9/03/21
Question 8 Fourier tutorials
Figure 7

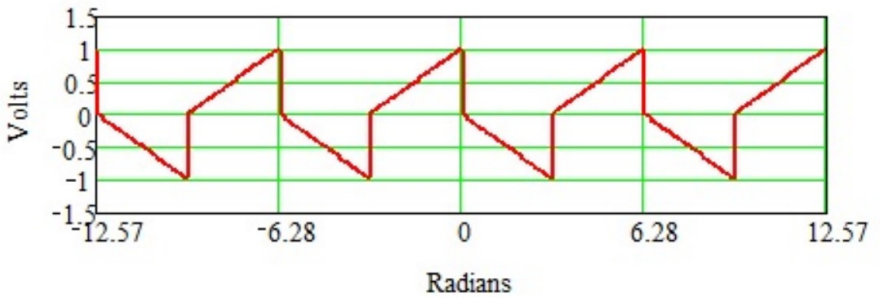

Figure 7 Half Wave Symmetrical Function

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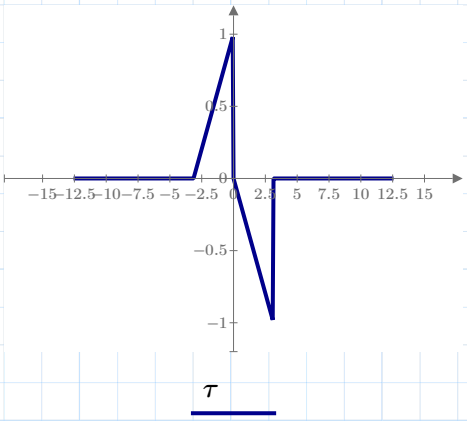
Check odd function for integrals :

$$f(t) := \begin{cases} \text{if } t < -\pi \\ \quad \text{return } 0 \\ \text{if } t < 0 \\ \quad \text{return } \left(\frac{t}{\pi} + 1\right) \\ \text{if } t < \pi \\ \quad \text{return } \frac{-t}{\pi} \\ 0 \end{cases}$$

$$\tau := -4 \cdot \pi, -4 \cdot \pi + \frac{2 \cdot \pi}{100} .. 4 \cdot \pi$$



Calculate Fourier coefficients :

$$P := 2 \cdot \pi$$

$$L := \frac{P}{2}$$

Fourier coefficients

$$a_n = \frac{2}{P} \int_P s(x) \cdot \cos\left(2\pi x \frac{n}{P}\right) dx$$

$$b_n = \frac{2}{P} \int_P s(x) \cdot \sin\left(2\pi x \frac{n}{P}\right) dx.$$

(Eq.1)

$$half\_a_0 := \frac{1}{2}\cdot\frac{2}{P}\cdot\left(\int_{-L}^{0}\left(\frac{t}{\pi}+1\right)\cdot\cos(0\cdot t)\,dt+\int_{0}^{L}\left(\frac{-t}{\pi}\right)\cdot\cos(0\cdot t)\,dt\right)\rightarrow 0$$

$$a(n) := \frac{2}{P}\cdot\left(\int_{-L}^{0}\left(\frac{t}{\pi}+1\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)dt+\int_{0}^{L}\left(\frac{-t}{\pi}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)dt\right)\xrightarrow{assume\,,\,n=integer\,,\,n\neq 0}\frac{-\left(2\cdot(-1)^n\right)+2}{n^2\cdot\pi^2}$$

$$b(n) := \frac{2}{P}\cdot\left(\int_{-L}^{0}\left(\frac{t}{\pi}+1\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)dt+\int_{0}^{L}\left(\frac{-t}{\pi}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)dt\right)\xrightarrow{assume\,,\,n=integer}\frac{(-1)^n-1}{n\cdot\pi}$$

Utilize coefficients to construct time signal :

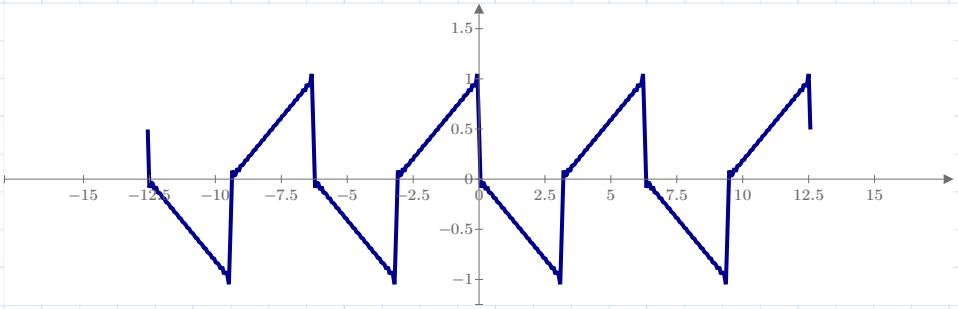
$n := 1\,..\,40$

Fourier series, sine-cosine form

$$s_N(x) = \frac{a_0}{2}+\sum_{n=1}^{N}\left(a_n\cos\left(\frac{2\pi nx}{P}\right)+b_n\sin\left(\frac{2\pi nx}{P}\right)\right). \quad \text{(Eq.2)}$$

$$S_N(t) := half\_a_0+\sum_n a(n)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)+\sum_n b(n)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{P}\cdot t\right)$$

$S_N(\tau)$



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