Introduction to POLES and ZEROS.

Response as a function of Omega (2 Pi f):
- In The Sinusoidal Steady State Response.

Pages 273-276 of Hyat and Kemerly 4th ed.

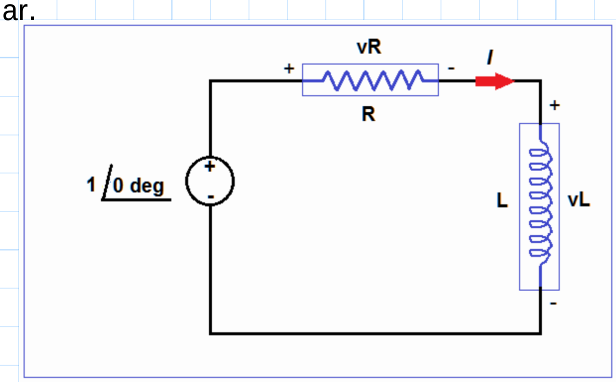
In the power industry the frequency is constant for 3 phase transmission. 50 Hz or 60 Hz, depending on region power company. Other wise frequency f and especially radian frequency Omega plays an important role in most areas of electrical, and mechanical engineering. Our study here under a sinusoidal source condition.

Less likely the radian frequency plays a similar role in an exponentially varying source the curve is not oscillating for one instance.

To keep it short as possible equations will be presented with short notes, you and I be able to follow thru.

$V_s = V_s \angle \theta$ <---Phasor form also polar.

$V_s = V_s \cos(\omega t + \theta)$



Series RL circuit:

$$I = \frac{V_s}{R + j\omega L}$$

Impedance: $$Z = \frac{V_s}{I}$$

Admittance: $$Y = \frac{I}{V_s} \qquad Y = \frac{\dfrac{V_s}{R + j\omega L}}{V_s}$$ <---Series RL circuit.

$$Y = \frac{1}{R + j\omega L}$$

<---This admittance can be interpretated as current produced by a voltage source of magnitude 1 at 0 deg.

Magnitude of the response: *Refer past notes.*

$$|Y| = \frac{1}{\sqrt{R^2 + j^2\omega^2 L^2}}$$

We seen this derivation in previous notes. j^2 = -1

$$|Y| = \frac{1}{\sqrt{R^2 - \omega^2 L^2}} \quad \text{--->} \quad |Y| = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

Right Angle - Pythogaras rule, Hypotenuse, place -wL if in -ve direction of graph. ***<--1***

Angle of the response:

$$\text{ang}Y = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

-wL/R results in 4th quadrant; -ve. ***<---2***

Equation 1 and 2 are mag and ph angle of response, both presented as a function of omega. Omega is the *format* we need to plot.

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From previous notes, time constants :
Current response for example: $i = Ae^{\frac{-t}{\tau\_RL}} = Ae^{\frac{-t}{\frac{L}{R}}} = Ae^{-\left(\frac{R}{L}\right)t}$

RL circuit time constant tau = L/R $\tau_{RL} = \frac{L}{R} \qquad \frac{1}{\tau_{RL}} = \frac{R}{L}$ <--- t - axis

To attain a plot lets give values to our components:

$R := 2 \quad L := 1 \quad V := 1\angle 0$ **deg**

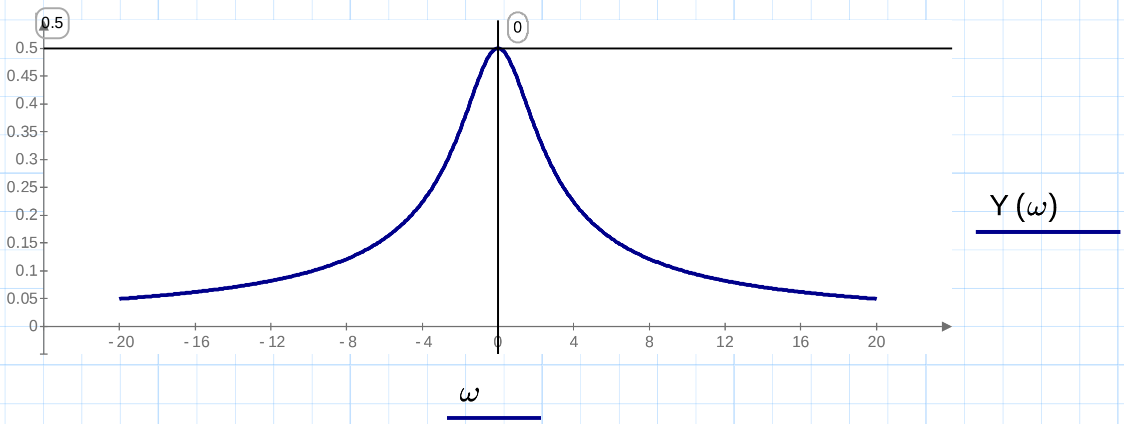
$Y(\omega) := \frac{1}{\sqrt{R^2 + \omega^2 \cdot L^2}}$

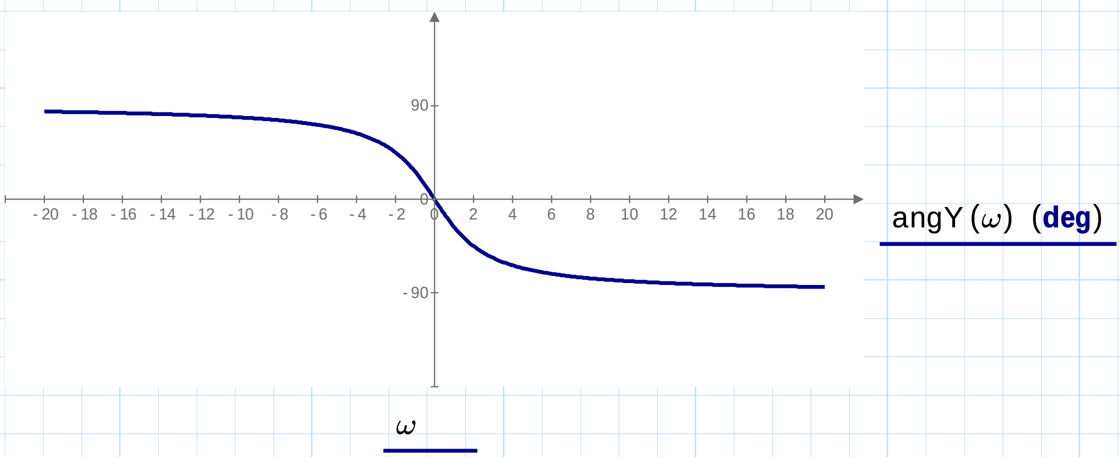
At w = 0 $Y(0) = \frac{1}{\sqrt{2^2 + 0^2 \cdot 1^2}} = 0.5$

At w = 0 $Y(0) = \frac{1}{R} = 0.5$

$angY(\omega) := -atan\left(\frac{\omega \cdot L}{R}\right)$

w: $\frac{R}{L} = 2 \quad \frac{2\ R}{L} = 4 \quad \frac{4\ R}{L} = 8 \quad \frac{6\ R}{L} = 12 \quad \frac{8\ R}{L} = 16 \quad \frac{10\ R}{L} = 20$





Exactly same in textbook plots above. We got the general idea. May seen this before in circuits course, if not its not out of ordinary hunderds of circuit plots have magnitude and angle plotted separately. *No where near a genius yet. A century away. Sorry.*

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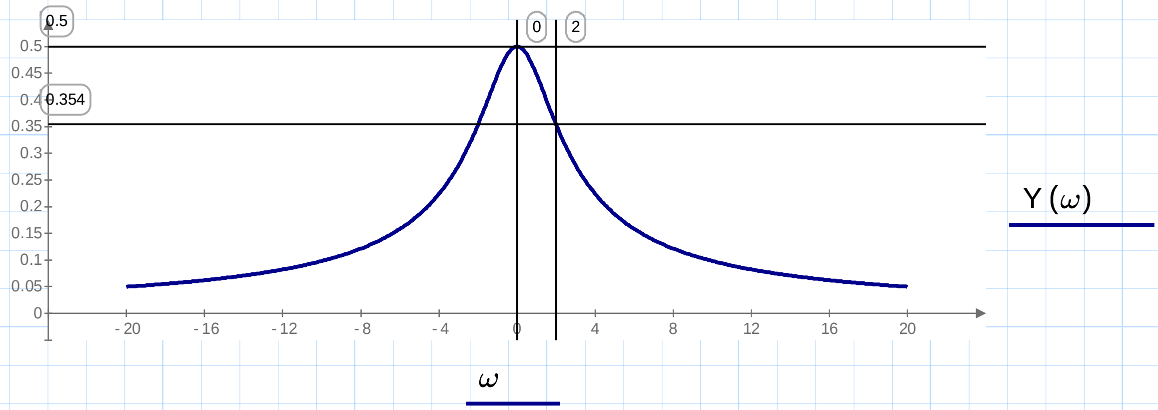


Magnitude vs radain frequency in the plot above is symmetrical.
w can be -ve.

v(t) = 50 cos(100t + 30 deg) ....w = 100
v(t) = 50 cos(-100t + 30 deg)....w = -100

At ***w =0*** the magnitude = 1/R, here 0.5 = 1/2 for plot above.
So you can conclude any sinusoidal response can be treated as discussed above for an RL circuit.

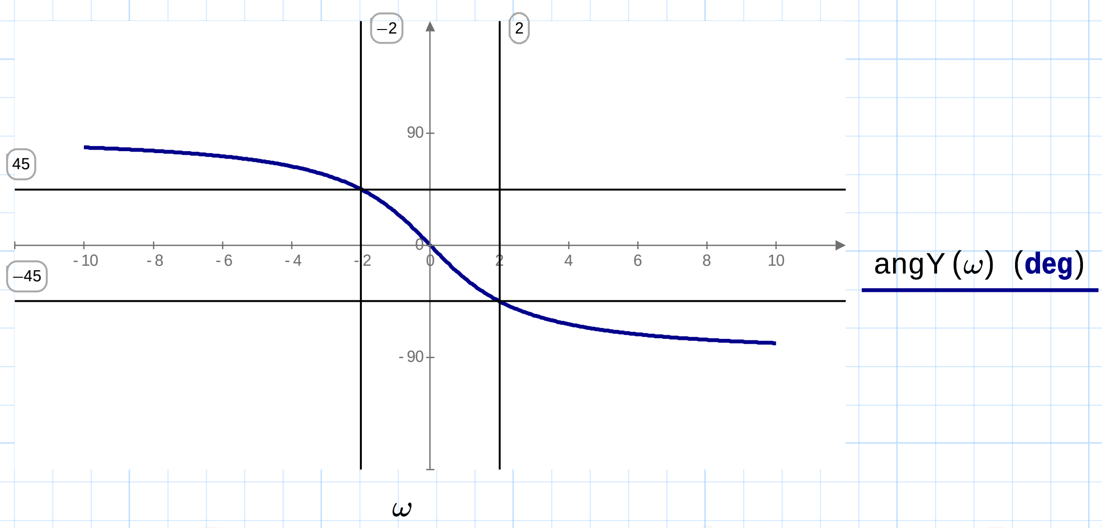
w1: $\frac{R}{L} = 2$ ***w2:*** $\frac{2\ R}{L} = 4$ ***w3:*** $\frac{4\ R}{L} = 8$ ***w4:*** $\frac{6\ R}{L} = 12$ ***w5:*** $\frac{8\ R}{L} = 16$

The magnitude of Y for w0 = 0: $\omega_0 = 0$ $Y_{\omega 0} := \frac{1}{R} = 0.5$

Multiply by 0.707: $\omega_1 = 2$ $Y_{\omega 1} = 0.707 \cdot (Y_{\omega 0}) = 0.354$

The phase angle for w1 at +/-2 is 45 degrees.
See plot below.

We saw this in plot. Unfortunatley it did not work for the other w2, w3,....



We seen some equations and plots but it did not have a written statement that went with it. Below from Hyat-Kemerly:

> 'The points w1 = R/L = 2 and -w1 = -R/L = -2 are marked on the plot. At these radian frequencies (2 and -2) the magnitude (Y) is 0.707 times the maximum magnitude at zero frequency, (w0 = 0; 2 Pi 0 = 0), and the phase angle has a magnitude of 45 degrees.'

From that paragraph above we progress to the following:

> 'At the frequency at which the admittance magnitude (Y) is 0.707 times its maximum value (Y = 0.5 at w0), the current magnitude is 0.707 times its maximum value, and the average power supplied by the source is 0.707^2 or 0.5 times its maximum value. *Comment: The maximum value will be on the power plot at w=0.*
>
> It is NOT very strange that w = R/L (here w = 2) is identified as a half power frequency.'

So, what they the engineers are saying is if we have a current versus radian frequency plot, at w=2 we have a current value which equals 0.707 times current value at w=0.

I = V/Z, Y = 1/Z, **I = Y V.**
So we see its possible for Y to provide a relationship to I because I = Y V.
Cleaver engineers!

Remember NOT for all radian frequencies on the plot ONLY for w1, and in our example w1 = R/L.

Some numbers you seen before:

$\sqrt{2} = 1.414 \quad \frac{\sqrt{2}}{2} = 0.707 \quad 0.707^2 = 0.5$ *Thats where 0.5 came about for the half power frequency.*

Power = VI.
The forcing function is v(t).
The forced response is i(t).
In the p(t) plot, where we have power as the y-axis, and w as the x-axis, at the half power frequency we have the average power supplied. Average power, not maximum, minimum,....but average. Got it! So we find what the half power frequency we got the average power supplied on the curve. Maximum power will be at w=0.

Next another example this time an LC circuit. New things to apply. LC circuit is a little harder more involved.

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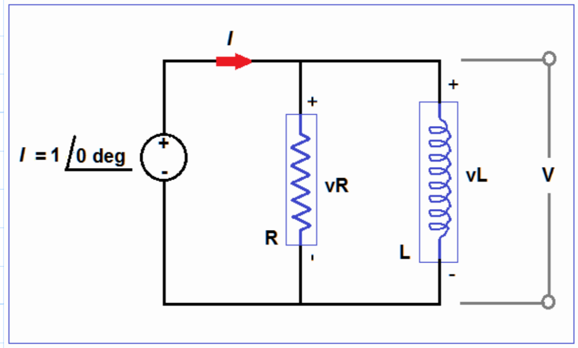
Parallel LC circuit:

Forcing function is current, we seek the voltage as forced response.

$I_s = I_s\angle 0$ deg <---Phasor form also polar.

Current can be leading or lagging, here leave it in phasor form at simple 0 deg. If leading voltage then v(t) need add a phase angle. Keep simple need not a sinusoidal form of I.



Paralle LC circuit:

$$V = I \cdot Z$$

$$Z_L = j\,\omega\,L$$

$$Z_C = \frac{1}{j\,\omega\,C}$$

$$Z_{total} = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{(j\,\omega)L\left(\frac{1}{j\,\omega\,C}\right)}{(j\,\omega)L + \left(\frac{1}{j\,\omega\,C}\right)} = \frac{(j\,\omega)L\left(\frac{1}{j\,\omega\,C}\right)}{(j\,\omega)L - j\cdot\left(\frac{1}{\omega\,C}\right)} = \frac{\frac{j}{j}\cdot\left(\frac{\omega\,L}{\omega\,C}\right)}{j\cdot\left(\omega\,L - \frac{1}{\omega\,C}\right)}$$

$$Z_{total} = \frac{\left(\frac{L}{C}\right)}{j\cdot\left(\omega\,L - \frac{1}{\omega\,C}\right)}$$ Next we try to factor for w.

$$Z_{total} = \frac{\left(\omega\cdot\frac{C}{L}\right)\cdot\left(\frac{L}{C}\right)}{j\cdot\left(\omega\cdot\frac{C}{L}\right)\cdot\left(\omega\,L - \frac{1}{\omega\,C}\right)} = \frac{(\omega)}{j\cdot\left(\omega^2\ C - \frac{1}{L}\right)} = \frac{(\omega)}{j\cdot C\cdot\left(\omega^2 - \frac{1}{LC}\right)}$$ Need to get t (1/LC) in denominator.

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_0^{\,2} = \frac{1}{LC}$$ In series and parallel circuit we have w0 = 1 / (SQRT LC)). We need to get it in that form.

$$Z_{total} = -j\cdot\frac{(\omega)}{C\cdot\left(\omega^2 - \omega_0^{\,2}\right)}$$

Did a substitution for w0^2, next we factor the omega parenthesis. And -j = (1/j).

$$(\omega - \omega_0)\ (\omega + \omega_0) = \omega^2 + \omega\,\omega_0 - \omega\,\omega_0 - w_0^{\,2}$$
$$= \omega^2 - \omega_0^{\,2}$$

$$Z_{total} = \frac{-j}{C}\cdot\frac{(\omega)}{(\omega - \omega_0)\ (\omega + \omega_0)}$$

How does the j get suppressed or disappear?

$$\frac{(\omega)}{(\omega - \omega_0)\ (\omega + \omega_0)}$$ <---The left most term is all radian frequency, and we have -j, and that makes it -jw.

$\frac{-1}{C} \cdot j \cdot \frac{(\omega)}{(\omega - \omega_0)\,(\omega + \omega_0)}$ in a form like this ---> $-\frac{1}{C} \cdot j\ \ \omega$

We seen -w +w on the x-axis, the j now says its an imaginary term, it can be on the +ve and -ve side of the axis.
'-jw 0 jw', and s = sigma + jw.
s can have roots s1 = sigma + jw
s2 = sigma - jw.
So the -jw in our expression is not a problem.
My first reaction each time I see -jw its how do I manipulate or work that, not a problem s = sigma +/- jw from our studies.

So next we take the absolute value of Z, this we seen in most our math and engineering course work.

$$|Z| = \left| \frac{1}{C} \cdot \frac{-j\,(\omega)}{(\omega - \omega_0)\,(\omega + \omega_0)} \right|$$

<---- What happens here?
On the plot axis -jw 0 jw
same as -w 0 w
Absolute value of -j takes it out of the expression.

$$j = \sqrt{-1}$$
$$-j = -\sqrt{-1}$$

$$(-j)^2 = \left(-\sqrt{-1}\right) \cdot \left(-\sqrt{-1}\right) = '+(-1)$$

$$(-j)^2 = -1$$

$$\left|(-j)^2\right| = 1$$

Sometimes you just take the -ve sign out but since its j, I did the square term first. You know that wasn't necessary.

$$|Z| = \frac{1}{C} \cdot \frac{(\omega)}{(\omega - \omega_0)\,(\omega + \omega_0)}$$

<---- By letting w0 = 1/SQRT(1/LC) and factoring the expression for the input impedance, the magnitude of the impedance may be writen in a form which enables those frequencies to be identified at which the response is zero or infinite - Hyat Kemerly

Key note: Frequencies for which the response is zero or infinite.

Such frequencies are termed critical frequencies.

Explanation on this coming.

Next, we proceed to create plots of |Z| and ang Z versus frequencies.

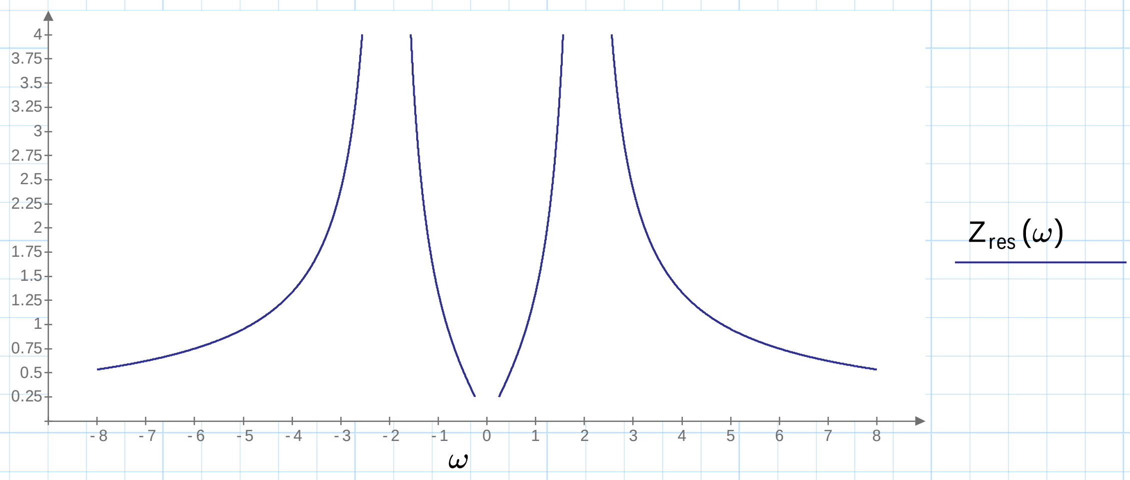
$R := 2 \quad L := 1 \quad C := 0.25 \quad \omega_0 := \frac{1}{\sqrt{L \cdot C}} = 2$ <---w0 not same to previous RL circuit.

w0: $\omega_0 = 2 \quad 2 \cdot \omega_0 = 4 \quad 3 \cdot \omega_0 = 6 \quad 4 \cdot \omega_0 = 8 \quad 5 \cdot \omega_0 = 10 \quad 6 \cdot \omega_0 = 12$

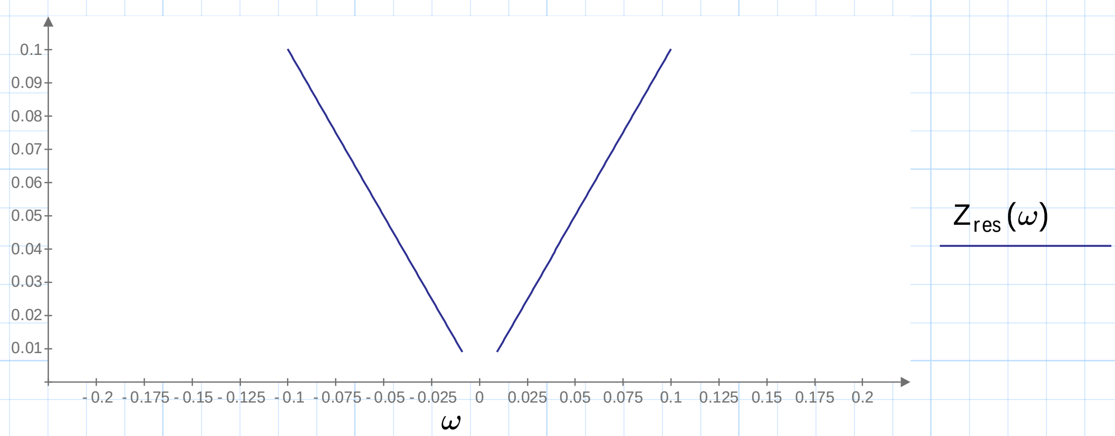
$\omega := -10.0, -9.999 .. 10.0$ <--- Initialisation for w

$Z(\omega) := \frac{1}{C} \cdot \frac{(\omega)}{(\omega - \omega_0)\,(\omega + \omega_0)}$ <--- Calculating Z(w)

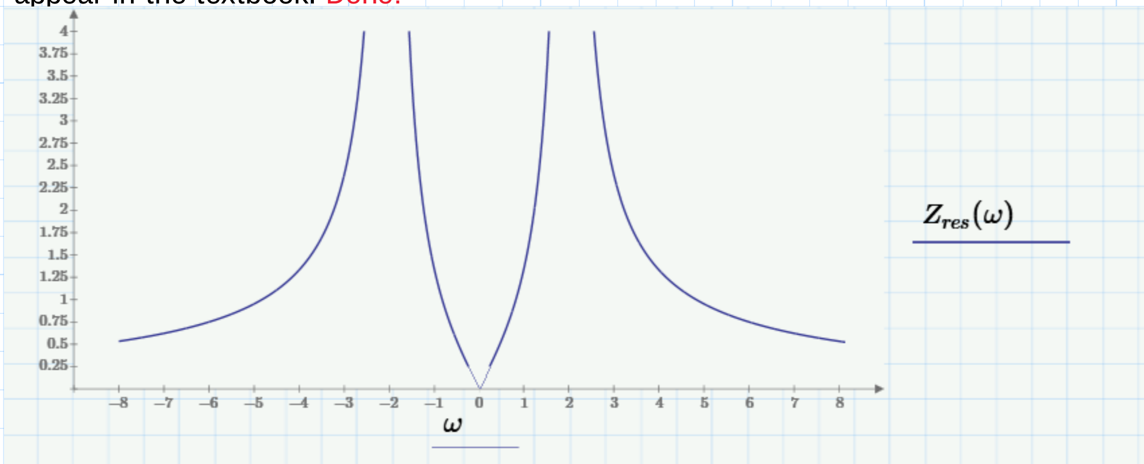
$Z_{res}(\omega) := |Z(\omega)|$ <--- Taking the magnitude of Z(w), otherwise we only get half the plot. The subscript res for response. Z response.



The middle part of plot at w=0 the 2 inner curves do not meet at 0 because of a division by zero when w=0 in Z(w). Shown below for clarity, when coming closer to w=0 by using a smaller interval. 0.01 closer to 0, compared to 0.25 to 0.



So in the plot below the part near to ***w=0*** has been sketched in just as it appear in the textbook. Done!



Next the angular plot for Z(w):

$$Z_{total} = \frac{\left(\frac{L}{C}\right)}{j\cdot\left(\omega L - \frac{1}{\omega C}\right)}$$

Lets try making the numerator 1, multiply by C/L.

Lets use Z now to represent Z total, we know its the circuit total impedance anyway.

$$Z = \frac{\left(\frac{C}{L}\right)\cdot\left(\frac{L}{C}\right)}{j\cdot\frac{C}{L}\cdot\omega L - \frac{C}{L}\cdot\frac{j}{\omega C}} = \frac{1}{j\cdot C\omega - \frac{j}{L\omega}}$$

$$Z = \left(\frac{1}{j}\right)\frac{1}{\left(C\omega - \frac{1}{L\omega}\right)} = -\frac{j}{\left(C\omega - \frac{1}{L\omega}\right)}$$

<----Phase angle plot this?

$$-\frac{j}{\left(C\omega - \frac{1}{L\omega}\right)}$$

<--- How do I get the phase angle
thru the inverse tangent expression?

$$angZ = -atan\left(\frac{\frac{1}{L\omega}}{C\omega}\right)$$

OR

$$angZ = -atan\left(\frac{C\omega}{\frac{1}{L\omega}}\right)$$

Either of these any correct? No.

I attempted several combinations all failed. According to the engineers this has to be done thru inspection. *Tan^-1 (y/x) = ? deg*. Does not exist you sketch it. You cant get tan^-1 to result in 90 degs. *Reason I emphasised on this was because the results are at 90 deg in the graph of the textbook.*

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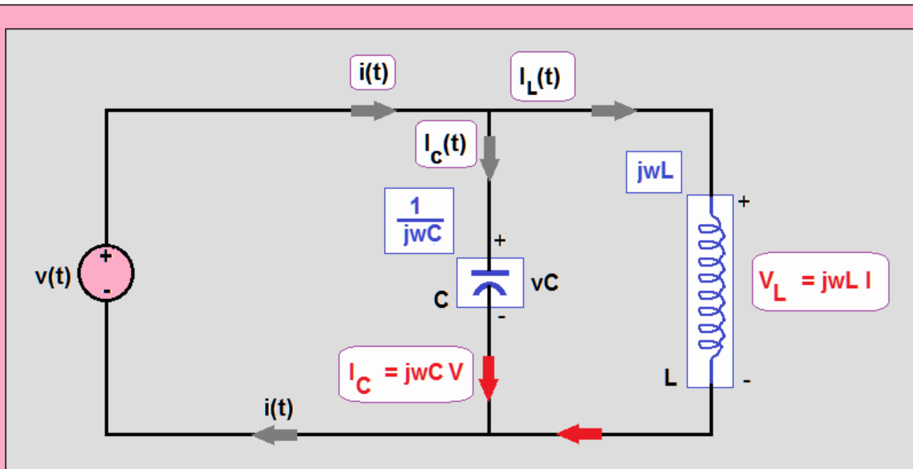
First I read-up on the lead or lag for the inductor and capacitor. Figure contents below may need correcting check with your textbook and course notes.



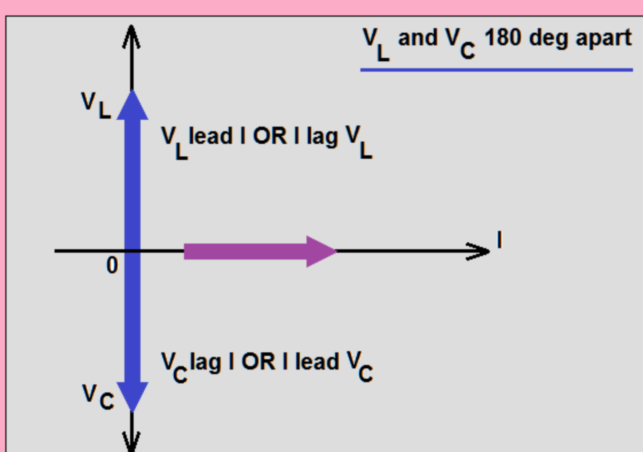
Rough Notes:

Inductor

1. **Angle of the factor jwL is exactly +90 deg, and I must therefore lag V by 90 deg in an inductor.**
   - Voltage V applied across the inductor terminals, it has to come ON first, V is leading, and eventually 90 degs later current I is appearing, potential V is needed to create the electric field which brings the current. 90 degs is a quarter cycle ahead.

Capacitor

2. **Angle of the factor jwC is -90 deg, and I leads V by 90 degs in a capacitor. Here the capacitor has to be charged-up, and charge relates to current. When discharging, current response is maximum due to increasing voltage that occured 90 degs earlier. So when current is travelling voltage was 90 degs behind. Tank filling up, current flowing in, current first, as level rises voltage rises. A tank filled up, full voltage, open the tap, current flow first. I here is 90 degs ahead of V.**



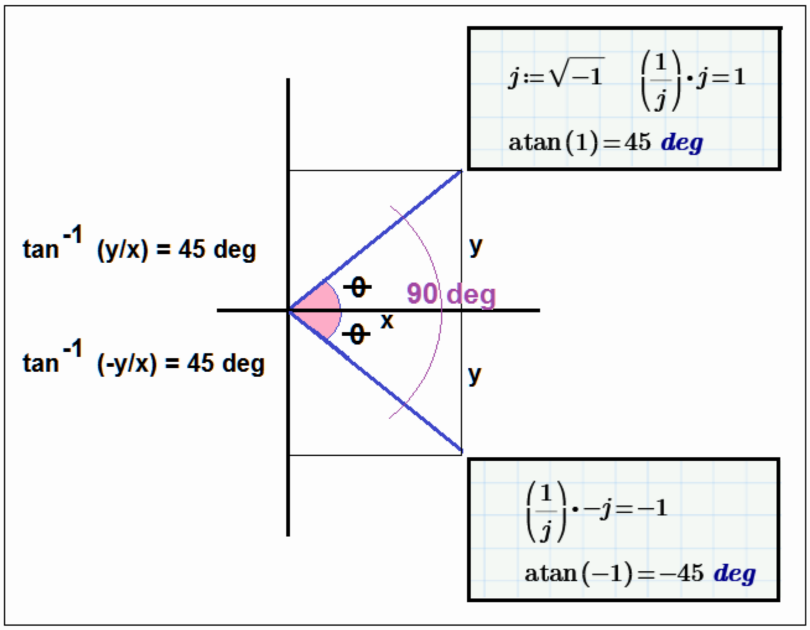
Thru inspection? I give it a try. Make 'fit-force' to match the answer.
You got a better solution go by it.

Takes me for-ever...thats okay I'm not interested in rebuilding Rome. 'Rome wasn't built in a day'. Not interested.

Math on j from college days, maybe this may help, only remember so much......You give it a better try. So I want to inspect it not evaluate.

$j := \sqrt{-1}$ $\quad \left(\frac{1}{j}\right) \cdot j = 1$ $\quad \left(\frac{1}{j}\right) \cdot -j = -1$

$\mathrm{atan}(1) = 45\ \mathbf{deg}$ $\quad \mathrm{atan}(-1) = -45\ \mathbf{deg}$

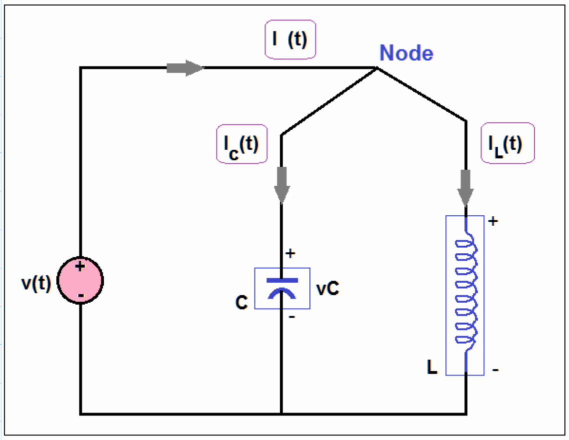


We can get the tangent of 45 degrees, and in this case can show a 90 degree between the two. Just in case if its needed in the inspection.

Not all lecturers will teach you that, some may not know depending on their experience, my UG ones were a little intuitive. Which may be bad because its a little harder to pass their test.

$-\frac{j}{\left(C\,\omega - \frac{1}{L\,\omega}\right)}$ <--- How do I get the phase angle thru the inverse tangent expression?

Continued next page.



Correct this discussion for any errors.

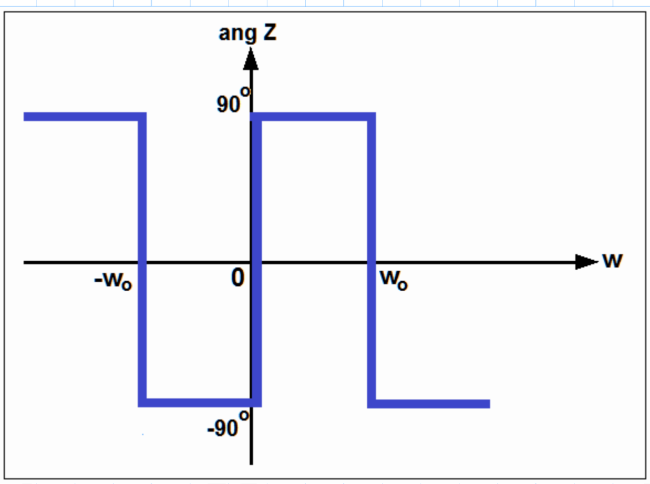
Lets assume the switch just got turned on, not shown here, so at time t<0 everything 0.

We are looking at t=0 the switch is ON.

Current starts to flow and comes to the node. Splits both directions, one to C and the other to L. We know how the capacitor and inductor work. Here the behaviour of the capacitor provide storage of charge to release as current at the right event. The inductor its a little more mysterious its v = L di/dt its providing voltage from the changing current. Both L and C have the same voltage across them in a parallel circuit. So I am saying the current I is the player here in this discussion.

Capacitor starts getting charged current is increasing and potential across its terminal rises. What is the condition here?
1. Ic leads v(t) which we now say v(t) is V (phasor form).
2. -90 degrees lead. -ve 90 meaning I was there 1/4 cycle first, to the right of t=0.

Inductor starts getting current its increasing and potential across its terminal rises. What is the condition here?
1. Ic lags v(t) which we now say v(t) is V (phasor form).
2. 90 degrees lag. +ve 90 meaning I was there 1/4 cycle late; to the left of t=0.



<----This is the answer the plot provided in textbook.

Lets explain the vertical line first.

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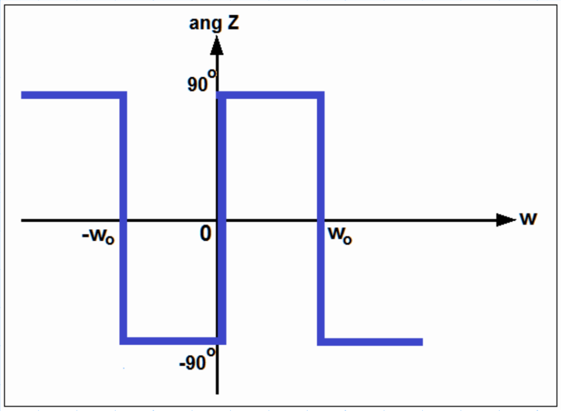
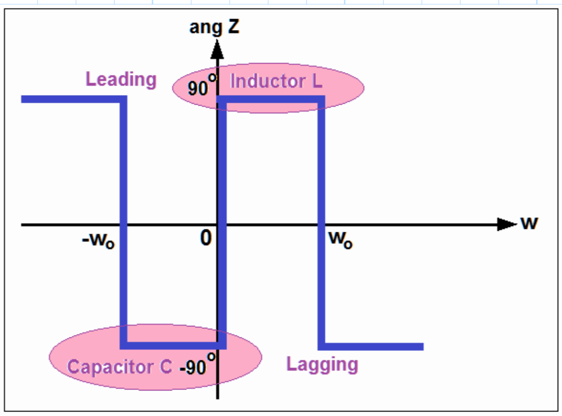


L and C get their current at the same time, when one is at 90 deg +ve because its lagging (L), the other is at -90 deg -ve because its leading (C).

All this at that very same w0 and w0 is 1/sqrt(LC).
So we cycle thru each multipe of w0 plot wise.



So maybe why the engineers, Hyat and Kemerly, highlighted only one time period before and after w=0. We have one leading C and one lagging L. We had in a previous figure V_L and V_C 180 degs apart, straight line. Vertical line. And lets say j for inductor +ve vertical half of vertical line and -j of capacitor -ve vertical half of vertical line.
Maybe you agree.

Next the horizontal line.

Between -w0 and 0 we have no change in angle, there is a group of frequencies, and the reaction/response is no change remains at -90 degs. This is whats expected of a capacitor stay at -90 deg. But why over an interval of frequencies?

$$Z_{total} = \frac{\left(\omega \cdot \frac{C}{L}\right) \cdot \left(\frac{L}{C}\right)}{j \cdot \left(\omega \cdot \frac{C}{L}\right) \cdot \left(\omega L - \frac{1}{\omega C}\right)} = \frac{(\omega)}{j \cdot \left(\omega^2 \ C - \frac{1}{L}\right)} = \frac{(\omega)}{j \cdot C \cdot \left(\omega^2 - \frac{1}{LC}\right)}$$

We had this expression in our earlier solution, term to the right may provide this answer.

$$\frac{(\omega)}{j \cdot C \cdot \left(\omega^2 - \frac{1}{LC}\right)} \text{ ---> } \frac{(\omega)}{j \cdot C \cdot \left(\omega^2 - \omega_0^{\ 2}\right)} \text{ ---> } \frac{1}{LC} \text{ ---> } \omega_0^{\ 2}$$

$\omega^2 - \omega_0^{\ 2}$ <--- This is that interval of frequency for L and C.

So to keep it tight on the proposed solution the loose ends you can tie up if any.
Check with your local engineer.

Zero and Pole:

We done the series RL and then did the parallel LC.

$$|Z| = \frac{1}{C} \cdot \frac{(\omega)}{(\omega - \omega_0)\ (\omega + \omega_0)}$$

<---- Going back a few pages we derivied this equation for the magnitude of Z.

By letting w0 = 1/SQRT(1/LC), and factoring the expression for the input impedance, the magnitude of the impedance may be writen in a form which enables those frequencies to be identified at which the response is zero or infinite - Hyat Kemerly

My/Our concern is with these frequencies where the response is zero or infinite.

Some frequencies give a zero response some frequencies give an infinite response.

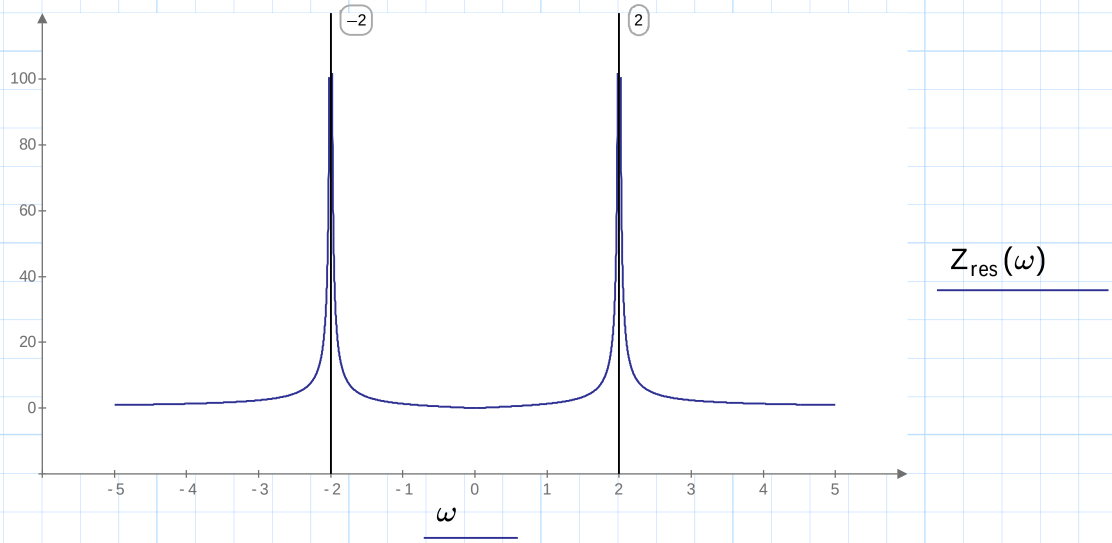
Such frequencies are termed critical frequencies, and their early identification simplifies the construction of the response curves.

We note first that the response has zero amplitude at w = 0; when this happens, we say that the response has a zero at w = 0, and we describe the frequency at which it occurs as a zero. **<---- Zero**

Discussion: *The opposite of zero response is maximum response or yet higher infinite response. We get infinite when we have something of value and divide it by something so small near 0, so let say its become zero, so 100/0 = infinite, which really was 100/0.0000001 but to get it so small its just the same as 0. But when we have 0/100 it equal 0 because we got nothing to begin with and if you have nothing, you divide nothing by 100 you got nothing. So we appreciate infinite more than? nothing.*

Continued next page with an adjustment on a previous plot.

We plotted this graph before now plotted with higher amplitude values.



We inserted vertical lines at w=-2 and w = 2.
Here the response is suddenly much higher than the other radian frequencies w.

We see the infinite amplitude in the plot at -w0 = -2 and w0 = 2, we got w0 = 1/sqrt(LC). For frequencies <-w0 and >w0 the amplitude approaches zero.

Response of infinite amplitude is noted at w = w0 and w = -w0; these frequencies are called poles, and the response is said to have a pole at each of these frequencies. **<---- Pole**

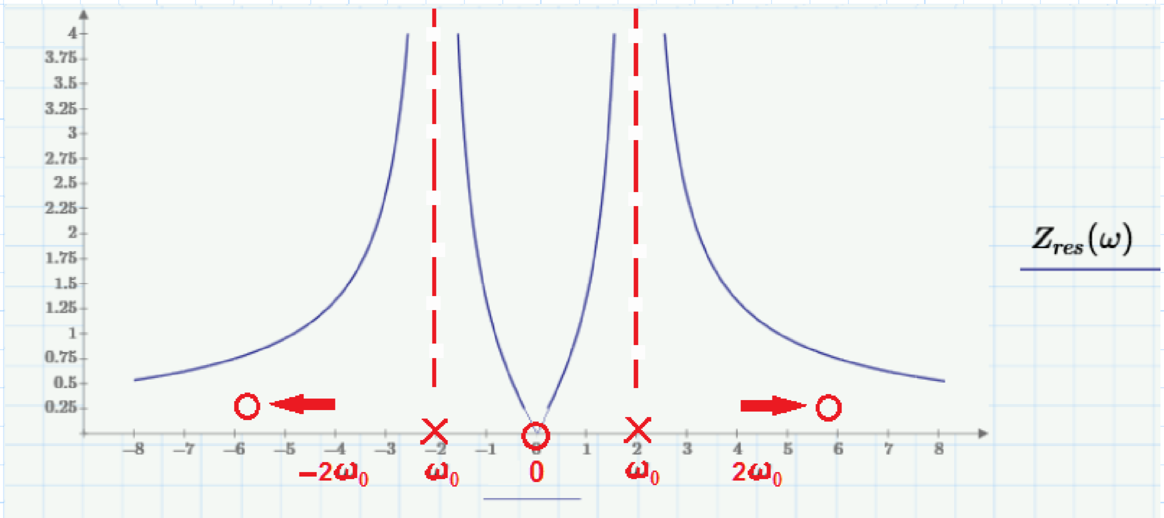
Finally, we note that the response approaches zero as w --> infinity and thus w = +/- infinity is also zero. **<---- Zero**

> *Note: It is customary to consider plus infinity and minus infinity as being the same point. The phase angle of the response at very large positive and negative values of w need not be the same however.*

Read a 2nd time, correct any errors, and if its satisfactory, then lets say we got zeros and poles identified.

In some textbooks zeros and poles are identified in a mathematical evaluation kind of explanation, if I am right thats mostly in controls course they assume yhou got this from a circuits course. So here we got some understanding on zeros and poles related to radian frequencies. Next how to identify them on a graph using markers/symbols.

POLES AND ZEROS.
Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill. Schaums Outline 6th Edition. Electric Circuits 6th Ed., Nahvi & Edminister. My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis.
Karl S. Bogha.



Location for critical frequencies are marked on the w-axis using small circles for zeros and crosses for poles. Poles or zeros at infinity frequencies should be indicated by an arrow near the axis, as shown figure above.

The actual drawings of the graph is made easier by adding broken vertical lines as asymptotes at each pole location. The completed graph of magnitude versus w (radian frequency) shown above where the slope at the origin is not zero.

Pages 273-276 of Hyat and Kemerly 4th ed.

Apologies in advance for any errors and omissions.