Solving Linear Ordinary Differential Equations

By including the below Mathcad sheet as a reference, it is possible to solve Linear Ordinary Differential Equations with constant coefficient: Note that an include sheet exists for Mathcad 11 (LODEsolve.mcd) separately from Mathcad 15 (LODEsolve.xmcd). The Mathcad 11 sheet v not work in Mathcad 15.

Reference:U:\Luc\MathCad\LODEsolve.mcd(R)

The function $LODEsolve(ode, f, y0, y, t)$ symbolically solves a (set of) linear Ordinary Differential Equation(s) (ODE) with constant coefficients.
The parameters to the function are:

ode Specifies the left-hand segment of the ODE(s), the part with the sought function(s) and its derivatives.
Write this left-hand segment as you would write the right-hand side of a function definition.

You cannot use prime notation ($y'(t)$ etc.), you must use $\frac{d}{dt}y(t)$ and $\frac{d^n}{dt^n}y(t)$ to describe derivatives.

Note that de ODE(s) must be Linear, *LODEsolve* cannot produce a correct solution if:
- powers of derivatives are included
- the coefficients of the derivatives depend on the independent variable (the last parameter to *LODEsolve*).

f Specifies the right hand segment of the ODE(s), the inhomogeneous part(s) or particular function(s).
The function(s) should be (a) mathematical function(s) for which the Laplace transform exists.
For for homogenous ODE(s), those without particular function(s), supply a 0.

y0 A vector (or matrix) with the initial conditions of the function(s) and its (their) derivatives.
In general for each function you must supply a number of initial conditions that is one less than the highest derivative that occurs for that function. Pad with 0's if necessary.
You cannot specify an initial condition for a value of the independent parameter other than 0.
See examples on how to deal with that situation.
LODEsolve will enlarge the vector (or matrix) with 0's for any initial condition not provided.

y The name of the sought function, or a vector of names in case of an ODE set, you can use any name (see notes below).
You MUST use the name(s) you give here as the name for the function(s) in *ode*.

t The name of the independent variable, you can use any name (see notes below).
You MUST use the name you give here as the name for the independent variable in *ode* and in *f*.

Notes: The function *LODEsolve* is a symbolic only function. This means that you cannot evaluate it numerically.
But you can assign its result to a function and use that numerically.

There are limits to the order of the ODE that can be solved. Symbolic solutions exist for polynomials up to order 4.
If the span of derivatives exceeds 3, it is well possible that no solution will be found.
For a set of ODE, derivatives up to the 9th can be handled (of course, without guarantee that a solution will be found).

Since LODEsolve is a symbolic function, you can use it to solve fully symbolic ODE's fully symbolically.

You should not use names that end in _$. Such names are used internally, and may cause unexpected results.

Examples:

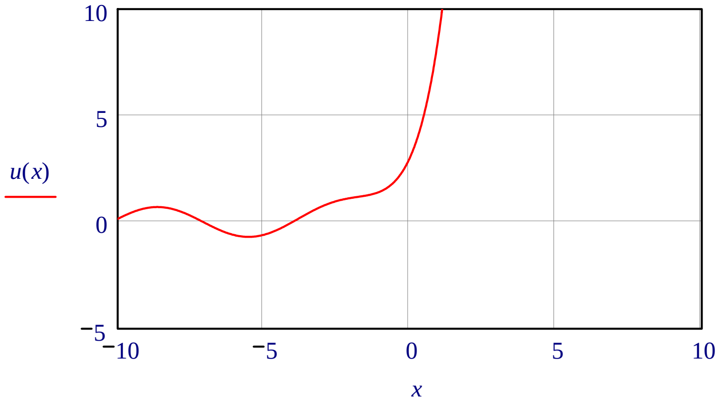
For a differential equation as: $\frac{d}{dx}u(x) - u(x) = sin(x)$ with initial condition: $u(0) = 3$.

The solution is: $u(x) := LODEsolve\left[\frac{d}{dx}u(x) - u(x), sin(x), (3), u, x\right] \rightarrow \frac{7}{2}\cdot exp(x) - \frac{1}{2}\cdot cos(x) - \frac{1}{2}\cdot sin(x)$

Proof: $\frac{d}{dx}u(x) - u(x) = sin(x) \rightarrow sin(x) = sin(x)$ $u(0) = 3$ The initial condition

Expressions on either side
of the = are the same

Use u(x) numerically, for plotting:



Note that $LODEsolve\left(\frac{d}{dtee}why(tee) - why(tee), sin(tee), 3, why, tee\right) \rightarrow \frac{7}{2}\cdot exp(tee) - \frac{1}{2}\cdot cos(tee) - \frac{1}{2}\cdot sin(tee)$

results in essentially the same function.

And also $f(t, f0, a, b, c) := LODEsolve\left(a\cdot\frac{d}{dt}f(t) + b\cdot f(t), c\cdot sin(t), f0, f, t\right) \rightarrow \frac{-c}{a^2 + b^2}\cdot a\cdot cos(t) + \frac{c}{a^2 + b^2}\cdot b\cdot sin(t) + \frac{1}{a^2 + b^2}\cdot ex$

It is a solution because $a\cdot\frac{d}{dt}f(t, f0, a, b, c) + b\cdot f(t, f0, a, b, c)\ simplify \rightarrow c\cdot sin(t)$ and $f(0, f0, a, b, c)\ simplify \rightarrow f0$

And filling in values for $f0, a, b, c$ such that the ODE matches the previous one, gives: $f(t, 3, 1, -1, 1) \rightarrow \frac{-1}{2}\cdot cos(t) - \frac{1}{2}\cdot sin(t) + \frac{7}{2}\cdot exp(t)$

Note that you cannot: $LODEsolve\left(t\cdot\frac{d}{dt}f(t), sin(t), f0, f, t\right) \rightarrow$ since the coefficient of the first derivative is not a constant

For a differential equation $LODE(\varphi,\tau) := 5\cdot\frac{d^5}{d\tau^5}\varphi(\tau) + 3\cdot\frac{d^4}{d\tau^4}\varphi(\tau) + 2\cdot\frac{d^3}{d\tau^3}\varphi(\tau) = e^{\tau}$ and $\varphi(0) = 1$ $\varphi'(0) = 0$ $\varphi''(0) = 0$ $\varphi'''(0) = 0$ $\varphi''''(0) = 0$

The solution is found with:

$$\varphi(\tau) := LODEsolve\left[5\cdot\frac{d^5}{d\tau^5}\varphi(\tau) + 3\cdot\frac{d^4}{d\tau^4}\varphi(\tau) + 2\cdot\frac{d^3}{d\tau^3}\varphi(\tau), e^{\tau}, \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \varphi, \tau\right] \rightarrow \frac{1}{10}\cdot exp(\tau) - \frac{1}{4}\cdot\tau^2 + \frac{1}{4}\cdot\tau + \frac{11}{8} - \frac{19}{40}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot cos\left(\frac{1}{10}\right.$$

then: $$\varphi'(\tau) := \frac{d}{d\tau}\varphi(\tau) \rightarrow \frac{1}{10}\cdot exp(\tau) - \frac{1}{2}\cdot\tau + \frac{1}{4} - \frac{7}{20}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot cos\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right) + \frac{59}{620}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot 31^{\frac{1}{2}}\cdot sin\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right)$$

$$\varphi''(\tau) := \frac{d}{d\tau}\varphi'(\tau) \rightarrow \frac{1}{10}\cdot exp(\tau) - \frac{1}{2} + \frac{2}{5}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot cos\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right) + \frac{1}{155}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot 31^{\frac{1}{2}}\cdot sin\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right)$$

$$\varphi'''(\tau) := \frac{d}{d\tau}\varphi''(\tau) \rightarrow \frac{1}{10}\cdot exp(\tau) - \frac{1}{10}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot cos\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right) - \frac{13}{310}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot 31^{\frac{1}{2}}\cdot sin\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right)$$

$$\varphi''''(\tau) := \frac{d}{d\tau}\varphi'''(\tau) \rightarrow \frac{1}{10}\cdot exp(\tau) - \frac{1}{10}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot cos\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right) + \frac{7}{310}\cdot exp\left(\frac{-3}{10}\cdot\tau\right)\cdot 31^{\frac{1}{2}}\cdot sin\left(\frac{1}{10}\cdot 31^{\frac{1}{2}}\cdot\tau\right)$$

Proof: $LODE(\varphi,\tau) \rightarrow exp(\tau) = exp(\tau)$ $\varphi(0) = 1$ $\varphi'(0) = 2.776\times 10^{-17}$ $\varphi''(0) = 0$ $\varphi'''(0) = 0$ $\varphi''''(0) = 0$

$\varphi'(0) \rightarrow 0$ yes, it's 0

Some examples from Spiegel, Murray R., ***Laplace transforms***, Chapter 3,

Solved problem 1:

$$ode_1(Y,t) := \frac{d^2}{dt^2}Y(t) + Y(t) \qquad f_1(t) := t \qquad y0_1 := \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad y_1 := Y$$

$$Y_1(t) := LODEsolve\left(ode_1(Y,t), f_1(t), y0_1, y_1, t\right) \rightarrow t + cos(t) - 3\cdot sin(t)$$

Check: $ode_1\left(Y_1,t\right) = f_1(t) \rightarrow t = t$ or: $ode_1\left(Y_1,t\right) - f_1(t) \rightarrow 0$ $\quad Y_1(0) \rightarrow 1$ $\quad \lim_{t \to 0} \frac{d}{dt}Y_1(t) \rightarrow -2$

Solved problem 2:

$$ode_2(Y,t) := \frac{d^2}{dt^2}Y(t) - 3\cdot\frac{d}{dt}Y(t) + 2\cdot Y(t) \qquad f_2(t) := 4\cdot e^{2\cdot t} \qquad y0_2 := \begin{pmatrix} -3 \\ 5 \end{pmatrix} \qquad y_2 := Y$$

$$Y_2(t) := LODEsolve\left(ode_2(Y,t), f_2(t), y0_2, y_2, t\right) \rightarrow 4\cdot t\cdot exp(2\cdot t) + 4\cdot exp(2\cdot t) - 7\cdot exp(t)$$

Check: $ode_2\left(Y_2,t\right) - f_2(t)$ simplify $\rightarrow 0$ $\quad Y_2(0) \rightarrow -3$ $\quad \lim_{t \to 0} \frac{d}{dt}Y_2(t) \rightarrow 5$

Solved problem 4:

$$ode_4(Y,t) := \frac{d^3}{dt^3}Y(t) - 3\cdot\frac{d^2}{dt^2}Y(t) + 3\cdot\frac{d}{dt}Y(t) - Y(t) \qquad f_4(t) := t^2\cdot e^t \qquad y0_4 := \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \qquad y_4 := Y$$

$$Y_4(t) := LODEsolve\left(ode_4(Y,t), f_4(t), y0_4, y_4, t\right) \rightarrow \frac{1}{60}\cdot t^5\cdot exp(t) - \frac{1}{2}\cdot t^2\cdot exp(t) - t\cdot exp(t) + exp(t)$$

Check: $ode_4\left(Y_4,t\right) - f_4(t)$ simplify $\rightarrow 0$ $\quad Y_4(0) \rightarrow 1$ $\quad \lim_{t \to 0} \frac{d}{dt}Y_4(t) \rightarrow 0$ $\quad \lim_{t \to 0} \frac{d^2}{dt^2}Y_4(t) \rightarrow -2$

Solved problem 5:

Same ODE, but now with symbolic initial conditions.................> $y0_5 := \begin{pmatrix} A \\ B \\ C \end{pmatrix}$

$$Y_5(t) := LODEsolve\left(ode_4(Y,t), f_4(t), y0_5, y_4, t\right) \text{ collect}, t, exp \rightarrow \frac{1}{60}\cdot t^5\cdot exp(t) + \left(-B + \frac{1}{2}\cdot A + \frac{1}{2}\cdot C\right)\cdot exp(t)\cdot t^2 + (-A + B)\cdot exp(t)\cdot t + A\cdot ex$$

It is normal for this expression to turn red, as some symbols are not numerically defined.

Check: $ode_4\left(Y_5,t\right) - f_4(t)$ simplify $\rightarrow 0$ $\quad Y_5(0) \rightarrow A$ $\quad \lim_{t \to 0} \frac{d}{dt}Y_5(t) \rightarrow B$ $\quad \lim_{t \to 0} \frac{d^2}{dt^2}Y_5(t) \rightarrow C$

Solved problem 7:

$$ode_7(Y,t) := \frac{d^2}{dt^2}Y(t) + a^2\cdot Y(t) \qquad f_7(t) := F(t) \qquad y0_7 := \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad y_7 := Y$$

$$Y_7(t) := LODEsolve\left(ode_7(Y,t), f_7(t), y0_7, y_7, t\right) \rightarrow \int_0^t \frac{F(\_U1)}{a}\cdot sin[a\cdot(t - \_U1)]\, d\_U1 - \frac{2}{a}\cdot sin(a\cdot t) + cos(a\cdot t)$$

It is normal for this expression to turn red, as some symbols are not numerically defined.

Check: $ode_7\left(Y_7,t\right) - f_7(t)$ simplify $\rightarrow 0$

$$Y_7(0) \rightarrow 1 \qquad \lim_{t \to 0} \frac{d}{dt}Y_7(t) \rightarrow \lim_{t \to 0} \left[\int_0^t F(\_U1)\cdot cos[a\cdot(t - \_U1)]\, d\_U1 - 2\cdot cos(a\cdot t) - sin(a\cdot t)\cdot a\right]$$

Note that both $\int_0^0 anything(u)\, du$ and $sin(a\cdot 0)$ should be 0, while $-2\cdot cos(a\cdot 0) = -2$.

Solved problem 12:

$$ode_{12}(X,Y,t) := \begin{pmatrix} \frac{d}{dt}X(t) - 2\cdot X(t) + 3\cdot Y(t) \\ \frac{d}{dt}Y(t) - Y(t) + 2\cdot X(t) \end{pmatrix} \qquad f_{12}(t) := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad y0_{12} := (8\ \ 3) \qquad y_{12} := \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$XY_{12}(t) := LODEsolve\left(ode_{12}(X,Y,t), f_{12}(t), y0_{12}, y_{12}, t\right)\ simplify \rightarrow \begin{bmatrix} (5\cdot exp(-5\cdot t) + 3)\cdot exp(4\cdot t) \\ (5\cdot exp(-5\cdot t) - 2)\cdot exp(4\cdot t) \end{bmatrix}$$

$X_{12}(t) := XY_{12}(t)_{ORIGIN}$

$Y_{12}(t) := XY_{12}(t)_{ORIGIN+1}$

Check: $ode_{12}\left(X_{12}, Y_{12}, t\right) - f_{12}(t)\ simplify \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad XY_{12}(0) \rightarrow \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Solved problem 13:

$$ode_{13}(X,Y,t) := \begin{pmatrix} \frac{d^2}{dt^2}X(t) + \frac{d}{dt}Y(t) + 3\cdot X(t) \\ \frac{d^2}{dt^2}Y(t) - 4\cdot\frac{d}{dt}X(t) + 3\cdot Y(t) \end{pmatrix} \qquad f_{13}(t) := \begin{pmatrix} 15\cdot e^{-t} \\ 15\cdot sin(2\cdot t) \end{pmatrix} \qquad y0_{13} := \begin{pmatrix} 35 & 27 \\ -48 & -55 \end{pmatrix} \qquad y_{13} := \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$XY_{13}(t) := LODEsolve\left(ode_{13}(X,Y,t), f_{13}(t), y0_{13}, y_{13}, t\right) \rightarrow \begin{pmatrix} 3\cdot exp(-t) - 15\cdot sin(3\cdot t) + 2\cdot cos(2\cdot t) + 30\cdot cos(t) \\ -3\cdot exp(-t) + 30\cdot cos(3\cdot t) + sin(2\cdot t) - 60\cdot sin(t) \end{pmatrix}$$

$X_{13}(t) := XY_{13}(t)_{ORIGIN}$

$Y_{13}(t) := XY_{13}(t)_{ORIGIN+1}$

Check: $ode_{13}\left(X_{13}, Y_{13}, t\right) - f_{13}(t)\ simplify \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad XY_{13}(0) \rightarrow \begin{pmatrix} 35 \\ 27 \end{pmatrix}$ $\qquad i := 0..1 \quad \lim_{t \to 0} \frac{d}{dt}XY_{13}(t)_i \rightarrow \begin{pmatrix} -48 \\ -55 \end{pmatrix}$

Solved problem 15:

$$ode_{15}(X,t) := m\cdot\frac{d^2}{dt^2}X(t) + k\cdot X(t) + \beta\cdot\frac{d}{dt}X(t) \qquad f_{15}(t) := 0 \qquad y0_{15} := \begin{pmatrix} X_0 \\ V_0 \end{pmatrix} \qquad y_{15} := X$$

$$X_{15}(t) := LODEsolve\left(ode_{15}(X,t), f_{15}(t), y0_{15}, y_{15}, t\right) \rightarrow 4\cdot\frac{exp\left(\frac{-1}{2}\cdot\frac{\beta}{m}\cdot t\right)}{4\cdot k\cdot m - \beta^2}\cdot k\cdot X_0\cdot m\cdot cos\left[\frac{1}{2}\cdot\left(\frac{4\cdot k\cdot m - \beta^2}{m^2}\right)^{\frac{1}{2}}\cdot t\right] - \frac{exp\left(\frac{-1}{2}\cdot\frac{\beta}{m}\cdot t\right)}{4\cdot k\cdot m - \beta^2}\cdot X_0\cdot\beta^2\cdot cos\Big[$$

It is normal for this expression to turn red, as some symbols are not numerically defined.

Check: $ode_{15}\left(X_{15}, t\right) - f_{15}(t)\ simplify \rightarrow 0$ $\qquad X_{15}(0)\ simplify \rightarrow X_0$ $\qquad \lim_{t \to 0} \frac{d}{dt}X_{15}(t) \rightarrow V_0$

or $\quad ode_{15.1}(X,t) := \frac{d^2}{dt^2}X(t) + 2\cdot\alpha\cdot\frac{d}{dt}X(t) + \omega^2\cdot X(t)$

$$X_{15.1}(t) := LODEsolve\left(ode_{15.1}(X,t), f_{15}(t), y0_{15}, y_{15}, t\right) \rightarrow -exp(-\alpha\cdot t)\cdot\frac{\omega^2}{(\alpha-\omega)\cdot(\alpha+\omega)}\cdot X_0\cdot cos\left[\left[-(\alpha-\omega)\cdot(\alpha+\omega)\right]^{\frac{1}{2}}\cdot t\right] - \frac{exp(-\alpha}{(\alpha-\omega)\cdot(\alpha}$$

It is normal for this expression to turn red, as some symbols are not numerically defined.

Check: $ode_{15.1}\left(X_{15.1}, t\right) - f_{15}(t)\ simplify \rightarrow 0$ $\qquad X_{15.1}(0)\ simplify \rightarrow X_0$ $\qquad \lim_{t \to 0} \frac{d}{dt}X_{15.1}(t) \rightarrow V_0$

Solved problem 17:

$$ode_{17}(I_1,I_2,t) := \begin{pmatrix} -5\cdot I_1(t) - \frac{d}{dt}I_1(t) + 2\cdot\frac{d}{dt}I_2(t) + 10\cdot I_2(t) \\ \frac{d}{dt}I_1(t) + 20\cdot I_1(t) + 15\cdot I_2(t) \end{pmatrix} \qquad f_{17}(t) := \begin{pmatrix} 0 \\ 55 \end{pmatrix} \qquad y0_{17} := (0 \;\; 0) \qquad y_{17} := \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$I_{17}(t) := LODEsolve(ode_{17}(I_1,I_2,t), f_{17}(t), y0_{17}, y_{17}, t) \rightarrow \begin{pmatrix} 2 - 2\cdot exp\left(\frac{-55}{2}\cdot t\right) \\ 1 - exp\left(\frac{-55}{2}\cdot t\right) \end{pmatrix}$$

$$I_1(t) := I_{17}(t)_{ORIGIN}$$

$$I_2(t) := I_{17}(t)_{ORIGIN+1}$$

Check: $ode_{17}(I_1,I_2,t) - f_{17}(t)\ simplify \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad I_{17}(0) \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Solved problem 18:

$$ode_{18}(Y,x) := \frac{d^4}{dx^4}Y(x) \qquad f_{18}(x) := \frac{W_0}{E\cdot I} \qquad y0_{18} := \begin{pmatrix} 0 \\ c1 \\ 0 \\ c2 \end{pmatrix} \qquad y_{18} := Y$$

$$Y_{18}(x,E,I,W_0,c1,c2) := LODEsolve(ode_{18}(Y,x), f_{18}(x), y0_{18}, y_{18}, x) \rightarrow \frac{1}{E\cdot I}\cdot\left(\frac{1}{24}\cdot W_0\cdot x^4 + \frac{1}{6}\cdot c2\cdot E\cdot I\cdot x^3 + c1\cdot E\cdot I\cdot x\right)$$

$$Y''_{18}(x,E,I,W_0,c1,c2) := \frac{d^2}{dx^2}Y_{18}(x,E,I,W_0,c1,c2)$$

$$(c1 \;\; c2) := \begin{pmatrix} Y_{18}(l,E,I,W_0,c1,c2) \\ Y''_{18}(l,E,I,W_0,c1,c2) \end{pmatrix} solve, c1, c2 \rightarrow \begin{pmatrix} \frac{1}{24}\cdot W_0\cdot\frac{l^3}{E\cdot I} & \frac{-1}{2}\cdot W_0\cdot\frac{l}{E\cdot I} \end{pmatrix}$$

Don't mind the red, $(c1 \;\; c2)$ do get defined symbolically.

$$Y_{18}(x) := Y_{18}(x,E,I,W_0,c1,c2)\ collect, W_0 \rightarrow \frac{1}{E\cdot I}\cdot\left(\frac{1}{24}\cdot x^4 - \frac{1}{12}\cdot l\cdot x^3 + \frac{1}{24}\cdot l^3\cdot x\right)\cdot W_0$$

It is normal for this expression to turn red, as some symbols are not numerically defined.

Check: $ode_{18}(Y_{18},x) - f_{18}(x)\ simplify \rightarrow 0$ $\qquad Y_{18}(0) \rightarrow 0$ $\qquad \lim_{x \to 0} \frac{d^2}{dx^2}Y_{18}(x) \rightarrow 0$

and at $x = l$: $\qquad Y_{18}(l) \rightarrow 0$ $\qquad \lim_{x \to l} \frac{d^2}{dx^2}Y_{18}(x) \rightarrow 0$

Miscellaneous problem 102: $X := X$ $Y := Y$ $Z := Z$ $t := t$

$$ode_{102}(t) := \begin{pmatrix} \frac{d}{dt}X(t) + \frac{d}{dt}Y(t) - Y(t) - Z(t) \\ \frac{d}{dt}Y(t) + \frac{d}{dt}Z(t) - X(t) - Z(t) \\ \frac{d}{dt}X(t) + \frac{d}{dt}Z(t) - X(t) - Y(t) \end{pmatrix} \qquad f_{102} := 0 \qquad y0_{102} := (2\ \ -3\ \ 1) \qquad y_{102} := \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$XYZ(t) := LODEsolve\left(ode_{102}(t), f_{102}, y0_{102}, y_{102}, t\right) \rightarrow \begin{pmatrix} 2\cdot exp\left(\frac{-1}{2}\cdot t\right)\cdot cos\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) - \frac{4}{3}\cdot exp\left(\frac{-1}{2}\cdot t\right)\cdot 3^{\frac{1}{2}}\cdot sin\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) \\ \frac{-1}{3}\cdot exp\left(\frac{-1}{2}\cdot t\right)\cdot 3^{\frac{1}{2}}\cdot sin\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) - 3\cdot exp\left(\frac{-1}{2}\cdot t\right)\cdot cos\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) \\ exp\left(\frac{-1}{2}\cdot t\right)\cdot cos\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) + \frac{5}{3}\cdot exp\left(\frac{-1}{2}\cdot t\right)\cdot 3^{\frac{1}{2}}\cdot sin\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) \end{pmatrix}$$

$X(t) := XYZ(t)_{ORIGIN}$ $Y(t) := XYZ(t)_{ORIGIN+1}$ $Z(t) := XYZ(t)_{ORIGIN+2}$

$$X(t)\ collect, exp \rightarrow \left(2\cdot cos\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right) - \frac{4}{3}\cdot 3^{\frac{1}{2}}\cdot sin\left(\frac{1}{2}\cdot 3^{\frac{1}{2}}\cdot t\right)\right)\cdot exp\left(\frac{-1}{2}\cdot t\right) \qquad X(0) = 2$$

$$\begin{pmatrix} \frac{d}{dt}X(t) + \frac{d}{dt}Y(t) - Y(t) - Z(t) \\ \frac{d}{dt}Y(t) + \frac{d}{dt}Z(t) - X(t) - Z(t) \\ \frac{d}{dt}X(t) + \frac{d}{dt}Z(t) - X(t) - Y(t) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad XYZ(0) = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$