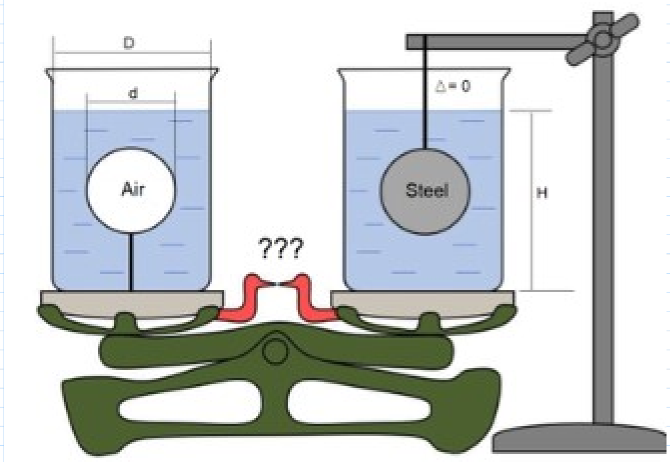
FWK

8/18/2020



1.225 kg/m3

The ISA or International Standard Atmosphere states the density of air is 1.225 **kg/m3** at sea level and 15 degrees C. The IUPAC uses an air density of 1.2754 **kg/m3** at 0 degrees C and 100 kPa for dry air. Density is affected not only by temperature and pressure but also by the amount of water vapor in the air. Feb 4, 2020

$$\rho_{Air} := 1.225\ \frac{kg}{m^3} = 0.07647\ \frac{lbm}{ft^3}$$

$$\rho_{Water} := 1000\ \frac{kg}{m^3} = 62.428\ \frac{lbm}{ft^3}$$

I'm used to the English units, just checking!

$D := 5\ cm$ $\quad d := 3\ cm$ $\quad H := 12\ cm$ $\quad r := \frac{d}{2}$ $\quad V := \frac{4}{3} \cdot \pi \cdot r^3$

This is an intriguing problem, that forces one to think about how to apply Newton's laws in a basic way. Here's my take on the problem.

First, I will assume the masses of the string, beaker, balloon, etc, are all zero. They do not add anything to the basic structure of the problem, and can only add unnecessary complication. Also, the nature of the problem is such that which side of the scale has the most force on it can be determined without any calculations in the idealized case.

Not sure what you nean, ". . .without any calculations. . ."

Consider the issue of buoyancy and the left side beaker. The water exerts an upward buoyant force Fb on the balloon (which has air in it, but not necessarily). By Newton's third law, the buoyancy creates an equal an opposite force on the bottom of the beaker, so that for the combined beaker, water, air balloon, string system, the net downward force remains the weight of the water plus the weight of the air. *I'm not convinced that the third law forces this!*

Archimedes' Principle states that the force due to buoyancy equals the weight of water displaced. We have seen that we can integrate the pressure distribution on the surface of the sphere to get that force. ***Note that the NET force on the sphere must subtract the weight of the air:*** *(* $p_{atm} := 1\ atm$ *, barometric pressure.) The net force is precisely the weight of air less than the buoyancy force.*

Pressure always acts normal to the surface. Because the sphere is symmetrical (about its vertical axis) the horizontal components of pressure will cancel. The vertical component is: $p(h) \cdot cos(\phi)$ *, where we recognize that pressure changes with depth.* $p(h) := p_{atm} + \rho_{Water} \cdot g \cdot h$ *, where* $g = 9.807\ \frac{m}{s^2}$ *, acceleration due to gravity. Let z be the distance from the free surface of the water to the top of the sphere. Then* $h(\phi, z) := z + r \cdot (1 - \cos(\pi - \phi))$ *.*

$$F_{buoy}(z) := \int_0^{\pi} p(h(\phi, z)) \cdot \cos(\phi) \cdot 2 \cdot \pi \cdot r^2 \cdot \sin(\phi)\, d\phi$$

and the net force on the object is

$$F_{net}(V, \rho_{ob}) := V \cdot (\rho_{Water} - \rho_{ob}) \cdot g$$

Applying numbers: (See, we do need to calculate!)

$F_{buoy}(4\ cm) = 0.1386382\ N$ $\quad F_{net}(V, \rho_{Air}) = 0.1384684\ N$ $\quad V \cdot \rho_{Air} \cdot g = 0.0001698\ N$

$$\frac{F_{buoy}(3\ cm) - F_{net}(V, \rho_{Air})}{V \cdot \rho_{Air} \cdot g} = 1$$

This net force must be reacted somewhere, or the sphere will move (Newton's First Law, "An object at rest will remain at rest unless acted on by an outside force . . ." Both the right and the left side spheres are supported by (let's call them "stings",) rods or strings. In the left beaker, this net force reaction is supplied by the sting which transfers this to the bottom of the left beaker.

Just for fun: The pressure acting on the bottom surface of the beaker is $P_{beaker} := p_{atm} - p(H) = -1.177\ kPa$. The pressure creates a force on the bottom of the beaker (pressure time area) of $F_{left\_water} := P_{beaker} \cdot \pi \cdot \left(\frac{D}{2}\right)^2 = -2.3106375\ N$

So the **NET** force on the left beaker is $F_{left\_net} := F_{left\_water} + F_{net}(V, \rho_{Air}) = -2.1721691\ N$

This is identically what we get if we do it the "old" way:

$$\left(\pi \cdot \left(\frac{D}{2}\right)^2 \cdot H - \frac{4}{3} \cdot \pi \cdot r^3\right) \cdot \rho_{Water} \cdot g + V \cdot \rho_{Air} \cdot g = 2.1721691\ N$$

It's my contention that integrating the pressures should satisfy ALL of the sources of forces that are transmitted through the water.

As a consequence, the total weight - downward force - on the left side of the scale is the weight of the water plus the weight of the air. This is also the result by isolating the beaker, water, air, balloon system. Except for the gravitational weights of the air and water, all other forces are internal to the isolated system, and thus have no effect of the overall downward force.

And we just demonstrated this!

We've discussed and analyzed the left beaker, and demonstrated that arrive at the same results via two different calculation schemes.

The right beaker is simpler because the sphere is supported from above the beaker; **any weight or force on the right sphere is not transmitted to the beaker**. The height (H) of water in the beaker is the same, so we can integrate the pressure (due to the "head" of water) over the area of the beaker and get the total force on the beaker: $F_{right\_water} := F_{left\_water} = -2.3106375\ N$. This is, however, the **total** contribution to the force, $F_{right\_net} := F_{right\_water}$, there are no other sources of force on this side.

For the system on the right, we may replace the steel ball and string with a rigid support and a ball of arbitrary material - heavier of lighter than water. the rigid support will be in tension or compression depending on whether the balloon material is heavier or lighter than water. In this case as before, the buoyancy has no net effect on the downward force against the scale. However, the external support provides the counter-force to the weight of the material inside the balloon. Hence, the net downward force on the right side scale is just the weight of the water outside the balloon.

This is NOT TRUE! Integration of the pressure gives a larger value because the displaced volume of the sphere raises H, the pressure head. Calculating the weight of the water remaining results in an erroneous result.