Piping-System Solutions Using Mathcad

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ABSTRACT: Mathcad procedures, in the form of worksheets, are presented and discussed for problems associated with piping systems. Examples include series piping systems, parallel piping systems, and piping networks. The Mathcad solution approaches differ significantly from conventional techniques and are more congruent with problem formulations. The use of Mathcad permits the students to be more concerned with problem formulations and results interpretations than with arithmetic, programming, and debugging issues. © 2002 Wiley Periodicals, Inc. Comput Appl Eng Educ 10: 59–78, 2002; Published online in Wiley InterScience (www.interscience.wiley.com.); DOI 10.1002/cae.10010

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INTRODUCTION

Two important, if not dominant, contemporary factors in the continuing evolution of engineering curricula and courses have been the increasing use and availability of computational assets and the continuous changes in the ABET requirements, especially ABET 2000. Generally, engineering curricula have been changed to integrate computer applications into more courses and to increase the number of courses containing legitimate "open-ended" and "integrated" engineering analysis and design experiences. Traditionally, such integration meant devoting time and effort to structured programming in higher-level languages (e.g., FORTRAN or C++). The thesis of this study is that the use of arithmetic systems, such as Mathcad, provides significant enhancements for the solution of piping problems in the context of engineering education and represents a more fruitful path that portends the future. That is to say, except for highly skilled engineering specialists with post-BS degree education, engineers are not likely to do much code development in their careers. Note that this does not imply that engineers would not use computers, only that applications, not code development, will be the engineering tasks.

This study examines the use of Mathcad to solve piping systems problems for a number of piping systems topics encountered in fluid mechanics and thermal systems courses, including some with significant design content. The basic goals of the courses and the physical principles remain essentially unchanged, but because of the use of Mathcad, the problems assigned can be more involved, more open-ended, and more integrated. A number of engineering education publications (e.g., [1,2]) including the third edition of a textbook [3], have explored the extensive use of Mathcad in a thermal systems design course which included significant piping problems.

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BACKGROUND

Piping systems are important components in many engineering systems. The importance of piping systems is reflected in engineering education by the number of disciplines (Aerospace, Agricultural, Biological, Chemical, Civil, Environmental, Mechanical, Nuclear, Petroleum, among others) that teach, with varying details and emphasis, some aspects of piping systems. However, no matter how complex the piping system, the basis of all analysis and design calculations for piping systems is the energy-equation applied over a segment of a pipe. Consider, for example, a portion of a series-piping segment as illustrated schematically in Figure 1. If the flow is from "a" to "b," then the energy equation becomes

$$\frac{P_a}{\gamma}+\frac{V_a^2}{2g}+z_a=\frac{P_b}{\gamma}+\frac{V_b^2}{2g}+z_b+\sum_{i=1}^{N}\frac{8}{\pi^2}\frac{Q^2}{gD_i^4}\left(f_i\frac{L_i}{D_i}+K_i+C_if_{T_i}\right)-W_s\frac{g_c}{g} \tag{1}$$

where N is the number of different pipes in the segment, K_i is the minor loss coefficient (expressed as a number), and $C_if_{T_i}$ is the minor loss coefficient (expressed as an equivalent-length times the fully rough friction factor) for pipe i. All variables are identified in the Nomenclature section.

The most convenient subdivision of piping systems is into series, parallel, and network. Series, as the name implies, consists of any number of piping elements (pipe segments, devices such as valves, and active elements such as pumps or turbines) arranged in series. In series systems, the flow rate through each element is the same and the head losses/gains are additive. Figure 1 illustrates such a system.

Parallel-piping systems are composed of piping elements in parallel arrangements. Figure 2a provides an example. In parallel systems, the head losses/gains through each leg are the same and the total system flow rate is the sum of the flow rates of the individual legs.

Piping networks, as illustrated in Figure 2b, are true networks that have some segments in parallel and some in series. For networks, the sum of the flow rates at a node, the juncture of two or more pipes, must be zero and the sum of the change in head (or pressure) around any arrangement of pipes forming a closed loop must likewise be zero.

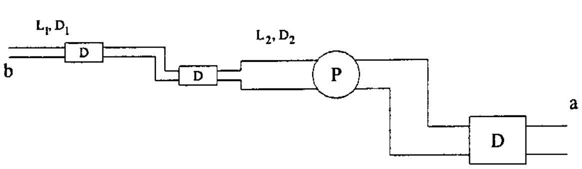

Figure 1 General piping system schematic.

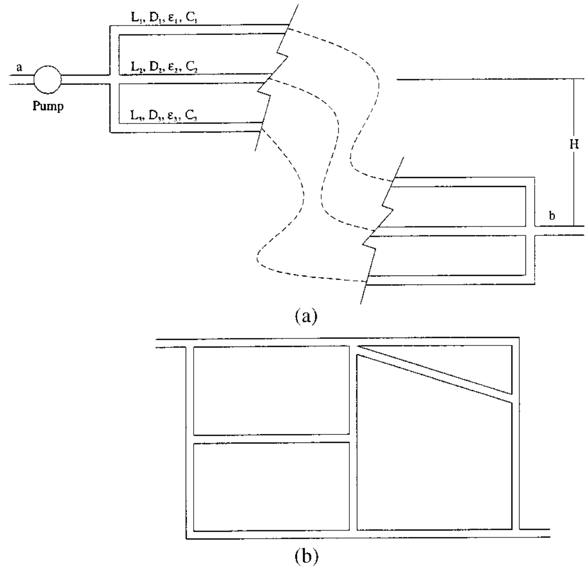

Figure 2 Parallel and network system representations. (a) Parallel piping system schematic; (b) Piping network schematic.

Mathcad solutions of problems for each of these three types of piping systems are considered in detail in the following sections. However, losses are important for any piping segment. Since, the major and minor losses formulations are the same for all types of piping systems, further examination of these are warranted. The Darcy friction factor, f, is typically explained in terms of the Moody diagram; however, the explicit expression of Haaland [4] for the turbulent friction factor is the more useful

$$f=\frac{0.3086}{\log\left[\frac{6.9}{Re}+\left(\frac{\varepsilon}{3.7D}\right)^{1.11}\right]^2}. \tag{2a}$$

The fully rough friction factor is the asymptotic values the friction factor as $Re\to\infty$, and from the above becomes

$$f_T=\frac{0.3086}{\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11}\right]^2}, \tag{2b}$$

For laminar flow, $f=64/Re$.

The minor losses are expressed in terms of a number, an entrance loss for example, or in terms of equivalent lengths of the fully rough friction factor.

The Crane Company Technical Report 410 [5] is a well respected and frequently cited source of minor loss coefficients. The friction factors and the minor loss coefficients are used in the following sections as series, parallel, and network piping system for which examples are presented and discussed.

SERIES PIPING SYSTEM EXAMPLE

As the name implies, a series piping system, illustrated in Figure 1, has elements in series. Three different categories of problems are associated with series piping systems: (1) Category I, in which the required increase in head, W_s, of the pump is the unknown; (2) Category II, in which the flow rate Q is the desired results; and (3) Category III, in which the pipe diameter is to be obtained. Category I problems are direct, but Category II and III problems are iterative. The same Mathcad approach can be used to solve all categories of series piping problems as well as for the operating point of a system with a specific pump. The most convenient form of the energy equation for series piping systems is to solve Equation 1 for the increase in head required of a pump

$$W_s \frac{g_c}{g} = \frac{P_b - P_a}{\gamma} + z_b - z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \frac{Q^2}{g D_i^4} \left[f_i \frac{L_i}{D_i} + C_i f_{T_i} + K_i \right] \quad (3)$$

Example 1 illustrates several of the series piping solution capabilities of the generalized Mathcad procedure proposed herein.

Example 1 Problem Statement

Water at 70 F flows from a reservoir through the series-piping system illustrated in Figure 3. For this system, determine the following:

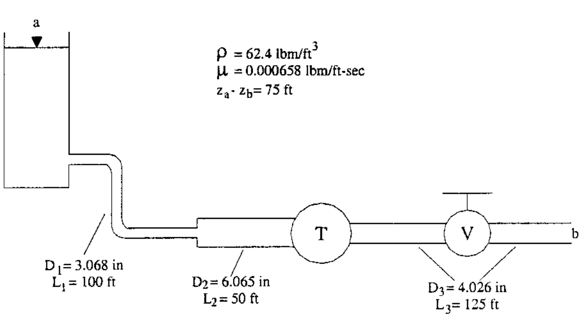

Figure 3 Example 1 series piping system schematic.

(1) The flow rate if the turbine were removed from the system;
(2) The power extracted by the turbine if the flow rate were 0.16 ft^3/sec;
(3) The flow rate if 2 hp were extracted by the turbine;
(4) The relationship between flow rate and power extracted for this system.

Example 1 Solution

The first step is to apply and reduce the energy equation for the system.

$$\frac{P_a}{\gamma} + \frac{V_a^2}{2g} + z_a = \frac{P_b}{\gamma} + \frac{V_b^2}{2g} + z_b + \frac{8}{\pi^2} \frac{Q^2}{g D_1^4} \left[K_{\text{ent}} + 2K_{\text{elbow}} + f_1 \frac{L_1}{D_1} \right] + \frac{8}{\pi^2} \frac{Q^2}{g D_2^4} \left[K_{\text{exp}} + f_2 \frac{L_2}{D_2} \right] + \frac{8}{\pi^2} \frac{Q^2}{g D_3^4} \left[K_{\text{gv}} + f_3 \frac{L_3}{D_3} \right] - W_s \frac{g_c}{g} \quad (4)$$

where W_s is positive for a pump and negative for a turbine. For this system,

$$P_a = P_b\,,\ V_a = 0, \quad \text{and} \quad \frac{V_b^2}{2g} = \frac{8}{\pi^2} \frac{Q^2}{g D_3^4}.$$

The Crane Company Technical Paper 410 [5] and Hodge and Taylor [3] provide information on minor loss coefficients. Accepted values of the minor loss coefficients for the system of Example 1 are $K_{\text{ent}} = 0.5$, $K_{\text{elbow}} = 30 f_T$, $K_{\text{exp}} = 9$ (for a diameter ratio of 1.98), and $K_{\text{gv}} = 55 f_T$. With the above values of the minor loss coefficients, the energy equation reduces to

$$W_s \frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2} \frac{Q^2}{g D_1^4} \left[0.5 + 60 f_{T_1} + f_1 \frac{L_1}{D_1} \right] + \frac{8}{\pi^2} \frac{Q^2}{g D_2^4} \left[9 + f_2 \frac{L_2}{D_2} \right] + \frac{8}{\pi^2} \frac{Q^2}{g D_3^4} \left[1.0 + 55 f_{T_3} + f_3 \frac{L_3}{D_3} \right] \quad (5)$$

Equation 5 will be used as the basis for solution for all parts of this problem. Of particular interests are the similarities of the Mathcad solution for each part and the general congruence of the solution of each part with the problem statement.

Part (1): The flow rate if the turbine were removed from the system.

This is a Category II problem, since the flow rate is to be found for $W_s = 0$ (no turbine or pump). The Mathcad worksheet for the solution is reproduced as Figure 4. This worksheet is the kernel for the solutions to all parts of this problem, so an examination is appropriate. The **solve-block** structure, shown near the bottom of the worksheet, is the key to all series piping problems solutions. The solve block is initialized by the **Given** statement and terminated by the **Find** command. The variables listed in the **Find** command are the unknowns to be solved from the equations contained between the **Given** and the **Find**; in this example, only a single equation, the energy equation, is within the solve block. The solve block permits the solution of all three categories of series piping problems to be obtained by simply indicating the required variable (unknown) in a **Find** command. The general rule for a solve block is that all variables must either be defined with values or given a guessed value. For this part of the problem, the unknown is the flow rate, Q. Hence, the first part of the worksheet specifies the values of the variables, constants, physical properties, and functional definitions for the Reynolds number and friction factors. For a Category I or III problem, only the required solution variable (and an initial guess) must be changed. The Mathcad procedure for the solution of any series-piping problem is to apply and reduce the energy equation, define the known variables in Equation 3, and specify the unknown. Thus, in the Mathcad approach, the solution algorithm is of little concern and the problem formulation and results interpretation become the center of activities. Units tracking is one important capability of Mathcad. All parts of this problem include units. For Part 1 of Example 1, the flow rate for the system with no turbine (or pump) is 1.022 ft^3/sec.

A number of different Category II problems are of interest in series piping systems. If a pump with a specified increase in head is inserted in the system, the resulting Category II problem can be solved by simply specifying the pump increase in head, W_s. For example, if a pump with an increase in head of 100 ft·lbf/lbm were placed in the system, the resulting flow rate would be 1.571 ft^3/sec with 5.673 hp (or 4.23 kW) being delivered to the fluid.

Part (2): The power extracted by the turbine if the flow rate were 0.16 ft^3/sec.

This is a Category I problem and is a direct calculation; however, the solve-block structure can be used by specifying an initial value of W_s and identifying it as the unknown in the **Find** statement. With W_s known, the power extracted from the fluid can be computed. Figure 5 presents that portion of the Mathcad worksheet utilizing the solve-block structure. The omitted part of the worksheet in Figure 5 is identical to the first part of the worksheet in Figure 4. The solve block provides the turbine decrease in head, 72.901 ft·lbf/lbm, from which the power extracted is computed to be 1.323 hp. The negative signs on the worksheet indicate a turbine and power extracted.

Part (3): The flow rate if 2 hp were extracted by the turbine.

This is more involved than either Parts 1 or 2. The simplest Mathcad solution approach is to define an equation for the power extracted by the turbine and to use a solve block with two equations (the power extracted by and the energy equation) to find both the flow rate and the turbine decrease in head. Figure 6 illustrates the relevant part of the Mathcad worksheet for this solution. As in Figure 5, the omitted part of the worksheet is identical to the first part of Figure 4. The solve block contains the two equations, and the **Find** command specifies the two unknowns for the system of two equations. The first solution, with an initial flow rate guess of 0.6 ft^3/sec, yields a flow rate of 0.871 ft^3/sec with a turbine decrease in head of 20.245 ft · lbf/lbm. As confirmation, the power is computed to be the required 2 hp. However, a second solve block execution with an initial flow rate guess of 0.2 ft^3/sec results in a flow rate of 0.252 ft^3/sec and a turbine decrease in head of 70.047 ft · lbf/lbm. For the second solve block, the power extracted is again 2 hp. Hence, the solution is double valued in flow rate for a given power extracted. This behavior is investigated further in Part 4.

The flexibility to add equations (and unknowns) to Mathcad solve blocks endows the series piping solution procedure with great utility. For example, the operating point of a specific pump in this system can be found by specifying the pump characteristics, "W_s vs. Q," as a second equation and using the solve block to obtain the flow rate and the pump increase in head. If the pump increase in head is $100 - 5Q - 8Q^2$, then the added equation (in the Mathcad format with appropriate units) becomes

$$W_s = 100\,\text{ft}\cdot\frac{\text{lbf}}{\text{lb}} - 5\,\text{ft}\cdot\frac{\text{lbf}}{\text{lb}\cdot\text{ft}^3}\cdot\text{sec}\cdot Q - 8\,\text{ft}\cdot\frac{\text{lbf}}{\text{lb}\cdot\text{ft}^6}\cdot\text{sec}^2\cdot Q^2. \qquad (6)$$

The resulting flow rate is 1.457 ft^3/sec which corresponds to 12.519 hp (9.335 kW) imparted to the fluid. The increase in head of the pump is 75.732 ft · lbf/lbm. Pump/system operating point determination in such an easy fashion is a very potent capability.

TITLE: Mathcad Implementation of Example 1.

ORIGIN:= 1 Set origin for counters to 1 from the default value of 0.

Input the pipe geometry: Diameter in inches, Length in feet, Roughness in feet:

Number of pipes

$$D := \begin{pmatrix} 3.068 \\ 6.065 \\ 4.026 \end{pmatrix} \cdot in \qquad L := \begin{pmatrix} 100 \\ 50 \\ 125 \end{pmatrix} \cdot ft \qquad \varepsilon := \begin{pmatrix} 0.00015 \\ 0.00015 \\ 0.00015 \end{pmatrix} \cdot ft$$

$$N := \text{length}(D)$$

Input the system boundary conditions: Pressures in psi, Elevations in feet:

$$\begin{pmatrix} P_a \\ P_b \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \frac{lbf}{in^2} \qquad \begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 75 \\ 0 \end{pmatrix} \cdot ft$$

Input the loss coefficients: K factor, Equivalent length

$$K := \begin{pmatrix} 0.50 \\ 9 \\ 1 \end{pmatrix} \qquad C := \begin{pmatrix} 60 \\ 0 \\ 55 \end{pmatrix}$$

Input the fluid properties: Density in lbm/ft^3, Viscosity in lbm/ft-s

$$\rho := 62.4 \frac{lb}{ft^3} \qquad \mu := 0.000658 \frac{lb}{ft \cdot sec}$$

Input the flow rate in cfs: $Q := 0.16 \dfrac{ft^3}{sec}$ (Initial guess on flow rate)

Define constants and adjust units for consistency: $g := 32.174 \dfrac{ft}{sec^2}$ $\qquad g_c := 32.174 \dfrac{ft \cdot lb}{lbf \cdot sec^2}$

Define the functions for Reynolds number, fully-rough friction factor, and friction factor:

$$Re(q,d) := \frac{4 \cdot \rho \cdot q}{\pi \cdot d \cdot \mu} \qquad f_T(d,\varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2}$$

$$f(q,d,\varepsilon) := \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,d)} + \left(\dfrac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2} & \text{if } Re(q,d) > 2300 \\ \dfrac{64}{Re(q,d)} & \text{otherwise} \end{cases}$$

Determination of flow rate when the turbine is removed. $W_s := 0 \cdot ft \cdot \dfrac{lbf}{lb}$ (No pump or turbine in system.)

Given

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$Q := \text{Find}(Q) \qquad Q = 1.022 ft^3 sec^{-1} \qquad Q = 458.79 \dfrac{gal}{min}$

Figure 4 The Mathcad worksheet for Part 1 of Example 1.

$$Q := 0.16 \frac{ft^3}{sec}$$

The generalized energy equation is:

$$W_s := -100\,ft \cdot \frac{lbf}{lb}$$ (Initial guess of turbine decrease in head.)

Given

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$$W_s := \mathrm{Find}(W_s) \qquad W_s = -72.901\,ft \cdot \frac{lbf}{lb}$$

Pump power (input to fluid):

$$Power := Q \cdot \rho \cdot W_s \qquad Power = -1.323\,hp \qquad Power = -1.491\,kW$$

Additional output of useful quantities:

$$i := 1 .. N \qquad V(q, d) := \frac{4 \cdot q}{\pi \cdot (d)^2}$$

$V(Q, D_i) =$ ($ft\,sec^{-1}$)	$Re(Q, D_i) =$	$f(Q, D_i, \varepsilon_i)$	$f_T(D_i, \varepsilon_i) =$
3.117	$7.556 \cdot 10^4$	0.021	0.017
0.798	$3.822 \cdot 10^4$	0.023	0.015
1.81	$5.758 \cdot 10^4$	0.022	0.016

Figure 5 A portion of the Mathcad worksheet for Part 2 of Example 1.

Part (4): The relationship between flow rate and power extracted for this system.

Consider how the system operates. Parts 1, 2, and 3 clearly indicate a relationship between the flow rate and the power extracted. If no power is extracted, as in Part 1, then the flow rate is a maximum. Part 1 is characterized by the maximum flow rate and a zero turbine decrease in head. If all the available head were extracted by the turbine, both the flow rate and the power extracted would be zero. In Parts 2 and 3, different flow rates and turbine decreases in head lead to different power extractions. Indeed, in Part 3, two different flow rates were found to yield 2 hp from the turbine. For the system, a decrease (a more negative number) in head by the turbine leads to a smaller flow rate, but the power contains the product of the flow rate and head decrease. Thus, a relative maximum or minimum is indicated. The Mathcad computations for power extracted as a function of flow rate are presented in Figure 7.

As in the previous figures, the omitted part of the worksheet is identical to the first part of Figure 1. A **range variable**, j, is established to permit the flow rate to be varied from zero to the maximum (determined in Part 1). The calculation of the decrease in head of the turbine, given the flow rate is a Category I problem and can be solved directly. Since the solution is direct, a solve block is not needed. The worksheet uses Q_j, where Q_j is defined using the range variable, to compute W_{s_j} as a function of the flow rate. The power is then calculated for each flow rate, and the results are displayed in graphical form. The maximum power, about 3.25 hp, extracted from the system occurs at a flow rate of $\sim$ 0.575 ft^3/sec. The double-valued, in-power extraction, nature of the system operation is well illustrated by the graph.

All parts of Example 1 used essentially the same energy equation, usually in a solve-block structure, and all parts featured the units capability of Mathcad. The series-piping examples demonstrate Mathcad solutions for the most common types of series piping problems. Somewhat more complex than series piping systems are parallel piping systems, which are examined next.

PARALLEL PIPING SYSTEM EXAMPLE

No better example exists for the effects of Mathcad on solution techniques than that for parallel piping

Determination of flow rate when the power extracted by the turbine is specified.

$W_s := -15 \cdot ft \cdot \frac{lbf}{lb}$ $\quad Q := .6 \cdot \frac{ft^3}{sec}$ (Initial guesses for specified power from system.)

Given

$-2 \cdot hp = Q \cdot \rho \cdot W_s$

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$\begin{pmatrix} Q \\ W_s \end{pmatrix} := Find(Q, W_s)$ $\quad Q = 0.871 ft^3 sec^{-1}$ $\quad W_s = -20.245 ft \cdot \frac{lbf}{lb}$

Pump power (input to fluid): $\quad Power := Q \cdot \rho \cdot W_s$ $\quad Power = -2\,hp$

But if a different initial guess, with a smaller flow rate, is used:

$W_s := -15 \cdot ft \cdot \frac{lbf}{lb}$ $\quad Q := .2 \cdot \frac{ft^3}{sec}$ (Initial guesses for specified power from system.)

Given

$-2 \cdot hp = Q \cdot \rho \cdot W_s$

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$\begin{pmatrix} Q \\ W_s \end{pmatrix} := Find(Q, W_s)$ $\quad Q = 0.252 ft^3 sec^{-1}$ $\quad W_s = -70.047 ft \cdot \frac{lbf}{lb}$

Pump power (input to fluid): $\quad Power := Q \cdot \rho \cdot W_s$ $\quad Power = -2\,hp$

Figure 6 A portion of the Mathcad worksheet for Part 3 of Example 1.

systems. Parallel piping systems, such as that illustrated in Figure 2b, have long been solved in iterative fashion by enforcing equality of change in head across each pipe and conservation of mass at the two nodes where two or more pipes intersect. The usual, pre-Mathcad procedure was to assume a flow rate in one pipe, compute the change in head in that pipe (a Category I problem), compute the flow rates in the remaining pipes by requiring their changes in head to be equal to that of the first pipe (Category II problems), and iterate the flow rates until convergence. In Mathcad, the solution procedure is more straightforward and closer to the formulation of the problem.

Conservation of mass for the parallel piping system of Figure 2b can be expressed as

$$Q_T = Q_1 + Q_2 + Q_3. \tag{7}$$

If the pump in Figure 2b is viewed as providing sufficient increase in head to make $P_a = P_b$, then the energy equations for the lines become

$$\begin{aligned} W_s \frac{g_c}{g} &= z_b - z_a + \frac{8}{\pi^2} \frac{Q_1^2}{g D_1^4} \left[f_1 \frac{L_1}{D_1} + C_1 f_{T_1} + K_1 \right], \\ W_s \frac{g_c}{g} &= z_b - z_a + \frac{8}{\pi^2} \frac{Q_2^2}{g D_2^4} \left[f_2 \frac{L_2}{D_2} + C_2 f_{T_2} + K_2 \right], \\ W_s \frac{g_c}{g} &= z_b - z_a + \frac{8}{\pi^2} \frac{Q_3^2}{g D_3^4} \left[f_3 \frac{L_3}{D_3} + C_3 f_{T_3} + K_3 \right]. \end{aligned} \tag{8}$$

Equation 7 and 8 form the system of equations that will be used in a Mathcad solve block to solve most parallel piping systems problems (with three parallel lines). The information required for the solve block will be entered in a fashion similar to that for series piping problems. The two most common parallel piping problem types are given W_s, find Q_T, Q_1, Q_2, and Q_3, or given Q_T, find W_s, Q_1, Q_2, and Q_3. Although the first type can be worked as a Category I problem for each line and the flow rates added, the solution technique presented herein makes use of the Mathcad solve-block procedure for all types of parallel piping

$j := 1..\ 17$ (Establish a range variable for a parametric study.)

$$Q_j := .000001\frac{ft^3}{sec} + (j-1)\cdot 1.022\frac{ft^3}{sec \cdot 16}$$ (Expression to vary the flow rate over the permissible range.)

The generalized energy equation is:

$$W_{s_j} := \frac{P_b - P_a}{\rho} + (Z_b - Z_a)\cdot\frac{g}{g_c} + \sum_{i=1}^{N} \frac{8}{\pi^2}\cdot\frac{(Q_j)^2}{g_c\cdot(D_i)^4}\cdot\left(f(Q_j, D_i, \varepsilon_i)\cdot\frac{L_i}{D_i} + K_i + C_i\cdot f_T(D_i, \varepsilon_i)\right)$$

$$Power_j := Q_j\cdot\rho\cdot W_{s_j}$$

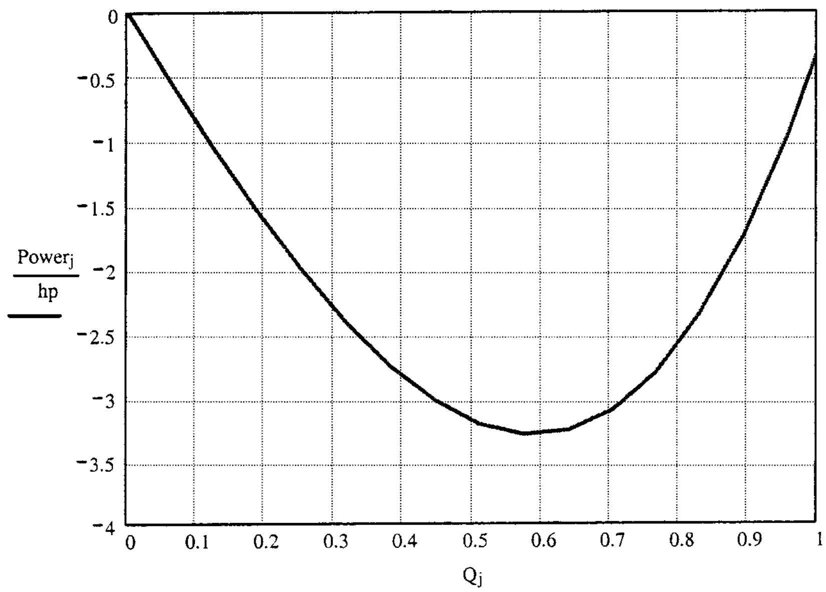

Figure 7 A portion of the Mathcad worksheet for Part 4 of Example 1.

problems. Consider the following example for a parallel-piping system.

Example 2 Problem Statement

A parallel piping system is composed of three pipes, as in Figure 2b. For this system, $z_a = z_b$ and the fluid properties are $\rho = 701$ kg/m^3 and $\mu = 0.00051$ N·s/m^2. Table 1 contains information on each pipe. For this system determine the following:

(1) The increase in head required and power imparted to the fluid for a total flow rate of 0.036 m^3/s;

Table 1 Pipe Characteristics for Example 2

Pipe	D (cm)	L (m)	ε (mm)	K	C
1	5	60	0.1	0	60
2	5	60	0.1	0	60
3	4	55	1.0	1.5	60

(2) The total flow rate if the increase in head of the pump were 500 N·m/kg;

(3) The power delivered to the fluid if a 10-kW booster pump were placed in line 3 and the total flow rate were maintained at 0.036 m^3/s.

Example 2 Solution

The usual first steps in solving a parallel piping system problem are to draw and label a schematic and apply and reduce the energy equation for each leg. Figure 2b is the schematic, and the problem statement essentially provides information for use directly in the energy equation.

Part (1): The increase in head required and power imparted to the fluid for a total flow rate of 0.036 m^3/sec.

Similar to Example 1, the same basic Mathcad formulation will be used for all three parts of Example 2, only the solve-block structure will be changed. Figure 8 contains the complete Mathcad worksheet for the solution of Part 1. This worksheet is the kernel for the solutions to all parts of this problem.

TITLE: Mathcad implementation of Example 2.

ORIGIN:= 1 Set origin for counters to 1 from default value of 0.

Input the pipe geometry and the elevation difference:

$$D := \begin{pmatrix} 5.0 \\ 5.0 \\ 4.0 \end{pmatrix} \cdot cm \quad L := \begin{pmatrix} 60 \\ 60 \\ 55 \end{pmatrix} \cdot m \quad \varepsilon := \begin{pmatrix} 0.1 \\ 0.1 \\ 1.0 \end{pmatrix} \cdot mm \quad \begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot m$$

Input the loss coefficients:

K factor Equivalent length Number of pipes

$$K := \begin{pmatrix} 0 \\ 0 \\ 1.5 \end{pmatrix} \quad C := \begin{pmatrix} 60 \\ 0 \\ 60 \end{pmatrix} \quad N := \text{length}(D)$$

Input the fluid properties:

Density in kg/m^3 Viscosity in N-s/m^2

$$\rho := 701 \cdot \frac{kg}{m^3} \qquad \mu := 0.000511 \cdot \frac{newton \cdot sec}{m^2}$$

Define constants and adjust units for consistency: $g := 9.807 \cdot \frac{m}{sec^2}$ $g_c := 1 \cdot \frac{m \cdot kg}{newton \cdot sec^2}$

Define the functions for the Reynolds number and the friction factors:

$$Re(q, D) := \frac{4 \cdot \rho \cdot q}{\pi \cdot D \cdot \mu} \qquad f_T(D, \varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot D}\right)^{1.11}\right]\right]^2}$$

$$f(q, D, \varepsilon) := \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,D)} + \left(\dfrac{\varepsilon}{3.7 \cdot D}\right)^{1.11}\right]\right]^2} & \text{if } Re(q,D) > 2300 \\ \dfrac{64}{Re(q,D)} & \text{otherwise} \end{cases}$$

Setup Solve Block by defining specified inputs and guessed values:

$$Q_T := 0.036 \frac{m^3}{sec} \quad Q1 := \frac{Q_T}{N} \quad Q2 := \frac{Q_T}{N} \quad Q3 := \frac{Q_T}{N} \quad W_s := 1000 m \cdot \frac{newton}{kg}$$

Given

$$Q_T = Q1 + Q2 + Q3$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q1)^2}{g \cdot (D_1)^4} \cdot \left(f(Q1, D_1, \varepsilon_1) \cdot \frac{L_1}{D_1} + K_1 + C_1 \cdot f_T(D_1, \varepsilon_1) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q2)^2}{g \cdot (D_2)^4} \cdot \left(f(Q2, D_2, \varepsilon_2) \cdot \frac{L_2}{D_2} + K_2 + C_2 \cdot f_T(D_2, \varepsilon_2) \right)$$

Figure 8 The Mathcad worksheet for Part 1 of Example 2.

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q3)^2}{g \cdot (D_3)^4} \cdot \left(f(Q3, D_3, \varepsilon_3) \cdot \frac{L_3}{D_3} + K_3 + C_3 \cdot f_T(D_3, \varepsilon_3) \right)$$

$$\begin{pmatrix} W_s \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} := \text{Find}(W_s, Q1, Q2, Q3) \qquad W_s = 858.4418\text{newton} \cdot \frac{m}{kg}$$

$$Q1 = 0.0149 m^3 s^{-1} \qquad Q2 = 0.0152 m^3 s^{-1} \quad Q3 = 0.0059 m^3 s^{-1}$$

$$\text{Power} := \rho \cdot Q_T \cdot W_s \qquad \text{Power} = 21.6636 kW$$

Additional output of useful quantities:

$$i := 1 .. N \qquad V(q, D) := \frac{4 \cdot q}{\pi \cdot (D)^2}$$

$$Q_1 := Q1 \qquad Q_2 := Q2 \qquad Q_3 := Q3$$

$V(Q_i, D_i) =$ (ms^{-1})	$Re(Q_i, D_i) =$	$f(Q_i, D_i, \varepsilon_i)$	$f_T(D_i, \varepsilon_i) =$
7.573	$5.2046 \cdot 10^5$	0.0238	0.0235
7.7588	$5.3323 \cdot 10^5$	0.0238	0.0235
4.6919	$2.5796 \cdot 10^5$	0.0533	0.0532

Figure 8 (*Continued*)

The first part of the worksheet specifies the values of the variables, constants, physical properties, and function definitions for the Reynolds number and friction factors. The solve-block structure, shown near the bottom of the worksheet, is the key to all parallel piping problems solutions. In Part 1 of this example, the solve block contains four equations, one conservation of mass and one energy equation for each leg of the parallel system. The solve block permits solution of both types of parallel piping problems by simply indicating the required variables (unknowns) in a **Find** command. All variables must either be defined with values or given guessed values. For this part of the problem, the unknowns are the flow rates in the individual legs and the increase in head required to make $P_a = P_b$. For other types of parallel piping problems, only the required solution variables (and the initial guesses) must be changed. As with series piping problems, the solution algorithm is of little concern and the problem formulation and results interpretation become the center of activities. Units tracking is also invoked in this example. For Part 1 of Example 2, the flow rates are 0.0149, 0.0152, and 0.0059 m^3/sec, respectively, for pipes 1, 2, and 3. The pump increase in head is 858.4 N · m/kg, and the power delivered to the fluid is 21.7 kW. One important salient feature of many Mathcad solutions is the general congruence of the problem formulation and the Mathcad implementation. In the case of parallel piping systems, this congruence is striking as the formulation process leads directly to the Mathcad input required for the solution.

Part (2): The total flow rate if the increase in head were 500 N · m/kg.

Figure 9 presents a portion of the Mathcad worksheet illustrating the solve-block arrangement required for the solution. Only the solve-block structure is presented in the figure as the first part of the worksheet for Part 2 is identical to that of Part 1. For Part 2, $W_s = 500$ N · m/kg is given and Q_T, Q_1, Q_2, Q_3, and Q_4 are the unknowns. For this part of Example 2, the total flow rate is 0.0274 m^3/sec, and the flow rates are 0.0113, 0.0116, and 0.0045 m^3/sec, respectively, for pipes 1, 2, and 3. The power delivered to the fluid is 9.61 kW.

Part (3): The power delivered to the fluid if a 10-kW booster pump were placed in line 3 and the total flow rate were maintained at 0.036 m^3/sec.

This is similar to Part 1, except that a 10-kW booster pump is placed in line 3. The Mathcad worksheet for this part of the problem is presented in Figure 10. The solve block is modified by the addition of one equation, 10 kW $= Q_3 \cdot \rho \cdot$ HP, that represents the power delivered to the fluid in line 3 and by the addition to the leg 3 energy equation of −HP, the head developed by the pump placed in leg 3. The **Find**

Setup Solve Block by defining specified inputs and guessed values:

$$Q_T := 0.036\frac{m^3}{sec} \qquad Q1 := \frac{Q_T}{N} \qquad Q2 := \frac{Q_T}{N} \qquad Q3 := \frac{Q_T}{N} \qquad W_s := 500\,m\cdot\frac{newton}{kg}$$

Given

$$Q_T = Q1 + Q2 + Q3$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q1)^2}{g\cdot(D_1)^4}\cdot\left(f(Q1,D_1,\varepsilon_1)\cdot\frac{L_1}{D_1} + K_1 + C_1\cdot f_T(D_1,\varepsilon_1)\right)$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q2)^2}{g\cdot(D_2)^4}\cdot\left(f(Q2,D_2,\varepsilon_2)\cdot\frac{L_2}{D_2} + K_2 + C_2\cdot f_T(D_2,\varepsilon_2)\right)$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q3)^2}{g\cdot(D_3)^4}\cdot\left(f(Q3,D_3,\varepsilon_3)\cdot\frac{L_3}{D_3} + K_3 + C_3\cdot f_T(D_3,\varepsilon_3)\right)$$

$$\begin{pmatrix} Q_T \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} := \mathrm{Find}(Q_T, Q1, Q2, Q3)$$

$Q_T = 0.0274 m^3 s^{-1}$ $Q1 = 0.0113 m^3 s^{-1}$ $Q2 = 0.0116 m^3 s^{-1}$ $Q3 = 0.0045 m^3 s^{-1}$

$Power := \rho\cdot Q_T\cdot W_s$ $\qquad Power = 9.6138 kW$

Additional output of useful quantities:

$i := 1..N \qquad V(q,D) := \frac{4\cdot q}{\pi\cdot(D)^2} \qquad Q_1 := Q1 \qquad Q_2 := Q2 \qquad Q_3 := Q3$

$V(Q_i,D_i) =$ ($m s^{-1}$)	$Re(Q_i,D_i) =$	$f(Q_i,D_i,\varepsilon_i) =$	$f_T(D_i,\varepsilon_i) =$
5.7687	$3.9645\cdot10^5$	0.0239	0.0235
5.9099	$4.0616\cdot10^5$	0.0239	0.0235
3.5795	$1.968\cdot10^5$	0.0533	0.0532

Figure 9 A portion of the Mathcad worksheet for Part 2 of Example 2.

command has HP added as an unknown (which requires a guessed value ahead of the solve block). For Part 3 of Example 2, the flow rates are 0.0131, 0.0135, and 0.0094 m^3/sec, respectively, for pipes 1, 2, and 3. The main pump increase in head is 670.667 N · m/kg, and the power delivered to the fluid is 16.9 kW. The power delivered to the fluid in leg 3 by the booster pump is confirmed to be 10 kW. This is a relatively difficult problem to work "by hand," but the Mathcad solution is simple, straight forward, and congruent with the problem formulation.

Piping networks, which contain both series and parallel components, are the next higher level of complexity for piping systems problems and are examined in the following section.

NETWORK PIPING SYSTEM EXAMPLE

Piping network analysis is built about the concepts of loops, a sequence of pipes that form a closed path, and nodes, a point where two or more lines are joined. Conservation of mass must be maintained at each node, and the pressure (or head) change around each loop must be zero. Using these concepts, a number of procedures can be devised to find the flow rate and change in pressure in each line. One approach is to mimic the procedure of parallel piping systems and write a system of non-linear algebraic equations expressing conservation of mass at each node and zero pressure change for each loop. However, even for a moderately complex piping network, obtaining a

Setup Solve Block by defining specified inputs and guessed values:

$$Q_T := 0.036\frac{m^3}{sec} \qquad Q1 := \frac{Q_T}{N} \qquad Q2 := \frac{Q_T}{N} \qquad Q3 := \frac{Q_T}{N}$$

$$W_s := 500 m\cdot\frac{newton}{kg} \qquad HP := 100 newton\cdot\frac{m}{kg}$$

Given

$$Q_T = Q1 + Q2 + Q3$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q1)^2}{g\cdot(D_1)^4}\cdot\left(f(Q1, D_1, \varepsilon_1)\cdot\frac{L_1}{D_1} + K_1 + C_1\cdot f_T(D_1, \varepsilon_1)\right)$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q2)^2}{g\cdot(D_2)^4}\cdot\left(f(Q2, D_2, \varepsilon_2)\cdot\frac{L_2}{D_2} + K_2 + C_2\cdot f_T(D_2, \varepsilon_2)\right)$$

$$W_s\cdot\frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2}\cdot\frac{(Q3)^2}{g\cdot(D_3)^4}\cdot\left(f(Q3, D_3, \varepsilon_3)\cdot\frac{L_3}{D_3} + K_3 + C_3\cdot f_T(D_3, \varepsilon_3)\right) - HP\cdot\frac{g_c}{g}$$

$$10\cdot kW = Q3\cdot\rho\cdot HP$$

$$\begin{pmatrix} W_s \\ Q1 \\ Q2 \\ Q3 \\ HP \end{pmatrix} := Find(W_s, Q1, Q2, Q3, HP)$$

$$W_s = 670.6675 newton\cdot\frac{m}{kg} \qquad HP = 1515.4763 newton\cdot\frac{m}{kg}$$

$$Q1 = 0.0131 m^3 s^{-1} \quad Q2 = 0.0135 m^3 s^{-1} \quad Q3 = 0.0094 m^3 s^{-1}$$

$$Power := \rho\cdot Q_T\cdot W_s \qquad Power = 16.925 kW$$

$$Power_b := \rho\cdot Q3\cdot HP \qquad Power_b = 10 kW$$

Additional output of useful quantities:

$$i := 1..N \qquad V(q, D) := \frac{4\cdot q}{\pi\cdot(D)^2} \qquad Q_1 := Q1 \qquad Q_2 := Q2 \qquad Q_3 := Q3$$

$V(Q_i, D_i) =$ ($m s^{-1}$)	$Re(Q_i, D_i) =$	$f(Q_i, D_i, \varepsilon_i) =$	$f_T(D_i, \varepsilon_i) =$
6.6883	$4.5966\cdot10^5$	0.0238	0.0235
6.8523	$4.7093\cdot10^5$	0.0238	0.0235
7.4907	$4.1184\cdot10^5$	0.0533	0.0532

Figure 10 A portion of the Mathcad worksheet for Part 3 of Example 2.

system of independent equations and generating acceptable initial guesses limit the utility of this approach. Most useful procedures for solving piping network problems use the fluid version of Kirchhoff's law for electric circuit analysis; see Jeppson [6] for a more complete discussion. The most common of these procedures is the Hardy–Cross technique; Jeppson [6] and Hodge and Taylor [3] provide details. The Hardy–Cross procedure was first devised for hand calculations, but its generality and systematic approach make it equally convenient for computer-based approaches.

Conservation of mass is initially established and enforced at each node and loop correction factors, ΔQ, are determined for each loop such that the change in pressure (or head) around a loop is zero. The change in head for a given line in a network is expressed in terms of the major and minor losses in the line in the same fashion as for series and parallel

HardyCross(h,dh,Q,N,tol)

This function executes the general Hardy-Cross solution algorithm. The return value is a vector of the flow rates. The argument list is described below.

h is a vector function of **Q** where each term defines the head loss for the corresponding pipe. This function can include pumps and devices in addition to major losses.

dh is a vector function of **Q** where each term defines the derivative of h_i with respect to Q_i. Appropriate derivatives must be defined for pumps and devices in each line.

Q is the vector of initial guesses for the flow rates--must satisfy conservation of mass

N is a matrix which sets the loop sign convention--rows = # pipes and cols = # loops
Fill the matrix by columns:
If the pipe does not fall in the loop enter 0;
Enter 1 when the assumed flow direction is positive;
Enter -1 when the assumed flow direction is negative.
The counterclockwise sense for a loop is the positive sign convention.

tol is the convergence tolerance--suggest 0.0001

```
HardyCross(h, dh, Q, N, tol) := | L ← cols(N)
                                | P ← rows(N)
                                | for l ∈ 1.. L
                                |   ΔQ_l ← 100
                                | while sqrt( Σ_{l=1}^{L} (ΔQ_l)^2 ) > tol
                                |   | for l ∈ 1.. L
                                |   |   ΔQ_l ← -1 · ( Σ_{i=1}^{P} N_{i,l}·h(Q)_i ) / ( Σ_{i=1}^{P} (N_{i,l})^2·dh(Q)_i )
                                |   | Q ← Q + N·ΔQ
                                | Q
```

Figure 11 Mathcad Hardy–Cross program element.

systems. To permit the inclusion of active devices such as pumps and turbines, the device head change, $\mathbf{h_d}$ is added to the line. The resulting expression for line l is

$$h_l(Q) = \frac{8}{\pi^2}\frac{Q_l^2}{gD_l^4}\left(f_l\frac{L_l}{D_l} + K_l + C_l f_{T_l}\right) + h_{d_l}(Q_l). \quad (9)$$

In Equation 9, the first term is the sum the major and minor losses for pipe l in the system; the second term, h_d is the change in head due to an active device (pump, turbine, or other device resulting in a change in head in line l). The usual Hardy–Cross sign convention is that head losses are positive and that counterclockwise flow in a loop is positive. Thus in Equation 9, the increase in head of a pump is a negative quantity.

The Mathcad procedure, **HardyCross (h,dh,Q, N,tol)**, for the Hardy–Cross iterative process is given in Figure 11, where **h(Q)** represents the change in head in a line and **dH(Q)** represents the change in head with respect to the flow rate Q. **N** is the connection matrix that describes the relationships between loops,

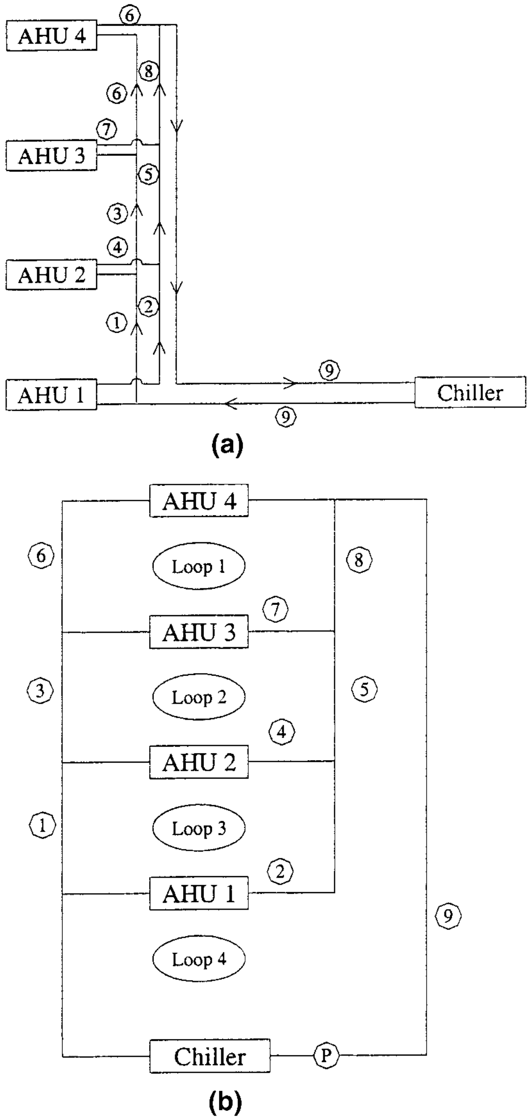

Figure 12 Physical Arrangement and Hardy–Cross schematic. (a) Physical layout; (b) Hardy–Cross schematic.

nodes, and flow directions. The procedure in Figure 11 illustrates an iterative Mathcad program element that uses the Hardy–Cross loop-correction expression to find the flow rate in each line. Convergence is attained when the root-sum-square of the loop corrections factors becomes less than an input tolerance, "**tol**." Major losses in piping networks can also be described in terms of the Hazen–Williams relation, $h_f = KQ^n$, or in terms of the Darcy friction factor. The Hazen–Williams representation is common in water systems, but the friction factor representation is the most general and will be used herein. To explain more fully the procedure, consider the following example.

Example 3 Problem Statement

The network solution part of a piping design for a building HVAC system will be considered. Cooling load requirements for each floor of a four-story office building were specified, along with the physical dimensions, the losses associated with the air handling units and the chiller, and the valve requirements. A maximum fluid velocity less than 10 ft/sec was mandated. The first part of the problem involved sizing the pipe and defining the layout. The physical results are presented in Figure 12a with individual line information tabulated in Table 2. The Hardy–Cross equivalent representation with the lines and loops defined is given in Figure 12b. The lines in Figure 12a, the physical representation, have a one-to-one correspondence with the lines in Figure 12b, the Hardy–Cross representation. Figure 12b is much easier to use in setting up the Hardy–Cross solution than is Figure 12a. Determine the pumping requirements for the system.

Table 2 Line Characteristics for the Network of Problem 3

Line	Length (ft)	Diameter (ft)	Q (ft^3/sec)	K	C	AHU	Chiller
1	15	1.4063	11	0	0	0	0
2	65	0.6651	3	0	100	$1.2Q^2$	0
3	15	1.4063	8	0	0	0	0
4	50	0.6651	3	0	100	$1.2Q^2$	0
5	15	1.4063	6	0	0	0	0
6	65	0.8350	5	0	100	$1.2Q^2$	0
7	50	0.6651	3	0	100	$1.2Q^2$	0
8	15	1.4065	9	0	0	0	0
9	200	1.4065	14	0	150	0	$0.04Q^2$

Example 3 Solution

The overhead with setting up the Hardy–Cross Mathcad solution has been done and was reported in the problem statement. Consider first, a network composed of just the three ''inner loops,'' with 14 ft^3/sec entering and 14 ft^3/sec exiting. The flow rates expected in the inner-loop arrangement with no booster pumps or valve ''turndowns,'' will first be determined. Then based on these results, the valves can be adjusted or booster pumps added to achieve the desired flow rates. Figure 13 is the worksheet required for the solution. The nomenclature for Figure 13 follows that of the earlier series and parallel systems worksheet solutions. All pipe diameters, lengths, and absolute roughness values are defined. The device head change vector, $\mathbf{h_d(Q)}$, and the derivative of the head change vector with respect to flow rate, $\mathbf{dh_d(Q)}$, are assembled. The initial flow rate guesses are included in the **Q** vector. The losses from the AHUs are included in $\mathbf{h_d(Q)}$. The same friction factor and Reynolds number expressions were used as in the series and parallel system solutions. The **K** and **C** vectors are formed. The valve losses are included in the **C** vector. Finally, **h(Q)** and **dh(Q)** are defined. The connection matrix, **N**, is formulated, and the Hardy–Cross procedure, **HardyCross.mcd**, is invoked. The connection matrix, N, contains one column for each loop, and each column contains an entry for each line. The matrix is generated by place a ''+1'' in the line position for a positive flow rate, a ''−1'' in the line position for a negative flow rate, and a ''0'' in the line position if the line is not in the loop. The connection matrix is

$$\mathbf{N} = \overset{\text{Loop}}{\begin{matrix} 1 & 2 & 3 \end{matrix}} \atop \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

The converged flow rates are the vector, **ans**, that is printed. These are the flow rates expected for the existing network (no valve turndowns or pumps). The flow rates do not meet the system requirements. The deviations of the computed flow rates from the required flow rates are calculated and presented in the vector, **percent**. The largest deviation, −25.86%, from the required flow rate occurs in line 6 that represents the fourth floor, the floor with the largest flow rate (and the highest pressure drop) requirement.

Two strategies are available to meet the flow rate requirements: (1) turndown valves in lines with excess flow rates to force more flow through lines with flow rates less than required, or (2) place booster pumps in lines with flow rates less than required. The next step in the problem solution is to place a booster pump in line 6. The increase in head of the booster pump is varied until the flow rate in line 6 is satisfactory; in this case, an increase in head of 9 ft · lbf/lbm is required to provide sufficient flow. The booster pump in line 6 yields acceptable results for all the required flow rates, so no additional booster pumps or valve turndowns are needed for the inner loops.

The final step in the solution of this example is to add the outer loop, loop 4, to the Hardy–Cross procedure. Since the inner loops have acceptable flow rates (with the addition of the booster pump in line 6), only the increase in head of the main pump (in line 9) needs to be determined. The increase in head of the main pump is varied until line 9 has the required flow rate of 14 ft^3/sec. Figure 14 presents the Mathcad worksheet (changes required from Fig. 13) for the solution of this example problem. The main pump is represented by $-\mathbf{HP_{main}}$ in position 9 of the $\mathbf{h_d(Q)}$ vector, and the booster pump is represented by $-\mathbf{HP_{boost}}$ in position 6 of the $\mathbf{h_d(Q)}$ vector. The outer loop has been added. A main pump increase in head of 27.5 ft · lbf/lb is satisfactory to provide the required flow rate in the main line, line 9, and since the inner loops had previously been satisfactory, no changes were needed in inner loops. The last part of the Figure 14 shows the power delivered to the fluid by the booster pump, 10.778 hp, and by the main pump, 43.68 hp. This is a relatively complex example, yet the Mathcad Hardy–Cross procedure is easy to apply and is used sequentially to solve the problem.

PEDAGOGICAL ISSUES

The examples have demonstrated the Mathcad techniques used to solve a variety of piping systems problems, but in an engineering education environment, pedagogical issues are as important as technical issues. This section delineates the authors' observations and experiences pertaining to student issues. Since, no formal evaluation instruments were devised and validated, the following comments are anecdotal in nature.

TITLE: Mathcad solution of Example 3
Friction-factor-based major loss representation.
ORIGIN≡ 1 Reset counter to start at 1 rather the default value of 0.

INNER LOOP CALCULATIONS:

Input the pipe geometry:
Diameter in feet:

$$D := (1.4063\ 0.6651\ 1.4063\ 0.6651\ 1.4063\ 0.835\ 0.6651\ 1.4063)^T$$

Length in feet:

$$L := (15\ 65\ 15\ 50\ 15\ 65\ 50\ 15)^T$$

Roughness in feet:

$$\varepsilon := (0.00015\ 0.00015\ 0.00015\ 0.00015\ 0.00015\ 0.00015\ 0.00015\ 0.00015)^T$$

Define constants and unit adjustments: $g := 32.174$

Define physical properties: $\nu := 0.000016$ $\rho := 62.4\frac{lb}{ft^3}$

Starting guess for flow rates in cfs--must satisfy the conservation of mass at each node:

$$Q := (11\ 3\ 8\ 3\ 6\ 5\ 3\ 9)^T$$

Define device head change vector:

$$h_d(q) := \left(0\ \ 1.2 \cdot q_2 \cdot |q_2|\ \ 0\ \ 1.2 \cdot q_4 \cdot |q_4|\ \ 0\ \ 1.2 \cdot q_6 \cdot |q_6|\ \ 1.2 \cdot q_7 \cdot |q_7|\ \ 0\right)^T$$

$$dh_d(q) := \left(0\ \ 2.4 \cdot |q_2|\ \ 0\ \ 2.4 \cdot |q_4|\ \ 0\ \ 2.4 \cdot |q_6|\ \ 2.4 \cdot |q_7|\ \ 0\right)^T$$

The usual functions for friction factor must be defined:

$$Re(qq, dd) := \frac{4 \cdot |qq|}{\pi \cdot dd \cdot \nu} \qquad f_T(dd, \varepsilon) := \frac{0.3086}{\log\left[\left(\frac{\varepsilon}{3.7 \cdot dd}\right)^{1.11}\right]^2}$$

$$f(qq, dd, \varepsilon) := \begin{cases} \begin{cases} \dfrac{0.3086}{\log\left[\dfrac{6.9}{Re(qq, dd)} + \left(\dfrac{\varepsilon}{3.7 \cdot dd}\right)^{1.11}\right]^2} & \text{if } Re(qq, dd) > 2300 \\ \dfrac{64}{Re(qq, dd)} & \text{otherwise} \end{cases} & \text{if } |qq| > 0 \\ 1 & \text{otherwise} \end{cases}$$

Define the minor loss coefficients K and the equivalent-lengths C:

$$K := (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$$

$$C := (0\ 100\ 0\ 100\ 0\ 100\ 100\ 0)^T$$

Figure 13 The Mathcad worksheet for the inner loop with no booster pump.

What Is the Student Mastery of Mathcad?

In the mechanical engineering (ME) program at Mississippi State University (MSU), Mathcad is the prime arithmetic engine used by the undergraduates. The College of Engineering at MSU has a Mathcad site license, so the software is available to all engineering students. An introductory numerical analysis/ Mathcad course entitled "Engineering Analysis" is required in the junior year and most junior/senior

Define the loss function for each line using the friction factor major loss expression:

$$h(Q) := \overrightarrow{\frac{8\cdot Q\cdot |Q|}{\pi^2\cdot g\cdot D^4}\cdot\left(f(Q,D,\varepsilon)\cdot\frac{L}{D} + K + C\cdot f_T(D,\varepsilon)\right)} + h_d(Q)$$

Define the derivative of the loss function:

$$dh(Q) := \overrightarrow{\frac{16\cdot |Q|}{\pi^2\cdot g\cdot D^4}\cdot\left(f(Q,D,\varepsilon)\cdot\frac{L}{D} + K + C\cdot f_T(D,\varepsilon)\right)} + dh_d(Q)$$

The Matrix **N** relates the assumed flow rate in each pipe to the counterclockwise loop sign convention.

$$N := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

➔ Reference:C:\old c drive\esd\HardyCross.mcd

ans := HardyCross (h, dh, Q, N, 0.001)

$$ans^T = (10.5964\ \ 3.4036\ \ 7.1516\ \ 3.4448\ \ 6.8484\ \ 3.707\ \ 3.4446\ \ 10.293)$$

$$percent := \left(\overrightarrow{\left(\frac{ans\cdot 100}{Q} - 100\right)}\right)^T$$

$$percent = (-3.669\ \ 13.453\ \ -10.605\ \ 14.828\ \ 14.141\ \ -25.86\ \ 14.819\ \ 14.367)$$

Figure 13 (*Continued*)

courses routinely use Mathcad. MSU ME undergraduates have good general mastery of Mathcad. Graduates students entering the MS or PhD programs from other institutions or programs may have never been exposed to Mathcad, but Mathcad is quite intuitive and the rudiments can be learned quickly. The availability of Mathcad worksheets on MSU ME course web sites provides a source of "debugged" Mathcad techniques and illustrates the capabilities of Mathcad in the ME arena. The rapidity with which students embrace Mathcad over programming language options indicates general student recognition of the utility of Mathcad.

How Is Mathcad Integrated in the Classroom?

All MSU ME classrooms have internet connections and power and are equipped with LCD/computer arrangements. The usual sequence is for the instructor present an example, develop the Mathcad capability needed to solve the example, and discuss the details in the worksheet. In many instances, students can download the worksheet and work collaboratively to master the procedure and understand the nuances of the example. Results of student surveys corroborate that students find the sequence of class presentation of Mathcad worksheets and then applications to meaningful problems in class or as homework to be a useful approach.

Does Mathcad Make a Difference in the "Engineering" Approach to Courses?

Using Mathcad permits an increase in the complexity and realism of homework assignments. Competency homework problem assignments can require parametric studies with more general inferences available to the students. The argument can be made that the use of Mathcad neglects the numerical analysis details and that, as a result, the students tend to be working with black boxes. To the authors, this does not appear to happen or to be a problem. Most of the applications software used in the engineering workplace could also be faulted with the same argument. Students have sufficient background to understand the general approach of what is occurring in the Mathcad procedures; in many instances, the students seem to

COMPLETE SYSTEM CALCULATIONS:

Input the pipe geometry:

Diameter in feet:

$$D := (1.4063\ \ 0.6651\ \ 1.4063\ \ 0.6651\ \ 1.4063\ \ 0.835\ \ 0.6651\ \ 1.4063\ \ 1.4063)^T$$

Length in feet:

$$L := (15\ \ 65\ \ 15\ \ 50\ \ 15\ \ 65\ \ 50\ \ 15\ \ 200)^T$$

Roughness in feet:

$$\varepsilon := (0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015\ \ 0.00015)^T$$

Starting guess for flow rates in cfs--must satisfy the conservation of mass at each node:

$$Q := (11\ \ 3\ \ 8\ \ 3\ \ 6\ \ 5\ \ 3\ \ 9\ \ 14)^T$$

Define device head change vector:

$$HP_{boost} := 19 \qquad HP_{main} := 27.5$$

$$h_d(q) := \left[0\ \ 1.2\cdot(q_2)^2\ \ 0\ \ 1.2\cdot(q_4)^2\ \ 0\ \ 1.2\cdot(q_6)^2 - HP_{boost}\ \ 1.2\cdot(q_7)^2\ \ 0\ \ 0.04\cdot(q_9)^2 - HP_{main}\right]^T$$

$$dh_d(q) := \left(0\ \ 2.4\cdot|q_2|\ \ 0\ \ 2.4\cdot|q_4|\ \ 0\ \ 2.4\cdot|q_6|\ \ 2.4\cdot|q_7|\ \ 0\ \ 0.08\cdot|q_9|\right)^T$$

Define the minor loss coefficients K and the equivalent-lengths C:

$$K := (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)^T$$

$$C := (0\ \ 100\ \ 0\ \ 100\ \ 0\ \ 100\ \ 100\ \ 0\ \ 150)^T$$

The Matrix **N** relates the assumed flow rate in each pipe to the counterclockwise loop sign convention.

$$N := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & -1 \end{pmatrix}^T$$

Reference:C:\old c drive\esd\HardyCross.mcd

$$ans := HardyCross(h, dh, Q, N, 0.001)$$

$$ans^T = (11.1261\ \ 3.0272\ \ 8.0666\ \ 3.0595\ \ 6.0867\ \ 5.011\ \ 3.0556\ \ 9.1423\ \ 14.1533)$$

$$percent := \left(\overrightarrow{\left(\frac{ans\cdot 100}{Q} - 100\right)}\right)^T$$

$$percent = (1.146\ \ 0.907\ \ 0.833\ \ 1.982\ \ 1.444\ \ 0.22\ \ 1.854\ \ 1.581\ \ 1.095)$$

$$Power_{boost} := \rho\cdot Q_6\cdot\frac{ft^3}{sec}\cdot HP_{boost}\cdot\frac{lbf\cdot ft}{lb} \qquad Power_{boost} = 10.778hp$$

$$Power_{main} := \rho\cdot Q_9\cdot\frac{ft^3}{sec}\cdot HP_{main}\cdot\frac{lbf\cdot ft}{lb} \qquad Power_{main} = 43.68hp$$

Figure 14 The Mathcad worksheet for the complete Example 3 solution.

develop a better understanding and appreciation of the engineering aspects of the problems. Hodge and Taylor (1) discuss in detail the effects Mathcad has had on a particular course.

CONCLUSIONS

The Mathcad procedures discussed herein illustrate a number of features of Mathcad and show how these features may be invoked to solve many different piping problems. The Mathcad procedures are more congruent with the solution formulation than the traditional procedures. The most significant outcome is that the use of Mathcad permits students to focus on engineering rather than computational aspects of problem solutions. The authors assess their experiences in using these Mathcad procedures in thermal/fluids courses as being successful and enhancing the abilities of students to work meaningful problems.

Arithmetic systems, such as Mathcad, offer a new paradigm for engineering calculations and for engineering education. This new paradigm, although not replacing any existing techniques, does offer another option for calculations with the important advantage that engineering tasks not programming tasks become the focus. The examples in this paper illustrate the potency of Mathcad, one of the arithmetic systems, in a variety of piping systems calculations of pedagogical interest.

NOMENCLATURE

C	equivalent lengths for minor loss coefficient
D	pipe diameter
f	Darcy friction factor
f_T	fully-rough friction factor
g	acceleration of gravity
g_c	conversion factor (English Engineering units), 32.174 ft-lbm/lbf-sec^2
h_d	head change due to a pump, turbine, or other active device
K	minor loss coefficient expressed as a number
L	pipe length
N	number of pipes, connection matrix
P	pressure
Q	flow rate
Re	Reynolds number, VD/ν
V	velocity
W_s	pump increase in head
o	elevation
γ	specific weight, ρg
ε	absolute roughness of pipe
μ	viscosity
ν	kinematic viscosity, μ/ρ
ρ	density

Subscripts

a	upstream location
b	downstream location
elbow	elbow
ent	entrance
exp	expansion
gv	gate valve
I	arbitrary pipe in a pipe network
i	counter
1	pipe 1
2	pipe 2
3	pipe 3

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BIOGRAPHIES

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