Second Order ODE that describes the physics: Laplace analysis of RLC electric circuit with using same values.

$$m \cdot \left(\frac{d^2}{d\tau^2} x(\tau)\right) + b \cdot \left(\frac{d}{d\tau} x(\tau)\right) + k \cdot x(\tau) = F_0 \cdot \cos(\omega_F \cdot \tau)$$

Where; $A_0 = \frac{F_0}{m}$ Note; $(\omega_0)^2 = \frac{k}{m}$ $\zeta := 1 \cdot 10^0$

Defining Inputs:

$$L \cdot \left(\frac{d^2}{dt^2} q(t)\right) + R \cdot \left(\frac{d}{dt} q(t)\right) + \frac{1}{C} \cdot q(t) = e(t)$$

$m := 60$ $k := 1 \cdot 10^1$ $\omega_0 := \sqrt{\frac{k}{m}} = 0.408$ $b := \zeta \cdot (2 \cdot m \cdot \omega_0) = 48.99$

$L := m$ $C := \frac{1}{k}$ $R := b$

$\omega_f := \omega_0 \cdot 1.0 = 0.408$ $A_0 := 100$ $init := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $b = 48.99$

$t1 := 0$ $n := 10$ $t2 := n \cdot \left(\frac{2 \cdot \pi}{\omega_f}\right)$ $npoints := 100 \cdot n = 1 \cdot 10^3$ $intvls := npoints - 1 = 999$

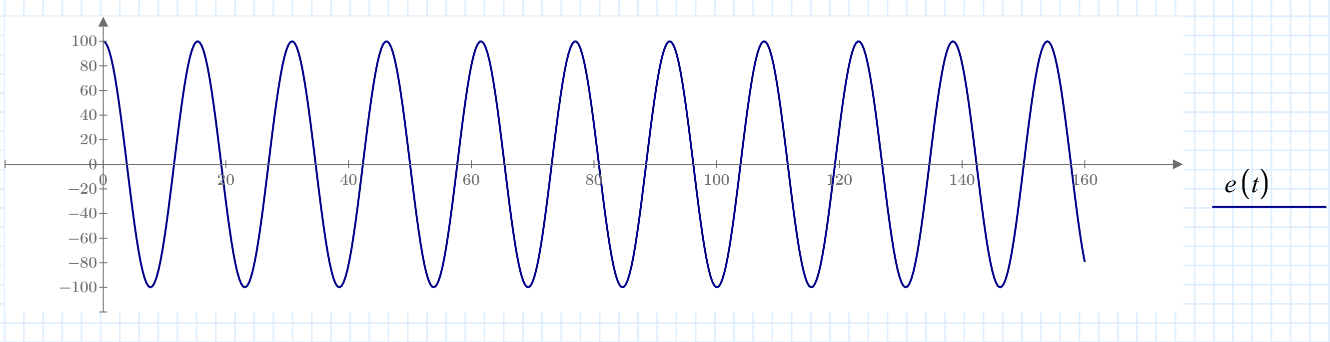
$Fs := \left(\frac{\omega_f}{2 \cdot \pi}\right) = 0.065$ $tmax := t2 = 153.906$ $T := tmax = 153.906$

$tstep := \frac{1}{100 \cdot Fs} = 0.154$ $d := 10\%$ $A := A_0 = 100$

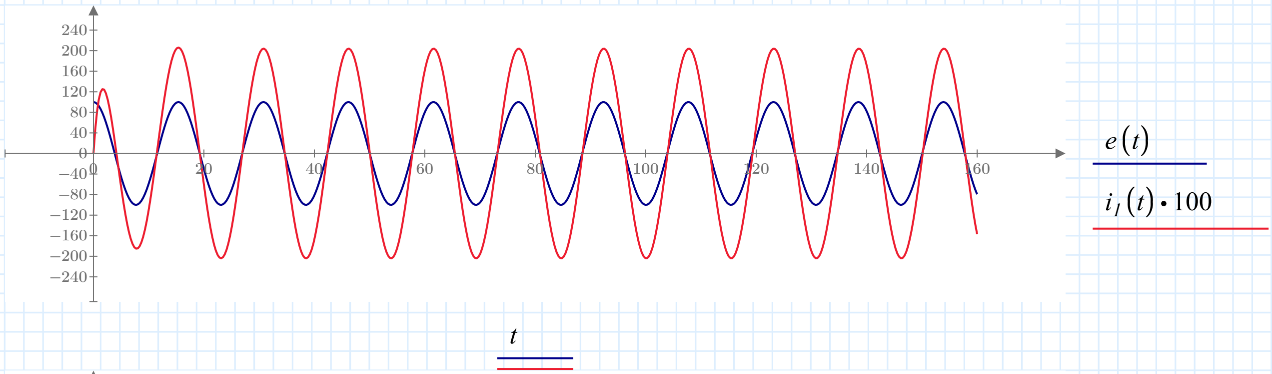
$E_m := A$ $\omega := \omega_f = 0.408$ $f := \frac{\omega}{2 \cdot \pi} = 0.065$ $T := \frac{1}{f} = 15.391$

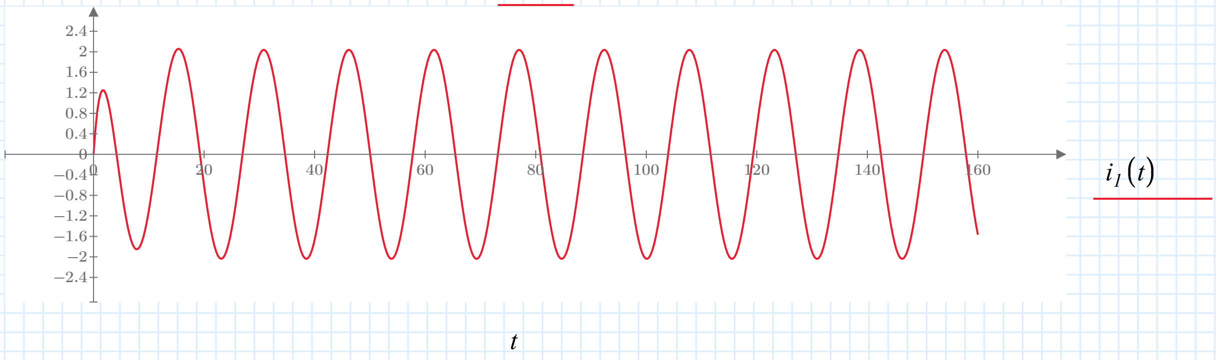
$u1(\tau) := A_0 \cdot \cos(\omega_f \cdot \tau)$ $e(t) := E_m \cdot \cos(\omega \cdot t)$ $T_{0.1} := \frac{T}{10} = 1.539$

$E_1(s) := e(t) \xrightarrow{laplace} \frac{100.0 \cdot s}{s^2 + 0.16666666666666666533}$ $E_1(s) := \frac{E_m \cdot s}{s^2 + \omega^2} \xrightarrow{float, 4} \frac{100.0 \cdot s}{s^2 + 0.1667}$ $\frac{1}{6} = 0.1666667$



$$i_1(t) := \frac{\frac{100 \cdot s}{s^2 + \frac{1}{6}}}{s \cdot L + R + \frac{1}{s \cdot C}} \xrightarrow{simplify,\ invlaplace,\ float, 3} 0.325 \cdot 10^9 \cdot e^{-0.408 \cdot t} \cdot \sin(0.256 \cdot 10^{-8} \cdot t) + (2.04 \cdot \cos(0.408 \cdot t) - 2.04 \cdot \cos(0.256 \cdot 10^{-8} \cdot t) \cdot e^{-0.408 \cdot t})$$

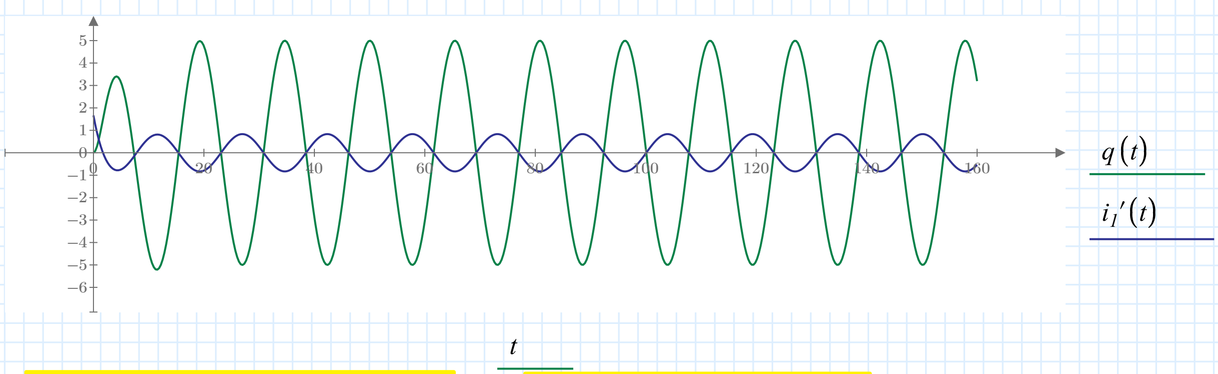




$$i_1(t) := 0.325 \cdot 10^{9} \cdot e^{-0.408 \cdot t} \cdot \sin\left(0.256 \cdot 10^{-8} \cdot t\right) + \left(2.04 \cdot \cos\left(0.408 \cdot t\right) - 2.04 \cdot \cos\left(0.256 \cdot 10^{-8} \cdot t\right) \cdot e^{-0.408 \cdot t}\right)$$

$$q(t) := \int^{t} i_1(t)\, \mathrm{d}t$$

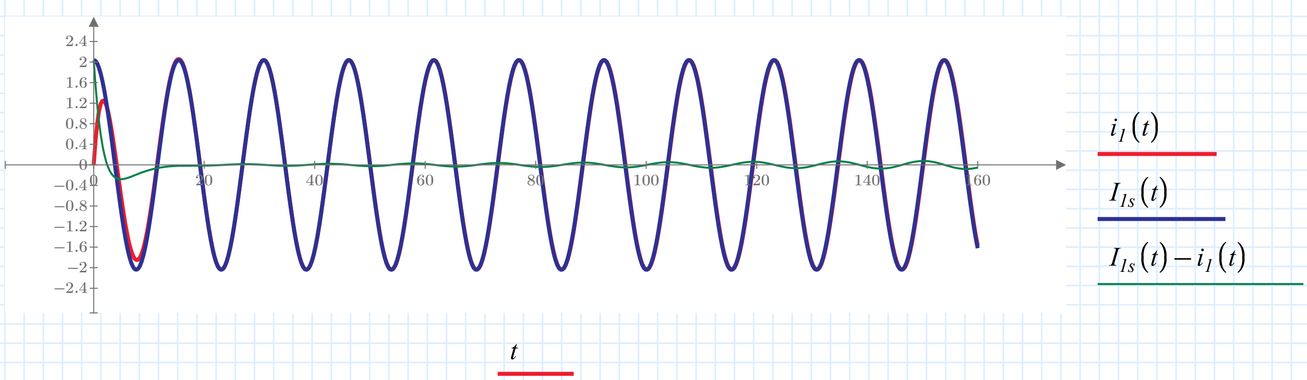
$$i_1'(t) \xrightarrow{float,3} \left(-\left(0.133 \cdot 10^{9}\right) \cdot \sin\left(0.256 \cdot 10^{-8} \cdot t\right) + 1.66 \cdot \cos\left(0.256 \cdot 10^{-8} \cdot t\right)\right) \cdot 2.72^{-0.408 \cdot t} - 0.832 \cdot \sin\left(0.408 \cdot t\right)$$



$$Z := \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2} = 48.99$$

$$\theta_Z := \operatorname{atan}\left(\frac{\omega \cdot L - \frac{1}{\omega \cdot C}}{R}\right) = 0$$

$$I_{1s}(t) := \frac{E_m \cdot \cos\left(\omega \cdot t - \theta_Z\right)}{Z}$$



$$u2(\tau) := A \cdot \left(\operatorname{mod}\left(\tau, \frac{1}{Fs}\right) \cdot Fs \le d\right) \cdot (\tau < tmax)$$

$$E_{21}(t) := E_m \cdot (\Phi(t) - \Phi(t - T_{0.1})) \xrightarrow{laplace} \frac{-100.0 \cdot e^{-1.5390597961942369 \cdot s} + 100.0}{s}$$

$$E_{21}(s) := \frac{E_m}{s} \cdot \left(1 - e^{-(s \cdot T_{0.1})}\right)$$

$$E_2(s) := \frac{\frac{E_m}{s} \cdot \left(1 - e^{-(s \cdot T_{0.1})}\right)}{1 - e^{-s \cdot T}} \xrightarrow{float, 3} \frac{100.0 \cdot 2.72^{-1.54 \cdot s} - 100.0}{s \cdot \left(2.72^{-15.4 \cdot s} - 1.0\right)}$$

$$e_2(t) := E_m \cdot (\Phi(t) - \Phi(t - T_{0.1}))$$

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e2(t) :=  while t > T
              t ← t − T
          if t < T
              e2(t)
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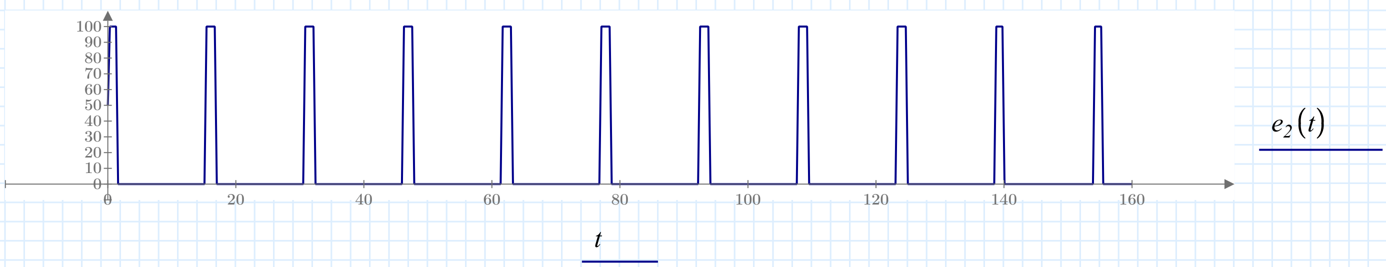
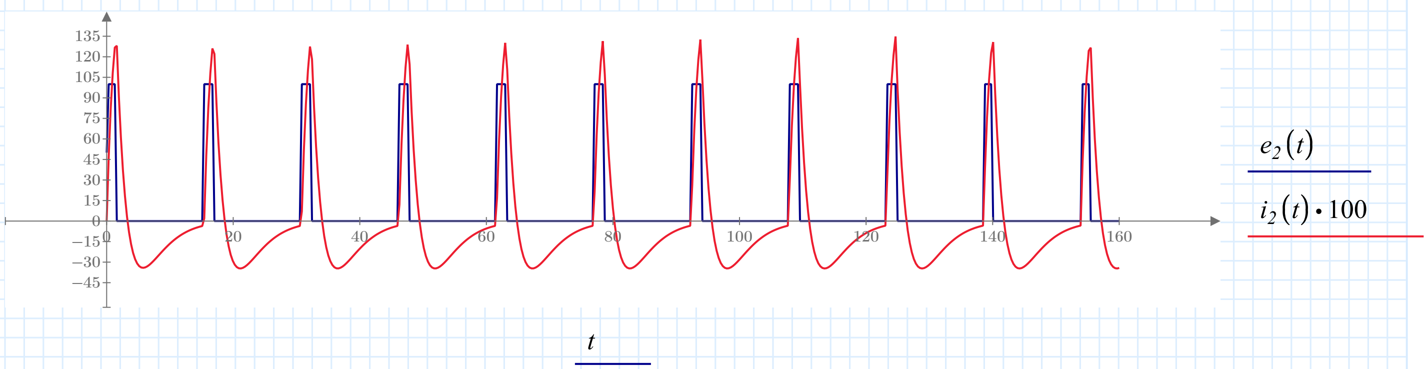
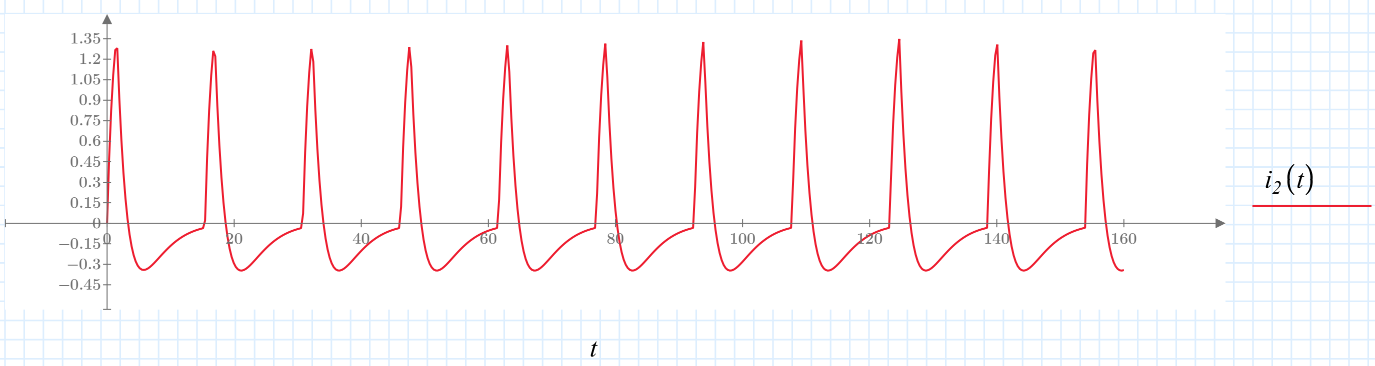


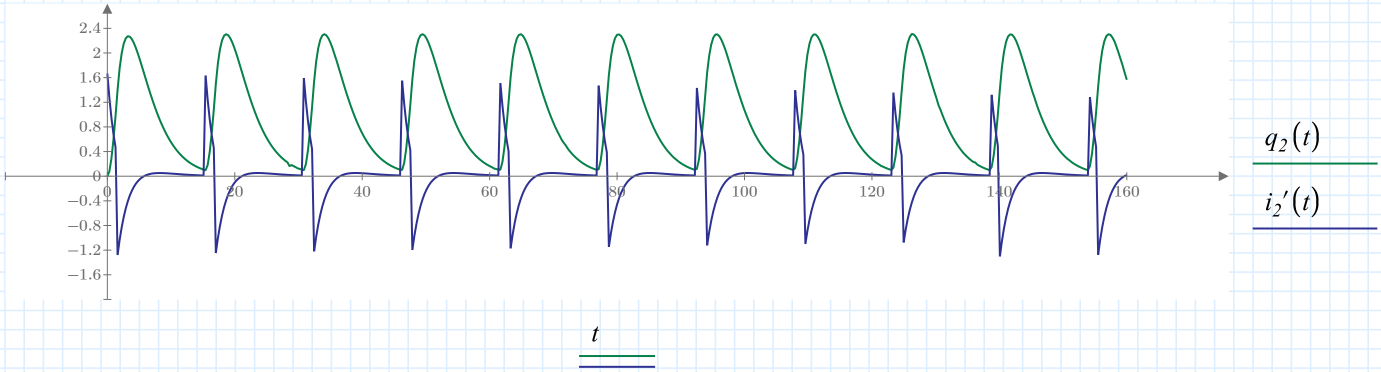
$$i_2(t) := \frac{\frac{E_m \cdot \left(1 - e^{-1.539 \cdot s}\right)}{s} \cdot \left(1 + e^{-15.39 \cdot s} + e^{-15.39 \cdot s \cdot 2} + e^{-15.39 \cdot s \cdot 3} + e^{-15.39 \cdot s \cdot 4} + e^{-15.39 \cdot s \cdot 5} + e^{-15.39 \cdot s \cdot 6} + e^{-15.39 \cdot s \cdot 7} + e^{-15.39 \cdot s \cdot 8} + e^{-15.39 \cdot s \cdot 9} + e^{-15.39 \cdot s \cdot 10}\right)}{s \cdot L + R + \frac{1}{s \cdot C}} \xrightarrow[invlaplace]{simplify}$$





$$q_2(t) := \int_0^t i_2(t)\,dt$$

$$i_2'(t) \rightarrow ?$$

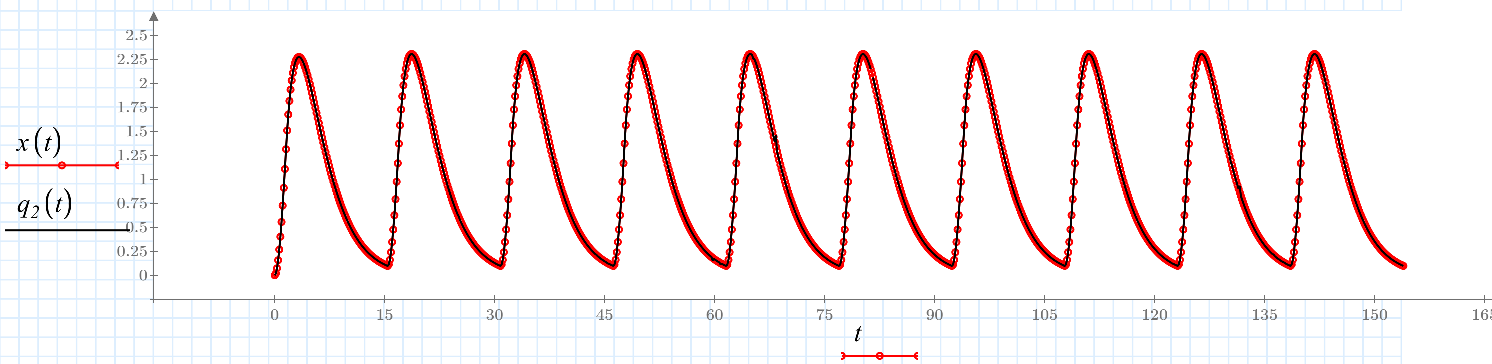


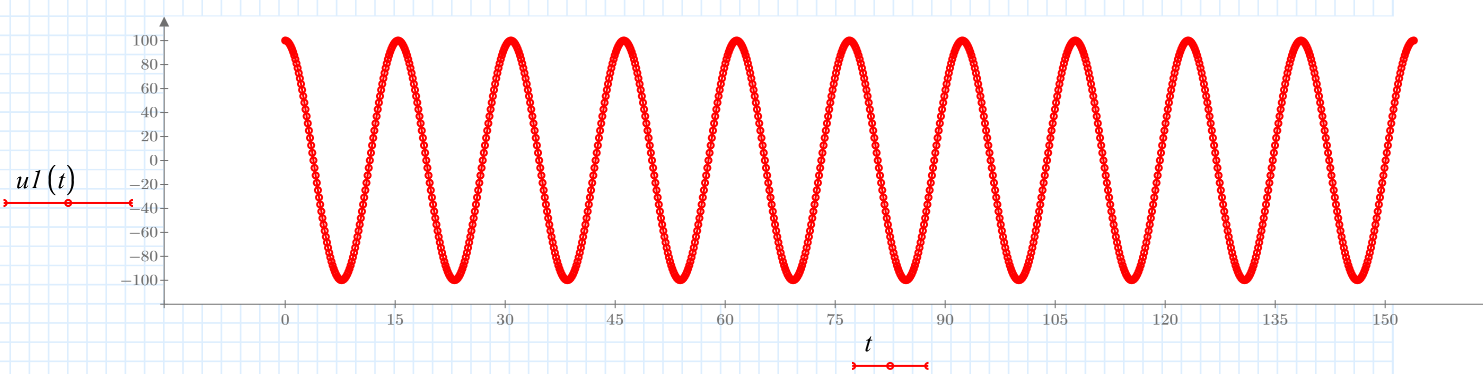
$$m \cdot \left(\frac{d^2}{d\tau^2} x(\tau)\right) + b \cdot \left(\frac{d}{d\tau} x(\tau)\right) + k \cdot x(\tau) = u2(\tau)$$

$$x(0) = 0 \qquad x'(0) = 0$$

$$x := \textbf{Odesolve}(x(\tau), tmax, intvls)$$

$$t := 0, tstep .. tmax$$





$v(t) := \frac{d}{dt} x(t)$ $ke(t) := .5 \cdot m \cdot v(t)^2$ $a(t) := \frac{d^2}{dt^2} x(t)$