Piping-System Solutions Using Mathcad

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NOMENCLATURE

C equivalent lengths for minor loss coefficient
D pipe diameter
f Darcy friction factor
f_T fully-rough friction factor
$g = 32.174 \frac{ft}{s^2}$ acceleration of gravity
g_c conversion factor (English Engineering units), $g_c := 32.174 \frac{ft \cdot lbm}{lbf \cdot sec^2} = 1$ (32.174 ft-lbm/lbf-sec2)
h_d head change due to a pump, turbine, or other active device
K minor loss coefficient expressed as a number
L pipe length
N number of pipes, connection matrix
P pressure
Q flow rate
Re Reynolds number, VD/n
V velocity
W_s pump increase in head
o elevation
γ specific weight, $\gamma := \rho \cdot g$
ε absolute roughness of pipe, $\varepsilon := 0.045\ mm$, steel pipe
μ viscosity, $\mu_w := 0.01\ poise = 0.000672 \frac{lbm}{ft \cdot sec}$, water at 20 deg centigrade
ν kinematic viscosity, $\frac{\mu}{\rho}$, $\nu := \frac{\mu_w}{\rho} = \left(1.077 \cdot 10^{-5}\right) \frac{ft^2}{s}$
ρ density, $\rho \equiv 62.4 \frac{lbm}{ft^3}$

Subscripts

a upstream location
b downstream location
elbow elbow
ent entrance
exp expansion
gv gate valve
I arbitrary pipe in a pipe network
i counter
1 pipe 1
2 pipe 2
3 pipe 3

no matter how complex the piping system, the basis of all analysis and design calculations for piping systems is the energy-equation applied over a segment of a pipe. Consider, for example, a portion of a series-piping segment as illustrated schematically in Figure 1. If the flow is from "a" to "b," then the energy equation becomes

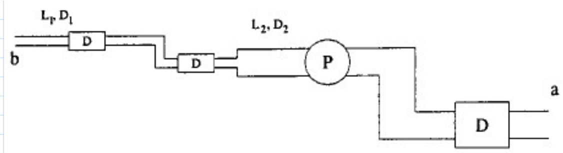


Figure 1 General piping system schematic.

$$\frac{P_a}{\gamma}+\frac{V_a^{\,2}}{2\cdot g}+z_a=\frac{P_b}{\gamma}+\frac{V_b}{2\cdot g}+z_b+\sum_{i=1}^{N}\frac{8\cdot Q^2}{\pi^2\cdot g\cdot D_i^{\,4}}\cdot\left(f_i\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_{T_i}\right)-W_s\cdot\frac{g_c}{g} \qquad (1)$$

It's instructive here to investigate the unit balance of this equation. Ascribe values with units:

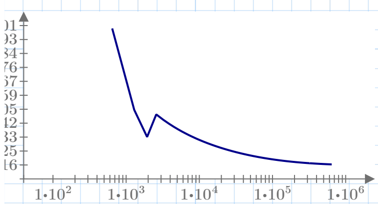
Approximately water $\begin{bmatrix}P_a\\P_b\end{bmatrix}:=\begin{bmatrix}15\\10\end{bmatrix}\cdot psi$ $\begin{bmatrix}V_a\\V_b\end{bmatrix}:=\begin{bmatrix}10\\15\end{bmatrix}\cdot\frac{ft}{sec}$

$D:=\begin{bmatrix}3\\3\\1\end{bmatrix}\cdot in$ $Q:=50\ gpm$ $L:=\begin{bmatrix}10\\5\\20\end{bmatrix}\cdot ft$ $\begin{bmatrix}z_a\\z_b\end{bmatrix}:=\begin{bmatrix}2\\1\end{bmatrix}\cdot ft$ ORIGIN ≡ 1

$i:=1..3$ $K_i:=1$ $C_i:=0.5$ $f=\dfrac{0.3086}{\log\left[\frac{6.9}{Re}+\left(\frac{\varepsilon}{3.7D}\right)^{1.11}\right]^2}$. For laminar flow, $f=64/Re$.

$$f(V,D):=\text{if}\left(\frac{\rho\cdot V\cdot D}{\mu_w}<2100,\frac{64\cdot\mu_w}{\rho\cdot V\cdot D},\frac{0.3086}{\log\left(\frac{6.9\cdot\mu_w}{\rho\cdot V\cdot D}+\left(\frac{\varepsilon}{3.7\cdot D}\right)^{1.11}\right)^2}\right)$$

$vv:=1\frac{ft}{min},2\frac{ft}{min}..1000\frac{ft}{min}$



$f(vv,5\ in)$ vs. $\dfrac{\rho\cdot vv\cdot 5\ in}{\mu_w}$

$$f_T(D):=\frac{0.3086}{\log\left(\left(\frac{\varepsilon}{3.7\cdot D}\right)^{1.11}\right)^2}$$

$f\left(V_a,D_1\right)=0.019$

$f_T\left(D_1\right)=0.017$

$$\frac{\begin{bmatrix}P_a\\P_b\end{bmatrix}}{\gamma}=\begin{bmatrix}34.615\\23.077\end{bmatrix}ft \quad \frac{\begin{bmatrix}V_a^{\,2}\\V_b^{\,2}\end{bmatrix}}{2\cdot g}=\begin{bmatrix}1.554\\3.497\end{bmatrix}ft \quad \begin{bmatrix}z_a\\z_b\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}ft \quad \sum_{i=1}^{3}\frac{8\cdot Q^2}{\pi^2\cdot g\cdot D_i^{\,4}}\cdot\left(f\left(V_a,D_i\right)\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_T\left(D_i\right)\right)=45.361\ ft$$

The paper assumed: $P_a=P_b$, $V_a=0$, and $\dfrac{V_b^2}{2g}=\dfrac{8}{\pi^2}\dfrac{Q^2}{gD_3^4}$

Then:

$$W_s\frac{g_c}{g}=z_b-z_a+\frac{8}{\pi^2}\frac{Q^2}{gD_1^4}\left[0.5+60f_{T_1}+f_1\frac{L_1}{D_1}\right]+\frac{8}{\pi^2}\frac{Q^2}{gD_2^4}\left[9+f_2\frac{L_2}{D_2}\right]+\frac{8}{\pi^2}\frac{Q^2}{gD_3^4}\left[1.0+55f_{T_3}+f_3\frac{L_3}{D_3}\right]$$

Then for unit balance:

$$W_s:=\frac{g}{g_c}\cdot z_a=64.348\ \frac{ft^2}{sec^2}$$

But W_s is pump increase in head

Start again

$$\frac{P_a}{\gamma}+\frac{{V_a}^2}{2\cdot g}+z_a=\frac{P_b}{\gamma}+\frac{{V_b}^2}{2\cdot g}+z_b+\sum_{i=1}^{N}\frac{8\cdot Q^2}{\pi^2\cdot g\cdot {D_i}^4}\cdot\left(f_i\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_{T_i}\right)-W_s\cdot\frac{g_c}{g} \qquad (1)$$

$\gamma=\rho\cdot g$ multiply through by γ

$$P_a+\frac{\rho\cdot {V_a}^2}{2}+z_a\cdot\rho\cdot g=P_b+\frac{\rho\cdot {V_b}^2}{2}+z_b\cdot\rho\cdot g+\sum_{i=1}^{N}\frac{8\cdot\rho\cdot Q^2}{\pi^2\cdot {D_i}^4}\cdot\left(f_i\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_{T_i}\right)-W_s\cdot\rho$$

$P_a=15\ psi$ $\quad \frac{\rho\cdot {V_a}^2}{2}=0.673\ psi$ $\quad z_a\cdot\rho\cdot g=0.867\ psi$

$$\sum_{i=1}^{3}\frac{8\cdot\rho\cdot Q^2}{\pi^2\cdot {D_i}^4}\cdot\left(f\left(V_a,D_i\right)\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_T\left(D_i\right)\right)=19.657\ psi$$

To balance units, equation (1) should be:

$$\frac{P_a}{\gamma}+\frac{{V_a}^2}{2\cdot g}+z_a=\frac{P_b}{\gamma}+\frac{{V_b}^2}{2\cdot g}+z_b+\sum_{i=1}^{N}\frac{8\cdot Q^2}{\pi^2\cdot g\cdot {D_i}^4}\cdot\left(f_i\cdot\frac{L_i}{D_i}+K_i+C_i\cdot f_{T_i}\right)-W_s \qquad (1)$$

Then the first effort, $W_s:=z_a=2\ ft$

And the second effort, $W_s\cdot\gamma=0.867\ psi$

Conclusions:

1. We really don't need the "gravitational constant," $g_c=1$ since it doesn't supply anything.
2. The useable form (for Mathcad) of equation (1) is the red form above, which has balanced units.