Reference:C:\Users\franc_000\Desktop\M15 Worksheets\global data.xmcd

Determinare tensioni e correnti istantanee nella seguente rete elettrica eccitata da una sorgente di corrente con andamento temporale a dente di sega bipolare simmetrico con frequenza 1MHz, valore della corrente massima di 5A e duty cycle del 30% .

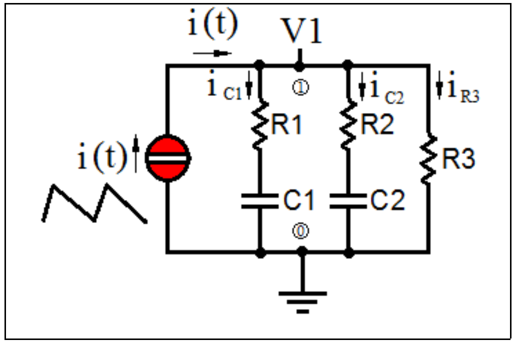


Fig.: 1)

Valori dei componenti passivi e reattivi:

$R1 := 1 \cdot k\Omega$ $\quad C1 := 22 \cdot nF$

$R2 := 2 \cdot k\Omega$ $\quad C2 := 33 \cdot nF$

$R3 := 3 \cdot k\Omega$

Valori picco-picco della corrente:..... $I_{pp} := 5.0 \cdot A$

Periodo del dente di sega periodico...........: $T := 1.0 \cdot \mu s$

Pulsazione del segnale..............................: $\omega_0 := \frac{2 \cdot \pi}{T}$ $\quad \omega_0 = 6.283 \cdot \frac{Mrads}{sec}$

Numero massimo di periodi da visualizzabili: $N := 20$

Duty-cycle...: $\delta_{cycl} := 30\,\%$

Durata della rampa con pendenza positiva : $\tau_{cy} := \delta_{cycl} \cdot T$ $\quad \tau_{cy} = 300 \cdot ns$

Reset simbolico delle costanti: $R1 := R1$ $\quad R2 := R2$

Funzione di eccitazione istantanea a dente di sega con duty-cycle regolabile

$I_{pp} = 5\,A$ $\quad N_{gd} = 50$

Definizione della funzione in un periodo T, dove Φ(t) sia il gradino unitario:

$$s1s2(t, T, \delta_{cycl}, Ampl) := \left[\frac{t \cdot \left(\Phi(t - T \cdot \delta_{cycl}) - \Phi(t) + \delta_{cycl} \cdot \Phi(t) - \delta_{cycl} \cdot \Phi(t - T)\right)}{T \cdot \delta_{cycl}} \ldots + \Phi(t - T) - \Phi(t - T \cdot \delta_{cycl}) \right] \cdot \frac{Ampl}{(\delta_{cycl} - 1)} \qquad 1)$$

Tale funzione è utilizzata per definire il segnale a dente di sega bipolare simmetrico e periodico seguente:

$$I_{s0}(t, T, \delta_{cycl}, I_{pp}, N_{gd}) := \sum_{k=0}^{N_{gd}} s1s2\left[t - (k - 1) \cdot T, T, \delta_{cycl}, I_{pp}\right] - \frac{I_{pp}}{2} \qquad 2)$$

$$I_s(t) := I_{s0}(t, T, \delta_{cycl}, I_{pp}, N_{gd}) \qquad 3)$$

$I_{pp} = 5\,A$ $\quad t := 0 \cdot T, 0 \cdot T + \frac{N \cdot T}{20000} .. N \cdot T$

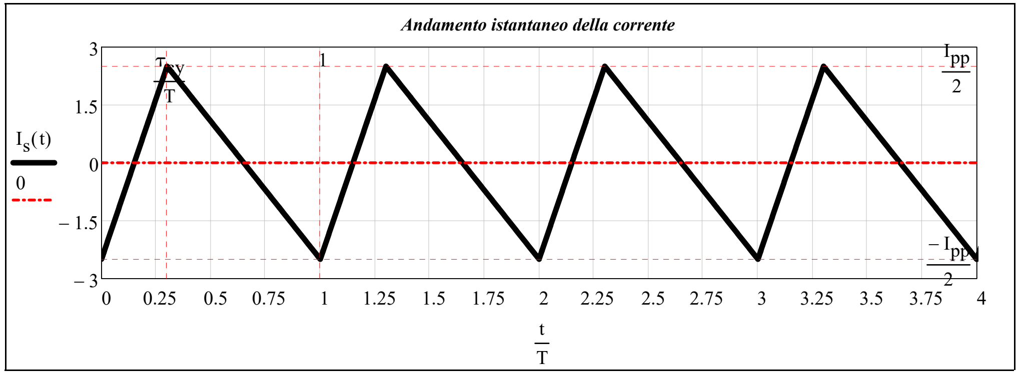


Fig.:2)

$N = 20$

Trasformata di Laplace del segnale:

$$\mathcal{L}\left\{ s1s2(t, T, \delta_{cycl}, Ampl) \right\} = \left[\frac{Ampl}{(\delta_{cycl} - 1)} \cdot \mathcal{L}\left\{ \left[\frac{t \cdot \left(\Phi(t - T \cdot \delta_{cycl}) - \Phi(t) + \delta_{cycl} \cdot \Phi(t) - \delta_{cycl} \cdot \Phi(t - T)\right)}{T \cdot \delta_{cycl}} \ldots + \Phi(t - T) - \Phi(t - T \cdot \delta_{cycl}) \right. \right. \right. ..$$

$$\mathcal{L}\left\{ \left[\frac{t \cdot \left(\Phi(t - T \cdot \delta_{cycl}) - \Phi(t) + \delta_{cycl} \cdot \Phi(t) - \delta_{cycl} \cdot \Phi(t - T)\right)}{T \cdot \delta_{cycl}} \ldots + \Phi(t - T) - \Phi(t - T \cdot \delta_{cycl}) \right] \right. = \frac{1}{(T \cdot \delta_{cycl})} \cdot \left[\mathcal{L}\left\{ t \cdot \Phi(t - T \cdot \delta_{cycl}) \right\} \ldots + \mathcal{L}\left\{ -t \cdot \Phi(t) \right\} \ldots + \delta_{cycl} \cdot \mathcal{L}\left\{ (t \cdot \Phi(t)) \right\} \ldots + \delta_{cycl} \cdot \mathcal{L}\left\{ t \cdot \Phi(t - T) \right\} \right] .. + \mathcal{L}\left\{ \Phi(t - T) \right\} \ldots + \mathcal{L}\left\{ \Phi(t - T \cdot \delta_{cycl}) \right\}$$

$T := T$ $\quad \delta_{cycl} := \delta_{cycl}$ $\quad t := t$

$$\mathcal{L}\left\{ t \cdot \Phi(t - T \cdot \delta_{cycl}) \right\} = \frac{e^{-T \cdot s \cdot \delta_{cycl}} \cdot (T \cdot s \cdot \delta_{cycl} + 1)}{s^2}$$

$$\mathcal{L}\left\{-t\cdot\Phi(t)\right\} = \frac{-1}{s^2} \qquad \mathcal{L}\left\{t\cdot\Phi(t)\right\} = \frac{1}{s^2}$$

$$\mathcal{L}\left\{t\cdot\Phi(t-T)\right\} = \frac{e^{-T\cdot s}\cdot(T\cdot s+1)}{s^2}$$

$$\mathcal{L}\left\{\Phi(t-T)\right\} = \frac{e^{-T\cdot s}}{s}$$

$$\mathcal{L}\left\{\Phi\left(t-T\cdot\delta_{cycl}\right)\right\} = \frac{e^{-T\cdot s\cdot\delta_{cycl}}}{s}$$

$$\frac{1}{\left(T\cdot\delta_{cycl}\right)}\cdot\left[\begin{array}{l}\mathcal{L}\left\{t\cdot\Phi\left(t-T\cdot\delta_{cycl}\right)\right\}\ \ldots\\ +\mathcal{L}\left\{-t\cdot\Phi(t)\right\}\ \ldots\\ +\delta_{cycl}\cdot\mathcal{L}\left\{(t\cdot\Phi(t))\right\}\ \ldots\\ +\delta_{cycl}\cdot\mathcal{L}\left\{t\cdot\Phi(t-T)\right\}\end{array}\right]\ \ldots \\ +\mathcal{L}\left\{\Phi(t-T)\right\}\ \ldots \\ +\mathcal{L}\left\{\Phi\left(t-T\cdot\delta_{cycl}\right)\right\} = \frac{1}{\left(T\cdot\delta_{cycl}\right)}\cdot\left[\begin{array}{l}\frac{e^{-T\cdot s\cdot\delta_{cycl}}\cdot\left(T\cdot s\cdot\delta_{cycl}+1\right)}{s^2}\ \ldots\\ +\frac{-1}{s^2}\ \ldots\\ +\delta_{cycl}\cdot\frac{1}{s^2}\ \ldots\\ +\delta_{cycl}\cdot\frac{e^{-T\cdot s}\cdot(T\cdot s+1)}{s^2}\end{array}\right]\ \ldots \\ +\frac{e^{-T\cdot s}}{s}\ \ldots \\ +\frac{e^{-T\cdot s\cdot\delta_{cycl}}}{s}$$

$$\mathcal{L}\left\{s1s2\right\} = \frac{\delta_{cycl}+2\cdot T\cdot s\cdot\delta_{cycl}}{T\cdot s^2\cdot\delta_{cycl}}\cdot e^{-T\cdot s} + \frac{\delta_{cycl}-1}{T\cdot s^2\cdot\delta_{cycl}} + \frac{e^{-T\cdot s\cdot\delta_{cycl}}\cdot\left(2\cdot T\cdot s\cdot\delta_{cycl}+1\right)}{T\cdot s^2\cdot\delta_{cycl}}$$

$$\mathcal{L}\left\{\left[\sum_{k=0}^{N_{gd}} s1s2\left[t-(k-1)\cdot T, T, \delta_{cycl}, I_{pp}\right] - \frac{I_{pp}}{2}\right]\right\} = \sum_{k=0}^{N_{gd}} \mathcal{L}\left\{s1s2\left[t-(k-1)\cdot T, T, \delta_{cycl}, I_{pp}\right]\right\} - \frac{I_{pp}}{2\cdot s}$$

$$\sum_{k=0}^{N_{gd}} \mathcal{L}\left\{s1s2\right\} = \sum_{k=0}^{N_{gd}} \mathcal{L}\left\{s1s2\right\}$$

Prima di determinare la risposta del sistema a tale eccitazione, calcolo l'impedenza del circuito ai morsetti.

Impedenza tra i nodi 1-0

Reset delle costanti $R1 := R1$ $R2 := R2$ $R3 := R3$ $C1 := C1$ $C2 := C2$ $s := s$ $t := t$

Considero il sistema trasformato secondo Laplace.

$$Z(s) = R3 \,||\, \left[\left(R1+\frac{1}{s\cdot C1}\right) \,||\, \left(R2+\frac{1}{s\cdot C2}\right)\right] \text{ simplify } \rightarrow Z(s) = \frac{R3\cdot(C1\cdot R1\cdot s}{C1\cdot R1\cdot s + C1\cdot R3\cdot s + C2\cdot R2\cdot s + C2\cdot R3\cdot s + C1\cdot C}$$

Impedenza tra i nodi 1 e 0:

$$Z(s) = \frac{R3\cdot(C1\cdot R1\cdot s+1)\cdot(C2\cdot R2\cdot s+1)}{C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot R3)\cdot s^2 + [C1\cdot(R1+R3)+C2\cdot(R2+R3)]\cdot s+1} \qquad 4)$$

Rielaborazione della impedenza:

$$Z(s) = \frac{R2\cdot R1\cdot R3}{(R1\cdot R2+R1\cdot R3+R2\cdot R3)}\cdot\frac{\left(s+\frac{1}{R1\cdot C1}\right)\cdot\left(s+\frac{1}{R2\cdot C2}\right)}{s^2+\frac{[C1\cdot(R1+R3)+C2\cdot(R2+R3)]}{[C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot R3)]}\cdot s+\frac{1}{[C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot F}}$$

in essa si individuano le seguenti costanti:

1) $Z_0 := \frac{R2\cdot R1\cdot R3}{(R1\cdot R2+R1\cdot R3+R2\cdot R3)}$ $\qquad Z_0 = 545.455\,\Omega$ 5)

e le seguenti pulsazioni:

2) $\omega_1 := \frac{1}{C1\cdot R1}$ 6)

3) $\omega_2 := \frac{1}{C2\cdot R2}$ 7)

4) $\omega_3 := \frac{1}{\sqrt{C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot R3)}}$ 8)

5) $\omega_4 := \frac{1}{C1\cdot(R1+R3)}$ 9)

6) $\omega_5 := \frac{1}{C2\cdot(R2+R3)}$ 10)

Sostituendo tali pulsazioni e Z_0 nella formula della impedenza, si ha:

$$\frac{C1\cdot(R1+R3)+C2\cdot(R2+R3)}{C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot R3)} = \omega_3^2\cdot\left(\frac{1}{\omega_4}+\frac{1}{\omega_5}\right) = \frac{\omega_3^2\cdot(\omega_4+\omega_5)}{\omega_4\cdot\omega_5} = \omega_{345} \qquad 11)$$

$$\omega_{345} := \frac{\omega_3^{\,2}\cdot(\omega_4+\omega_5)}{\omega_4\cdot\omega_5} \qquad 12)$$

$\omega_1 = 0.045\cdot\frac{Mrads}{sec}$ $\quad \omega_2 = 0.015\cdot\frac{Mrads}{sec}$ $\quad \omega_3 = 0.011\cdot\frac{Mrads}{sec}$ $\quad \omega_{345} = 0.032\cdot\frac{Mrads}{sec}$

L'imtedenza quindi assume la forma: $$Z(s) := Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{s^2+\omega_{345}\cdot s+\omega_3^{\,2}} \qquad 13)$$

Poli e zeri di Z(s)

$z_0 := -\omega_1 \qquad z_1 := -\omega_2$

$z_0 = -45.455\cdot\frac{krads}{sec} \qquad z_1 = -15.152\cdot\frac{krads}{sec}$

Reset delle costanti simboliche:

$\omega := \omega \quad \omega_1 := \omega_1 \quad \omega_2 := \omega_2 \quad \omega_{345} := \omega_{345} \quad \omega_3 := \omega_3 \quad \omega_0 := \omega_0$

Calcolo dei poli di: $$Z(s) = Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{s^2+\omega_{345}\cdot s+\omega_3^{\,2}} \qquad 14)$$

Poli: $$p_0 := \frac{1}{2}\cdot\left(\sqrt{\omega_{345}^{\,2}-4\cdot\omega_3^{\,2}}-\omega_{345}\right) \qquad p_1 := \frac{-1}{2}\cdot\left(\omega_{345}+\sqrt{\omega_{345}^{\,2}-4\cdot\omega_3^{\,2}}\right) \qquad 15)$$

$$p_0 = -4.629\cdot\frac{krads}{sec} \qquad p_1 = -27.052\cdot\frac{krads}{sec} \qquad 16)$$

$$\omega_{p0} := |p_0| \qquad \omega_{p1} := |p_1| \qquad 17)$$

$$\omega_{p0} = 4.629\cdot\frac{krads}{sec} \qquad \omega_{p1} = 27.052\cdot\frac{krads}{sec} \qquad 18)$$

Media geometrica: $$\omega_{p0p1} := \sqrt{\omega_{p0}\cdot\omega_{p1}} \qquad 19)$$

$$\omega_{p0p1} = 11.19\cdot\frac{krads}{sec} \qquad 20)$$

Quindi si può scrivere: $$Z(s) := Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}$$

Diagrammi del modulo e della fase di Z(s)

$\frac{\omega_3}{\omega_1} = 0.246 \qquad \omega := \frac{\omega_3}{100},\frac{\omega_3}{100}+\frac{\omega_1\cdot 100-\frac{\omega_3}{100}}{100000}\,..\,\omega_1\cdot 100 \qquad \frac{\omega_{p0p1}}{\omega_1} = 0.246$

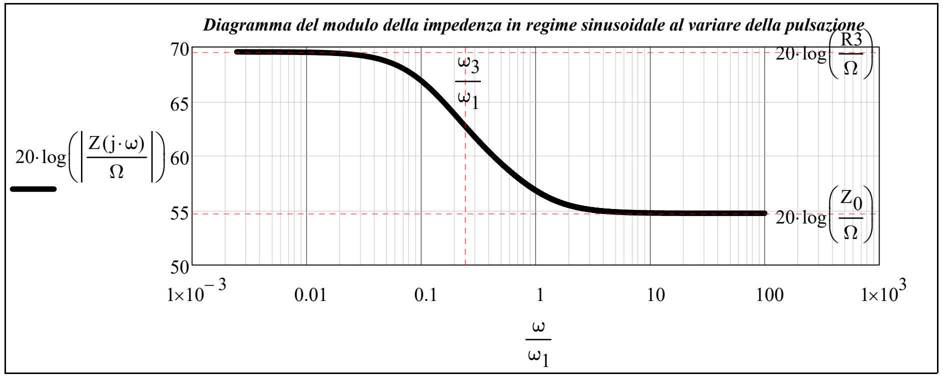


Fig.:3)

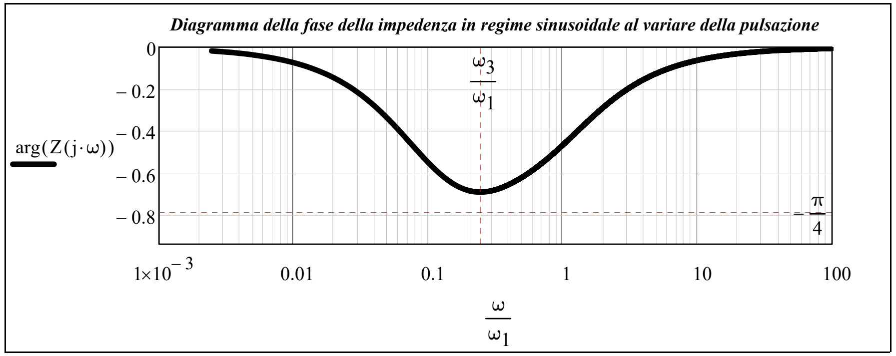


Fig.:4)

Calcolo delle tensioni e correnti istantanee nella rete

Data l'impedenza del bipolo equivalente: $$Z(s) = Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)} \qquad 21)$$

La trasformata di Laplace della tensione ai morsetti di Z(s), noto la trasformata di Laplace della corrente, è dato dalla legge di Ohm generalizzata:

$$V1(s) = I(s)\cdot Z(s) \qquad 22)$$

per cui la trasformata di Laplace della corrente che fluisce in C1 è:

$$V1(s) = I(s)\cdot Z(s) = I(s)\cdot\left[Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}\right] \qquad 23)$$

così pure le trasformate di Laplace della correnti nei dispositivi passivi presenti nella rete, sono dati da:

La trasformata di Laplace della corrente in C1:

$$I_{C1}(s) = \frac{I(s)\cdot Z(s)}{R1+\frac{1}{s\cdot C1}} = \frac{V1(s)}{R1+\frac{1}{s\cdot C1}} \qquad 24)$$

sostituzione della 23) nella 24):

$$I_{C1}(s) = \frac{I(s)}{\frac{R1}{s}\cdot(s+\omega_1)}\cdot Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)} = \frac{I(s)}{R1}\cdot Z_0\cdot\frac{s\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)} \qquad 25)$$

La trasformata di Laplace della corrente in C2:

$$I_{C2}(s) = \frac{I(s)\cdot Z(s)}{R2+\frac{1}{s\cdot C2}} = \frac{V1(s)}{R2+\frac{1}{s\cdot C2}} \qquad 26)$$

sostituzione della 23) nella 26):

$$I_{C2}(s) = \frac{I(s)\cdot Z(s)}{R2+\frac{1}{s\cdot C2}} = \frac{I(s)}{\frac{R2}{s}\cdot(s+\omega_2)}\cdot\left[Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}\right] = \frac{I(s)}{R2}\cdot Z_0\cdot\frac{s\cdot(s+\omega_1)}{(s-p_0)\cdot(s-p_1)} \qquad 27)$$

La trasformata di Laplace della corrente in R3: $I3(s) = \frac{I(s)\cdot Z(s)}{R3} = \frac{V1(s)}{R3}$ 28)

sostituzione della 23) nella 28): $I3(s) = \frac{I(s)}{R3}\cdot\left[Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}\right]$ 29)

il fasore della corrente prodotta dal generatore è datodalla somma dei tre fasori:

$$I(s) = \mathscr{L}\left(I\_source(t)\right) = I_{C1}(s) + I_{C2}(s) + I3(s) \qquad 30)$$

Calcolo

$$V1(s) = I(s)\cdot Z(s) = I(s)\cdot Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}$$

$$V1(s) = I(s)\cdot Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)} \qquad W1(s) = Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}$$

$$W1(s) = Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)} = Z_0\cdot\left[-\frac{(\omega_1+p_1)\cdot(\omega_2+p_1)}{(p_0-p_1)\cdot(s-p_1)} + \frac{(\omega_1+p_0)\cdot(\omega_2+p_0)}{(p_0-p_1)\cdot(s-p_0)} + 1\right]$$

$$W1(s) := Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{(s-p_0)\cdot(s-p_1)}$$

$$V1(j\cdot\omega) = Z_0\cdot I(j\cdot\omega)\cdot\frac{(j\cdot\omega+\omega_1)\cdot(j\cdot\omega+\omega_2)}{(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2}$$

$$\left|\frac{(j\cdot\omega+\omega_1)\cdot(j\cdot\omega+\omega_2)}{(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2}\right| = \frac{\sqrt{\omega_1^2+\omega^2}\cdot\sqrt{\omega_2^2+\omega^2}}{\sqrt{\left(\omega_3^2-\omega^2\right)^2+\left(\omega_{345}\cdot\omega\right)^2}}$$

$$\lim_{\omega\to\omega_0}\frac{\sqrt{\omega_1^2+\omega^2}\cdot\sqrt{\omega_2^2+\omega^2}}{\sqrt{\left(\omega_3^2-\omega^2\right)^2+\left(\omega_{345}\cdot\omega\right)^2}} \to \frac{\sqrt{\omega_0^2+\omega_1^2}\cdot\sqrt{\omega_0^2+\omega_2^2}}{\sqrt{\omega_0^4-2\cdot\omega_0^2\cdot\omega_3^2+\omega_0^2\cdot\omega_{345}^2+\omega_3^4}}$$

$$\frac{\sqrt{\omega_1^2+\omega_0^2}\cdot\sqrt{\omega_2^2+\omega_0^2}}{\sqrt{\left(\omega_3^2-\omega_0^2\right)^2+\left(\omega_{345}\cdot\omega_0\right)^2}} = 1$$

$$\varphi_1(\omega) := \operatorname{atan}\left(\frac{\omega}{\omega_1}\right) + \operatorname{atan}\left(\frac{\omega}{\omega_2}\right) - \left(\operatorname{atan}\left(-\frac{\omega}{p_0}\right) + \operatorname{atan}\left(-\frac{\omega}{p_1}\right)\right)$$

$$V1(\omega) = Z_0\cdot\left|I(j\cdot\omega)\right|\cdot\frac{\sqrt{\omega_1^2+\omega^2}\cdot\sqrt{\omega_2^2+\omega^2}}{\sqrt{\left(\omega_3^2-\omega^2\right)^2+\left(\omega_{345}\cdot\omega\right)^2}}\cdot e^{j\cdot(\varphi_1(\omega)+\varphi_s(\omega))}$$

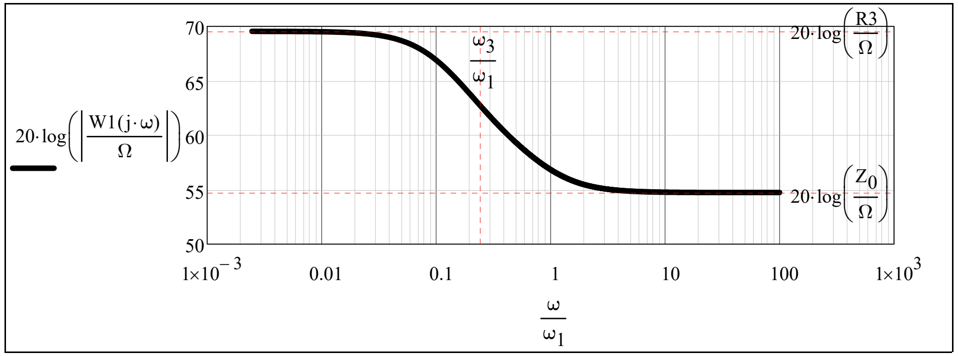


Fig.:5

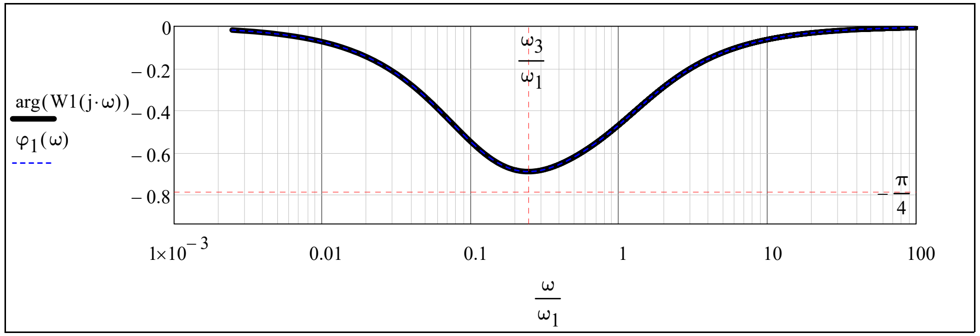


Fig.:6)

Calculation of the voltage time profile as the inverse Laplace transform of V1(s)

$$s := s \quad \omega_1 := \omega_1 \quad \omega_2 := \omega_2 \quad \omega_3 := \omega_3 \quad \omega_{345} := \omega_{345}$$

calcolo

$$a := \frac{\omega_{345}}{2} \qquad b := \frac{\sqrt{\omega_{345}^2 - 4\cdot\omega_3^2}}{2} \qquad c_0 := \left(2\cdot\omega_3^2 - \omega_{345}^2 - 2\cdot\omega_1\cdot\omega_2 + \omega_1\cdot\omega_{345} + \omega_2\cdot\omega_{345}\right)$$

$$d := \left(\omega_1 + \omega_2 - \omega_{345}\right) \qquad e_0 := \frac{c_0}{2\cdot b\cdot d} \qquad e_0 = -0.325$$

$$a = 0.016\cdot\frac{\text{Mrads}}{\text{sec}} \qquad b = 0.011\cdot\frac{\text{Mrads}}{\text{sec}} \qquad c_0 = -210.596\cdot\frac{\text{Mrads}}{\text{sec}^2} \qquad d = 0.029\cdot\frac{\text{Mrads}}{\text{sec}} \qquad e_0 = -0.325$$

$$w_1(t) := \Delta(t) + \left(1 + e_0\right)\cdot e^{-a\cdot t}\cdot\left[e^{-b\cdot t} + \left[\frac{1 - e_0}{\left(1 + e_0\right)}\right]\cdot e^{b\cdot t}\right]\cdot\frac{d}{2}$$

$$t := 0, \frac{1000\cdot T}{10^5} .. 1000\cdot T$$

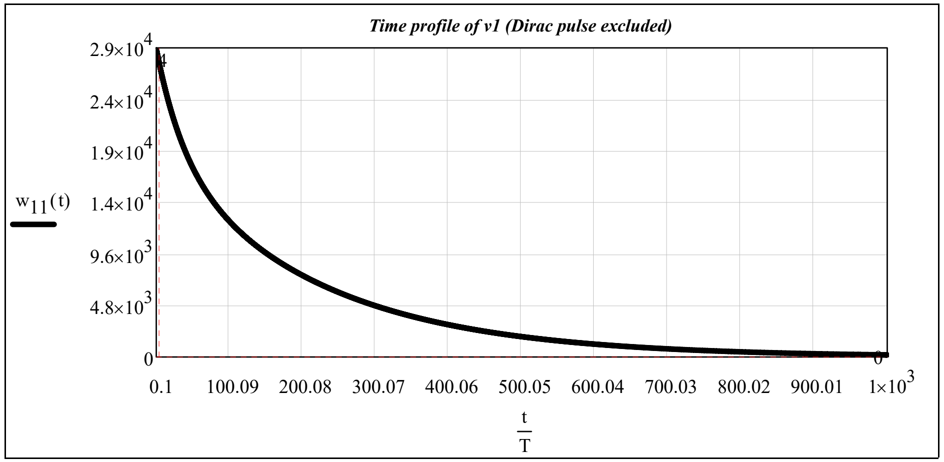


Fig.:7)

$$v1(t) = Z_0\cdot\mathcal{L}^{-1}\left[I(s)\cdot\frac{\left(s + \omega_1\right)\cdot\left(s + \omega_2\right)}{s^2 + \omega_{345}\cdot s + \omega_3^2}\right]$$

$$v1(t) := Z_0\cdot\int_0^t I_s(t - \tau)\cdot w_1(\tau)\, d\tau$$

$$Z_0\cdot\int_0^t I_s(t - \tau)\cdot w_1(\tau)\, d\tau = Z_0\cdot\int_0^t I_s(\tau)\cdot w_1(t - \tau)\, d\tau$$

$$t := 0, \frac{8\cdot T}{10^2} .. 8\cdot T$$

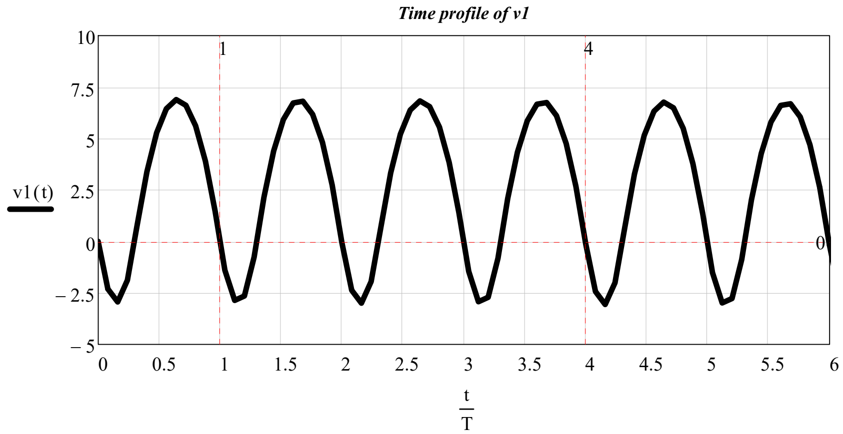


Fig.:8

Calculation of I_{C1}

$$Z(s) = Z_0\cdot\frac{\left(s + \omega_1\right)\cdot\left(s + \omega_2\right)}{s^2 + \omega_{345}\cdot s + \omega_3^2} \quad V1(s) = I(s)\cdot Z(s) \quad I(s) = \mathcal{L}(\text{I\_source}(t))$$

$$I_{C1}(s) = \frac{I(s)}{R1}\cdot Z_0\cdot\frac{s\cdot\left(s + \omega_2\right)}{s^2 + \omega_{345}\cdot s + \omega_3^2} \qquad K11 = \frac{Z_0}{R1}$$

$$K11 := \frac{R2\cdot R3}{R1\cdot R2 + R1\cdot R3 + R2\cdot R3} \qquad W(s) := K11\cdot\frac{s\cdot\left(s + \omega_2\right)}{\left(s^2 + \omega_{345}\cdot s + \omega_3^2\right)}$$

$K11 = 0.5\,\omega_1 = 0.045\cdot\frac{Mrads}{sec}$ $\quad \omega_2 = 0.015\cdot\frac{Mrads}{sec}$ $\quad \omega_3 = 0.011\cdot\frac{Mrads}{sec}$ $\quad \omega_{345} = 0.032\cdot\frac{Mrads}{sec}$

$$I_{C1}(s) = I(s)\cdot K11\cdot\frac{s\cdot(s+\omega_2)}{\left(s^2+\omega_{345}\cdot s+\omega_3^2\right)} = I(s)\cdot W(s)$$

Bode plots of W(s)

$$s^2+\omega_{345}\cdot s+\omega_3^2 \;\Big|\; \text{solve}, s \;\; \text{simplify} \rightarrow \begin{pmatrix} \frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}-\frac{\omega_{345}}{2} \\ -\frac{\omega_{345}}{2}-\frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2} \end{pmatrix}$$

$$p_1 := \frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}-\frac{\omega_{345}}{2} \qquad p_2 := -\frac{\omega_{345}}{2}-\frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}$$

$$p_1 = -4.629\times10^3\,\frac{1}{s} \qquad p_2 = -2.705\times10^4\,\frac{1}{s}$$

$$I_{c1}(\omega) = K11\cdot|I(s)|\cdot\left|\frac{j\cdot\omega\cdot(j\cdot\omega+\omega_2)}{\left[(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2\right]}\right|\cdot e^{j\cdot(\varphi_2(\omega)+\varphi_s(\omega))}$$

$$\left|\frac{j\cdot\omega\cdot(j\cdot\omega+\omega_2)}{\left[(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2\right]}\right| = \frac{\omega\cdot\sqrt{\omega^2+\omega_2^2}}{\sqrt{\left(\omega_3^2-\omega^2\right)^2+\left(\omega_{345}\cdot\omega\right)^2}}$$

$$\frac{\omega_0\cdot\sqrt{\omega_0^2+\omega_2^2}}{\sqrt{\left(\omega_3^2-\omega_0^2\right)^2+\left(\omega_{345}\cdot\omega_0\right)^2}} = 1$$

$$\varphi_2(\omega) := \frac{\pi}{2}+\text{atan}\left(\frac{\omega}{\omega_2}\right)-\left(\text{atan}\left(-\frac{\omega}{p_1}\right)+\text{atan}\left(-\frac{\omega}{p_2}\right)\right) \qquad \varphi_2(\omega_0) = 2.631\times10^{-3}$$

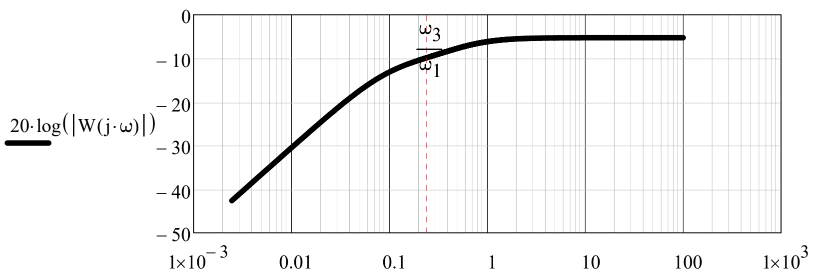


Fig.:9) $\frac{\omega}{\omega_1}$

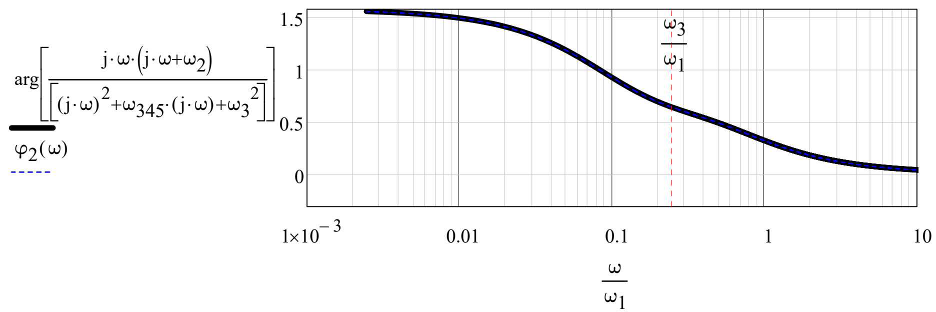


Fig.:10)

Calculation of the current I_{C1} time profile as the inverse Laplace transform of $I_{C1}(s)$

$$s := s \quad \omega_2 := \omega_2 \quad \omega_3 := \omega_3 \quad \omega_{345} := \omega_{345}$$

$$\frac{s\cdot(s+\omega_2)}{\left(s^2+\omega_{345}\cdot s+\omega_3^2\right)} \;\text{invlaplace}, s \rightarrow$$

$$w(t) := \Delta(t)+e^{-\frac{t\cdot\omega_{345}}{2}}\cdot(\omega_2-\omega_{345})\cdot\left[\cosh\left(\frac{t\cdot\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}\right)-\frac{\sinh\left(\frac{t\cdot\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}\right)\cdot\left(2\cdot\omega_3^2-\omega_{345}^2\cdot\right.}{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}\cdot(\omega_2-\omega_{345})}\right.$$

$$i_{C1}(t) := K11\cdot\int_0^t I_s(t-\tau)\cdot w(\tau)\,d\tau$$

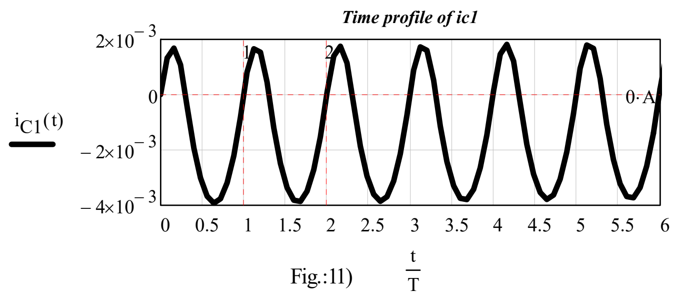


Fig.:11)

$$W(s) = \frac{R2\cdot R3}{R1\cdot R2+R1\cdot R3+R2\cdot R3}\cdot s\cdot\frac{\left[s+\frac{1}{(C2\cdot R2)}\right]}{\left[s^2+\left[\frac{C1\cdot(R1+R3)+C2\cdot(R2+R3)}{C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2\cdot R3)}\right]\cdot s+\frac{1}{[C1\cdot C2\cdot(R1\cdot R2+R1\cdot R3+R2}\right.}$$

$$W(s) = K11\cdot\frac{s\cdot(s+\omega_2)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}$$

$$K11 := \frac{R2\cdot R3}{R1\cdot R2+R1\cdot R3+R2\cdot R3} \qquad I_{C1}(s) = I(s)\cdot W(s) = I(s)\cdot K11\cdot\frac{s\cdot(s+\omega_2)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}$$

$$w(t) = \mathscr{L}^{-1}\left[\frac{s\cdot(s+\omega_2)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}\right]$$

$$W(s) = K11\cdot\mathscr{L}(w(t))$$

$$i_{C1}(t) = K11\cdot\mathscr{L}^{-1}\left[I(s)\cdot\frac{s\cdot(s+\omega_2)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}\right] = K11\cdot\int_0^t I_s(\tau)\cdot w(t-\tau)\,d\tau$$

Calculation of I_{C2}

$$I_{C2}(s) = \frac{I(s)\cdot Z(s)}{R2+\frac{1}{s\cdot C2}} \qquad Z(s) = Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{s^2+\omega_{345}\cdot s+\omega_3^2} \qquad Z_0 = \frac{R2\cdot R1\cdot R3}{(R1\cdot R2+R1\cdot R3+R2\cdot R3)}$$

$$I_{C2}(s) = \frac{I(s)}{R2}\cdot Z_0\cdot\frac{s\cdot(s+\omega_1)}{s^2+\omega_{345}\cdot s+\omega_3^2} \qquad K2 := \frac{R1\cdot R3}{(R1\cdot R2+R1\cdot R3+R2\cdot R3)}$$

$K2 = 0.27$ $\quad \omega_1 = 4.545\times10^4\cdot\frac{rad}{sec}$ $\quad \omega_2 = 1.515\times10^4\,\frac{1}{s}$ $\quad \omega_3 = 1.119\times10^4\cdot\frac{rad}{sec}$ $\quad \omega_{345} = 3.168\times10^4\cdot\frac{rad}{sec}$

$$I_{C2}(s) = I(s)\cdot\frac{K2\cdot s\cdot(s+\omega_1)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}$$

Bode plots of W2(s)

$$W2(s) := \frac{K2\cdot s\cdot(s+\omega_1)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}$$

$$p_1 := \frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}-\frac{\omega_{345}}{2} \qquad p_2 := -\frac{\omega_{345}}{2}-\frac{\sqrt{\omega_{345}^2-4\cdot\omega_3^2}}{2}$$

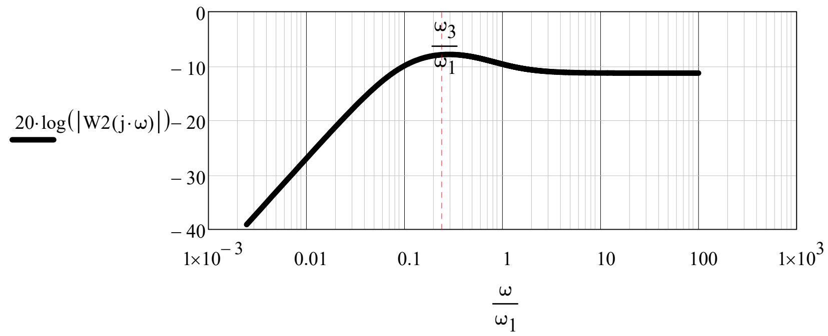
$$p_1 = -4.629\times10^3\,\frac{1}{s} \qquad p_2 = -2.705\times10^4\,\frac{1}{s}$$

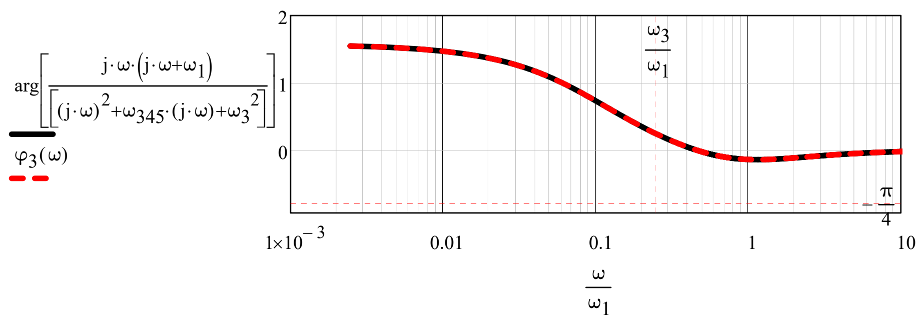
$$I_{c2}(\omega) = K2\cdot|I(s)|\cdot\left|\frac{j\cdot\omega\cdot(j\cdot\omega+\omega_1)}{\left[(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2\right]}\right|\cdot e^{j\cdot(\varphi_2(\omega)+\varphi_s(\omega))}$$

$$\left|\frac{j\cdot\omega\cdot(j\cdot\omega+\omega_1)}{\left[(j\cdot\omega)^2+\omega_{345}\cdot(j\cdot\omega)+\omega_3^2\right]}\right| = \frac{\omega\cdot\sqrt{\omega^2+\omega_1^2}}{\sqrt{(\omega_3^2-\omega^2)^2+(\omega_{345}\cdot\omega)^2}}$$

$$\frac{\omega_0\cdot\sqrt{\omega_0^2+\omega_1^2}}{\sqrt{(\omega_3^2-\omega_0^2)^2+(\omega_{345}\cdot\omega_0)^2}} = 1$$

$$\varphi_3(\omega) := \frac{\pi}{2}+\operatorname{atan}\left(\frac{\omega}{\omega_1}\right)-\left(\operatorname{atan}\left(-\frac{\omega}{p_1}\right)+\operatorname{atan}\left(-\frac{\omega}{p_2}\right)\right) \qquad \varphi_2(\omega_0) = 2.631\times10^{-3}$$





Calculation of the current I_{C2} time profile as the inverse Laplace transform of $I_{C2}(s)$

$s := s \quad \omega_1 := \omega_1 \quad \omega_2 := \omega_2 \quad \omega_3 := \omega_3 \quad \omega_{345} := \omega_{345}$

$$\frac{s\cdot(s+\omega_1)}{(s^2+\omega_{345}\cdot s+\omega_3^2)}\ \text{invlaplace} \rightarrow$$

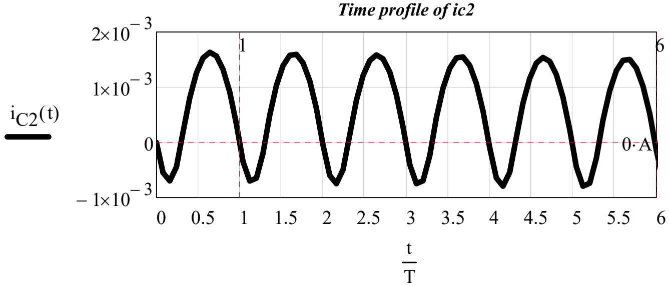
$\omega_6 := \omega_1 - \omega_{345}$ $\qquad \omega_7 := \sqrt{\omega_{345}^2 - 4\cdot\omega_3^2}$ $\qquad \omega_8 := \sqrt{2\cdot\omega_3^2 - \omega_{345}^2 + \omega_1\cdot\omega_{345}}$

$$w_2(t) := \Delta(t) + e^{-\frac{t\cdot\omega_{345}}{2}}\cdot\omega_6\cdot\left(\cosh\left(\frac{t\cdot\omega_7}{2}\right) - \frac{\sinh\left(\frac{t\cdot\omega_7}{2}\right)\cdot\omega_8^2}{\omega_7\cdot\omega_6}\right)$$

$$i_{C2}(t) = \mathscr{L}^{-1}\left(I_{C2}(s)\right)$$

$$i_{C2}(t) := K2\cdot\int_0^t I_s(t-\tau)\cdot w_2(\tau)\,d\tau$$

Time profile of ic2



Calculation of I3

$$I3(s) = \frac{I(s)}{R3}\cdot Z_0\cdot\frac{(s+\omega_1)\cdot(s+\omega_2)}{s^2+\omega_{345}\cdot s+\omega_3^2} \qquad Z_0 = \frac{R2\cdot R1\cdot R3}{(R1\cdot R2 + R1\cdot R3 + R2\cdot]}$$

$$\frac{Z_0}{R3} = \frac{R2\cdot R1}{(R1\cdot R2 + R1\cdot R3 + R2\cdot R3)}$$

$$K3 := \frac{R1\cdot R2}{R1\cdot R2 + R1\cdot R3 + R2\cdot R3}$$

Bode plots of W2(s)

$$W3(s) = \frac{K3\cdot(s+\omega_1)\cdot(s+\omega_2)}{\left(s^2+\omega_{345}\cdot s+\omega_3^2\right)}$$

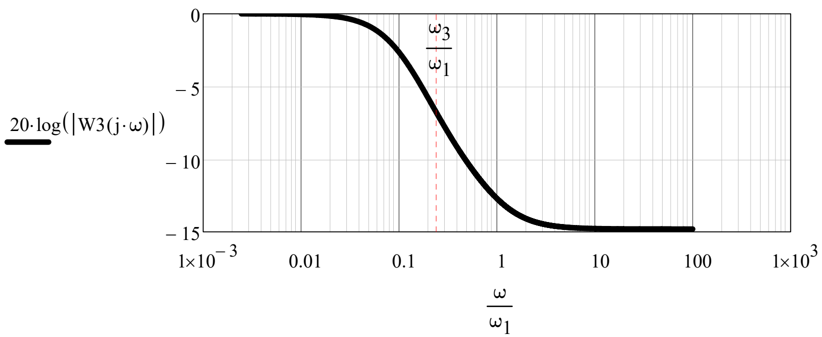
$$p_1 := \frac{\sqrt{\omega_{345}^2 - 4\cdot\omega_3^2}}{2} - \frac{\omega_{345}}{2} \qquad p_2 := -\frac{\omega_{345}}{2} - \frac{\sqrt{\omega_{345}^2 - 4\cdot\omega_3^2}}{2}$$

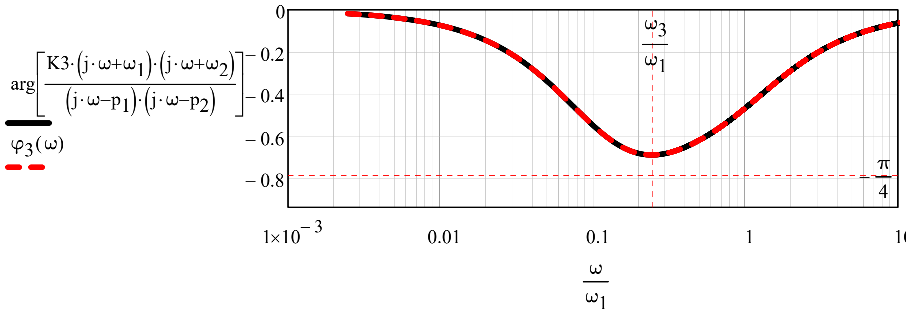
$$p_1 = -4.629\times10^{-3}\cdot\frac{Mrads}{s} \qquad p_2 = -0.027\cdot\frac{Mrads}{s}$$

$$W3(s) := \frac{K3\cdot(s+\omega_1)\cdot(s+\omega_2)}{(s-p_1)\cdot(s-p_2)}$$

$$W3(j\cdot\omega) = \frac{K3\cdot(j\cdot\omega+\omega_1)\cdot(j\cdot\omega+\omega_2)}{(j\cdot\omega-p_1)\cdot(j\cdot\omega-p_2)}$$

$$\varphi_3(\omega) := \operatorname{atan}\left(\frac{\omega}{\omega_1}\right) + \operatorname{atan}\left(\frac{\omega}{\omega_2}\right) - \left(\operatorname{atan}\left(-\frac{\omega}{p_1}\right) + \operatorname{atan}\left(-\frac{\omega}{p_2}\right)\right)$$





Calculation of the current I_{C3} time profile as the inverse Laplace transform of $I_{C3}(s)$

$$\frac{(s+\omega_1)\cdot(s+\omega_2)}{\left(s^2+\omega_{345}\cdot s+\omega_3^2\right)} \text{ invlaplace } \rightarrow$$

$\omega_7 := \sqrt{\omega_{345}^2 - 4\cdot\omega_3^2}$ $\qquad \omega_9 := \sqrt{\omega_{345}\cdot(\omega_1+\omega_2-\omega_{345}) + 2\cdot\omega_3^2 - 2\cdot\omega_1\cdot\omega_2}$

$\omega_{10} := \omega_7\cdot(\omega_1+\omega_2-\omega_{345})$ $\qquad \omega_{11} := \omega_1+\omega_2-\omega_{345}$

$$w_3(t) := K3 \cdot \left[\Delta(t) + e^{-\frac{t \cdot \omega_{345}}{2}} \cdot \left(\cosh\left(\frac{t \cdot \omega_7}{2}\right) - \frac{\sinh\left(\frac{t \cdot \omega_7}{2}\right) \cdot \omega_9^{\,2}}{\omega_{10}} \right) \cdot \omega_{11} \right]$$

$$i_{R3}(t) := K3 \cdot \int_0^t I\_source(t - \tau) \cdot w_3(\tau)\, d\tau$$

$$i_{R3}(t) := I_s(t) - i_{C1}(t) - i_{C2}(t)$$

$$R3 = 3 \times 10^3\, \Omega$$