Zeros and Poles of 2-nodes circuits

$Z(s) := s$ Zeros=0 $L_1 := 1$

$Z(s) := \frac{1}{s}$ Poles=0 $C_1 := 1$

$Z(s) := \frac{(s^2+1)}{s} \xrightarrow{parfrac} s + \frac{1}{s}$ $L_1 := 1$ $C_2 := 1$ ← Series

$Z(s) := \frac{s}{(s^2+1)}$ Parallel

$\frac{1}{s+\frac{1}{s}} \xrightarrow{factor} \frac{s}{s^2+1}$ $C_1 := 1$ $L_1 := 1$

$Z(s) := \frac{(s^2+1)}{s \cdot (s^2+2)} \xrightarrow{parfrac} \frac{s}{2 \cdot (s^2+2)} + \frac{1}{2 \cdot s}$ $C_1 := 2$ Parallel $L_2 := \frac{1}{2}$ $C_2 := 2$ ← Series

$\frac{1}{2 \cdot s+\frac{2}{s}} \xrightarrow{factor} \frac{s}{2 \cdot (s^2+1)}$

Zeros $\omega = 0, \omega = \sqrt{2}$

$Z(s) := \frac{s \cdot (s^2+2)}{(s^2+1)} \xrightarrow{parfrac} s + \frac{s}{s^2+1}$ $L_1 := 1$ Parallel $C_2 := 1$ $L_2 := 1$ ← Series

Poles $\omega = \sqrt{1}$

Zeros $\omega = \sqrt{1}, \omega = \sqrt{3}$

$Z(s) := \frac{(s^2+1) \cdot (s^2+3)}{s \cdot (s^2+2)} \xrightarrow{parfrac} s + \frac{s}{2 \cdot (s^2+2)} + \frac{3}{2 \cdot s}$

Poles $\omega = 0, \omega = \sqrt{2}$

Parallel

$L_1 := 1$ $L_2 := \frac{1}{2}$ $C_2 := 2$ $C_3 := \frac{2}{3}$ ← Series

Zeros $\omega = 0, \omega = \sqrt{2}$

$$Z(s) := \frac{s \cdot (s^2 + 2)}{(s^2 + 1) \cdot (s^2 + 3)} \xrightarrow{parfrac} \frac{s}{2 \cdot (s^2 + 1)} + \frac{s}{2 \cdot (s^2 + 3)}$$

Poles $\omega = \sqrt{1}, \omega = \sqrt{3}$

Parallel	Parallel	
$L_1 := \frac{1}{2}$	$C_2 := 1$	← Series
$C_1 := 2$	$L_2 := \frac{1}{6}$	

Zeros $\omega = \sqrt{1}, \omega = \sqrt{3}$

$$Z(s) := \frac{(s^2 + 1) \cdot (s^2 + 3)}{s \cdot (s^2 + 2) \cdot (s^2 + 4)} \xrightarrow{parfrac} \frac{s}{4 \cdot (s^2 + 2)} + \frac{3 \cdot s}{8 \cdot (s^2 + 4)} + \frac{3}{8 \cdot s}$$

Poles $\omega = 0, \omega = \sqrt{2}, \omega = \sqrt{4}$

Parallel	Parallel		
$L_1 := \frac{1}{8}$	$L_2 := \frac{3}{32}$	$C_3 := \frac{8}{3}$	← Series
$C_1 := 4$	$C_2 := \frac{8}{3}$		

Zeros $\omega = 0, \omega = \sqrt{2}, \omega = \sqrt{4}$

$$Z(s) := \frac{s \cdot (s^2 + 2) \cdot (s^2 + 4)}{(s^2 + 1) \cdot (s^2 + 3)} \xrightarrow{parfrac} s + \frac{3 \cdot s}{2 \cdot (s^2 + 1)} + \frac{s}{2 \cdot (s^2 + 3)}$$

Poles $\omega = \sqrt{1}, \omega = \sqrt{3}$

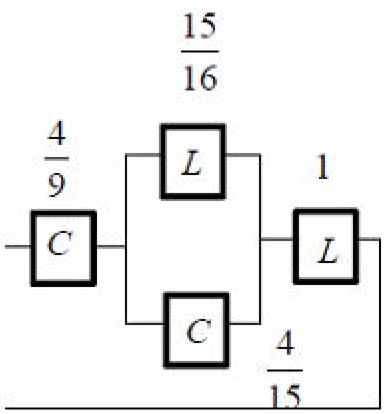
	Parallel	Parallel	
$L_1 := 1$	$C_2 := \frac{2}{3}$	$C_3 := 2$	← Series
	$L_2 := \frac{3}{2}$	$L_3 := \frac{1}{6}$	

9. LC 4 elements circuits

F1

$$Z(s) := \frac{(s^2+1)}{s} \cdot \frac{(s^2+9)}{(s^2+4)} \xrightarrow{parfrac} s + \frac{15 \cdot s}{4 \cdot (s^2+4)} + \frac{9}{4 \cdot s}$$

$$Z(s) := s + \frac{\frac{15}{4} \cdot s}{s^2+4} + \frac{9}{4 \cdot s} \qquad C_0 := \frac{4}{9} \qquad C := \frac{4}{15} \qquad L := \frac{15}{16} \qquad L\infty := 1$$



a

$$\text{clear}(s, C, L)$$

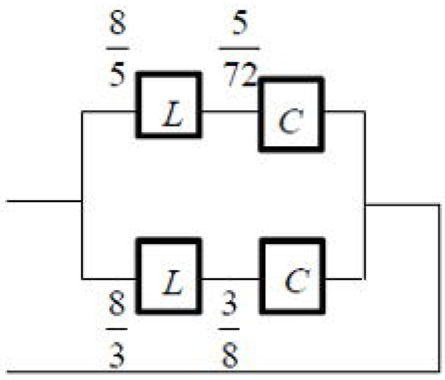
$$\frac{s \cdot L \cdot \frac{1}{s \cdot C}}{s \cdot L + \frac{1}{s \cdot C}} \xrightarrow{simplify} \frac{L \cdot s}{C \cdot L \cdot s^2 + 1}$$

$$\frac{L \cdot s}{C \cdot L \cdot s^2 + 1} = \frac{s \cdot \frac{1}{C}}{s^2 + \frac{1}{L \cdot C}}$$

F2

$$Y(s) := \frac{s}{(s^2+1)} \cdot \frac{(s^2+4)}{(s^2+9)} \xrightarrow{parfrac} \frac{3 \cdot s}{8 \cdot (s^2+1)} + \frac{5 \cdot s}{8 \cdot (s^2+9)}$$

$$Y(s) := \frac{\frac{3}{8} \cdot s}{s^2+1} + \frac{\frac{5}{8} \cdot s}{s^2+9} \qquad L_1 := \frac{8}{3} \qquad C_1 := \frac{3}{8} \qquad L_2 := \frac{8}{5} \qquad C_2 := \frac{5}{72}$$



c

$$\text{clear}(s, C, L)$$

$$\frac{s \cdot C \cdot \frac{1}{s \cdot L}}{s \cdot C + \frac{1}{s \cdot L}} \xrightarrow{simplify} \frac{C \cdot s}{C \cdot L \cdot s^2 + 1}$$

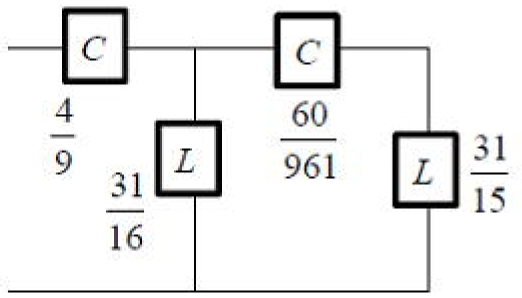
$$\frac{C \cdot s}{C \cdot L \cdot s^2 + 1} = \frac{s \cdot \frac{1}{L}}{s^2 + \frac{1}{L \cdot C}}$$

C1

$$Z(s) := \frac{(s^2+1)}{s}\cdot\frac{(s^2+9)}{(s^2+4)} \xrightarrow{parfrac} s+\frac{15\cdot s}{4\cdot(s^2+4)}+\frac{9}{4\cdot s}$$

$$s+\frac{15\cdot s}{4\cdot(s^2+4)} \xrightarrow{confrac,fraction} \cfrac{s}{\cfrac{16}{31}+\cfrac{s^2}{\cfrac{961}{60}+\cfrac{s^2}{\cfrac{15}{31}}}}$$

$$C_1 := \frac{4}{9}$$



b

$$\frac{1}{s\cdot C_1}+\cfrac{1}{\cfrac{1}{s\cdot L_1}+\cfrac{1}{\cfrac{1}{s\cdot C_2}+s\cdot L_2}}$$

$$L_1 := \frac{31}{16}$$

$$C_2 := \frac{60}{961} \qquad L_2 := \frac{31}{15}$$

$$\frac{s}{(s^2+1)}\cdot\frac{(s^2+4)}{(s^2+9)} \xrightarrow{confrac} \begin{bmatrix} 0 & s \\ \frac{9}{4} & s^2 \\ \frac{16}{31} & s^2 \\ \frac{961}{60} & 0 \end{bmatrix}$$

$$\frac{(s^2+1)}{s}\cdot\frac{(s^2+9)}{(s^2+4)} \xrightarrow{parfrac} s+\frac{15\cdot s}{4\cdot(s^2+4)}+\frac{9}{4\cdot s}$$

$$\left(s+\frac{15\cdot s}{4\cdot(s^2+4)}\right)^{-1} \xrightarrow{parfrac} \frac{16}{31\cdot s}+\frac{60\cdot s}{31\cdot(4\cdot s^2+31)}$$

$$\left(\frac{60\cdot s}{31\cdot(4\cdot s^2+31)}\right)^{-1} \xrightarrow{parfrac} \frac{31\cdot s}{3\cdot 5}+\frac{961}{60\cdot s}$$

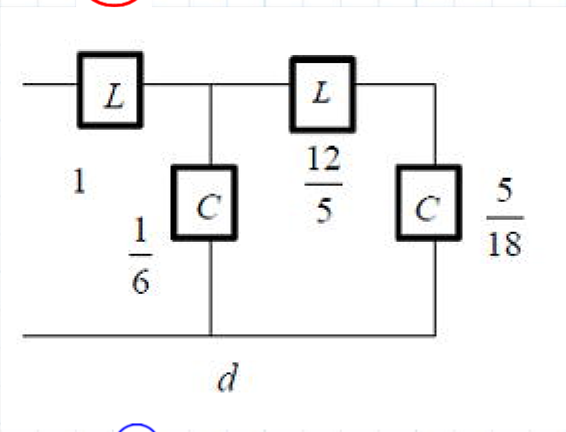
C2

$$Z(s) := \frac{(s^2+1)}{s}\cdot\frac{(s^2+9)}{(s^2+4)} \xrightarrow{parfrac} s+\frac{15\cdot s}{4\cdot(s^2+4)}+\frac{9}{4\cdot s}$$

$$\cfrac{1}{\cfrac{15\cdot s}{4\cdot(s^2+4)}+\cfrac{9}{4\cdot s}} \xrightarrow{parfrac} \frac{s}{2\cdot 3}+\frac{5\cdot s}{6\cdot(2\cdot s^2+3)}$$

$$\frac{1}{\frac{5\cdot s}{6\cdot\left(2\cdot s^2+3\right)}} \xrightarrow{parfrac} \frac{12\cdot s}{5}+\frac{18}{5\cdot s}$$

$L_1 := 1$



$$s\cdot L_1+\frac{1}{s\cdot C_1+\frac{1}{s\cdot L_2+\frac{1}{s\cdot C_2}}}$$

$L_2 := \frac{12}{5}$ $C_2 := \frac{5}{18}$

$C_1 := \frac{1}{6}$

$$Z(s) := \frac{\left(s^2+1\right)}{s}\cdot\frac{\left(s^2+9\right)}{\left(s^2+4\right)} \xrightarrow{parfrac} s+\frac{15\cdot s}{4\cdot\left(s^2+4\right)}+\frac{9}{4\cdot s}$$

$$\left(\frac{15\cdot s}{4\cdot\left(s^2+4\right)}+\frac{9}{4\cdot s}\right)^{-1} \xrightarrow{parfrac} \frac{s}{2\cdot 3}+\frac{5\cdot s}{6\cdot\left(2\cdot s^2+3\right)}$$

$$\left(\frac{5\cdot s}{6\cdot\left(2\cdot s^2+3\right)}\right)^{-1} \xrightarrow{parfrac} \frac{12\cdot s}{5}+\frac{18}{5\cdot s}$$