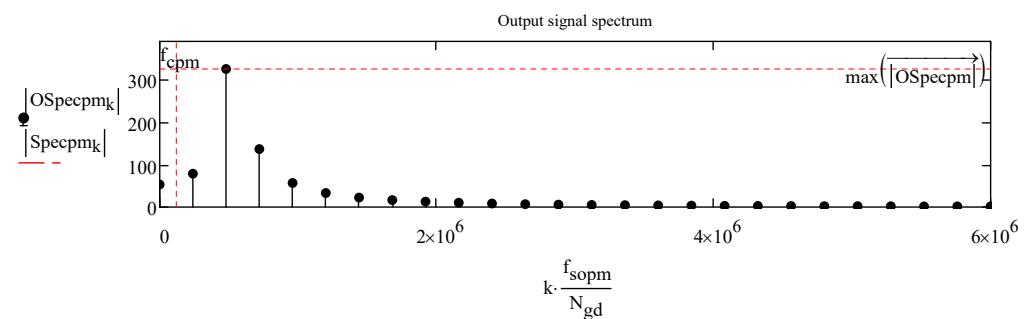


Fig.: (4.2.7.10)

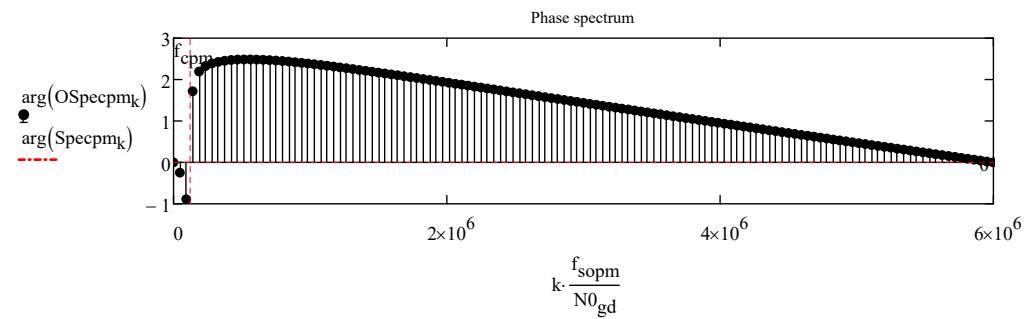


Fig.: (4.2.7.11)

$$W_{db}\omega_c := 20 \cdot \log(|W_{hp}(j\cdot\omega_{cpm})|) \quad W_{db}\omega_c = 8.573 \text{ dB} \quad (4.2.7.21)$$

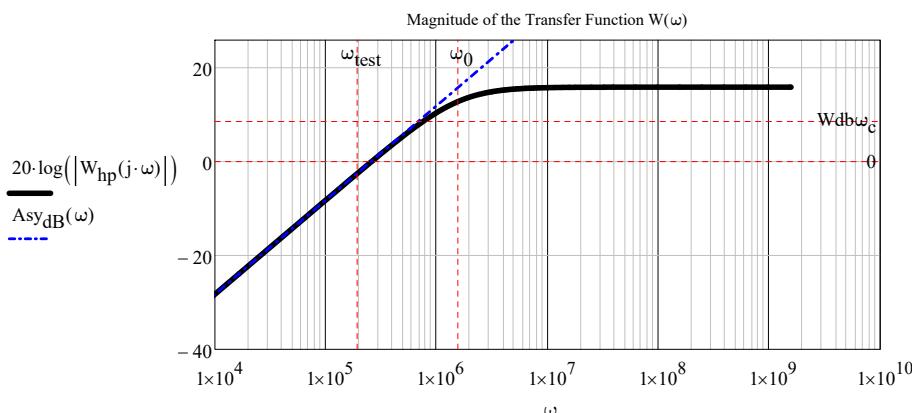


Fig.: (4.2.7.12)

4.3

Equivalent Digital High Pass Filter (I° order)

4.3.1) Z-transfer function of the I° Order High Pass Digital Filter

Given the transfer function: $W_{hp}(s) = \frac{A_0 \cdot s}{s + \omega_0}$, the corresponding z-transform can be calculated with the

$$\text{change of variable: } s = \frac{1 - z^{-1}}{T_{smp}}, \quad (4.3.1.1)$$

where T_{smp} is the sampling period depending on the signal bandwidth.

(To allow the symbolic calculation, I choose a value of T_{smp} among those previously calculated (sinusoidal signal)). Later, however, it will be calculated for each input signal.

$$T_{smp} := T_{s3} \quad \omega_{smp} := \frac{2 \cdot \pi}{T_{smp}} \quad \omega_{smp} = 5.498 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \omega_0 = 1.571 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$A_0 := A_0 \quad T_{smp} := T_{smp} \quad \omega_{smp} := \omega_{smp}$$

$$H1_{hp}(z) := \frac{A_0 \cdot s}{s + \omega_0} \left| \begin{array}{l} \text{substitute, } s = \frac{1}{T_{smp}} \cdot (1 - z^{-1}) \\ \text{collect, } z \\ \text{collect, } A_0 \end{array} \right. \rightarrow \frac{z - 1}{z \cdot (T_{smp} \cdot \omega_0 + 1) - 1} \cdot A_0 \quad (4.3.1.2)$$

and after some algebraic manipulation and the definition of the following *z transfer function*:

$$\frac{z - 1}{z \cdot (T_{smp} \cdot \omega_0 + 1) - 1} = \frac{(1 - z^{-1}) \cdot A_0}{1 + T_{smp} \cdot \omega_0 - z^{-1}} = \frac{A_0}{(1 + T_{smp} \cdot \omega_0)} \cdot \frac{(1 - z^{-1})}{\left[1 - \frac{z^{-1}}{(1 + T_{smp} \cdot \omega_0)} \right]}$$

$$\text{and its coefficients: } u0 := \frac{A_0}{(1 + T_{smp} \cdot \omega_0)} \quad v0 := \frac{1}{(1 + \omega_0 \cdot T_{smp})} \quad (4.3.1.3)$$

$$u0 = -2.146540144 \quad v0 = 0.357756691 \quad f_{smp} := \frac{1}{T_{smp}}$$

I get the following result for the t. f. as a function of z^{-1} :

$$H_{hp}(z) := u0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v0} \quad (4.3.1.4)$$

$$\omega_0 = 1.571 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \omega_{test} = 0.196 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$20 \cdot \log \left(\left| H_{hp} \left(e^{j \cdot \omega_{test} \cdot T_{smp}} \right) \right| \right) = -2.702 \text{ dB}$$

$$HdB1(\omega) := 20 \cdot \log \left(\left| H_{hp} \left(e^{j \cdot \omega \cdot T_{smp}} \right) \right| \right) \quad \varphi 1(\omega) := \arg \left(H_{hp} \left(e^{j \cdot \omega \cdot T_{smp}} \right) \right) \quad (4.3.1.5)$$

$$HdBc := 20 \cdot \log \left(\left| H_{hp} \left(e^{j \cdot \omega_{test} \cdot T_{smp}} \right) \right| \right) \quad \varphi 1c := \arg \left(H_{hp} \left(e^{j \cdot \frac{2\pi}{T_{test}} \cdot T_{smp}} \right) \right) \quad (4.3.1.6)$$

$$\omega := \frac{\omega_0}{8 \cdot U_0}, \frac{\omega_0}{8 \cdot U_0} + \frac{\omega_{smp} \cdot 20 \cdot U_0 - \omega_0}{8 \cdot U_0} \dots 20 \cdot U_0 \cdot \omega_{smp} \quad \frac{\omega_{smp}}{\omega_{test}} = 28$$

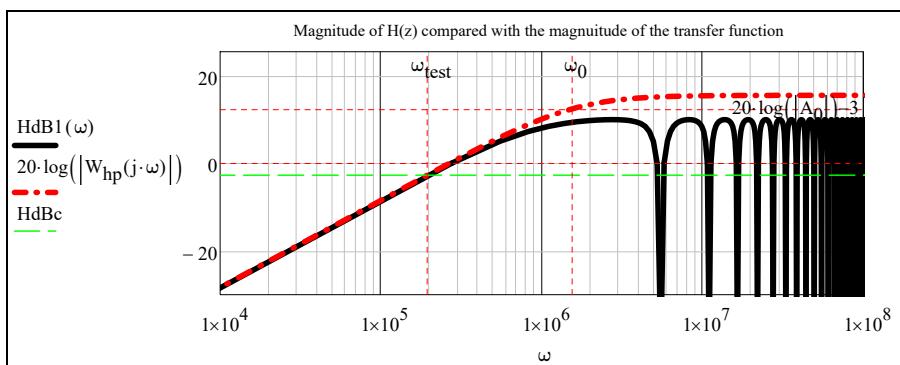


Fig.: (4.3.1.1)

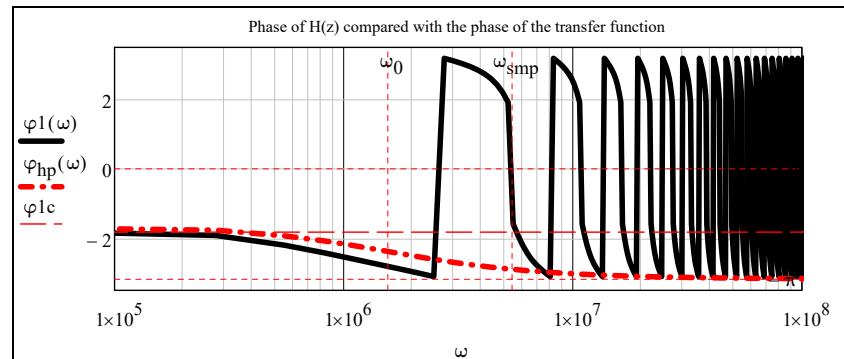


Fig.: (4.3.1.2)

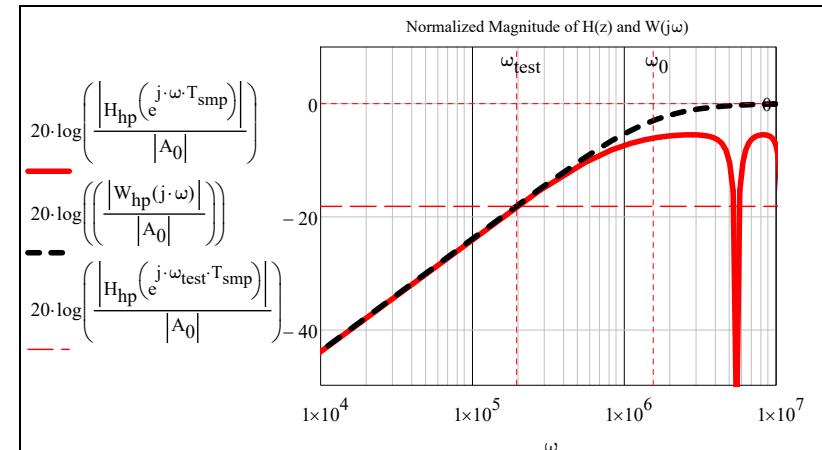


Fig.: (4.3.1.3)

4.3 Equivalent Digital High Pass Filter (I^oorder)

4.3.2) Difference equations (HIGH PASS FILTER (I^oorder)) Canonical form

After having defined the coefficients:

$$u_0 = \frac{A_0}{(1 + T_{\text{smp}} \cdot \omega_0)} \quad v_0 = \frac{1}{(1 + \omega_0 \cdot T_{\text{smp}})}$$

the z transfer function $H_{hp}(z) = u_0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v_0}$, can be decomposed in this way:

divide and multiply the t. f. by $W_{-}(z)$ and split the fraction in two:

$$H_{hp}(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W_{-}(z)} \cdot \frac{W_{-}(z)}{X(z)} \quad (4.3.2.1)$$

$$\text{resulting: } \frac{Y(z)}{W_{-}(z)} \cdot \frac{W_{-}(z)}{X(z)} = u_0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v_0} \quad (4.3.2.2)$$

$$\text{define: } \frac{Y(z)}{W_{-}(z)} = u_0 \cdot (1 - z^{-1}) \quad (4.3.2.3)$$

$$\text{so that: } Y(z) = u_0 \cdot W_{-}(z) - u_0 \cdot z^{-1} \cdot W_{-}(z) \quad (4.3.2.4)$$

$$\text{whose inverse z transform is: } y(n) = u_0 \cdot w(n) - u_0 \cdot w(n-1) \quad (4.3.2.5)$$

$$\text{Now define: } \frac{W_{-}(z)}{X(z)} = \frac{1}{1 - z^{-1} \cdot v_0} \quad (4.3.2.6)$$

$$\text{from which obtain: } X(z) = (1 - z^{-1} \cdot v_0) \cdot W_{-}(z) = W_{-}(z) - z^{-1} \cdot v_0 \cdot W_{-}(z) \quad (4.3.2.7)$$

$$\text{whose inverse z transform is: } x(n) = w(n) - v_0 \cdot w(n-1) \quad (4.3.2.8)$$

Ultimately the corresponding set of **difference equations** is:

$$1) w(n) = x(n) + v_0 \cdot w(n-1) \quad (4.3.2.9)$$

$$2) y(n) = u_0 \cdot (w(n) - w(n-1)) \quad (4.3.2.10)$$

they give the output sequence $y(n)$ knowing the input one $x(n)$.

Calculation of th initial and final values.

◻ Symbolic initialization of z, u_0, v_0

$$\text{Z T. Initial value theorem: } \lim_{z \rightarrow \infty} \left(u_0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v_0} \right) \rightarrow u_0 \quad u_0 = -2.147$$

$$\text{Z T. Final value theorem: } \lim_{z \rightarrow 0} \left(u_0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v_0} \right) \rightarrow \begin{cases} \frac{u_0}{v_0} & \text{if } v_0 \neq 0 \\ \text{undefined} & \text{otherwise} \end{cases} \quad \frac{u_0}{v_0} = -6$$

4.3 Equivalent Digital High Pass Filter (I^oorder)

4.3.2.1) Sequence of the sinusoidal voltage response

$$x1_i(n) := u_3 n \quad (4.3.2.1.1)$$

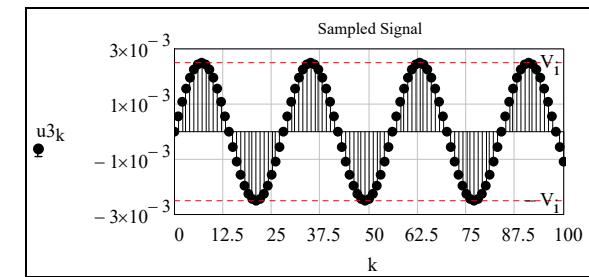


Fig.: (4.3.2.1.1)

$$\omega_0 = 1.571 \cdot \frac{\text{Mrad/s}}{\text{s}}$$

$$\text{Coefficients: } u_0 := \frac{A_0}{(1 + T_{\text{smp}} \cdot \omega_0)} \quad v_0 := \frac{1}{(1 + \omega_0 \cdot T_{\text{smp}})} \quad (4.3.2.1.2)$$

$$\omega_{s3} := \frac{2 \cdot \pi}{T_{\text{smp}}} \quad u_0 = -2.146540144 \quad v_0 = 0.357756691 \quad f_{s3} := \frac{1}{T_{\text{smp}}} \quad (4.3.2.1.2)$$

$$1) w1(n) := \begin{cases} x1_i(n) + v_0 \cdot w1(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.1.3)$$

$$2) y1(n) := \begin{cases} u_0 \cdot (w1(n) - w1(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.1.4)$$

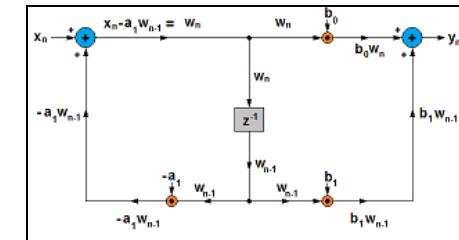


Fig.: (4.3.2.1.2)

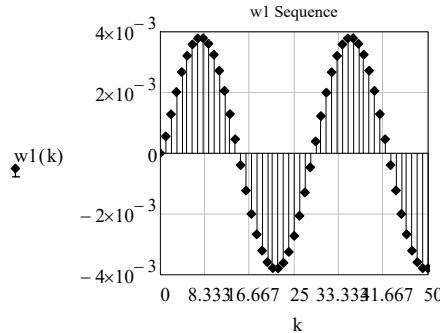


Fig.: (4.3.2.1.3)

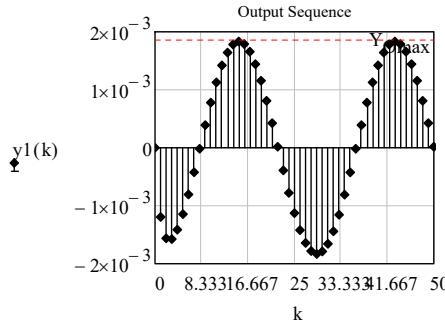
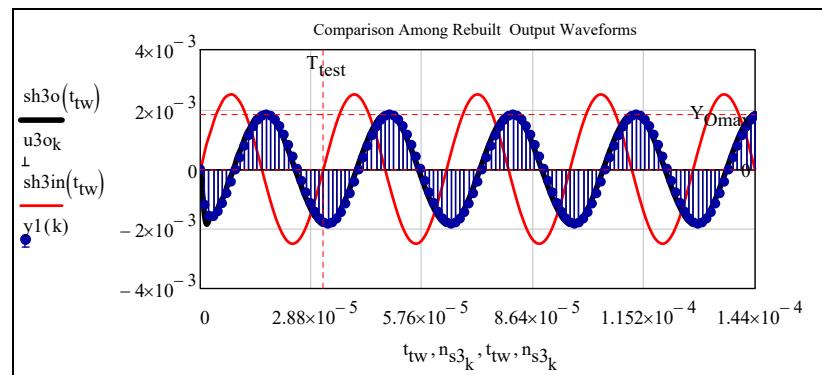


Fig.: (4.3.2.1.4)



$$\text{Sampled signal: } v1x_k := y1(k) \quad (4.3.2.1.5)$$

$$\text{Spec1x} := \text{FFT}(v1x) \quad (4.3.2.1.6)$$

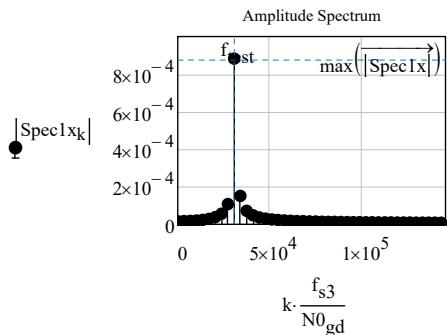


Fig.: (4.3.2.1.5)

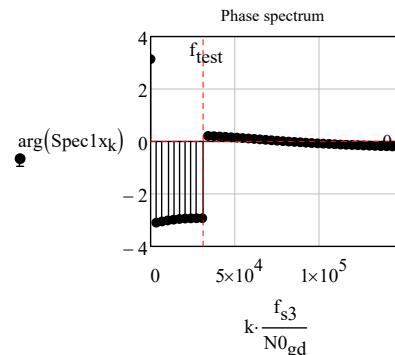


Fig.: (4.3.2.1.6)

4.3 Equivalent Digital High Pass Filter (I^oorder)

4.3.2.2) Sequence of the Voltage Pulse response

Digital first order High pass filter difference equations:

$$x2_i(n) := u_{-4n} \quad (4.3.2.2.1)$$

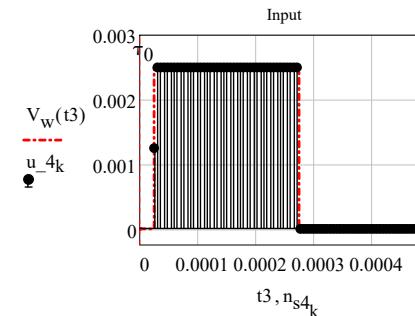


Fig.: (4.3.2.2.1)

$$\text{Coefficients: } u00 := \frac{A_0}{(1 + T_{s4} \cdot \omega_0)} \quad v00 := \frac{1}{(1 + \omega_0 \cdot T_{s4})}$$

$$1) \quad w2(n) := \begin{cases} x2_i(n) + v00 \cdot w2(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.2.2)$$

$$2) \quad y2(n) := \begin{cases} u00 \cdot (w2(n) - w2(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.2.3)$$

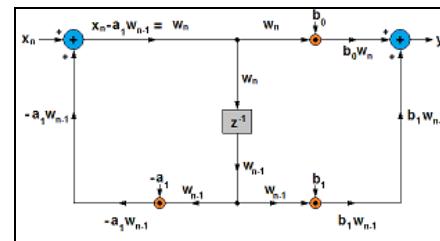


Fig.: (4.3.2.2.1)

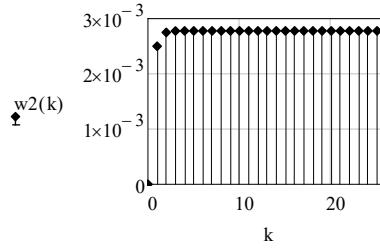


Fig.: (4.3.2.2)

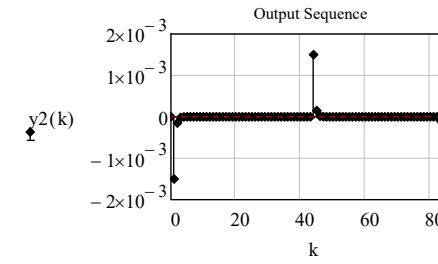


Fig.: (4.3.2.3)

$$\text{Sampled signal: } v2x_k := y2(k)$$

$$\text{Spec2x} := \text{fft}(v2x)$$

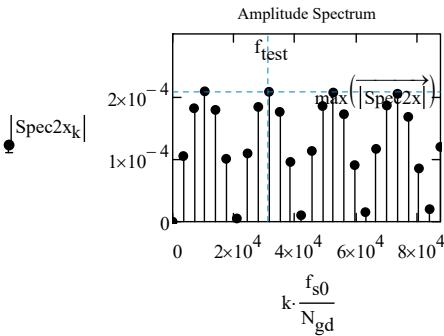


Fig.: (4.3.2.4)

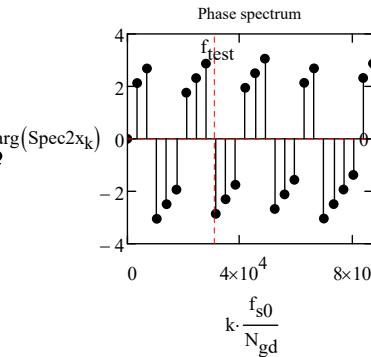


Fig.: (4.3.2.5)

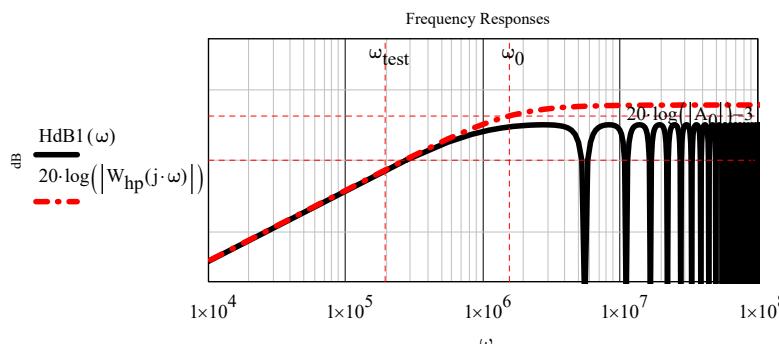


Fig.: (4.3.2.6)

4.3 Equivalent Digital High Pass Filter (I^o order)

4.3.2.3) Sequence of the Bipolar Triangular wave response

Digital first order High pass filter difference equations:

$$x3_1(n) := u_{-5n} \quad (4.3.2.3.1)$$

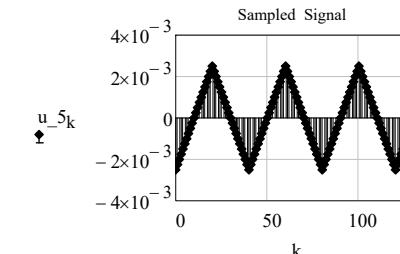


Fig.: (4.3.2.3.1)

$$\text{Coefficients: } u01 := \frac{A_0}{(1 + T_{\text{stri}} \cdot \omega_0)} \quad v01 := \frac{1}{(1 + \omega_0 \cdot T_{\text{stri}})}$$

$$1) \quad w3(n) := \begin{cases} x3_1(n) + v01 \cdot w3(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.3.2)$$

$$2) \quad y3(n) := \begin{cases} u01 \cdot (w3(n) - w3(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.3.2.3.3)$$

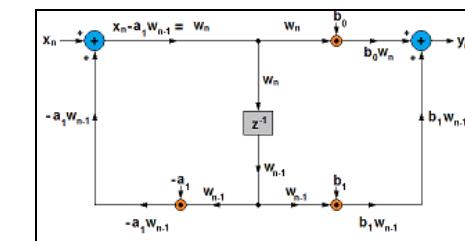


Fig.: (4.3.2.3.2)

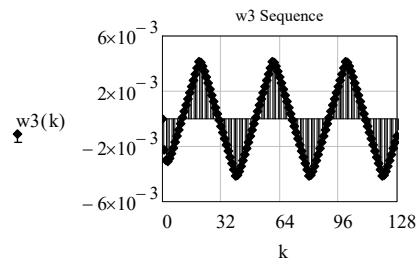


Fig.: (4.3.2.3.3)

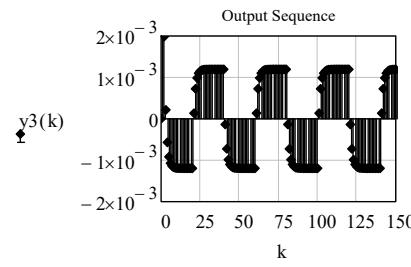


Fig.: (4.3.2.3.4)

$$\text{Sampled signal: } v3x_k := y3(k) \quad (4.3.2.3.4)$$

$$\text{Spec3x} := \text{FFT}(v3x) \quad (4.3.2.3.5)$$

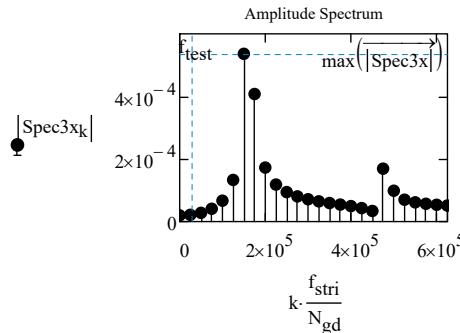


Fig.: (4.3.2.3.5)

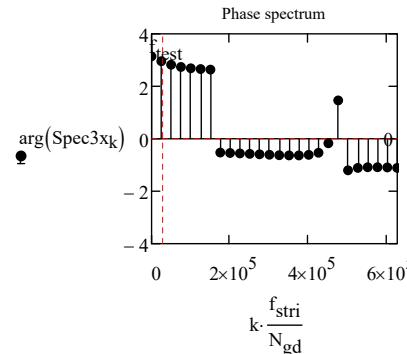


Fig.: (4.3.2.3.6)

4.3 Equivalent Digital High Pass Filter (I^oorder)

4.3.2.4) Sequence of the Sawtooth wave response:

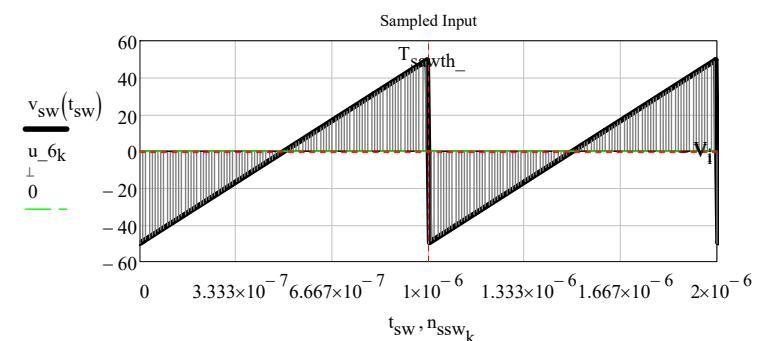


Fig.: (4.3.2.4.1)

$$\text{Coefficients: } u02 := \frac{A_0}{(1 + T_{ssw} \cdot \omega_0)} \quad v02 := \frac{1}{(1 + \omega_0 \cdot T_{ssw})} \quad (4.3.1.3)$$

$$u02 = -5.903405749 \quad v02 = 0.983900958$$

Digital first order High pass filter difference equations:

$$x4_1(n) := u_6n$$

$$1) \quad w4(n) := \begin{cases} x4_1(n) + v02 \cdot w4(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y4(n) := \begin{cases} u02 \cdot (w4(n) - w4(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

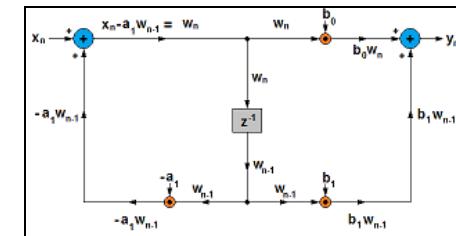


Fig.: (4.3.2.4.2)

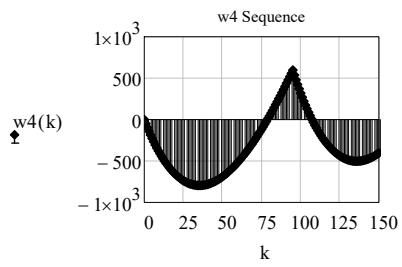


Fig.: (4.3.2.4.3)

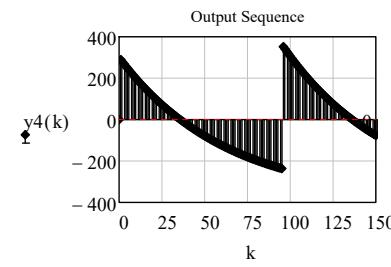


Fig.: (4.3.2.4.4)

Sampled signal: $v4x_k := y4(k)$

$\text{Spec4x} := \text{fft}(v4x)$

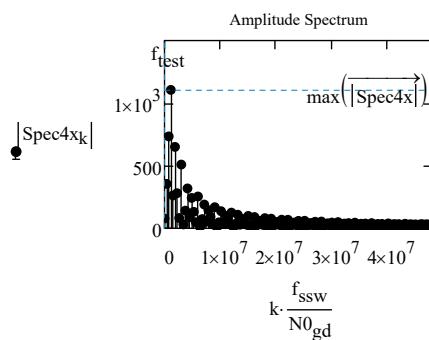


Fig.: (4.3.2.4.5)

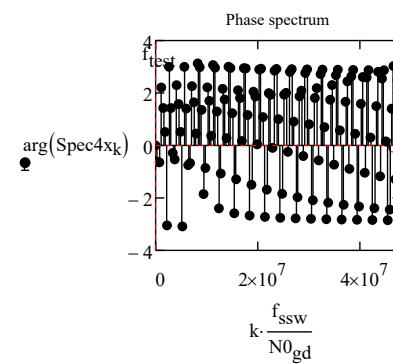


Fig.: (4.3.2.4.6)

4.3 Equivalent Digital High Pass Filter (I^oorder)

4.3.2.5) Sequence of the AM Signal response:

$$u_{-7k} := \frac{v2_{am}(nam_k)}{\text{volt}}$$

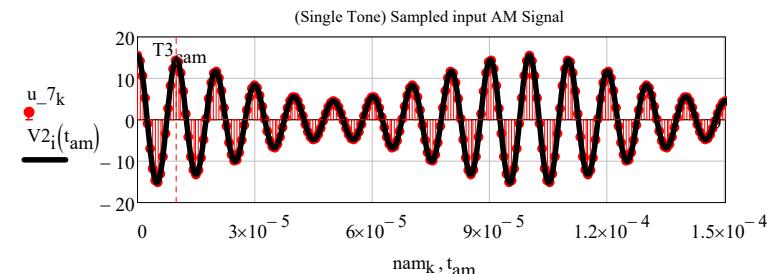


Fig.: (4.3.2.5.1)

Digital first order High pass filter difference equations:

Coefficients: $u03 := \frac{A_0}{(1 + T_{\text{sam}} \cdot \omega_0)}$ $v03 := \frac{1}{(1 + \omega_0 \cdot T_{\text{sam}})}$

$$u03 = -2.970300537 \quad v03 = 0.49505009$$

$$x5_1(n) := u_{-7n}$$

$$1) \quad w5(n) := \begin{cases} x5_1(n) + v03 \cdot w5(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y5(n) := \begin{cases} u03 \cdot (w5(n) - w5(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

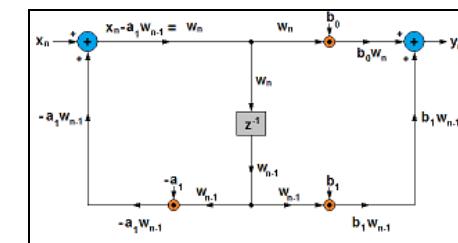


Fig.: (4.3.2.5.2)

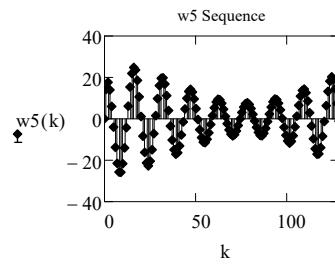


Fig.: (4.3.2.5.3)

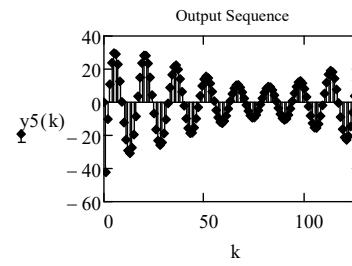


Fig.: (4.3.2.5.4)

Sampled signal: $v5x_k := y5(k)$

$\text{Spec5x} := \text{FFT}(v5x)$

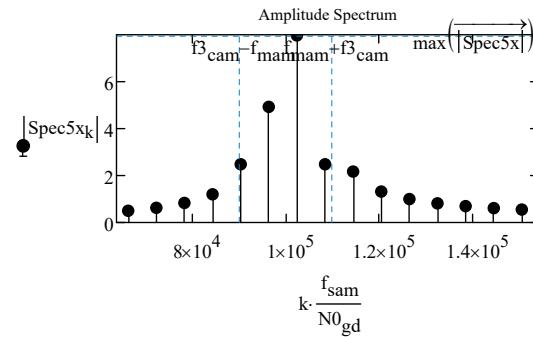


Fig.: (4.3.2.5.5)

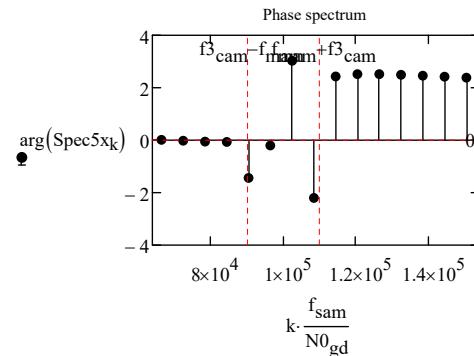


Fig.: (4.3.2.5.6)

4.3 Equivalent Digital High Pass Filter (I^o order)

4.3.2.6) Sequence of the Frequency Modulated signal response

Digital first order High pass filter difference equations:

$$u_{-8k} = \frac{V_{fm}(nfm_k)}{\text{volt}}$$

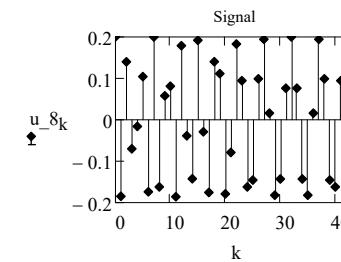


Fig.: (4.3.2.6.1)

Coefficients: $u04 := \frac{A_0}{(1 + T_{sfm} \cdot \omega_0)}$ $v04 := \frac{1}{(1 + \omega_0 \cdot T_{sfm})}$

$u04 = -5.15630205$ $v04 = 0.859383675$

$x6_i(n) := u_{-8n}$

1) $w6(n) := \begin{cases} x6_i(n) + v04 \cdot w6(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$

2) $y6(n) := \begin{cases} u04 \cdot (w6(n) - w6(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$

$\omega_{cfm} = 18.85 \cdot \frac{\text{Mrads}}{\text{sec}}$

$\omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}}$ $A_0 = -6$ $m_{fm} = 8$

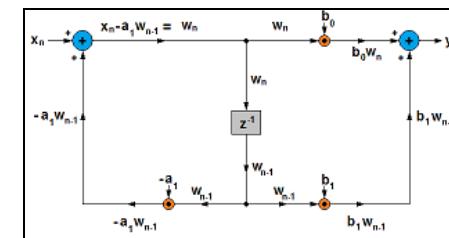


Fig.: (4.3.2.6.2)

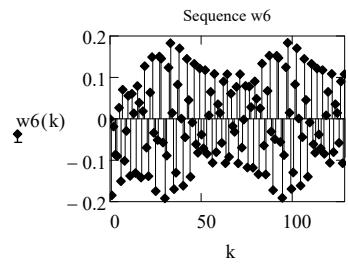


Fig.: (4.3.2.6.3)

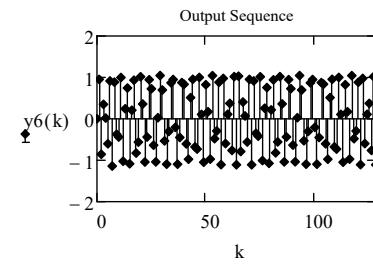


Fig.: (4.3.2.6.4)

Sampled signal: $v6x_k := y6(k)$

$\text{Spec6x} := \text{FFT}(v6x)$

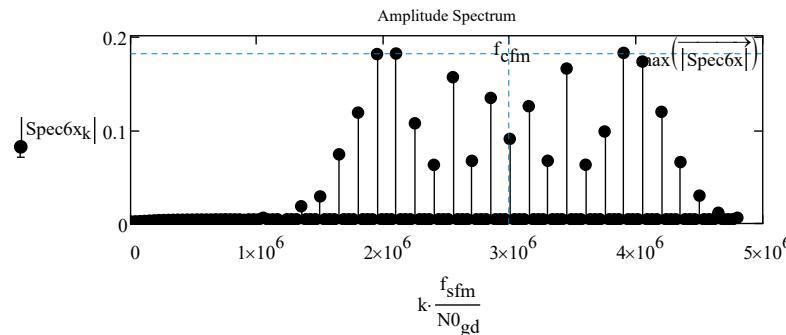


Fig.: (4.3.2.6.5)

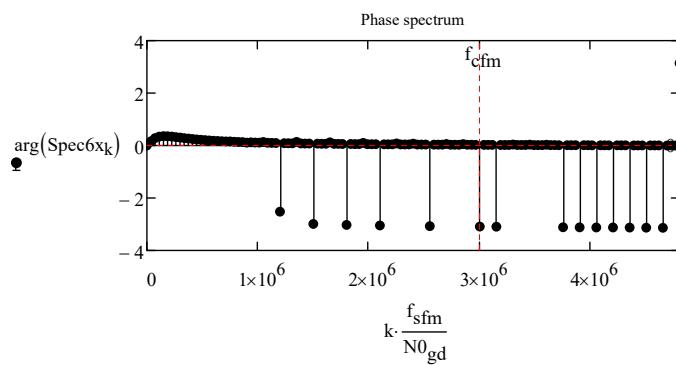


Fig.: (4.3.2.6.6)

4.3 Equivalent Digital High Pass Filter (I^o order)

4.3.2.7) Sequence of the Phase Modulated signal response

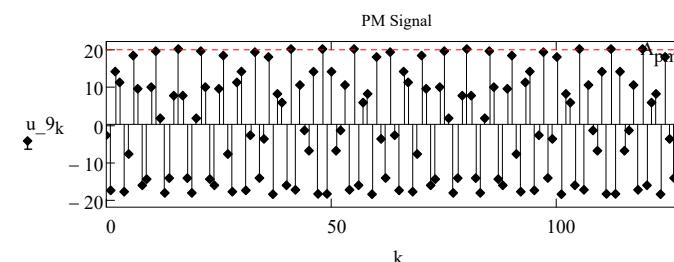


Fig.: (4.3.2.7.1)

Digital first order High pass filter difference equations:

$$\text{Coefficients: } u05 := \frac{A_0}{(1 + T_{\text{spm}} \cdot \omega_0)} \quad v05 := \frac{1}{(1 + \omega_0 \cdot T_{\text{spm}})}$$

$$u05 = -1.217781852 \quad v05 = 0.202963642$$

$$x7_1(n) := u_9n$$

$$1) \quad w7(n) := \begin{cases} x7_1(n) + v05 \cdot w7(n-1) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y7(n) := \begin{cases} u05 \cdot (w7(n) - w7(n-1)) & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_{\text{cpm}} = 0.785 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\omega_{\text{pm}} = 0.039 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$A_0 = -6$$

$$m_{\text{pm}} = 8$$

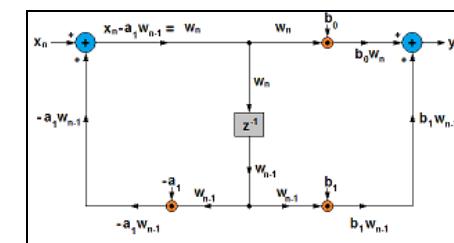


Fig.: (4.3.2.7.2)

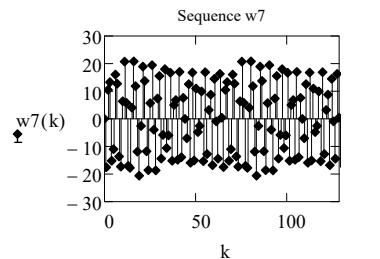


Fig.: (4.3.2.7.3)

Sampled signal: $v7x_k := y7(k)$

$\text{Spec7x} := \text{FFT}(v7x)$

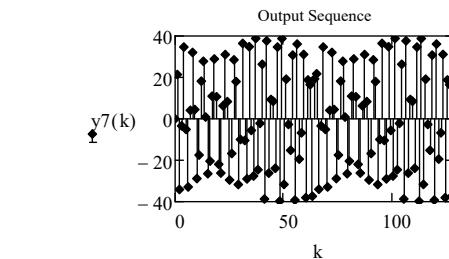


Fig.: (4.3.2.7.4)

Spec7x := FFT(v7x)

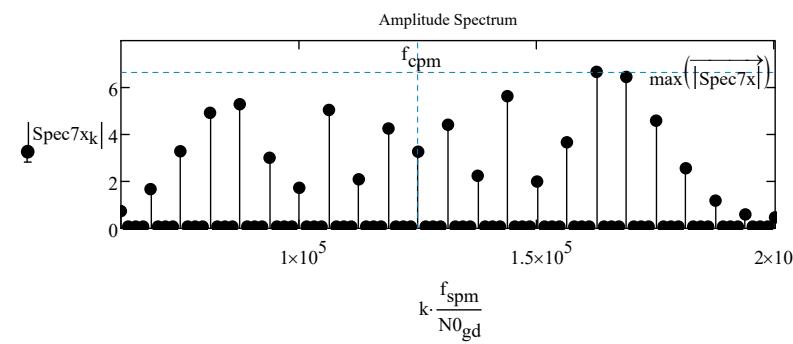


Fig.: (4.3.2.7.5)

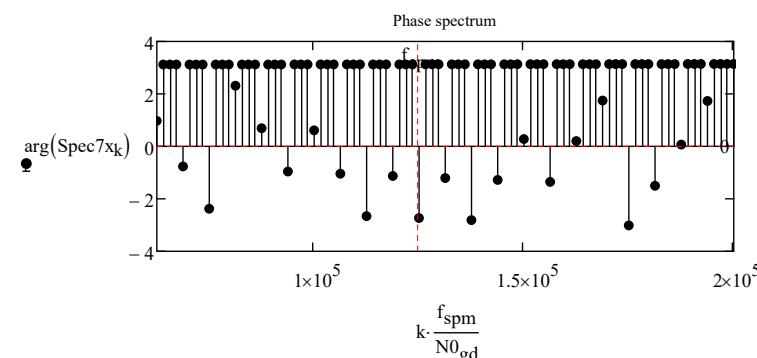


Fig.: (4.3.2.7.6)

4.4 Transfer Function Sequence. Convolution Output

$$T_{\text{smp}} := T_{\text{smp}} \quad \omega_0 := \omega_0 \quad \alpha_0 := \alpha_0 \quad \beta_0 := \beta_0$$

$$H_{\text{hp}}(z) = u_0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v_0}$$

$$\text{Numerator degree} \quad N_n := 1 \quad \text{Denominator degree} \quad M_d := 1$$

$$N1 := N_n + M_d \quad N0_{\text{gd}} = 256 \quad h1_k := 0$$

A generic first order transfer function in the z domain takes this form:

$$H_{\text{hp}}(z) = \frac{b_0 + b_1 \cdot z^{-1}}{a_0 + a_1 \cdot z^{-1}}$$

The coefficients of the numerator and denominator can be defined as the elements of two vectors, namely a and b, hence:

Coefficients:	$u_0 := \frac{A_0}{(1 + T_{\text{smp}} \cdot \omega_0)}$	$v_0 := \frac{1}{(1 + \omega_0 \cdot T_{\text{smp}})}$
$\omega_{\text{smp}} := \frac{2 \cdot \pi}{T_{\text{smp}}}$	$u_0 = -2.146540144$	$v_0 = 0.357756691$
	$f_{\text{smp}} := \frac{1}{T_{\text{smp}}}$	

(4.4.1)

Numerator coeffs.

$$n1 := 1 .. N0_{\text{gd}} - 1 \quad b_{n1} := 0.0 \quad a_{n1} := 0.0$$

$$b_0 := u_0 \quad a_0 := 1$$

$$b_1 := -u_0 \quad a_1 := -v_0$$

and divide the two polynomials by means of the following algorithm:

$$N1 = 2 \quad h1_0 := \frac{b_0}{a_0} \quad h1_{n1} := \frac{1}{a_0} \cdot \left[b_{n1} - \sum_{i=1}^{n1} (h1_{n1-i} \cdot a_i) \right]$$

T. F. Numerator coefficients:

$$a^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & -0.358 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & -2.147 & 2.147 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence of the Impulse response:

$$h1^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

| 0 | -2.1465 | 1.3786 | 0.4932 | 0.1764 | 0.0631 | 0.0226 | ... |

The whole procedure to obtain $h1$ is implemented by the program polyalg().

polyalg

Stability (SI< ∞):

$$S1 := \sum_{k=0}^{\text{rows}(h1)-1} |h1_k| \quad S1 = 4.293$$

Energy of the sequence $h1$:

$$E1 := \sum_{k=0}^{\text{rows}(h1)-1} (|h1_k|)^2 \quad E1 = 6.787$$

$$t := 0 \cdot \tau_0, \frac{20 \cdot \tau_0}{1000} \dots 20 \cdot \tau_0$$

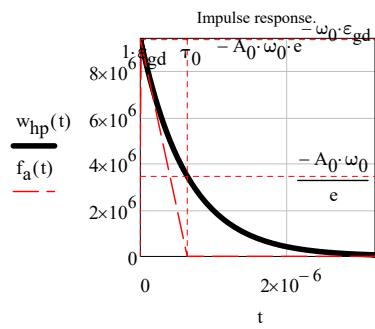


Fig.: (4.4.1)

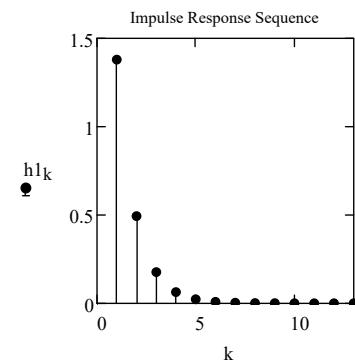


Fig.: (4.4.2)

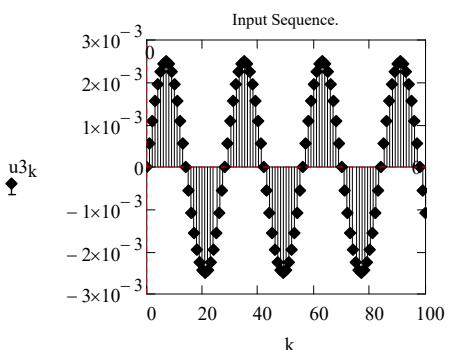


Fig.: (4.4.1.1)

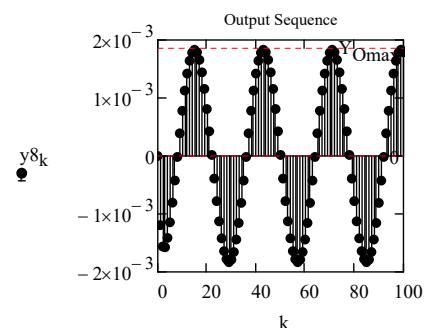


Fig.: (4.4.1.2)

Sampled signal:

$\text{Spec8x} := \text{FFT}(y8)$

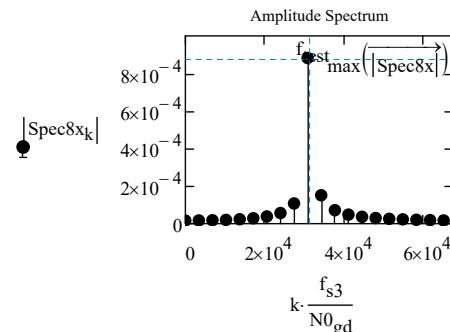


Fig.: (4.4.1.3)

$$f_{\text{test}} = 31.25 \text{ kHz}$$

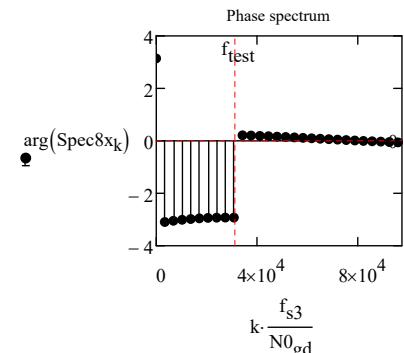


Fig.: (4.4.1.4)

4.4 Transfer Function Sequence obtained by an Iterative Algorithm. Convolution Output

4.4.1) Sequence of the sinusoidal voltage response

$$\nu := n1$$

$$y8_\nu := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h1_k \cdot u3_{\nu-k}, 0))$$

4.4 Transfer Function Sequence, Convolution Output

4.4.2) Sequence of the Voltage Pulse response

$$\text{Out1} := \text{polyalg}(A_0, \omega_0, T_{s4}, N_{gd})$$

$$h11 := (\text{Out1}^{(2)})_0$$

Coefficients: $u00 := (\text{Out1}^{(0)})_0$ $v00 := (\text{Out1}^{(1)})_0$

$$\omega_{s4} := \frac{2\pi}{T_{s4}}$$

$$u00 = -0.600038765$$

$$f_{s0} := \frac{1}{T_{s4}}$$

Numerator coeffs.

Denominator coeffs.

$$n1 := 1..N_{gd}-1 \quad b_{n1} = 0.0$$

$$b_0 = u00$$

$$a_{n1} = 0.0$$

$$a_0 = 1$$

$$b_1 = -u00$$

$$a_1 = -v00$$

and divide the two polynomials by means of the following algorithm:

$$N1 = 2 \quad h10 = \frac{b_0}{a_0} \quad h1n1 = \frac{1}{a_0} \left[b_{n1} - \sum_{i=1}^{n1} (h1_{n1-i} \cdot a_i) \right]$$

T. F. Numerator coefficients:

$$a^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & 1 & -0.358 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... \\ \hline \end{array}$$

T. F. Denominator coefficients:

$$b^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & -2.147 & 2.147 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... \\ \hline \end{array}$$

Sequence of the Impulse response:

$$h11^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & -0.6 & 0.54 & 0.054 & 5.401 \cdot 10^{-3} & 5.4014 \cdot 10^{-4} & ... \\ \hline \end{array}$$

Stability ($S1 < \infty$):

$$S1 := (\text{Out1}^{(3)})_0$$

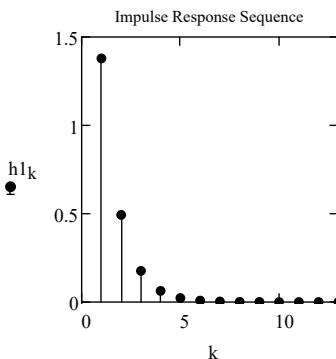
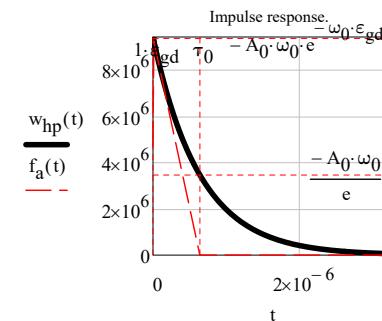
$$S1 = 1.2$$

Energy of the sequence $h1$:

$$E1 := (\text{Out1}^{(4)})_0$$

$$E1 = 0.655$$

$$t := 0 \cdot \tau_0, \frac{20 \cdot \tau_0}{1000} .. 20 \cdot \tau_0$$



$$y9_v := \sum_{k=0}^v (\text{if}(v-k \geq 0, h11_k \cdot u_{-4v-k}, 0))$$

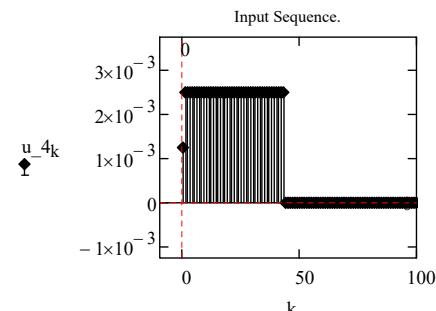


Fig.: (4.4.2.1)

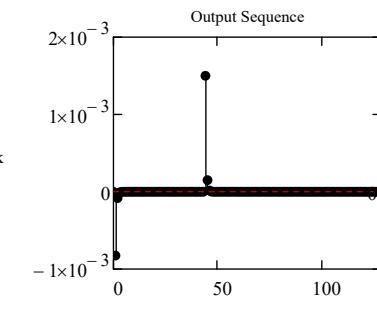


Fig.: (4.4.2.2)

Sampled signal:

$$\text{Spec9x} := \text{fft}(y9)$$

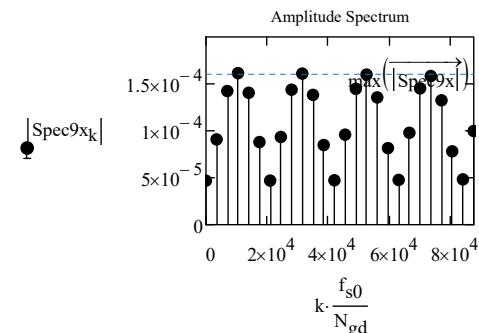


Fig.: (4.4.2.3)

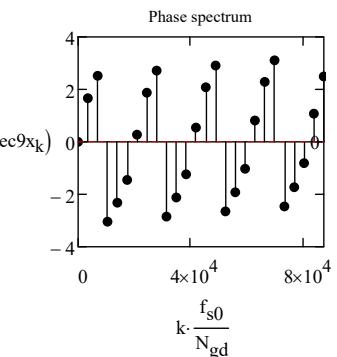


Fig.: (4.4.2.4)

4.4 Transfer Function Sequence, Convolution Output

4.4.3) Sequence of the Bipolar Triangular wave response

$$\text{Out2} := \text{polyalg}(A_0, \omega_0, T_{\text{stri}}, N_{0\text{gd}})$$

$$h2 := (\text{Out2}^{(2)})_0$$

$$\text{Coefficients: } u0 := (\text{Out2}^{(0)})_0$$

$$v0 := (\text{Out2}^{(1)})_0$$

$$\omega_{\text{smp}} := \frac{2 \cdot \pi}{T_{\text{smp}}}$$

$$u0 = -2.658823655$$

$$v0 = 0.443137276$$

$$\omega_{\text{smp}} := \frac{1}{T_{\text{smp}}}$$

Sequence of the Impulse response:

	0	1	2	3	4	5	...
0	-2.6588	1.4806	0.6561	0.2907	0.1288	...	

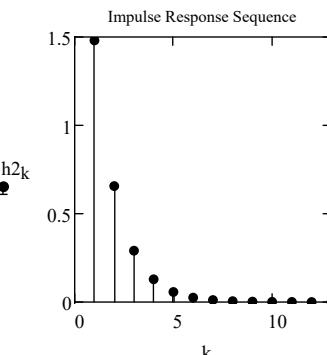
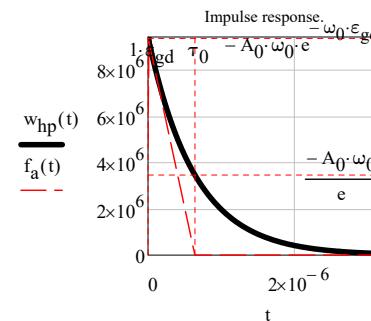
Stability ($S1 < \infty$):

$$S2 := \sum_{k=0}^{\text{rows}(h1)-1} |h2_k| \quad S2 = 5.318$$

Energy of the sequence h1:

$$E2 := \sum_{k=0}^{\text{rows}(h1)-1} (|h2_k|)^2 \quad E2 = 9.797$$

$$t := 0 \cdot \tau_0, \frac{20 \cdot \tau_0}{1000} \dots 20 \cdot \tau_0$$



$$y10_{\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h2_k \cdot u_{-5\nu-k}, 0))$$

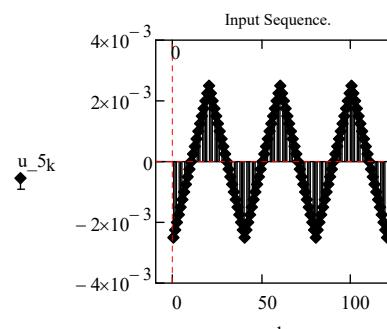


Fig.: (4.4.3.1)

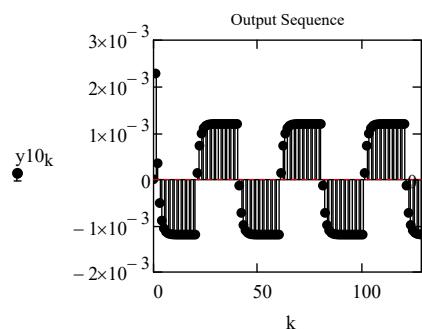


Fig.: (4.4.3.2)

Sampled signal:

$$\text{Spec10x} := \text{fft}(y10)$$

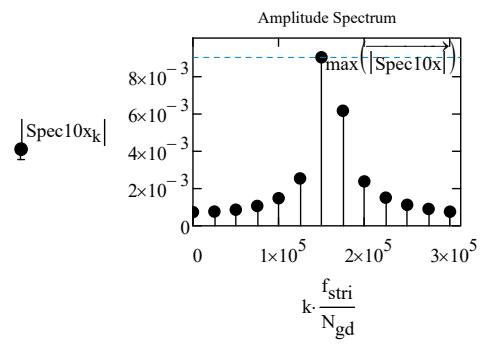


Fig.: (4.4.3.3)

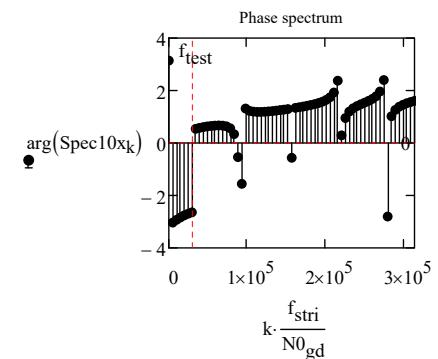


Fig.: (4.4.3.4)

4.4 Transfer Function Sequence. Convolution Output

4.4.4) Sequence of the Sawtooth wave response

$$\text{Out3} := \text{polyalg}(A_0, \omega_0, T_{\text{ssw}}, N_0, \text{gd})$$

$$h3 := (\text{Out3}^{(2)})_0$$

Coefficients:

$$u_{00} := (\text{Out3}^{(0)})_0$$

$$v_{00} := (\text{Out3}^{(1)})_0$$

$$\omega_{\text{ssw}} := \frac{2\pi}{T_{\text{ssw}}}$$

$$u_{00} = -5.903405749$$

$$f_{\text{ssw}} := \frac{1}{T_{\text{ssw}}}$$

$$y_{11\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h3_k \cdot u_{-6\nu-k}, 0))$$

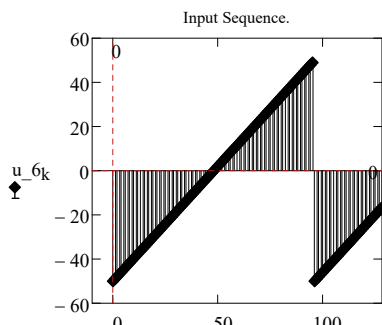


Fig.: (4.4.1b)

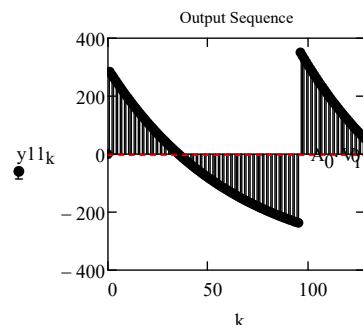


Fig.: (4.4.2)

$$\text{Spec11x} := \text{fft}(y_{11})$$

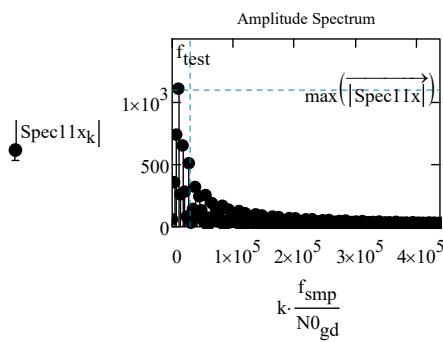


Fig.: (4.4.3)

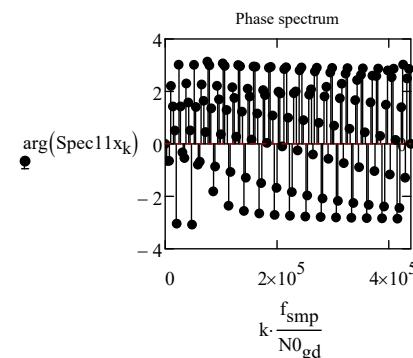


Fig.: (4.4.4)

4.4 Transfer Function Sequence, Convolution Output

4.4.5) Sequence of the (Single tone) AM Signal response

$$\text{Out4} := \text{polyalg}(A_0, \omega_0, T_{\text{sam}}, N_{0\text{gd}})$$

$$h4 := (\text{Out4}^{(2)})_0$$

$$y_{12v} := \sum_{k=0}^v (\text{if}(v-k \geq 0, h4_k \cdot u_{-7v-k}, 0))$$

Input Sequence.

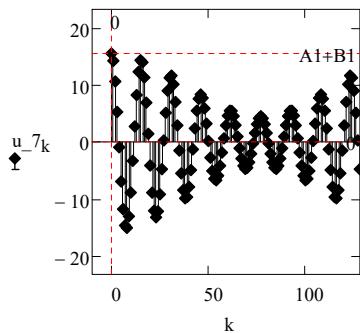


Fig.: (4.4.5.1)

Output Sequence

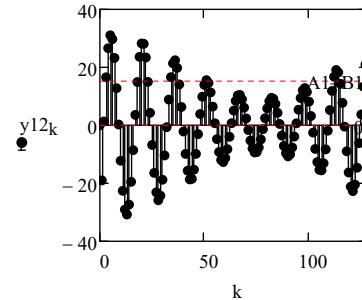


Fig.: (4.4.5.2)

Sampled signal:

$$\text{Spec12x} := \text{fft}(y_{12})$$

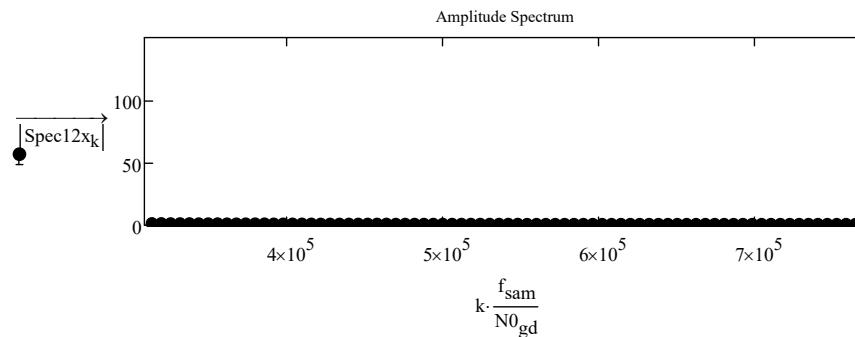


Fig.: (4.4.5.3)

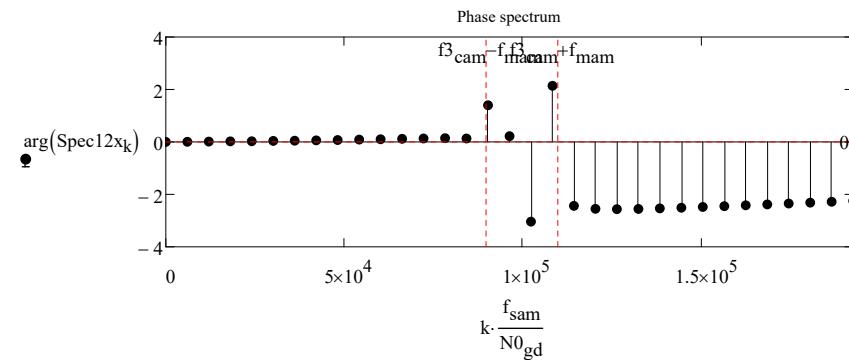


Fig.: (4.4.5.4)

4.4 Transfer Function Sequence, Convolution Output

4.4.6) Sequence of the (Single tone) Frequency Modulated carrier response

$$\text{Out5} := \text{polyalg}(A_0, \omega_0, T_{\text{sfm}}, N_{0\text{gd}})$$

$$h5 := (\text{Out5}^{(2)})_0$$

$$\text{Coefficients: } u_{00} := (\text{Out5}^{(0)})_0$$

$$v_{00} := (\text{Out5}^{(1)})_0$$

$$y_{13\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h5_k \cdot u_{8\nu-k}, 0))$$

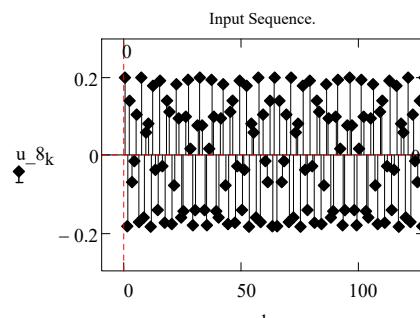


Fig.: (4.4.6.1)

Sampled signal: $m_{\text{fm}} = 8$ $\text{Spec13x} := \text{fft}(y_{13})$

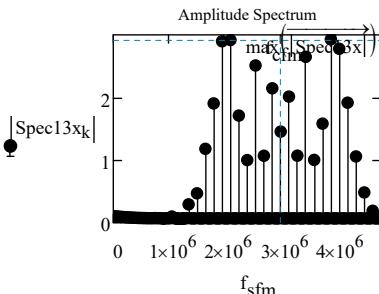


Fig.: (4.4.6.3)

$$\omega_{\text{cfm}} = 18.85 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$\omega_{\text{fmm}} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}}$$

Fig.: (4.4.6.2)

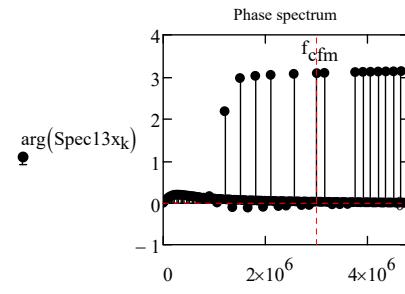


Fig.: (4.4.6.4)

$$A_0 = -6 \quad m_{\text{fm}} = 8$$

4.4 Transfer Function Sequence, Convolution Output

4.4.7) Sequence of the (Single tone) Phase Modulated carrier response

$$\text{Out6} := \text{polyalg}(A_0, \omega_0, T_{\text{spm}}, N_{0\text{gd}})$$

$$h6 := (\text{Out6}^{(2)})_0$$

$$\text{Coefficients: } u_{00} := (\text{Out6}^{(0)})_0 \quad v_{00} := (\text{Out6}^{(1)})_0$$

$$y_{14\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h6_k \cdot u_{9\nu-k}, 0))$$

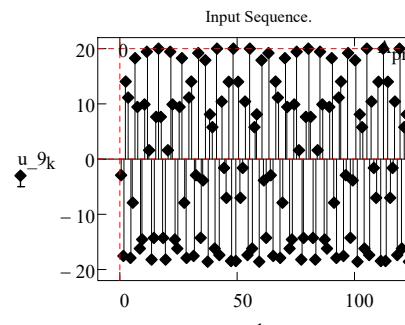


Fig.: (4.4.7.1)

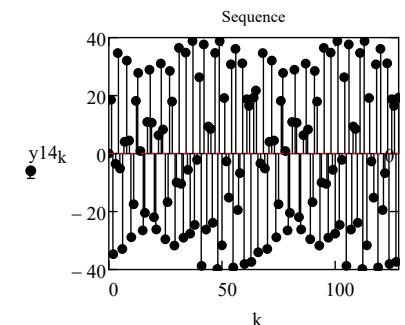


Fig.: (4.4.7.2)

Sampled signal: $m_{\text{pm}} = 8$ $\text{Spec14x} := \text{fft}(y_{14})$

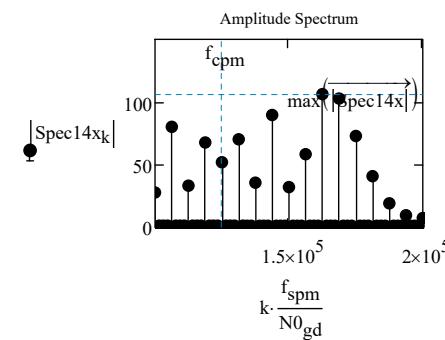


Fig.: (4.4.7.3)

$$\omega_{\text{cpm}} = 0.785 \cdot \frac{\text{Mrads}}{\text{sec}} \quad \omega_{\text{pmm}} = 0.039 \cdot \frac{\text{Mrads}}{\text{sec}}$$

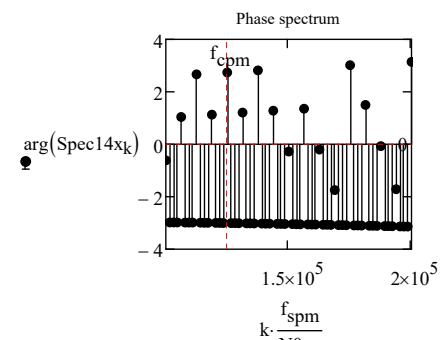


Fig.: (4.4.7.4)

$$A_0 = -6 \quad m_{\text{pm}} = 8$$

4.5

Search of the time sequence output by a discrete convolution

The sequence corresponding to the transfer function, can be found applying the "invztrans" MATHCAD's operator as follows:

$$u0 := u0 \quad v0 := v0 \quad k := k$$

$$h1_k := u0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v0} \text{ invztrans}, z, k \rightarrow \frac{u0 \cdot (v0^{k+1} + \delta(k, 0) - v0^k)}{v0}$$

The result is the sequence of the impulse response, here depicted:

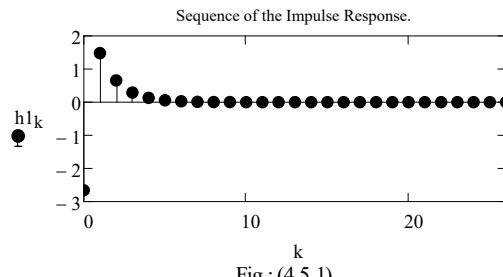


Fig.: (4.5.1)

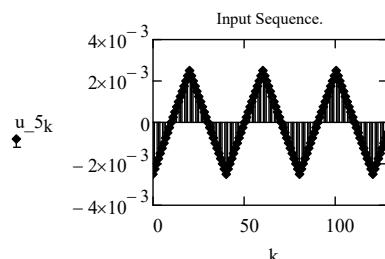
The Output of the Digital System is given by the **discrete convolution** between the discrete-time input signal (the discrete time-sequence of the *triangular wave* for this example) and the discrete impulse response of the System:

$$\text{Out7} := \text{polyalg}(A_0, \omega_0, T_{\text{stri}}, N0_{\text{gd}})$$

$$h7 := (\text{Out7}^{(2)})_0$$

$$\text{Coefficients: } u00 := (\text{Out7}^{(0)})_0 \quad v00 := (\text{Out7}^{(1)})_0$$

$$y15n1 := \sum_{k=0}^{n1} (\text{if}(n1 - k \geq 0, h7_k \cdot u_{-5n1-k}, 0))$$



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Fig.: (4.5.2)

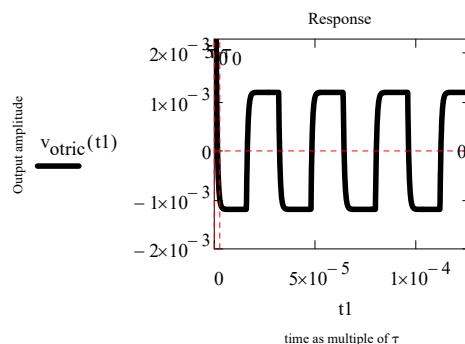


Fig.: (4.5.3)

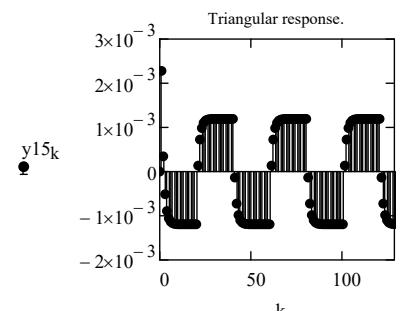


Fig.: (4.5.4)

Knowing the sequences of any input and of the impulse response and the relative Z transforms, the **z-antitransform of the product** of the two z functions can be determined , it corresponds to the convolution of the two sequences, as follows:

$$X_{hp}(z) := \sum_{n=0}^{N1-1} (u1_n \cdot z^{-n})$$

$$H_{hps}(z) := \sum_{n=0}^{N0_{\text{gd}}-1} (h1_n \cdot z^{-n})$$

$$Y_{hp}(z) := H_{hps}(z) \cdot X_{hp}(z)$$

$$\text{input signal: } V_i := V_i, n := n, \quad V_i \text{ ztrans} \rightarrow \frac{V_i z}{(z-1)^2}$$

System output corresponding to the z-inverse transform of the product:

$$y16k := u0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v0} \cdot \frac{V_i \cdot z}{(z-1)^2} \quad \left| \begin{array}{l} \text{invztrans}, z, k \\ \text{simplify} \end{array} \right. \rightarrow \frac{V_i \cdot u0 \cdot (v0^k - 1)}{v0 - 1}$$

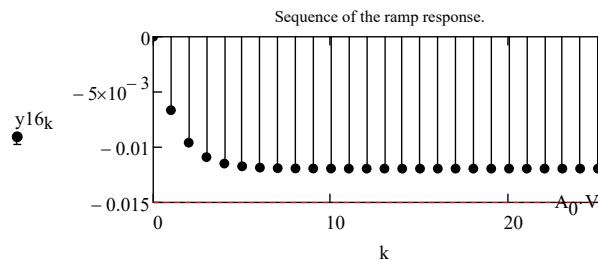


Fig.: (4.5.5)

sampling frequency: $f_{\text{smp}} = 0.875 \cdot \text{MHz}$

sampling period: $T_{\text{smp}} = 1.143 \times 10^3 \cdot \text{ns}$,

sampling angular frequency: $\omega_{\text{smp}} = 5.498 \frac{\text{Mrads}}{\text{sec}}$

$$\Delta T := \frac{T_{\text{test}}}{N_0 g d}$$

$$T_{\text{test}} = 32 \cdot \mu\text{s}$$

$$t := T_{\text{test}} \cdot 0, T_{\text{test}} \cdot 0 + \frac{T_{\text{test}}}{100} \dots 10 \cdot T_{\text{test}}$$

$$n_k := \frac{k}{f_{\text{smp}}}$$

Example: Sinusoidal input

Z transform of the input signal: $\omega := \omega_{\text{test}}$ $\omega = 0.196 \frac{\text{Mrads}}{\text{sec}}$

$$\Delta T = 125 \cdot \text{ns}$$

$$\Delta T := \Delta T \quad \omega := \omega \quad V_i = 2.5 \times 10^{-3} \text{ V} \quad n := n \quad V_i := V_i$$

$V_i \cdot \sin(\omega \cdot n \cdot \Delta T)$	$\xrightarrow{\text{ztrans}}$	$\frac{V_i \cdot z \cdot \sin(\omega \cdot \Delta T)}{z^2 - 2 \cdot \cos(\omega \cdot \Delta T) \cdot z + 1}$
---	-------------------------------	---

I place: $K2 := \sin(\Delta T \cdot \omega)$ $\cos(\Delta T \cdot \omega) = \sqrt{1 - K2^2}$ $\sqrt{1 - K2^2} = 1$

$$K2 := K2$$

$$\text{poles1} := 1 - 2\sqrt{1 - K2^2} \cdot z^{-1} + z^{-2} \text{ solve}, z \rightarrow \begin{pmatrix} \sqrt{1 - K2^2} + K2 \cdot j \\ \sqrt{1 - K2^2} - K2 \cdot j \end{pmatrix}$$

$$p1_0 := \text{poles1}_0 \quad p1_0 = 1 + 0.025j \quad p1_1 := \text{poles1}_1 \quad p1_1 = 1 - 0.025j$$

$$\frac{V_i \cdot z \cdot K2}{z^2 - 2\sqrt{1 - K2^2} \cdot z + 1} = \frac{K2 \cdot V_i \cdot z}{(p1_0 - z) \cdot (p1_0 - \overline{p1_0})} - \frac{K2 \cdot V_i \cdot z}{(p1_0 - \overline{p1_0}) \cdot (z - \overline{p1_0})}$$

Computing the corresponding sequence the result returned for the symbolic operation is too large to be displayed, but can be used for other calculations if assigned to a function.

$$y17_k := u0 \cdot \frac{1 - z^{-1}}{1 - z^{-1} \cdot v0} \cdot \left[\frac{K2 \cdot V_i \cdot z}{(p1_0 - z) \cdot (p1_0 - \overline{p1_0})} \dots \right] \xrightarrow[\text{simplify}]{} \left[\frac{-K2 \cdot V_i \cdot z}{(p1_0 - \overline{p1_0}) \cdot (z - \overline{p1_0})} \dots \right]$$

$$y17_k := K2 \cdot V_i \cdot u0 \cdot \left[\frac{[(1 - p1_0) \cdot (p1_0)^k + v0^k \cdot (v0 - 1)]}{(p1_0 - \overline{p1_0}) \cdot (v0 - \overline{p1_0})} \dots \right] + \frac{v0^k \cdot (v0 - 1) - (\overline{p1_0} - 1) \cdot (\overline{p1_0})^k}{(v0 - \overline{p1_0}) \cdot (\overline{p1_0} - p1_0)}$$

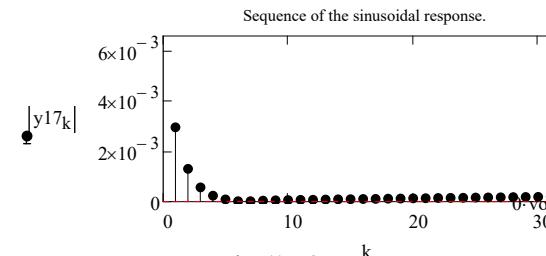


Fig.: (4.5.6)

$$1.1 \cdot \left| \frac{K2 \cdot V_i \cdot u0}{(p1_0 - \overline{p1_0}) \cdot (v0 - \overline{p1_0})} \right| = 6.562 \times 10^{-3} \text{ V}$$

$$A_0 = -6 \quad 20 \cdot \log \left(\left| W_{hp} \left(j \cdot \sqrt{\omega_0 \cdot \omega_{\text{smp}}} \right) \right| \right) = 14.472$$

$$\omega := \frac{\omega_0}{U_0}, \frac{\omega_0}{U_0} + \frac{\omega_{\text{smp}} \cdot U_0 - \frac{\omega_0}{U_0}}{4 \cdot U_0^2} \cdot \frac{U_0}{4} \cdot \omega_{\text{smp}}$$

BODE Plots of $H(z)$ compared with that of $W(j\omega)$

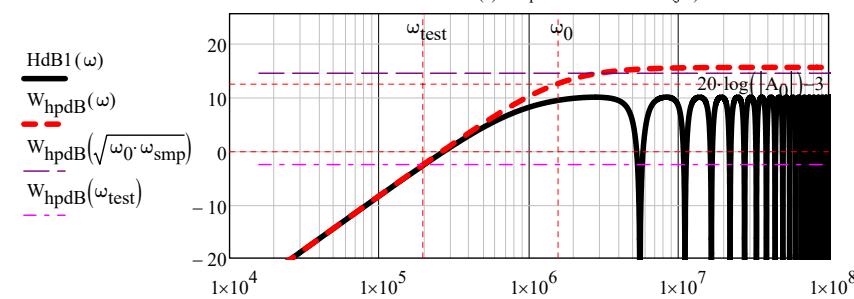


Fig.: (4.5.7)

4.6 The bilinear transformation

4.6.1) Z-transfer function of the I^o Order High Pass Digital Filter

$$s = \frac{2}{T_{\text{smp}}} \cdot \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right), \quad 4.6.1.1)$$

the amplitude response of the analog function is preserved.

$$A_0 := A_0 \quad \omega_0 := \omega_0 \quad \omega_{\text{smp}} := \omega_{\text{smp}} \quad \omega_{\text{smp}} = \frac{2 \cdot \pi}{T_{\text{smp}}} \quad \frac{2}{T_{\text{smp}}} = \frac{\omega_{\text{smp}}}{\pi}$$

$$H11_t(z) := \frac{A_0 \cdot s}{s + \omega_0} \begin{cases} \text{substitute, } s = \frac{\omega_{\text{smp}}}{\pi} \cdot \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right) \\ \text{collect, } z \\ \text{collect, } A_0 \cdot \omega_{\text{smp}} \end{cases} \rightarrow \quad 4.6.1.2)$$

$$\frac{1 - z^{-1}}{\omega_{\text{smp}} + \pi \cdot \omega_0 - z^{-1} \cdot (\omega_{\text{smp}} - \pi \cdot \omega_0)} \cdot (A_0 \cdot \omega_{\text{smp}}) = \frac{1 - z^{-1}}{1 - z^{-1} \cdot \left(\frac{\omega_{\text{smp}} - \pi \cdot \omega_0}{\omega_{\text{smp}} + \pi \cdot \omega_0} \right)} \cdot \left(\frac{A_0 \cdot \omega_{\text{smp}}}{\omega_{\text{smp}} + \pi \cdot \omega_0} \right)$$

$$H11(z) = \frac{1 - z^{-1}}{1 - \left(\frac{\omega_{\text{smp}} - \pi \cdot \omega_0}{\omega_{\text{smp}} + \pi \cdot \omega_0} \right) \cdot z^{-1}} \cdot \left[\frac{A_0 \cdot \omega_{\text{smp}}}{(\omega_{\text{smp}} + \pi \cdot \omega_0)} \right] \quad 4.6.1.3)$$

The following new *parameters* are necessary for the design of the digital filter:

$$\delta_0 := \frac{\omega_{\text{smp}} - \pi \cdot \omega_0}{\omega_{\text{smp}} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{\text{smp}}}{(\omega_{\text{smp}} + \pi \cdot \omega_0)}, \quad 4.6.1.4)$$

$$\omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_0 = 0.053964066, \quad \chi_0 = -3.161892199$$

the new t. f. is:

$$H11(z) := \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \quad 4.6.1.5)$$

$$\text{Z T. Initial value theorem: } \lim_{z \rightarrow \infty} \left(\chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \right) \rightarrow \chi_0 \quad \chi_0 = -3.162 \quad 4.6.1.6)$$

$$\text{Z T. Final value theorem: } \lim_{z \rightarrow 0} \left(\chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \right) \rightarrow \begin{cases} \frac{\chi_0}{\delta_0} & \text{if } \delta_0 \neq 0 \\ \text{undefined} & \text{otherwise} \end{cases} \quad 4.6.1.7)$$

$$H11\text{NdB}(\omega) := 20 \cdot \log \left(\frac{|H11(e^{j \cdot \omega \cdot T_{\text{smp}}})|}{|A_0|} \right) \quad W_{hp}\text{NdB}(\omega) := 20 \cdot \log \left(\frac{|W_{hp}(j \cdot \omega)|}{|A_0|} \right) \quad 4.6.1.8)$$

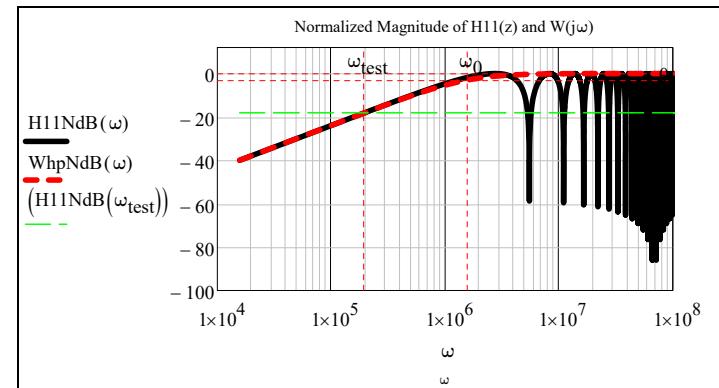


Fig.: (4.6.1.1)

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.2) Difference equations (High Pass filter(I^o order)).

Canonical form

$$H11(z) = \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} \quad 4.6.2.1$$

$$\frac{Y(z)}{W(z)} = \chi_0 \cdot (1 - z^{-1}) \quad 4.6.2.2$$

$$Y(z) = \chi_0 \cdot W(z) - \chi_0 \cdot z^{-1} \cdot W(z) \quad 4.6.2.3$$

$$y(\nu) = \chi_0 \cdot (w(\nu) - w(\nu - 1)) \quad 4.6.2.4$$

$$\frac{W(z)}{X(z)} = \frac{1}{(1 - \delta_0 \cdot z^{-1})} \quad 4.6.2.5$$

$$X(z) = (1 - \delta_0 \cdot z^{-1}) \cdot W(z) = W(z) - \delta_0 \cdot z^{-1} \cdot W(z) \quad 4.6.2.6$$

$$w(n) = x(n) + \delta_0 \cdot w(n - 1) \quad 4.6.2.7$$

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.3) Sequence of the sinusoidal voltage response

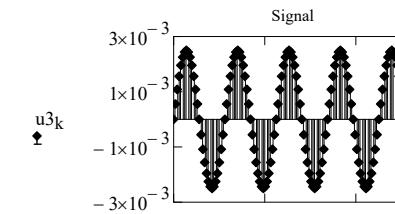


Fig.: (4.6.2.1.1)

The following new *parameters* are necessary for the design of the digital filter:

$$\delta_0 := \frac{\omega_{s3} - \pi \cdot \omega_0}{\omega_{s3} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{s3}}{(\omega_{s3} + \pi \cdot \omega_0)}, \quad 4.6.2.1.1$$

$$\omega_{s3} = 5.498 \cdot \frac{\text{Mrads}}{\text{s}}, \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_0 = 0.053964066, \quad \chi_0 = -3.161892199$$

The corresponding set of difference equations:

$$v_{ni}(\nu) := \frac{u_{3\nu}}{\text{volt}} \quad 4.6.2.1.1$$

$$1) \quad \underline{w}(\nu) := \begin{cases} v_{ni}(\nu) + \delta_0 \cdot \underline{w}(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases} \quad 4.6.2.1.2$$

$$2) \quad \underline{y}(\nu) := \begin{cases} [\chi_0 \cdot (\underline{w}(\nu) - \underline{w}(\nu - 1))] & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases} \quad 4.6.2.1.3$$

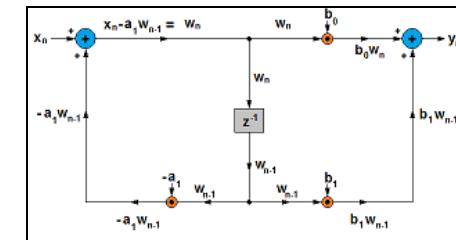


Fig.: (4.6.2.1.2)

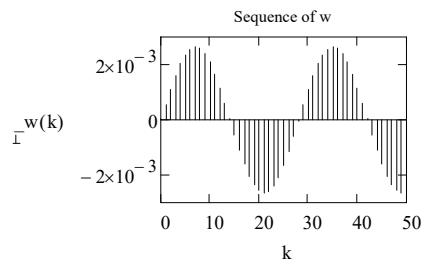


Fig.: (4.6.2.1.3)

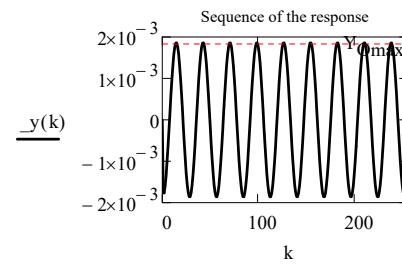


Fig.: (4.6.2.1.4)

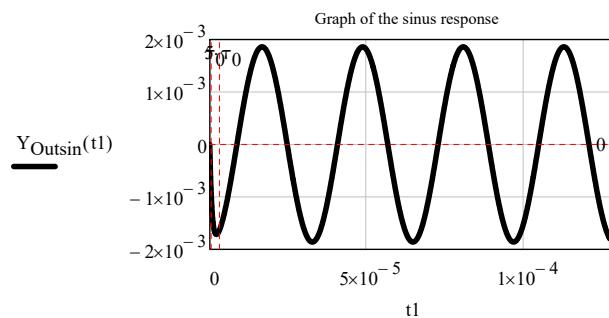


Fig.: (4.6.2.1.5)

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.4) Sequence of the Voltage Pulse response

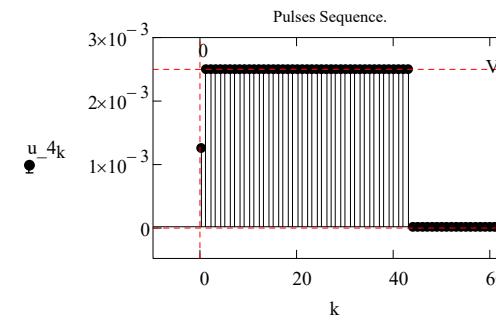
$$T_{\text{test}} = 3.2 \times 10^4 \cdot \text{ns}$$

$$\tau_0 = 0.637 \cdot \mu\text{s}$$

$$\text{Chosen test signal period, } T_{s4} = 5.729 \cdot \mu\text{s}$$

$$\frac{1}{T_{\text{test}}} = 0.031 \cdot \text{MHz}$$

Short pulse sequence of amplitude V_i :



$$\omega_{s4} := 2 \cdot \pi \cdot f_{s0}$$

Fig.: (4.6.2.2.1)

The following new *parameters* are necessary for the design of the digital filter:

$$\delta_0 := \frac{\omega_{s4} - \pi \cdot \omega_0}{\omega_{s4} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{s4}}{(\omega_{s4} + \pi \cdot \omega_0)}, \quad 4.6.2.1.1$$

$$\omega_{s4} = 1.097 \cdot \frac{\text{Mrads}}{\text{s}}, \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta_0 = -0.636342278, \quad \chi_0 = -1.0909731652$$

Digital first order High pass filter difference equations:

$$\text{dimensionless input signal: } v_{i18}(\nu) := \frac{u_{-4\nu}}{\text{volt}}$$

$$1) \quad w_{18}(\nu) := \begin{cases} v_{i18}(\nu) + \delta_0 \cdot w_{18}(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y_{18}(\nu) := \begin{cases} \chi_0 \cdot (w_{18}(\nu) - w_{18}(\nu - 1)) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

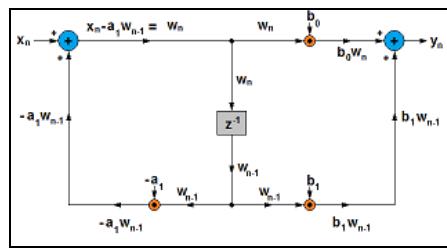


Fig.: (4.6.2.2.2)

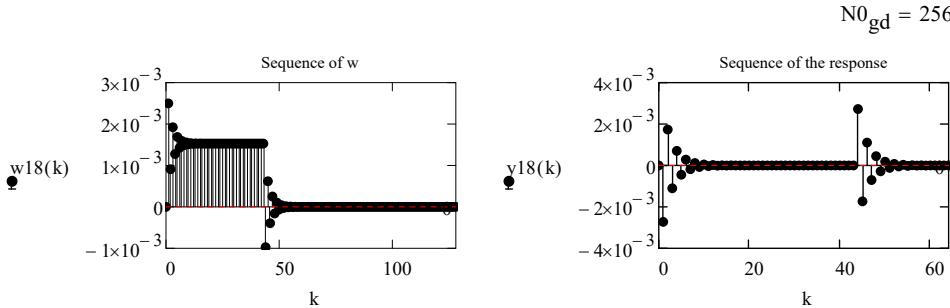


Fig.: (4.6.2.2.3)

Fig.: (4.6.2.2.4)

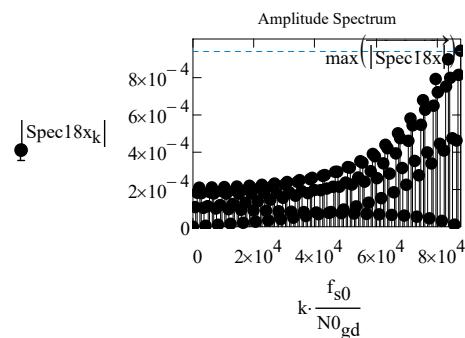


Fig.: (4.6.2.2.7)

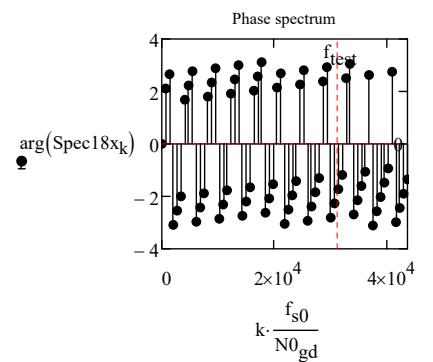


Fig.: (4.6.2.2.8)

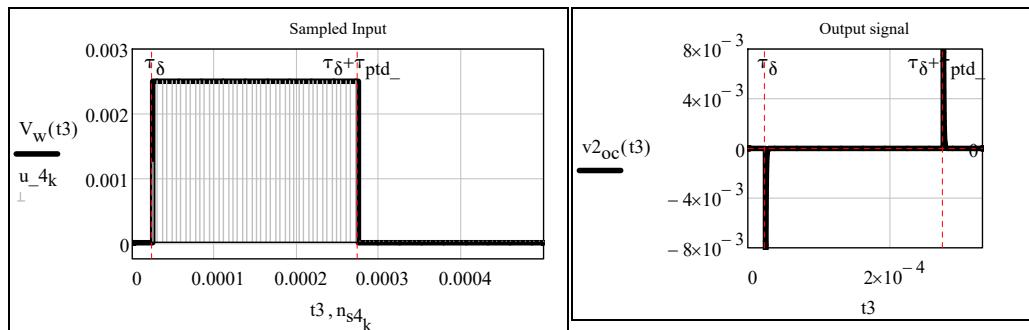


Fig.: (4.6.2.2.5)

Fig.: (4.6.2.2.6)

Sampled signal: $v18x_k := y18(k)$
 $\text{Spec18x} := \text{fft}(v18x)$

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.5) Sequence of the Bipolar Triangular wave response:

$$T_{\text{test}} = 3.2 \times 10^4 \cdot \text{ns} \quad T_{\text{stri}} = 800 \cdot \text{ns} \quad \tau_0 = 0.637 \cdot \mu\text{s}$$

$$\text{Chosen test signal period, } T_{\text{test}} = 3.2 \times 10^4 \cdot \text{ns} \quad \frac{1}{T_{\text{test}}} = 0.031 \cdot \text{MHz}$$

Triangular wave sequence:

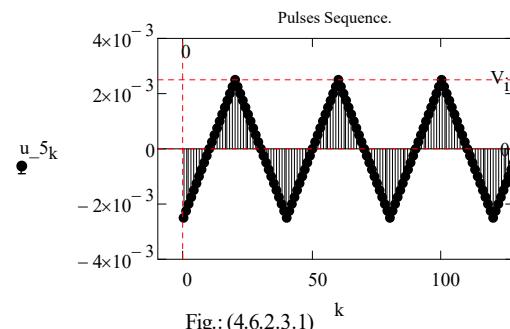


Fig.: (4.6.2.3.1)

The following new parameters are necessary for the design of the digital filter:

$$\omega_{\text{stri}} := 2 \cdot \pi \cdot f_{\text{stri}} \quad \delta 0 := \frac{\omega_{\text{stri}} - \pi \cdot \omega_0}{\omega_{\text{stri}} + \pi \cdot \omega_0}, \quad \chi 0 := \frac{A_0 \cdot \omega_{\text{stri}}}{(\omega_{\text{stri}} + \pi \cdot \omega_0)}, \quad 4.6.2.1.1$$

$$\omega_{\text{stri}} = 7.854 \cdot \frac{\text{Mrads}}{\text{s}} \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta 0 = 0.22826091, \quad \chi 0 = -3.6847827294$$

Digital first order High pass filter recurrence relations:

$$\text{dimensionless input signal: } v19_i(\nu) := u_5\nu \quad \frac{f_{\text{stri}}}{f_{\text{test}}} = 40$$

$$1) \quad w19(\nu) := \begin{cases} v19_i(\nu) + \delta 0 \cdot w19(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y19(\nu) := \begin{cases} \chi 0 \cdot (w19(\nu) - w19(\nu - 1)) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

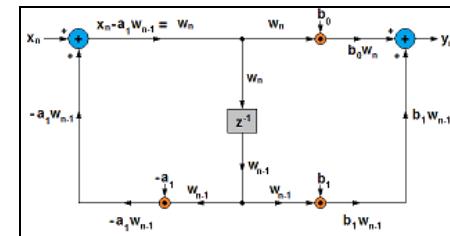


Fig.: (4.6.2.3.2)

$$N0_{\text{gd}} = 256$$

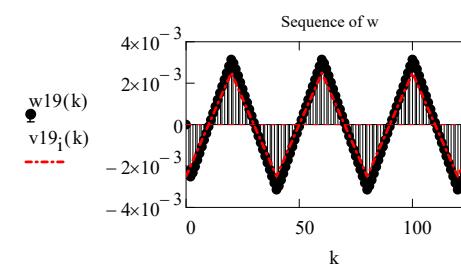


Fig.: (4.6.2.3.3)

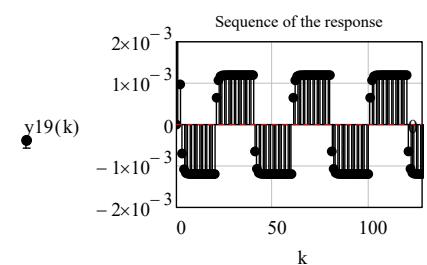


Fig.: (4.6.2.3.4)

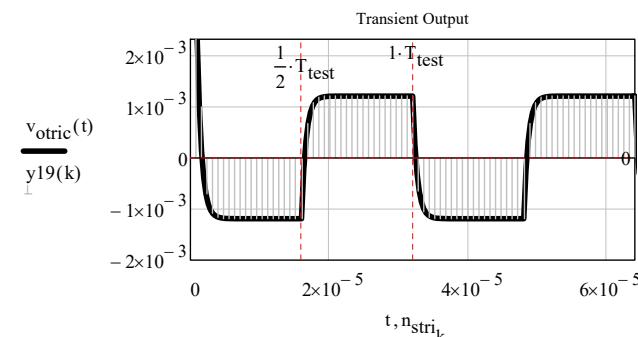


Fig.: (4.6.2.3.5)

Sampled signal: $v19x_k := y19(k)$

$\text{Spec19x} := \text{fft}(v19x)$

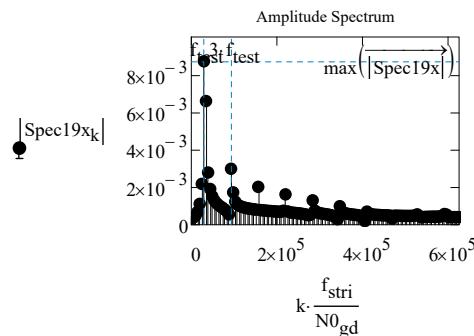


Fig.: (4.6.2.3.6)

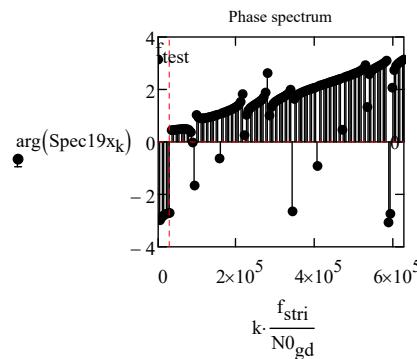


Fig.: (4.6.2.3.7)

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.6) Sequence of the Sawtooth wave response

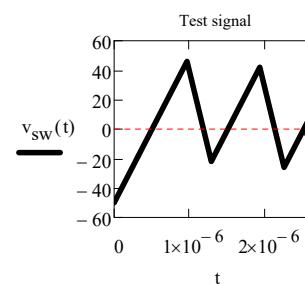


Fig.: (4.6.2.4.1)

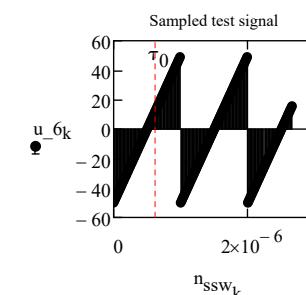


Fig.: (4.6.2.4.2)

The following new *parameters* are necessary for the design of the digital filter:

$$\omega_{ssw} := 2 \cdot \pi \cdot f_{ssw}, \quad \delta 0 := \frac{\omega_{ssw} - \pi \cdot \omega_0}{\omega_{ssw} + \pi \cdot \omega_0}, \quad \chi 0 := \frac{A_0 \cdot \omega_{ssw}}{(\omega_{ssw} + \pi \cdot \omega_0)}, \quad 4.6.2.1.1$$

$$\omega_{ssw} = 603.186 \frac{\text{Mrads}}{\text{s}}, \quad \omega_0 = 1.571 \times 10^3 \frac{\text{krad}}{\text{s}}, \quad \delta 0 = 0.983770317, \quad \chi 0 = -5.9513109511$$

Step sequence of amplitude V_i :

Digital first order High pass filter recurrence relations:

$$\text{dimensionless input signal: } v20_i(\nu) := u_{-6\nu}$$

$$\frac{f_{ssw}}{f_{test}} = 3.072 \times 10^3$$

$$1) \quad w20(\nu) := \begin{cases} v20_i(\nu) + \delta 0 \cdot w20(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y20(\nu) := \begin{cases} \chi 0 \cdot (w20(\nu) - w20(\nu - 1)) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

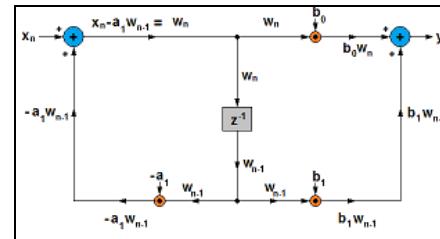


Fig.: (4.6.2.4.3)

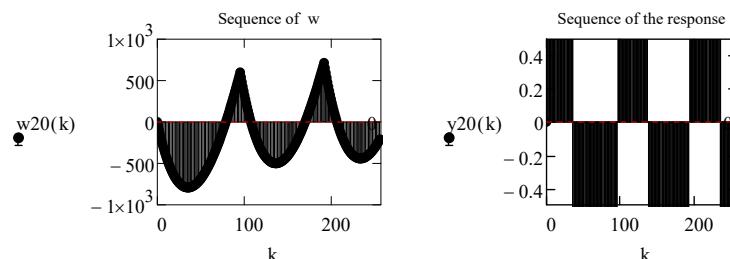
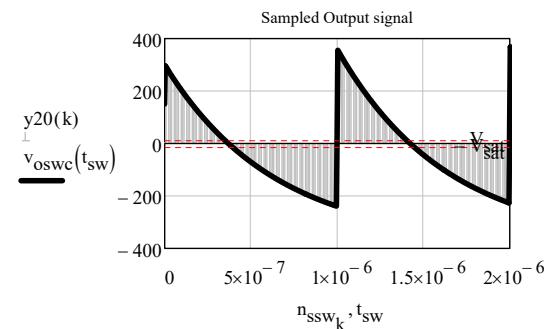


Fig.: (4.6.2.4.4)

Fig.: (4.6.2.4.5)



Sampled signal: $v20x_k := y20(k)$ Spec20x := fft(v20x)

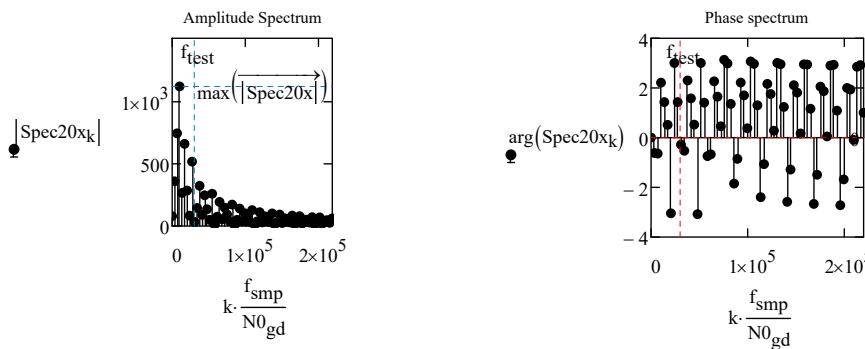


Fig.: (4.6.2.4.6)

Fig.: (4.6.2.4.7)

$$\frac{\max(|\cdot|)}{\max(\overrightarrow{|\text{Spec20x}|})} = \blacksquare$$

$$\max(\overrightarrow{|\text{Spec20x}|}) = 1.114 \times 10^9 \frac{1}{\mu\text{s}}$$

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.7 Sequence of the (Single tone) AM Signal response

AM input-signal sequence of amplitude V_i :

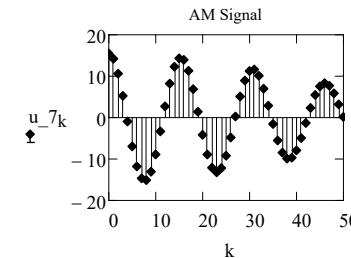


Fig.: (4.6.2.5.1)

The following new parameters are necessary for the design of the digital filter:

$$\omega_{\text{sam}} := 2 \cdot \pi \cdot f_{\text{sam}} \quad \delta 0 := \frac{\omega_{\text{sam}} - \pi \cdot \omega_0}{\omega_{\text{sam}} + \pi \cdot \omega_0}, \quad \chi 0 := \frac{A_0 \cdot \omega_{\text{sam}}}{(\omega_{\text{sam}} + \pi \cdot \omega_0)}, \quad 4.6.2.5.1)$$

$$\omega_{\text{sam}} = 9.676 \cdot \frac{\text{Mrads}}{\text{s}} \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{k.radians}}{\text{s}}, \quad \delta 0 = 0.324504357, \quad \chi 0 = -3.9735130721$$

Digital first order High pass filter recurrence relations:

dimensionless input signal: $v21_i(\nu) := u_{-7\nu}$

$$\frac{f_{\text{sam}}}{f_{\text{mam}}} = 154$$

$$1) \quad w21(\nu) := \begin{cases} v21_i(\nu) + \delta 0 \cdot w21(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y21(\nu) := \begin{cases} \chi 0 \cdot (w21(\nu) - w21(\nu - 1)) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

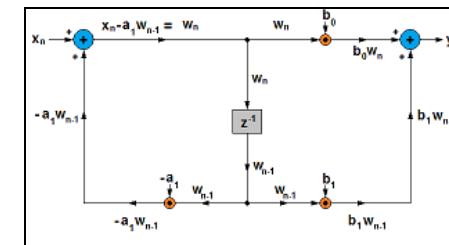


Fig.: (4.6.2.5.2)

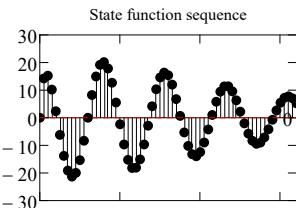


Fig.(4.6.2.5.3)

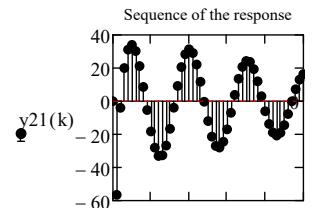


Fig.(4.6.2.5.4)

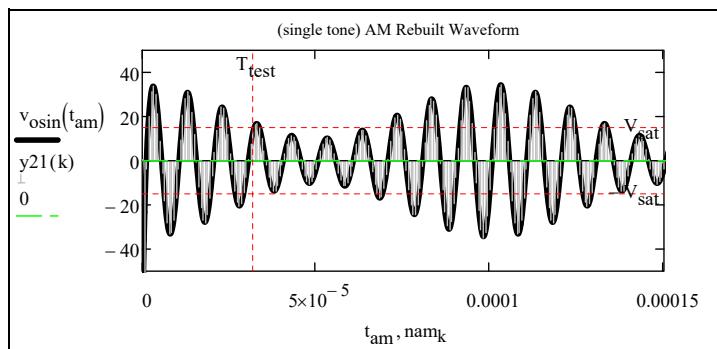
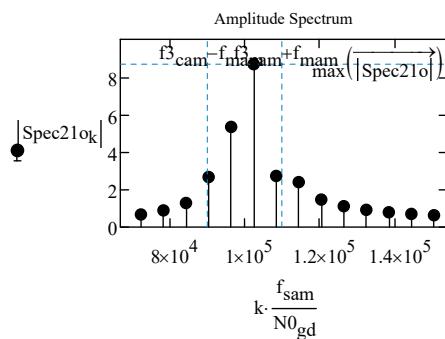
Sampled signal: $v_{21x_k} := y_{21}(k)$ $\text{Spec21o} := \text{FFT}(v_{21x})$ 

Fig.(4.6.2.5.5)

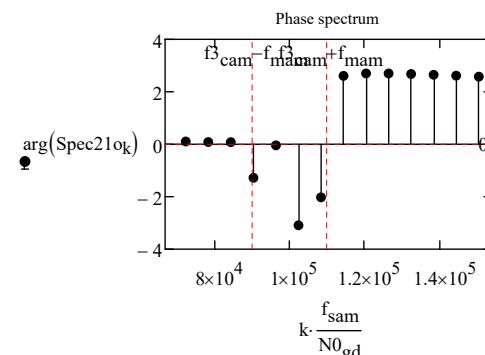


Fig.(4.6.2.5.6)

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.8) Sequence of the (Single tone) Frequency Modulated carrier response $m_{fm} = 8$

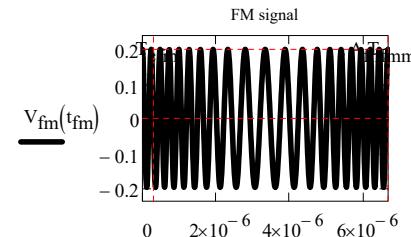


Fig.(4.6.2.6.1)

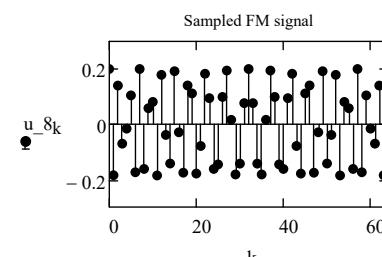


Fig.(4.6.2.6.3)

$$\max(\overrightarrow{|\text{OSpecfm}|}) = 3.373$$

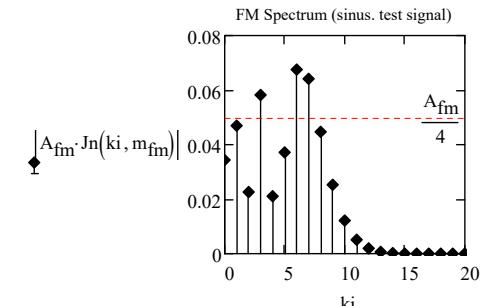


Fig.(4.6.2.6.2)

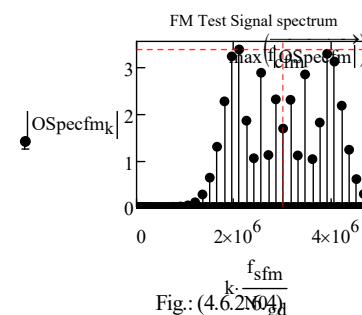


Fig.(4.6.2.6.4)

$$\frac{f_{sfm}}{f_{cfm}} = 3.2$$

The following new *parameters* are necessary for the design of the digital filter:

$$\omega_{sfm} := 2 \cdot \pi \cdot f_{sfm}, \quad \delta_0 := \frac{\omega_{sfm} - \pi \cdot \omega_0}{\omega_{sfm} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{sfm}}{(\omega_{sfm} + \pi \cdot \omega_0)}, \quad 4.6.2.5.1)$$

$$\omega_{sam} = 9.676 \cdot \frac{\text{Mrads}}{\text{s}}, \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_0 = 0.848749533, \quad \chi_0 = -5.5462485977$$

Digital first order High pass filter difference relations:

dimensionless input signal: $v_{22i}(\nu) := u_{-8\nu}$

$$1) \quad w_{22}(\nu) := \begin{cases} v_{22i}(\nu) + \delta_0 \cdot w_{22}(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$2) \quad y_{22}(v) := \begin{cases} \chi_0 \cdot (w_{22}(v) - w_{22}(v-1)) & \text{if } v > 0 \\ 0 & \text{otherwise} \end{cases}$$

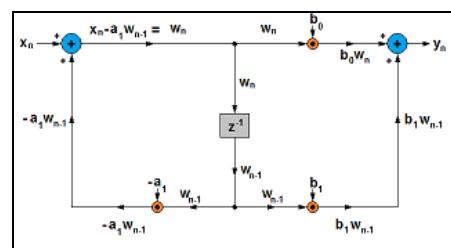


Fig.: (4.6.2.6.5)

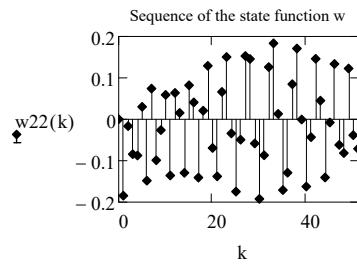


Fig.: (4.6.2.6.6)

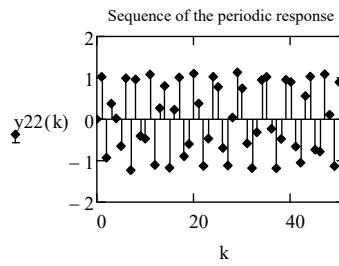


Fig.: (4.6.2.6.7)

$$f_{cfm} = 3 \cdot \text{MHz} \quad \frac{f_{smp}}{f_{cfm}} = 0.292$$

$$U_{22k} := y_{22}(k)$$

$$\text{Spec22} := \text{fft}(U_{22k}) \quad m_{fm} = 8 \quad \omega_{fmm} = 0.942 \cdot \frac{\text{Mrads}}{\text{sec}}$$

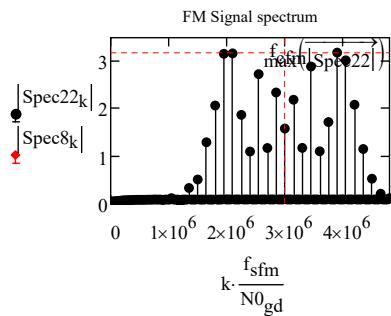


Fig.: (4.6.2.6.8)

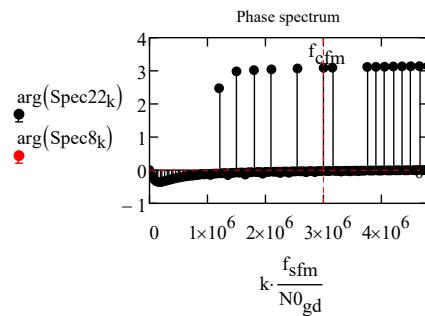


Fig.: (4.6.2.6.9)

$$\max(\overrightarrow{|\text{Spec8}|}) = ■$$

$$\max(\overrightarrow{|\text{Spec22}|}) = 3.162$$

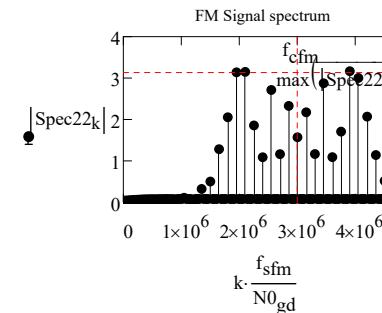


Fig.: (4.6.2.6.10)

$$\frac{\max(|\text{Spec8}|)}{\max(|\text{Spec22}|)} = ■$$

$$\max(|\text{Spec22}|) = 9.538$$

4.6 Equivalent Digital High Pass Filter (I^o order) - The bilinear transformation

4.6.9) Sequence of the (Single tone) Phase Modulated carrier response

$$m_{pm} = 8$$

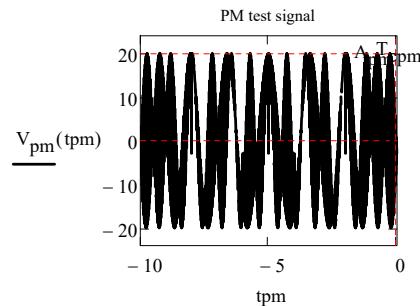


Fig.(4.6.2.7.1)

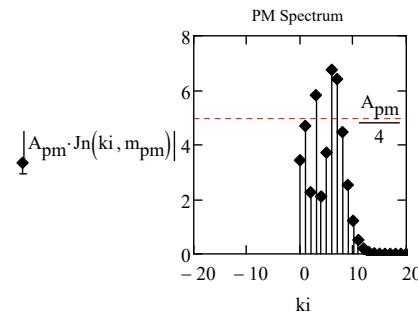


Fig.(4.6.2.7.2)

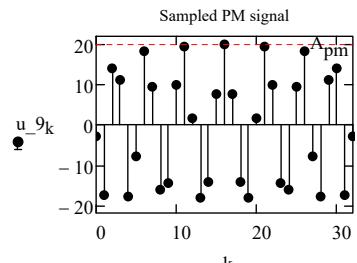


Fig.(4.6.2.7.3)

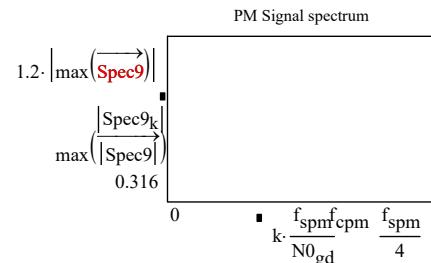


Fig.(4.6.2.7.4)

The following new *parameters* are necessary for the design of the digital filter:

$$\omega_{\text{spm}} := 2 \cdot \pi \cdot f_{\text{spm}} \quad \delta_0 := \frac{\omega_{\text{spm}} - \pi \cdot \omega_0}{\omega_{\text{spm}} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{\text{spm}}}{(\omega_{\text{spm}} + \pi \cdot \omega_0)}, \quad 4.6.2.5.1$$

$$\omega_{\text{sam}} = 9.676 \cdot \frac{\text{Mrads}}{\text{s}} \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krads}}{\text{s}}, \quad \delta_0 = -0.325121276, \quad \chi_0 = -2.0246361721$$

dimensionless input signal: $v_{23i}(\nu) := u_{9\nu}$

$$1) \quad w_{23}(\nu) := \begin{cases} v_{23i}(\nu) + \delta_0 \cdot w_{23}(\nu - 1) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases} \quad \delta_0 = -0.325121276$$

$$2) \quad y_{23}(\nu) := \begin{cases} \chi_0 \cdot (w_{23}(\nu) - w_{23}(\nu - 1)) & \text{if } \nu > 0 \\ 0 & \text{otherwise} \end{cases} \quad \chi_0 = -2.024636$$

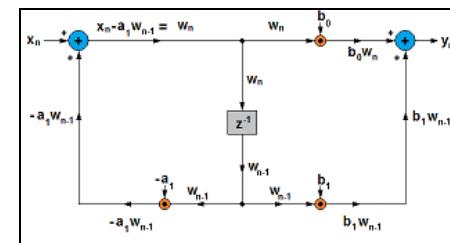


Fig.(4.6.2.7.5)

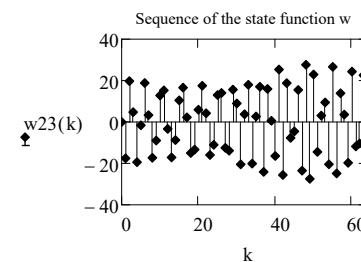


Fig.(4.6.2.7.6)

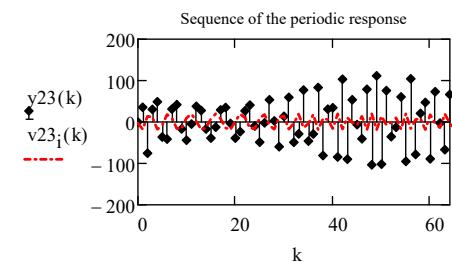


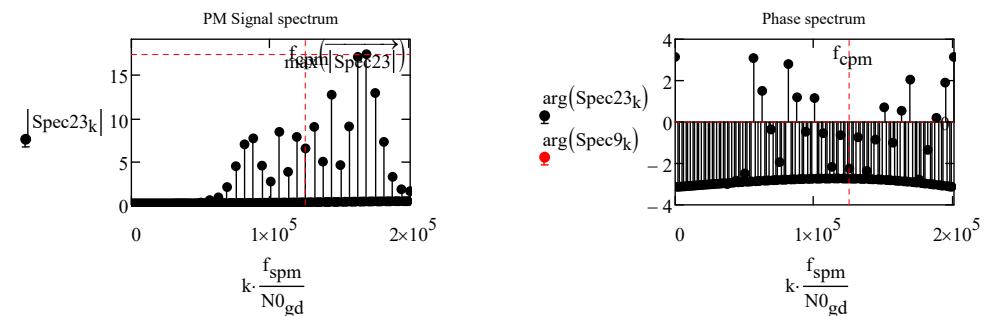
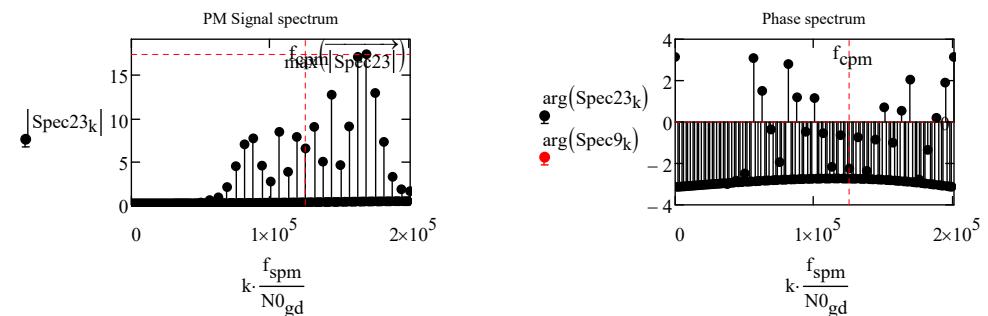
Fig.(4.6.2.7.7)

$$U_{23k} := y_{23}(k)$$

$$\text{Spec23} := \text{FFT}(U_{23})$$

$$m_{pm} = 8$$

$$\omega_{\text{pmm}} = 0.039 \cdot \frac{\text{Mrads}}{\text{sec}}$$



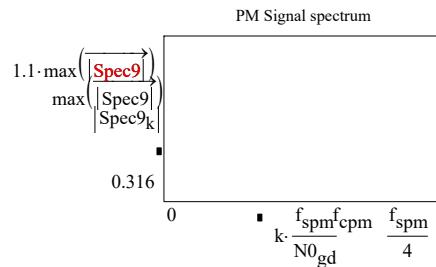


Fig.: (4.6.2.7.10)

$$\frac{\max(|\text{Spec9}|)}{\max(|\text{Spec23}|)} = \frac{1}{17.345}$$

$$\max(|\text{Spec23}|) = 17.345$$

4.7 *Synthetic division algorithm (considering the bilinear transformation)*

The following new *parameters* of the z transfer function are necessary for the design of the digital filter with sinusoidal input:

$$\delta_0 := \frac{\omega_{s3} - \pi \cdot \omega_0}{\omega_{s3} + \pi \cdot \omega_0}, \quad \chi_0 := \frac{A_0 \cdot \omega_{s3}}{(\omega_{s3} + \pi \cdot \omega_0)}, \quad 4.7.1)$$

$$\omega_{s3} = 5.498 \cdot \frac{\text{Mrads}}{\text{s}} \quad \omega_0 = 1.571 \times 10^3 \cdot \frac{\text{krad}}{\text{s}}, \quad \delta_0 = 0.053964066, \quad \chi_0 = -3.161892199$$

The sequence corresponding to the following t. f. :

$$m_{pm} = 8 \quad H11(z) = \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \quad z_0 := -1 \quad p_0 := \delta_0 \quad p_0 = 0.054$$

is realized using the following method:

Numerator degree $Nu_n := 1$ Denominator degree $Md_d := 1$

$$N2 := Nu_n + Md_d \quad N0_{gd} = 256 \quad h11_k := 0$$

$$N2 = 2 \quad H11(z) = \frac{b_0 + b_1 \cdot z^{-1}}{a_0 + a_1 \cdot z^{-1}}$$

I can define the coefficients of the numerator and denominator as elements of two vectors, namely a and b:

Numerator coeffs. Denominator coeffs.

$$b_k := 0.0 \quad a_k := 0.0$$

$$b_0 := \chi_0 \quad a_0 := 1$$

$$b_1 := -\chi_0 \quad a_1 := -\delta_0$$

and divide the two polynomials by means of the following algorithm:

$$h11_0 := \frac{b_0}{a_0} \quad h11_\nu := \frac{1}{a_0} \cdot \left[b_\nu - \sum_{i=1}^{\nu} (h11_{\nu-i} \cdot a_i) \right]$$

T. F. Numerator coefficients:

$$a^T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & -0.054 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

T. F. Denominator coefficients:

$$b^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & -3.162 & 3.162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

Sequence Impulse Response:

$$h11^T = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 \\ 0 & -3.1619 & 2.9913 & 0.1614 & 8.7109 \cdot 10^{-3} & \dots \end{bmatrix}$$

Stability ($S_1 < \infty$):

$$S_2 := \sum_{k=0}^{\text{rows}(h11)-1} |h11_k| \quad S_2 = 6.324$$

$$\text{Energy of the sequence } h11: \quad E11 := \sum_{k=0}^{\text{rows}(h11)-1} (|h11_k|)^2 \quad E11 = 18.971$$

The whole procedure to obtain $h1$ is implemented by the program **Bpolyalg()**.

$$\tau_0 = 0.637 \cdot \mu\text{s} \quad 100 \cdot T_{\text{test}} = 3.2 \times 10^3 \cdot \mu\text{s} \quad t4 := 0 \cdot T_{\text{test}}, \frac{T_{\text{test}}}{100} \dots 1000 \cdot T_{\text{test}} \quad T_{\text{test}} = 32 \cdot \mu\text{s}$$

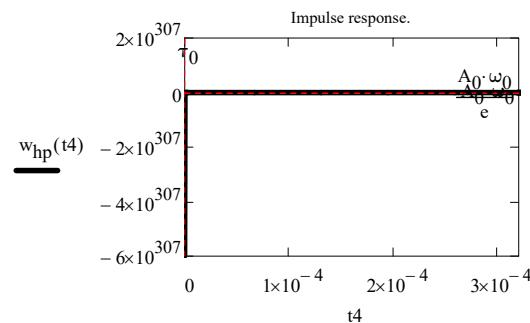


Fig.: (4.7.1)

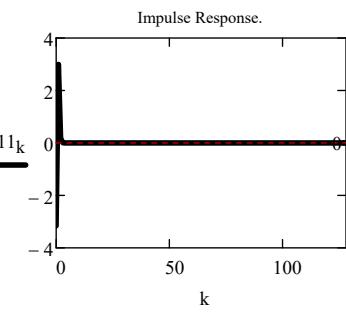


Fig.: (4.7.2)

$$f_{\sin} = \boxed{\text{MHz}}$$

$$\frac{f_{\text{smp}}}{f_c} = \boxed{\text{}}$$

4.7 Iterative algorithm (considering the bilinear transformation)

4.7.1) Sequence of the sinusoidal voltage response

$$t := 0 \cdot \tau_0, \frac{400 \cdot \tau_0}{10000} \dots 400 \cdot \tau_0 \quad y24_\nu := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h11_k \cdot u3_{\nu-k}, 0))$$

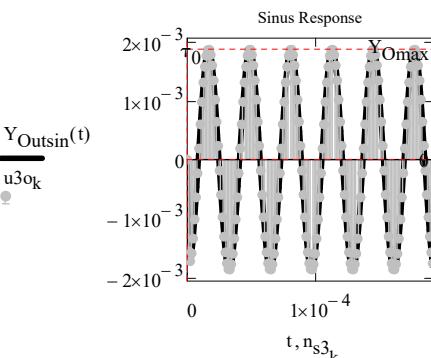


Fig.: (4.7.1.1)

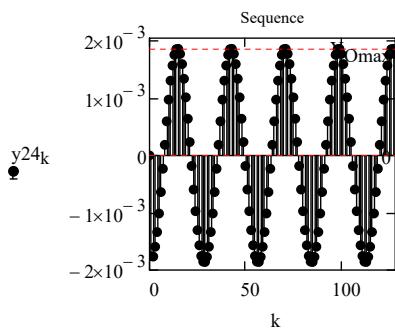


Fig.: (4.7.1.2)

4.7 Iterative algorithm (considering the bilinear transformation)

4.7.2) Sequence of the Voltage Pulse response

Bpolyalg

$$BOut1 := Bpolyalg(A_0, \omega_0, T_{s4}, N0_{gd})$$

$$h11 := (BOut1^{(2)})_0$$

$$y25_\nu := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h11_k \cdot u_{-4\nu-k}, 0))$$

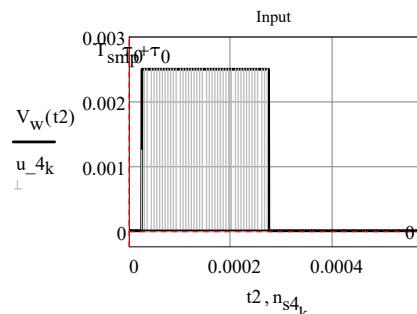


Fig.: (4.7.2.1)

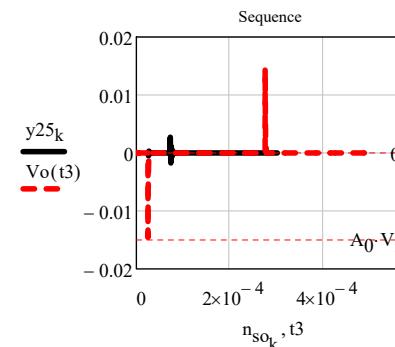


Fig.: (4.7.2.2)

4.7 Iterative algorithm (considering the bilinear transformation)

4.7.3) Sequence of the triangular wave response

$$BOut2 := Bpolyalg(A_0, \omega_0, T_{stri}, N0_{gd})$$

$$h12 := (BOut2^{(2)})_0$$

$$y_{26\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h12_k \cdot u_{-5\nu-k}, 0))$$

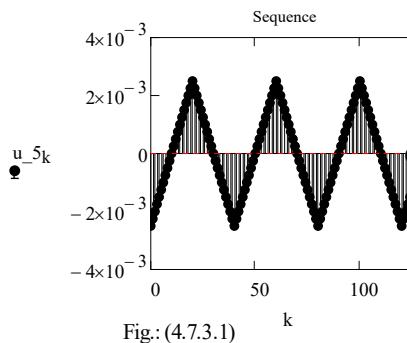


Fig.: (4.7.3.1)

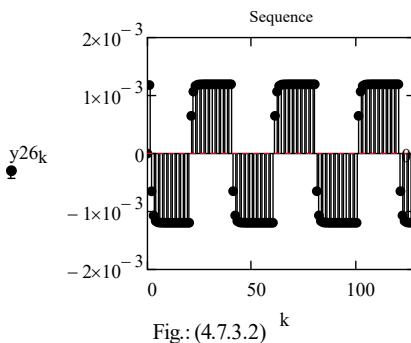


Fig.: (4.7.3.2)

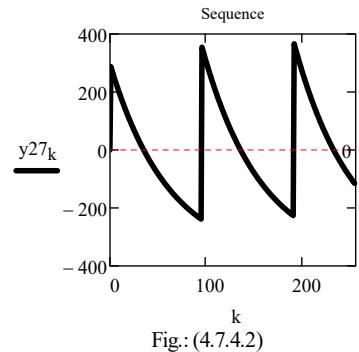
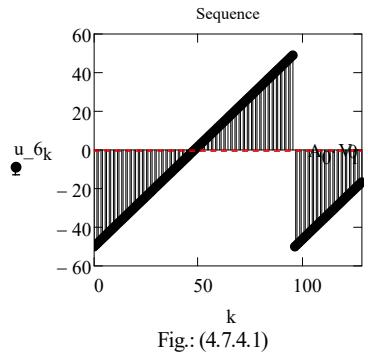
4.7 Iterative algorithm (considering the bilinear transformation)

4.7.4) Sequence of the Sawtooth wave response

$$BOut3 := Bpolyalg(A_0, \omega_0, T_{ssw}, N0_{gd})$$

$$h13 := (BOut3^{(2)})_0$$

$$y_{27\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h13_k \cdot u_{-6\nu-k}, 0))$$



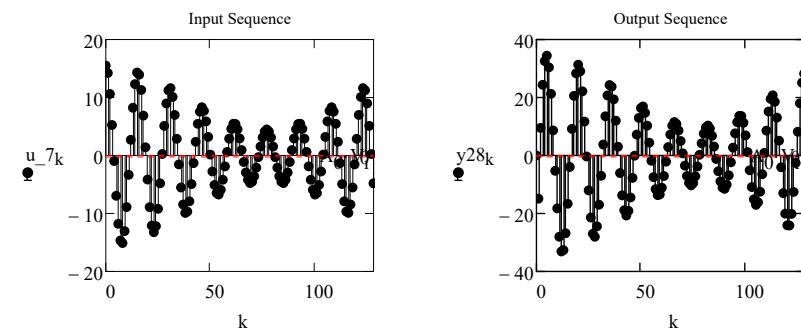
4.7 Iterative algorithm (considering the bilinear transformation)

4.7.5) Sequence of the (Single tone) AM Signal response

$$BOut4 := Bpolyalg(A_0, \omega_0, T_{sam}, N_{gd})$$

$$h14 := (BOut4^{(2)})_0$$

$$y28v := \sum_{k=0}^v (\text{if}(v-k \geq 0, h14_k \cdot u_{7v-k}, 0))$$



4.7 Iterative algorithm (considering the bilinear transformation)

4.7.6) Sequence of the (Single tone) Frequency Modulated carrier response

$$BOut5 := Bpolyalg(A_0, \omega_0, T_{sfm}, N_{gd})$$

$$h15 := (BOut5^{(2)})_0$$

$$y29v := \sum_{k=0}^v (\text{if}(v-k \geq 0, h15_k \cdot u_{8v-k}, 0))$$

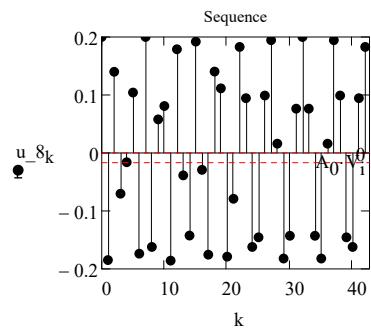


Fig.: (4.7.6.1)

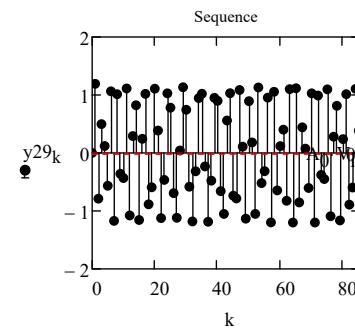


Fig.: (4.7.6.2)

4.7 Iterative algorithm (considering the bilinear transformation)

4.7.7) Sequence of the (Single tone) Phase Modulated carrier response

$$\begin{aligned} \text{BOut6} &:= \text{Bpolyalg}\left(A_0, \omega_0, T_{\text{spm}}, N_{\text{gd}}\right) \\ h16 &:= \left(\text{BOut6}^{(2)}\right)_0 \end{aligned}$$

$$y30\nu := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h16_k \cdot u_{-9\nu-k}, 0))$$

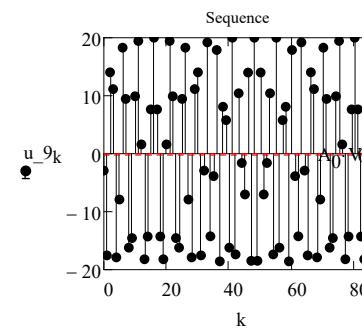


Fig.: (4.7.7.1)

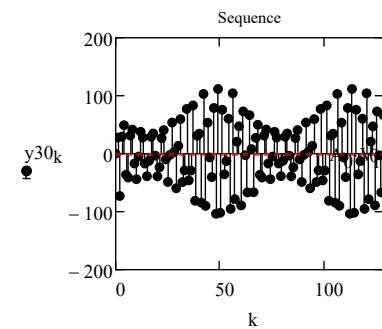


Fig.: (4.7.7.2)

4.8

Analytical search of the output sequence by means of the residues method (considering the bilinear transformation)

$$\delta_0 := \delta_0 \quad \chi_0 := \chi_0 \quad z := z$$

Poles and zeroes of $H11(z) = \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}}$:

$$v := \text{numer}(H11(z)) \text{ coeffs}, z \rightarrow \begin{pmatrix} 0 \\ A_0 \cdot \omega_{\text{smp}}^2 + \pi \cdot A_0 \cdot \omega_0 \cdot \omega_{\text{smp}} \\ -A_0 \cdot \omega_{\text{smp}}^2 - \pi \cdot A_0 \cdot \omega_0 \cdot \omega_{\text{smp}} \end{pmatrix}$$

$$\text{zeros} := \text{polyroots}(v) \quad \text{zeros} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$v := \text{denom}(H11(z)) \text{ coeffs}, z \rightarrow \begin{pmatrix} 0 \\ \omega_{\text{smp}}^2 - \pi^2 \cdot \omega_0^2 \\ -\pi^2 \cdot \omega_0^2 - 2 \cdot \pi \cdot \omega_0 \cdot \omega_{\text{smp}} - \omega_{\text{smp}}^2 \end{pmatrix}$$

$$\text{poles} := \text{polyroots}(v) \quad \text{poles} = \begin{pmatrix} 0 \\ 0.054 \end{pmatrix}$$

$$\delta_0 = 0.054, \quad \chi_0 = -3.162$$

The calculation gives:

$$h11_k :=$$

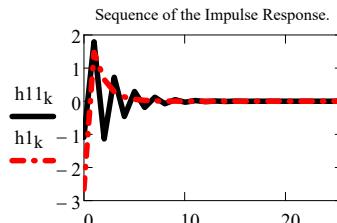


Fig.: (4.8.1)

1.6 iii) By using the "invztrans" operator:

$$\chi_0 := \chi_0 \quad \delta_0 := \delta_0 \quad \nu := \nu$$

$$h101_\nu := \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \text{ invztrans}, z, \nu \rightarrow \frac{\chi_0 \cdot (\delta_0^{\nu+1} + \delta(\nu, 0) - \delta_0^\nu)}{\delta_0}$$

$$t4 := 0 \cdot \tau, \frac{\tau}{1000} .. 50 \cdot \tau$$

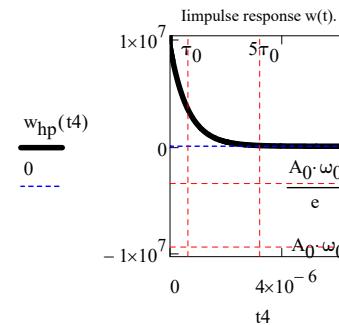


Fig.: (4.8.2)

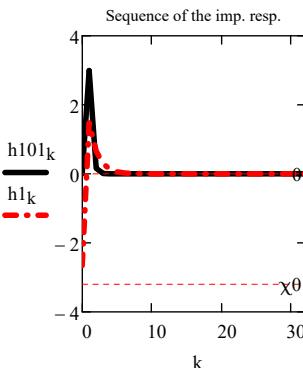


Fig.: (4.8.3)

Stability ($S11 < \infty$):

$$S11 := \sum_{j=0}^{\text{rows}(h101)-1} |h101_j| \quad S11 = 3.162$$

Energy of the sequence $h11$:

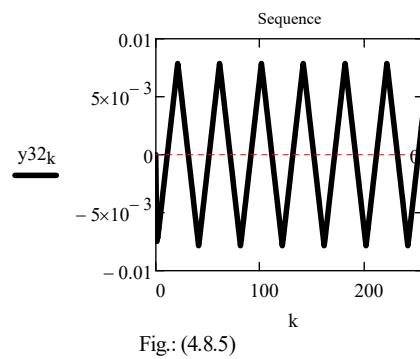
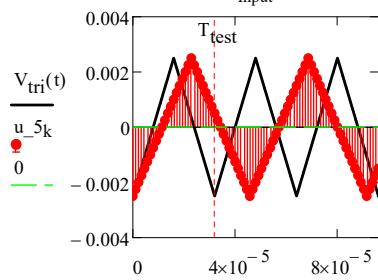
$$E101 := \sum_{j=0}^{\text{rows}(h101)-1} (|h101_j|)^2 \quad E101 = 8.974$$

The Output of the Digital System is given by the discrete convolution between the input signal (in this case the sequence of a step function) and the impulse response of the System:

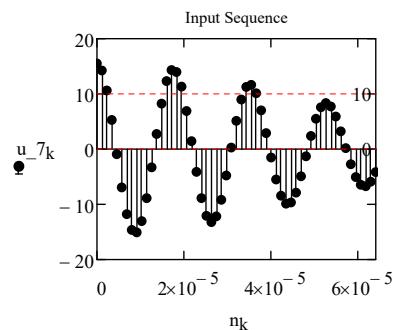
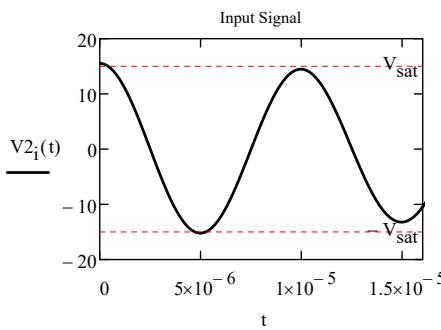
$$y31_\nu := \sum_{j=0}^{\nu} (\text{if}(\nu - j \geq 0, h1_j \cdot u50_{\nu-j}, 0))$$

$$y_{32\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h_{101k} \cdot u_{-5\nu-k}, 0))$$

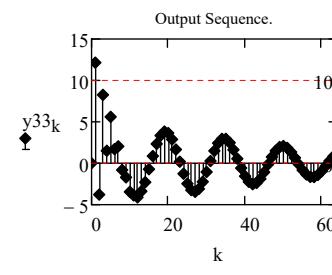
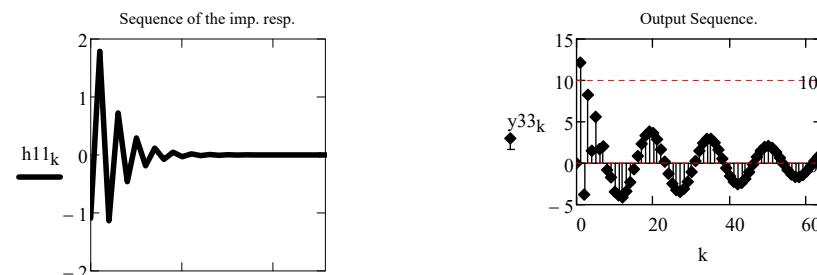
Example 1) Triangular wave



Example 2) AM Signal input:



$$y_{33\nu} := \sum_{k=0}^{\nu} (\text{if}(\nu - k \geq 0, h_{11k} \cdot u_{-7\nu-k}, 0))$$



Knowing the sequences of any input and of the impulse response and the relative Z transforms, I can determine the inverse of the product of the two Z transformed, corresponding to the convolution of the two sequences, as follows:

$$X_4(z) := \sum_{n=0}^{N-1} (u_{1n} z^{-n})$$

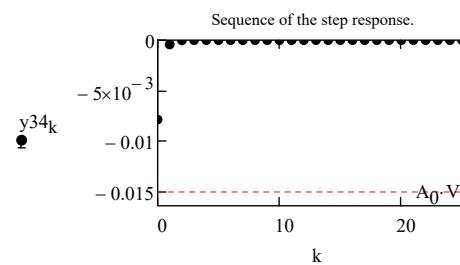
$$H_4(z) := \sum_{n=0}^{N-1} (h_{11n} z^{-n})$$

$$Y_4(z) := H_4(z) \cdot X_4(z)$$

$$V_i := V_i \quad V_i \text{ ztrans } \rightarrow \frac{V_i z}{z-1}$$

$$\delta_0 = 0.053964066$$

$$y_{34k} := \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \cdot \frac{V_i \cdot z}{z-1} \quad \begin{array}{l} \text{invztrans}, z, k \\ \text{simplify} \end{array} \rightarrow V_i \cdot \chi_0 \cdot \delta_0^k$$



$$n := n \quad V_i = 2.5 \times 10^{-3} \sqrt{\Delta T} = 125 \cdot \text{ns}$$

$$\text{Example 3)} \omega_2 := \omega_{\text{test}}. \text{ System Input: } x_{2k} := \frac{V_i}{\text{volt}} \cdot \sin(k \cdot \omega_2 \cdot \Delta T)$$

$$V_i := V_i \quad \Delta T := \Delta T \quad \omega_2 := \omega_2$$

$$K_3 := \sin(\Delta T \cdot \omega_2) \quad \sqrt{1 - K_3^2} = 1 \quad K_3 = 0.025 \quad \omega_{0\text{dB}} = 0.266 \cdot \frac{\text{Mrads}}{\text{sec}}$$

$$X_{\sin}(z) := \frac{V_i \cdot z \cdot K_3}{z^2 - 2 \cdot \sqrt{1 - K_3^2} \cdot z + 1} \quad K_3 := K_3$$

$$\text{poles1} := z^2 - 2\sqrt{1 - K_3^2} \cdot z + 1 \text{ solve, } z \rightarrow \begin{pmatrix} \sqrt{1 - K_3^2} + K_3 \cdot j \\ \sqrt{1 - K_3^2} - K_3 \cdot j \end{pmatrix}$$

$$\text{poles1} = \begin{pmatrix} 1 + 0.025j \\ 1 - 0.025j \end{pmatrix}$$

$$\frac{V_i \cdot z \cdot K_3}{z^2 - 2\sqrt{1 - K_3^2} \cdot z + 1} = \frac{V_i \cdot z \cdot K_3}{(z - \text{poles1}_0) \cdot (z - \text{poles1}_1)}$$

$$r := 1.0 \quad p2_0 := \text{poles1}_0 \quad p2_0 = 1 + 0.025j$$

$$p2_1 := \text{poles1}_1 \quad p2_1 = 1 - 0.025j$$

$$\xi(t5) := r \cdot \cos(t5)$$

$$\psi(t5) := r \cdot \sin(t5)$$

$$\zeta_1(t5) := \xi(t5) + j \cdot \psi(t5)$$

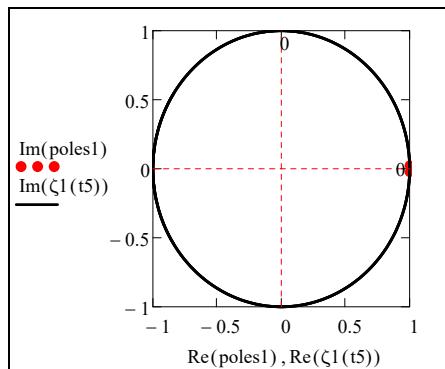


Fig.(4.8.11)

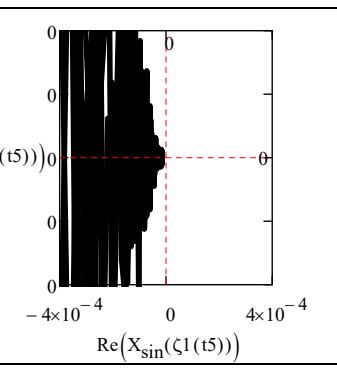


Fig.(4.8.12)

The sequence of the result returned for the symbolic operation is too large to be displayed.
It require some seconds.

$$y12_k := \chi_0 \cdot \frac{1 - z^{-1}}{1 - \delta_0 \cdot z^{-1}} \cdot \left[\frac{K_2 \cdot V_i \cdot z}{(p2_0 - z) \cdot (p2_0 - \overline{p2_0})} \dots \right] \Big|_{\substack{\text{invztrans}, z, \text{using, n} = k \\ \text{simplify}}}$$

$$+ (-1) \cdot \frac{K_2 \cdot V_i \cdot z}{(p2_0 - \overline{p2_0}) \cdot (z - \overline{p2_0})}$$

$$\frac{K_2 \cdot V_i \cdot \chi_0}{\text{volt}} \cdot \left[\delta_0 \cdot (\overline{p2_0})^k - p2_0^k \cdot \overline{p2_0} - p2_0 \cdot (\overline{p2_0})^k + \delta_0^k \cdot \overline{p2_0} + p2_0 \cdot \delta_0^{k+1} \dots \right]$$

$$+ p2_0^{k+1} \cdot \delta_0 - 2 \cdot \delta_0^{k+1} - 2 \cdot \delta_0^{k+2} - p2_0 \cdot (\overline{p2_0})^{k+1} - p2_0^{k+1} \cdot \overline{p2_0} \dots$$

$$+ \delta_0 \cdot (\overline{p2_0})^{k+1} + \delta_0^{k+1} \cdot \overline{p2_0} + p2_0 \cdot \delta_0^k + p2_0^k \cdot \delta_0 \Big]$$

$$y12_k := \frac{(\overline{p2_0} - \delta_0) \cdot (p2_0 - \overline{p2_0}) \cdot (\delta_0 - \overline{p2_0})}{(p2_0 - \delta_0) \cdot (p2_0 - \overline{p2_0}) \cdot (\delta_0 - \overline{p2_0})}$$

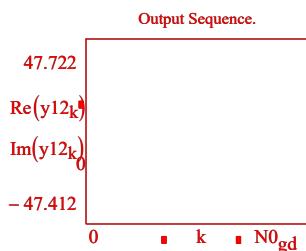


Fig.: (4.8.13)

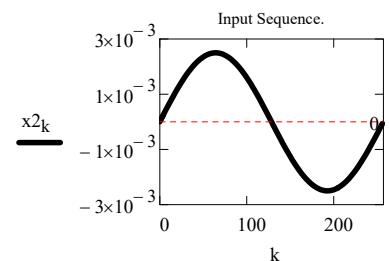


Fig.: (4.8.14)

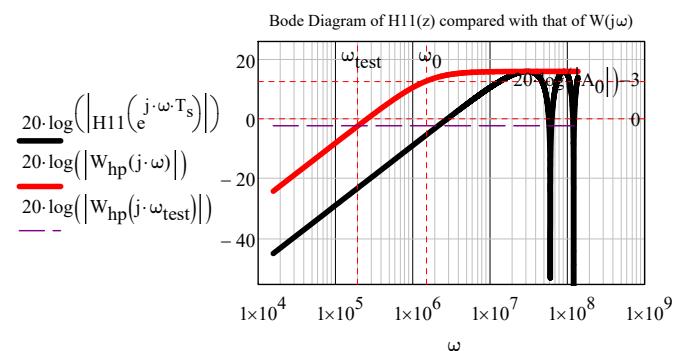


Fig.(4.8.15)

$$20 \cdot \log \left(\left| H11 \left(e^{j \cdot \omega \cdot T_s} \right) \right| \right) = -2.53 \cdot \text{dB}$$

$$20 \cdot \log \left(\left| W_{hp} \left(j \cdot \omega_{test} \right) \right| \right) = -2.566 \cdot \text{dB}$$

