Climate Change Basic Modeling. Basic Models On Climate Change and Climate Modeling.
Textbook Resources: 1). Energy Systems Engineering (Vanek Albright & Angenent) - Primary.
2). Other references indicated in notes.
Notes by **Karl Bogha**. My Climate Engineering Homework.

Climate Engineering For Engineers and Students In Renewable - Mechanical/Civil/Electrical Engineering - Physics.

The skills to gain here are:

1. Understanding climate change variables in engineering perspective.
2. Gain a connection from the science side of things to the engineering side of climate modeling.
3. Study notes on the starter model of climate modeling thru the energy balance.
4. Go thru a few example-exercises.
5. Build a foundation on climate modeling.
6. Know where to go for further depth and detail.
7. Be able to apply climate modeling for engineering work place and engineering class rooms.
8. Study-Skill based on Chapter 4 3rd edition of Energy Systems Engineering (Vanek Albright Angenent). Engineer-Student is encouraged to get a copy of this valuable textbook for practice and reference.

Relative Strengths of Warming and Cooling Influences

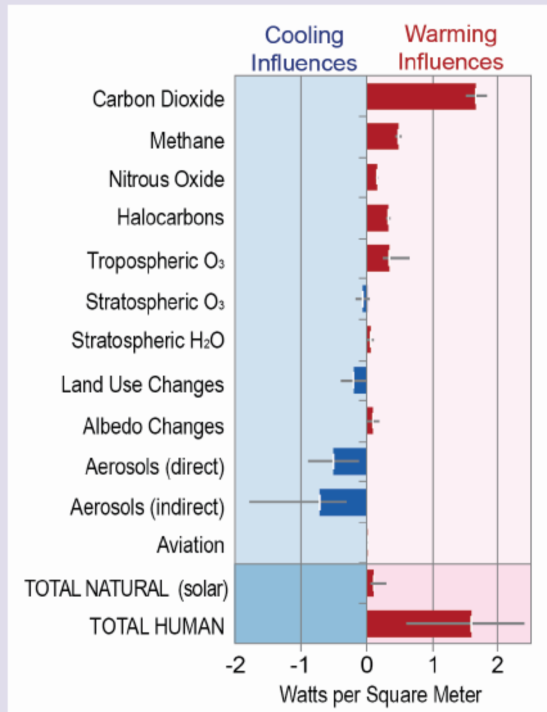


Figure 6. Different factors have exerted a warming influence (red bars) or a cooling influence (blue bars) on the planet. The warming or cooling influence of each factor is measured in terms of the change in radiative forcing in watts per square meter by 2005 relative to 1750. This figure includes all the

Source: US Global Change Research Program

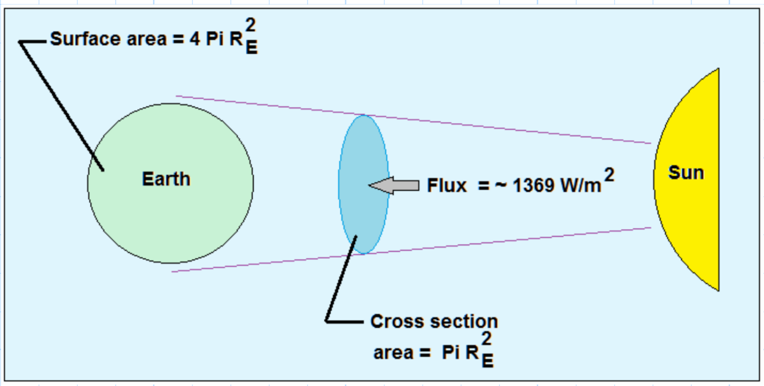
(Figure source: *Climate Change 2007: The Physical Science Basis. Working Group I Contribution to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Figure 2.20 (A), Cambridge University Press[15]).

Apologies in advance for any omissions and errors.

Climate Change Basic Modeling. Basic Models On Climate Change and Climate Modeling.
Textbook Resources: 1). Energy Systems Engineering (Vanek Albright & Angenent) - Primary.
2). Other references indicated in notes.
Notes by **Karl Bogha**. My Climate Engineering Homework.

Notes made from Chapter 4 Climate Change and Climate Modeling of Energy Systems Engineering 3rd Ed by Vanek, Albright, and Angenent (VAA).

Equations to apply for basic modeling:



Equations here come from course work in
1. heat transfer.
2. field and waves.
3. physics.

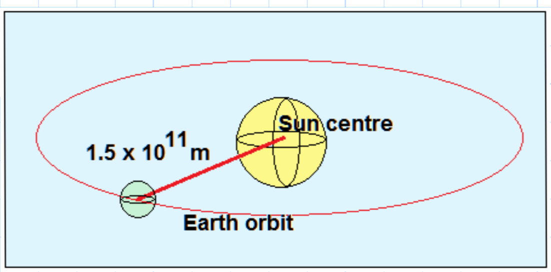
The energy flux from black body radiation, F_BB, is a function of the fourth power of temperature T:

$$F_{BB} = \sigma T^4 \ \frac{W}{m^2}$$ *Temperature T in K.*

where $\sigma = 5.67 \cdot 10^{-8} \ \frac{W}{m^2 \ K}$ <--- Stephen Boltzman constant.

Sun observed temperature: 5770 K

$$F_{BB} := (5.67 \cdot 10^{-8}) \cdot (5770)^4 = 6.285 \cdot 10^7 \ \frac{W}{m^2}$$



Sun radius: $7 \cdot 10^8$m
Average distance between centre of sun to earth's orbit: $1.5 \cdot 10^{11}$ m
Intensity of energy flux per area decays with square of distance.

Average intensity of energy flux at the distance from sun to earth: $F_{BB\_eart} \cdot \left(\frac{Sun\_radius}{Dist\_Sun\_Earth}\right)^2$

$$(6.285 \cdot 10^7) \cdot \left(\frac{7 \cdot 10^8}{1.5 \cdot 10^{11}}\right)^2 = 1368.73$$

$S_0 = 1369 \ \frac{W}{m^2}$ <--- Solar constant. *Widely used in PV system installation.*

Energy balance between energy reaching the earth thru solar radiation Ein and the energy flowing out due to black body radiation Eout can be equated as follows:

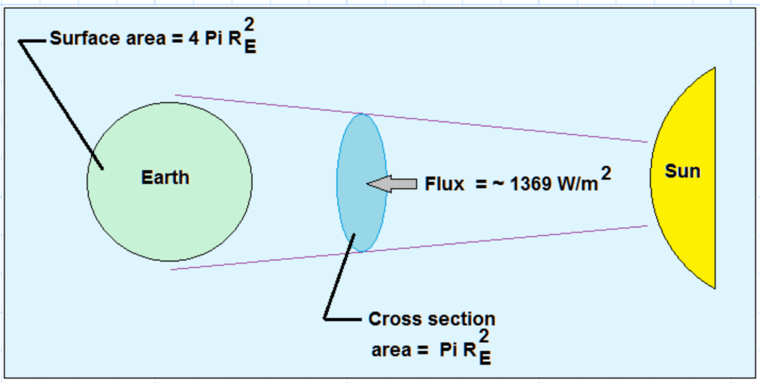
$$E_{in} = E_{out}$$ <--- Starting point to get to an applicable expression.

$$F_{sun}A_{disc} = F_{BB}A_{earth}$$

$$S_0 \cdot (1-\alpha) \cdot \pi \cdot R_{earth}^{\ 2} = \sigma \cdot T_{earth}^{\ 4} \cdot 4 \cdot \pi \cdot R_{earth}^{\ 2}$$

$$S_0 \cdot (1-\alpha) = \sigma \cdot T_{earth}^{\ 4} \cdot 4$$

$$\frac{S_0 \cdot (1-\alpha)}{4} = \sigma \cdot T_{earth}^{\ 4}$$



α is the reflection of solar energy of earth surface, albedo, typical value 0.3.
Next solving for T gives the earth's surface temperature in degrees Kelvin.

$$T_{earth}^{\ 4} = \frac{S_0 \cdot (1-\alpha)}{4 \cdot \sigma} = \frac{S_0 \cdot (1-0.3)}{4 \cdot 5.67 \cdot 10^{-8}} = \left(\frac{0.7}{4 \cdot 5.67 \cdot 10^{-8}}\right) S_0$$

$$T_{earth}^{\ 4} = 3.0864 \cdot 10^{6} \cdot 1369 = 4.2253 \cdot 10^{9}$$

$$T_{earth} = \left(4.2253 \cdot 10^{9}\right)^{\frac{1}{4}} = 254.956 \text{ K}$$

$$T_{earth} = 255 \text{ K}$$ <--- Basically average temperature represents temperature of a planet orbitting our sun on same orbit and radius as earth.
255K will be seen often for various reasons.
288K also seen often is the observed average temperature in absence of warming in the period 1980-2000.

Wavelength - Energy Flux - Absorption.

From modern college engineering physics or modern physics: Energy contained in a wave is a function of frequency f.

Where f = c / lambda.

Lamba is wavelength and c the speed of light.

Energy is released in waves when electons drop to a lower energy state.

Change in energy E: $\Delta E = h \cdot f$ Plank's constant h = 6.626 x 10^-34 Js

Since $f = \frac{c}{\lambda}$ $\Delta E = h \cdot \frac{c}{\lambda}$

Black body energy flux equation **--->** $$F_{BB} = \frac{C_1 \left(\lambda \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda \cdot 10^6\right) \cdot T}\right)}\right) - 1} \quad \frac{W}{m^2}$$

Where

$$C_1 = 3.742 \cdot 10^8 \; \frac{W m^3}{K} \quad \text{and}$$

$$C_2 = 1.4387 \cdot 10^4 \; mK$$

Integrating F_BB above for all values of lambda = 0 to 8 for a given temperature T gives F_BB equal:

$F_{BB} = \sigma \cdot T^4$ *<---As on previous page.*

Based on F_BB equation above, wavelength lambda_max where F_BB is maximum ---> with lambda_max given in units of metre.

$$\lambda_{max} = \frac{2.897 \cdot 10^{-3}}{T} \; m$$

Continued on next page with example.

Example 4-1 Wavelength - Energy Flux - Absorption $(\lambda - F_{BB} - \alpha)$

Using a surface temperature value of 288K,
evaluate F_BB(*lambda*) at *lambda_max*,
and also at *lambda* = 5*um* (5x10^-6 m) and 25 *um*.

Solution:

Given: $T := 288$ K. *This is NOT the same temperature calculated previously ie 255K. Rather here it is 288K for earth's surface temperature.*

a).

Calculate lambda_max using the equation $\lambda_{max} = \dfrac{2.897 \cdot 10^{-3}}{T}$ m

$$\lambda_{max} := \frac{2.897 \cdot 10^{-3}}{288} = 0.00001006 \quad m$$

OR 10.06 x 10^-6 m OR 1.006 x 10^-5 m

Now calculate the black body/object radiant energy flux:

$C_1 := 3.742 \cdot 10^8$ $\dfrac{Wm^3}{K}$ $\qquad C_2 := 1.4387 \cdot 10^4$ mK

$$F_{BB} := \frac{C_1 \cdot \left(\lambda_{max} \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_{max} \cdot 10^6\right) \cdot T}\right)}\right) - 1}$$

<--- Black object radiation.

Break the above expression to parts, may help form a better understanding of the parts relative to the whole expression's solution.

$$Part_{num} := C_1 \cdot \left(\lambda_{max} \cdot 10^6\right)^{-5} = 3.633 \cdot 10^3 \qquad epwr := \left(\frac{C_2}{\left(\lambda_{max} \cdot 10^6\right) \cdot T}\right) = 4.966$$

$$F_{BB} := \frac{Part_{num}}{e^{epwr} - 1} = 25.502$$

$$F_{BB\_max} := \frac{C_1 \cdot \left(\lambda_{max} \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_{max} \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 25.502 \quad \frac{W}{m^2}$$

Answer.

b). $\lambda_b := 5 \cdot 10^{-6}$ m

$$F_{BB\_b} := \frac{C_1 \cdot \left(\lambda_b \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_b \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 5.486 \quad \frac{W}{m^2}$$ Answer.

c). $\lambda_c := 25 \cdot 10^{-6}$ m

$$F_{BB\_c} := \frac{C_1 \cdot \left(\lambda_c \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_c \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 6.01 \quad \frac{W}{m^2}$$ Answer.

$\lambda_{max} = 10 \cdot 10^{-6}$ $\quad F_{BB} = 25.5$
$\lambda_c = 25 \cdot 10^{-6}$ $\quad F_{BB} = 6.01$
$\lambda_b = 5 \cdot 10^{-6}$ $\quad F_{BB} = 5.49$

Notice placing a black object radiation F_BB value in descending order does not match likewise for the wavelength lambda in similar order, for the same temperature T = 288K.

There is an outcome from the above observation thru a plot.

Its a real world problem, at T=288K, need another 2 values to make good the curve to be plotted in Excel. So this curve start from near zero and end at near zero.

$\lambda_o := 1 \cdot 10^{-6}$ m $$F_{BB\_d} := \frac{C_1 \cdot \left(\lambda_o \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_o \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 0 \quad \frac{W}{m^2}$$ Shorter wavelength.

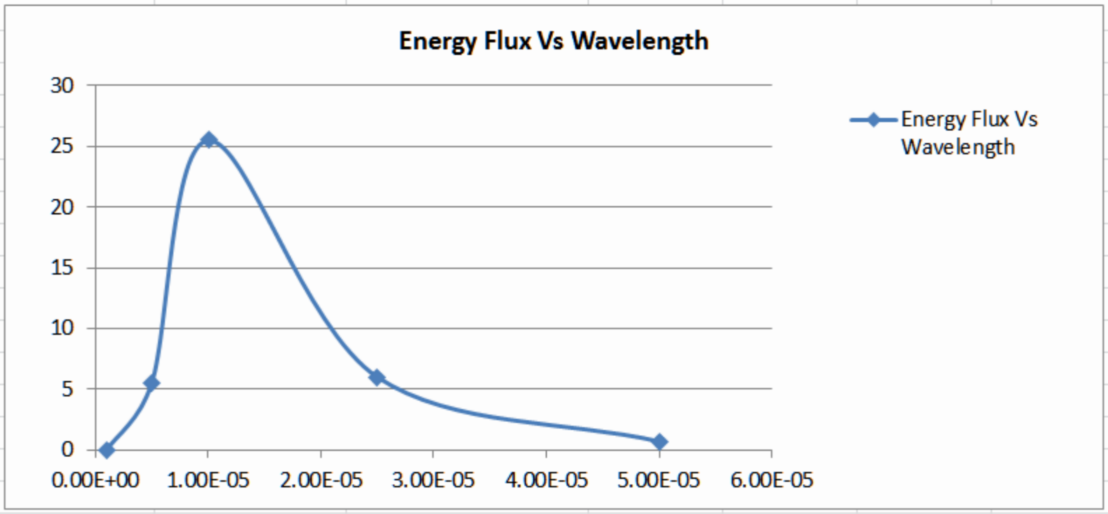
$\lambda_d := 50 \cdot 10^{-6}$ m $$F_{BB\_d} := \frac{C_1 \cdot \left(\lambda_d \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_d \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 0.698 \quad \frac{W}{m^2}$$ Longer wavelength.

Next a plot on energy flux vs wavelength.
Use Excel instead because the calculations were not done using a function here, and tabulations can be meaningfull too.

Continued next page.

Results from Excel posted below.

Earth surface temperature: 288K		
	Wavelength	F_BB Radiation
λ-o	1.00E-06	0
λ-b	5.00E-06	5.49
λ-a-max	1.01E-05	25.5
λ-c	2.50E-05	6.01
λ-d	5.00E-05	0.7



Conclusion from VAA:

The plot above shows as the wavelength moves away on either side of the maximum wavelength (10.06 um) the energy flux declines.

Comment:

At a given earth surface temperature a maximum wavelength can be calculated which results in a maximum black object-body radiation, and wavelenghts on either side decline to zero.

Of course the role of the exponential term in the formulae of F_BB impacts the results in a way to lend-support the behaviour of energy flux to wavelength.

Lets do this exercise again for earth calculated surface temperature of 255K, and do a plot again.

clear $\langle F_{BB}, T\rangle$ $T := 255$

$$\lambda_{emax} := \frac{2.897 \cdot 10^{-3}}{255} = 0.00001136 \text{ m}$$

OR 11.36 x 10^-6 m

$$F_{BB\_max} := \frac{C_1 \cdot \left(\lambda_{emax} \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_{emax} \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 13.878 \ \frac{W}{m^2}$$

mamium wavelength at 255 gives maximum energy flux.

$\lambda_a := 1 \cdot 10^{-6}$

$$F_{BB\_a} := \frac{C_1 \cdot \left(\lambda_a \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_a \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 0 \quad \text{W/m^2.}$$

$\lambda_b := 5 \cdot 10^{-6}$

$$F_{BB\_b} := \frac{C_1 \cdot \left(\lambda_b \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_b \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 1.5 \quad \text{W/m^2.}$$

$\lambda_c := 25 \cdot 10^{-6}$

$$F_{BB\_c} := \frac{C_1 \cdot \left(\lambda_c \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_c \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 4.5 \quad \text{W/m^2.}$$

$\lambda_d := 50 \cdot 10^{-6}$

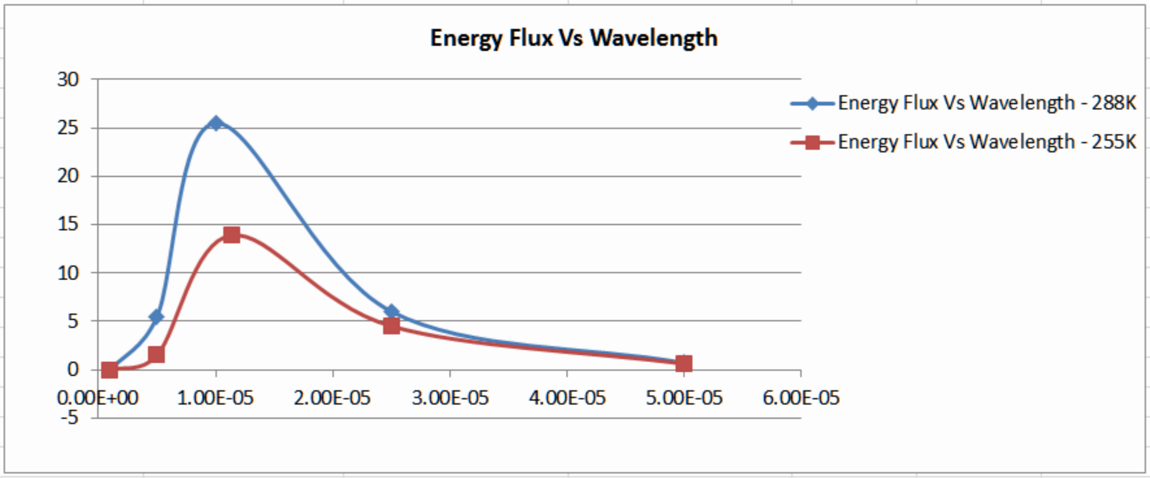
$$F_{BB\_d} := \frac{C_1 \cdot \left(\lambda_d \cdot 10^6\right)^{-5}}{\left(e^{\left(\frac{C_2}{\left(\lambda_d \cdot 10^6\right) \cdot T}\right)}\right) - 1} = 0.6 \quad \text{W/m^2.}$$

Next a plot on energy flux vs wavelength for both 255K and 288K.

Continued next page.

Results from Excel posted below.

Earth surface temperature:		288K	255K
	Wavelength	F_BB Radiation	F_BB Radiation
λ-o	1.00E-06	0	0
λ-b	5.00E-06	5.49	1.5
λ-a-max	1.14E-05	25.5	13.9
λ-c	2.50E-05	6.01	4.5
λ-d	5.00E-05	0.7	0.6



Comments:

The maximum energy flux sees a decline at maximum wavelength at T = 255K compared to T=288K.

The decline in energy flux is 25.5 - 13.9 = 11.6 W/m^2.

Percentage increase of 255K ? $\left(\frac{11.6}{13.9}\right) = 83.5\%$ *83.5% increase since -18C (255K)*

$288 - 273 = 15$ 288 K is 15 deg C. Maybe average temperatures in some parts of earth.

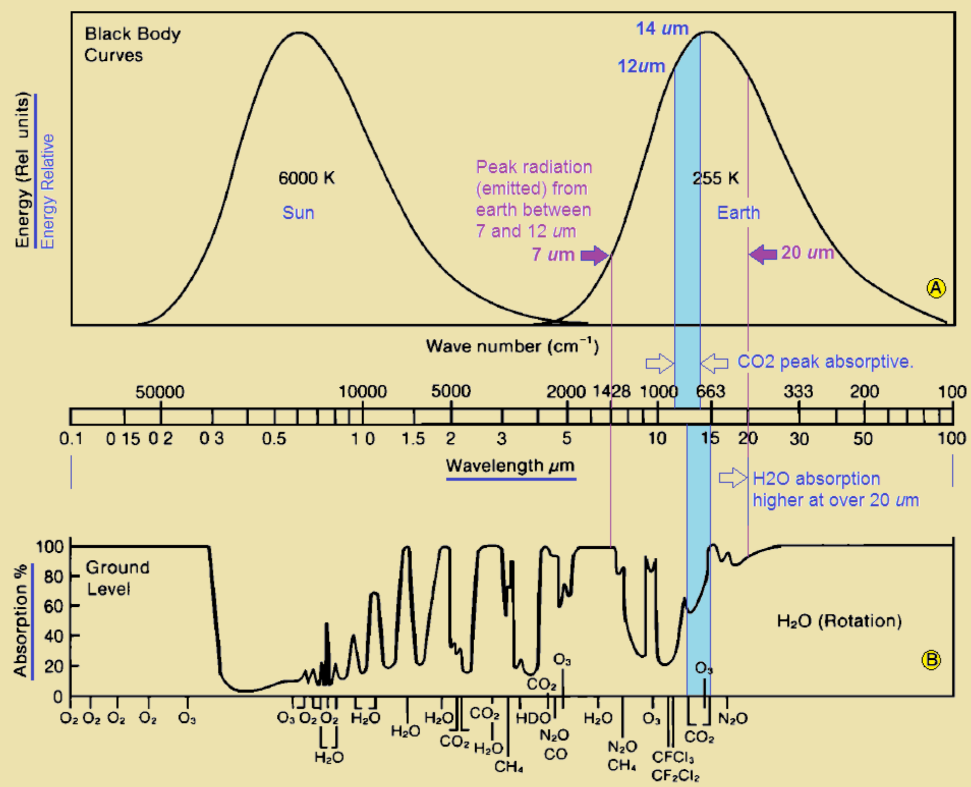
$255 - 273 = -18$ 255 K is -18 deg C. Not average temperatures in most parts of earth, maybe at the poles.

Discussion:

At albedo 0.3 the earth surface temperature is 255K which is -18 deg C.
How do you intepret that?
Average expected based on a scientific equation so its the ice age.
At 288K which is more realistic, 15 deg C, we have a higher maximum wavelength and energy flux based on higher temperature.

Contribution of specific wavelengths and gases to greenhouse effect.

This plot below of wavelengths and percent energy absorption is a step futher in the science of climate change and modeling. This figure provided in VAA was creditted to American Geophysical Union and John F.B. Mitchell who wrote the paper in 1989. *It is noted in Mitchell's paper the figure was from MacCracken and Luther 1985.* Posted here from Mitchell's 1989 paper titled The "Greenhouse" Effect And Climate Change. *Public domain.*



(*a*) Spectral distribution of long-wave emission from blackbodies at 6000 K and 255 K, corresponding to the mean emitting temperatures of the Sun and Earth, respectively, and (*b*) percentage of atmospheric absorption for radiation passing from the top of the atmosphere to the surface. Notice the comparatively weak absorption of the solar spectrum and the region of weak absorption from 8 to 12 μm in the long-wave spectrum [from *MacCracken and Luther*, 1985].

(A). Distribution of relative energy of emissions from sun and earth black bodies by wavelength
(B). and percent absorption of greenhouse gases. Source:Mitchell 1989.Pub:Amer Geophysical Union.

Color markup included in figure above in this study-skill notes.

To understand the extent of climate warming, we must consider two factors:

1). the ability of different molecules to absorp radiation at different wavelengths
2). the extent to which radiation is already absorped by preexisting levels of gases when concentration of some gases increase. - VAA.

Point 2 is on having existing levels quantified so future increases can be compared to a reference.
Obtain current figures similar to the above from other sources.

Notes here from VAA refer to figure on previous page.

Diatomic molecules: CO.
Triatomic molecules: H2O or CO2.
Both di and triatomic molecules have the ability to absorb small increments of energy.
Their molecular bonding structure gives them that ability.
Allowing them to absorb energy in wavelengths between 8 *u*m and 30 *u*m.
Diatomic molecules with the same atom like O2, N2,... here these molecules only absorb energy at higher waves.

CO2 molecules have a peak absorptive capacity in the range from 12 *u*m to 14 *u*m - thats the bottom plot, and coincides with the peaking of the energy flux based on Earth's black body temperature - thats the top plot.

At the wavelength 20*u*m and above, a 100% of the earth surface black body radiation is absorped. Figure on previous page shows H2O absorbs that 100% of energy.

Wavelength from the sun is smaller compared to from earth. Both emit radiant energy, as both are black bodies. The absorption for both is in earth's atmosphere by the gases and the ground. The ground is absorbing most low wavelength energy from the sun, shown to the left of the plot. To the right H2O absorption in vapor and ocean water.

At the sun side at its lower wavelenths, 0.7 *u*m to 1.5 *u*m, H2O absorbed radiant energy here too. H2O has a high absorption across the spectrum of wavelenghts but highest at the earth side at above 20 um.

Between 7 and 14 *u*m wavelenghts rising black body radiation from earth (emitted out from earth into atmosphere), with a fraction of radiation remaining unabsorbed. Here CO2 molecules have peak absorptive capacity between 12-14 *u*m.

Fossil fuel activity increases the release of CO2 into the earth's atmosphere, *CO2 emissions*, allowing CO2 molecules to absorb radiant energy resulting in temperature increases.

Increasing CO2 emission increase CO2 concentration increase percent absorption resulting in an increasing average temperature of atmosphere and earth's surface.

Continuing next with a table from VAA, Table 4-2, and followed by an example.

Comment: The figure on the radiation and absorption is intimidating. I admit. That field is in analytical-instrumentation chemistry. What am I to do? Take a series of courses to get there? Of course not. Engineering work is applying scientific discoveries and principles into products and services. I can live with that. You could look into instrumentation further. At best great majority of renewable engineers are using data from measurements to build something. The measurement taken by instruments, analog and digital, the data used to make decisions.

Gas	Mass Percent	Contribution to Warming (W/m^2)	Wavelength Range for Peak Contribution* (10^-6 m)
H2O	0.23%	~100	>20
CO2	0.051%	~50	12-14
CH4	<0.001%	~5	3,8
Other (For N2O, O3, CFC11,CFC12, and so on.	<0.001% for each	<2 for each	For N2O:16 For CFCs: 11 For O3: Several

Total atmospheric mass = 5.3 x 10^18 kg.
Note: Mass percent and contribution for CO2 shown is at 750 Gt mass of carbon C in atmosphere (2750 Gt CO2). As CO2 concentration increases both mass percent and contribution to warming increase.

Table. Mass Percent and Contribution to Atmospheric Warming of Major Greenhouse Gases.
- Chapter 4, 3rd Ed, VAA.

H2O and CO2 combines, 150+50, totals 150W per square metre.

This 150W accounts for most of the 33K increase in surface temperature.
What caused it? Greenhouse effect caused the increase in temperature.

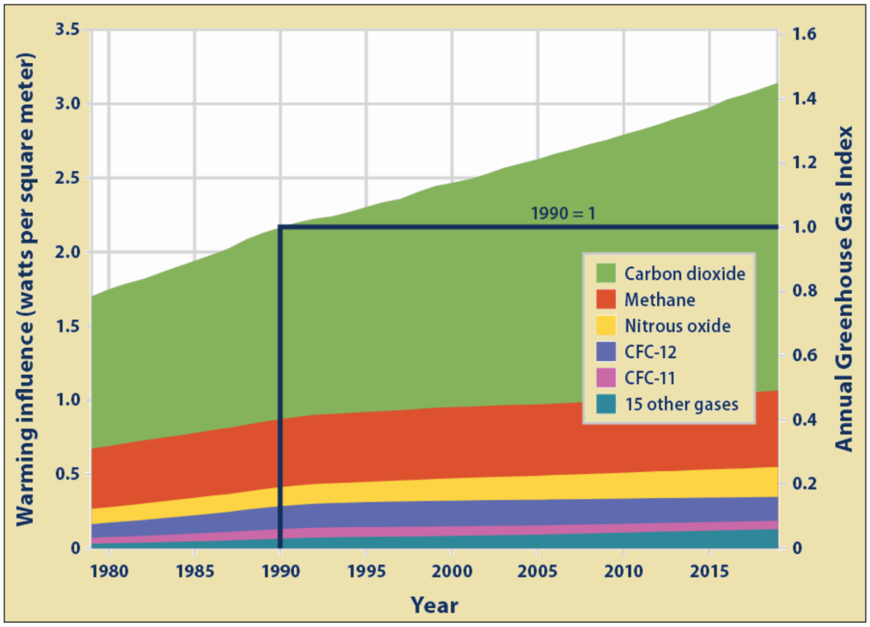
Where did the 33K come from?
In the previous pages, check your available professional textbook, two temperatures were predominant which were 255K and 288K. The difference between the two, 288-255, results in 33K.

The last column in the table above relates to the figure on energy flux emissions and absorption specifically to the wavelengths.

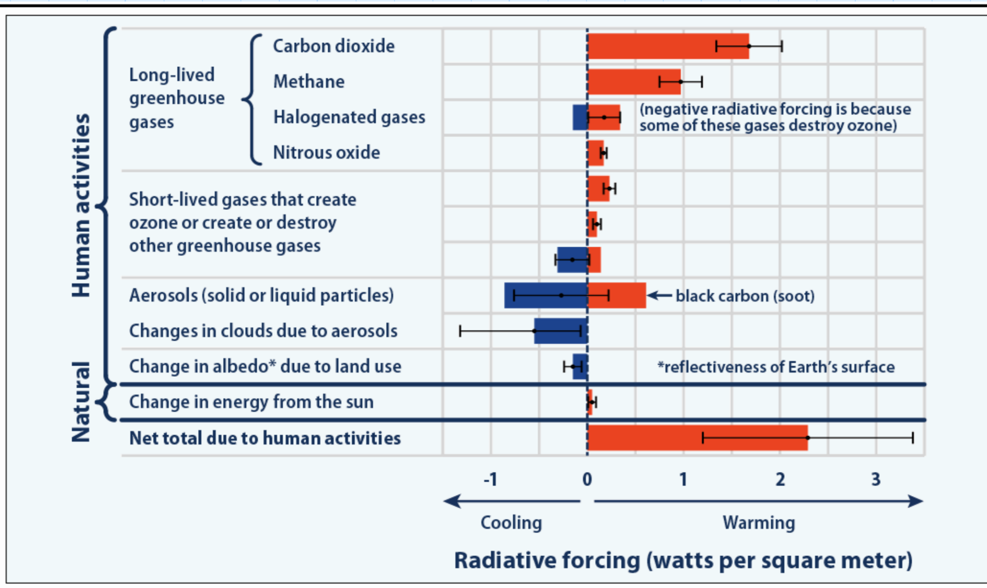
Two gases methane CH4 and N2O have a high energy trapping potential. Their concentration in air maybe low but their energy trapping gives it a valued feature for warming. Methane primarily released thru dairy farms where cows release this gas thru their digestive system. N2O maybe thru industrial processes.

Data on temperature is logged daily around the world, rising averages were observed. The concentration of a certain gas compound may be very low in the air but their contibution or impact on temperature rise is high.

For recent data/charts/graphs the next page has two each from NOAA and IPCCC.
These were from US EPA webpage 30th May 2021. PDF files are downloadable there.



This figure shows the amount of radiative forcing caused by various greenhouse gases, based on the change in concentration of these gases in the Earth's atmosphere since 1750. Radiative forcing is calculated in watts per square meter, which represents the size of the energy imbalance in the atmosphere. On the right side of the graph, radiative forcing has been converted to the Annual Greenhouse Gas Index, which is set to a value of 1.0 for 1990. Data source: NOAA, 2020[1] *Web update: April 2021*



This figure shows the total amount of radiative forcing caused by human activities—including indirect effects—between 1750 and 2011. Radiative forcing is calculated in watts per square meter, which represents the size of the energy imbalance in the atmosphere. Each colored bar represents scientists' best estimate, while the thin black bars indicate the likely range of possibilities. The natural change in the energy received from the sun over this time period is provided for reference. Data source: IPCC, 2013[2] *Web update: May 2014*

Example 4-2: GHG Temperature.

Suppose that water H2O and CO2 were the only GHGs in the atmosphere.
Calculate the resulting temperature in each case.

Solution:

Only CO2 and H2O are present rest of GHGs are not present in atmosphere. In this case with only carbon dioxide and water, assume neglecting totality of GHGs effect, with energy flux reaching earth resulting in earth temperature of 255K be used.

H2O contributes 100W/m2.

CO2 contributes 50W/m2.

Using energy flux from a black object radiation as a function of fourth power of temperature:

$$F_{BB} = \sigma \cdot T^4$$

$\sigma := 5.67 \cdot 10^{-8}$ Boltzman constant W / m^2 K^4.

$T := 255$ K, neglecting GHGs effect.

Now using the equation above calculate the black object radiation on earth's surface.

$$F_{BB} = \sigma \cdot (T^4) = 240 \quad \text{W/m^2.}$$

240 W/m2 is the energy at 255K and does not include the burden caused by the increasing water/water vapor and carbon dioxide in the atmosphere.

To calculate the temperature caused by each gas or water rearrange expression and add the burden wattage of each in the expression:

$$T = \left(\frac{F_{BB} + (\text{Gas\_water\_burden})}{\sigma} \right)^{\frac{1}{4}}$$

Continued next page.

Temperature in K caused by H2O:

$$T_{H2O} = \left(\frac{240 + (100)}{\sigma}\right)^{\frac{1}{4}} = 278$$

Difference in K from 255K caused by H2O:

$$278 - 255 = 23\ K$$

Temperature in K caused by CO2:

$$T_{H2O} = \left(\frac{240 + (50)}{\sigma}\right)^{\frac{1}{4}} = 267$$

Difference in K from 255K caused by CO2:

$$267 - 255 = 12\ K$$

Which compound caused a bigger difference in temperature rise ?

H2O caused a bigger difference compared to CO2.

This result is surprising since water is seen as a cooling agent.
In the air-atmosphere H2O existence is vapour not liquid,
other than when there is rain.
The ocean/seas/lakes are a huge reservoir for releasing H2O into the atmosphere, they are also a reservoir for absorbing carbon.

VAA:
Note that these values add up to more than the total temperature change *Delta-T* due to the greenhouse effect, which must be calculated by first calculating the total contribution to warming from all GHGs before calculating the value of Delta-T.

It may be too simple a calculation for some, however these are the typical equations used on GHGs. H2O would be more susceptible to trap heat in a vapor state compared to aa all gas state of CO2.

Discussion topic suggestion:

Vapour (H2O) having the additional conductive heat transfer compared to CO2 (gas) only having radiation to increase its temperature. Can be discussed in group/ work place.

Before getting into the topic of energy balance for climate modeling, a few sketches to help visualisation.

Energy balance is primarily used in the sciences and mechanical engineering subjects such process-system and power plant analysis.

These figures were modified from those provided in VAA Chapter 4 for assisting in the modeling process. This thru some colouring and sketching.

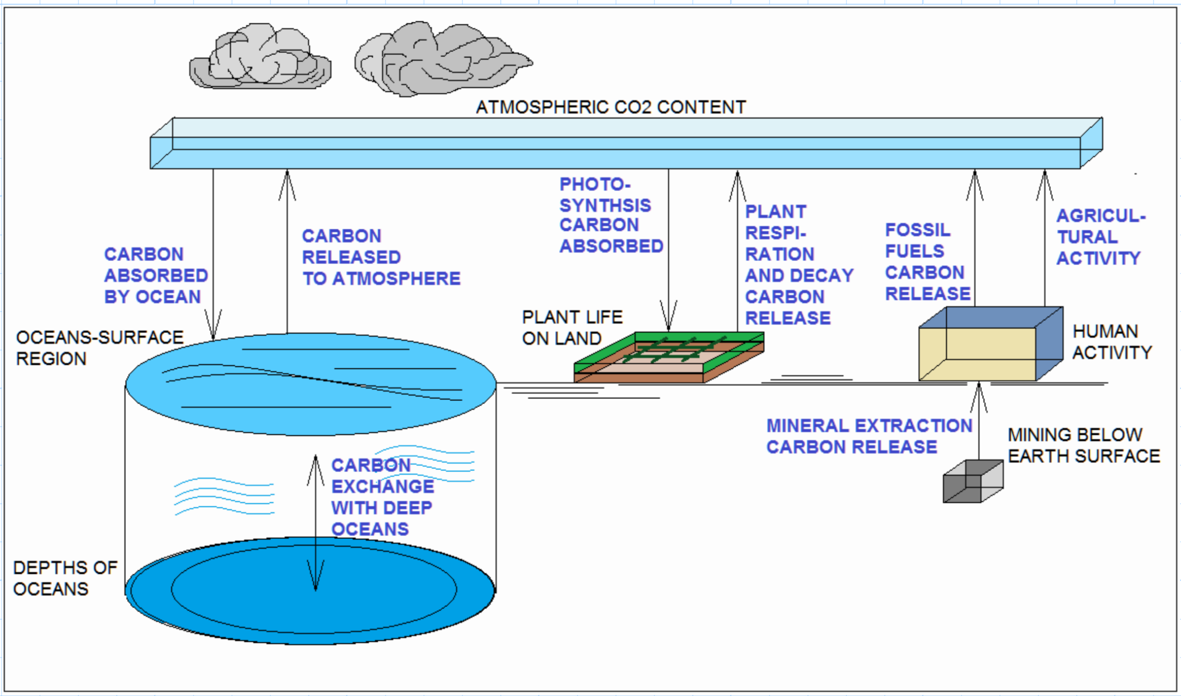


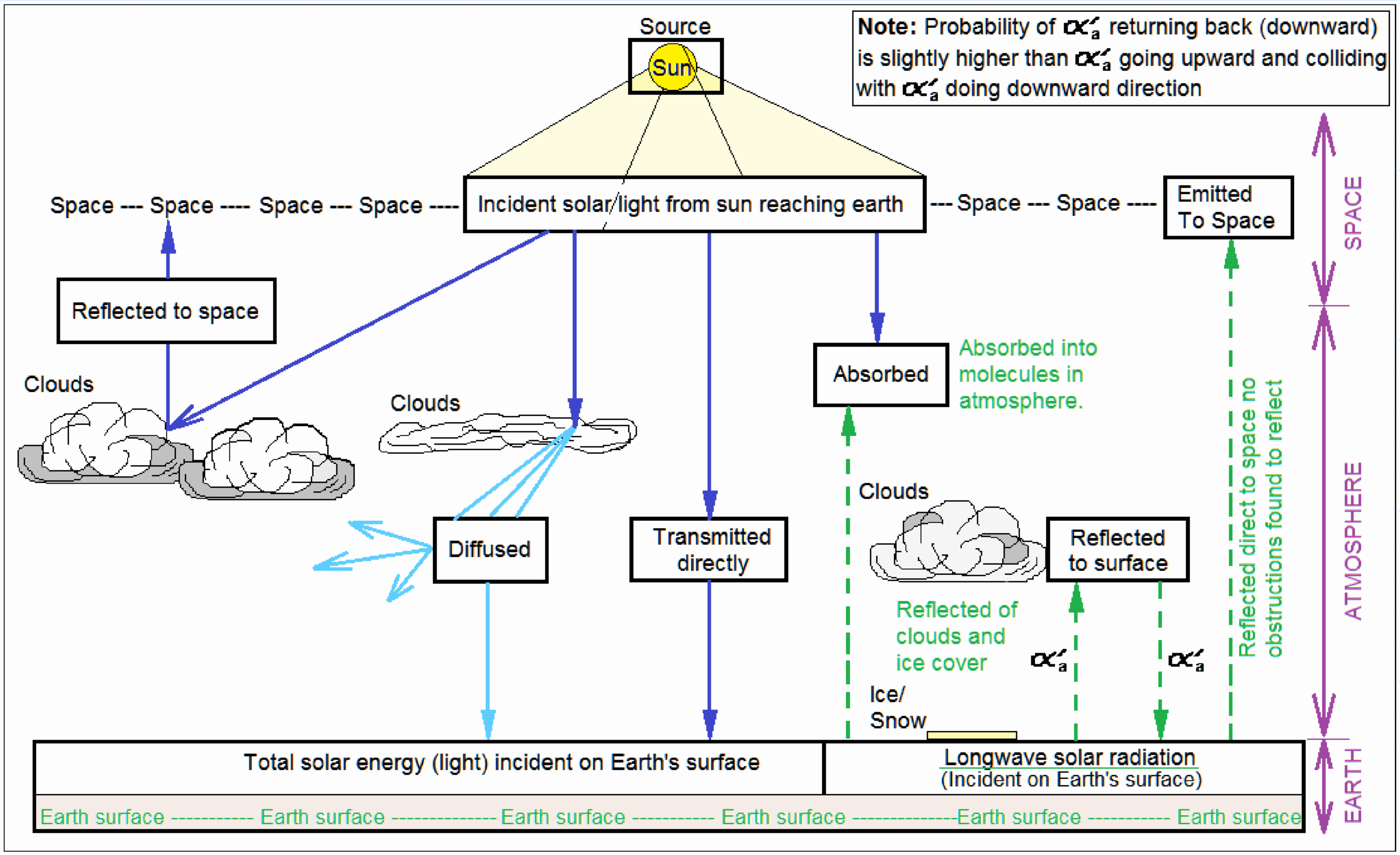
Figure above shows the carbon flow between human activity, atmosphere, land based plant life, and oceans.

For next page turn your PDF view to landscape just for the next page.

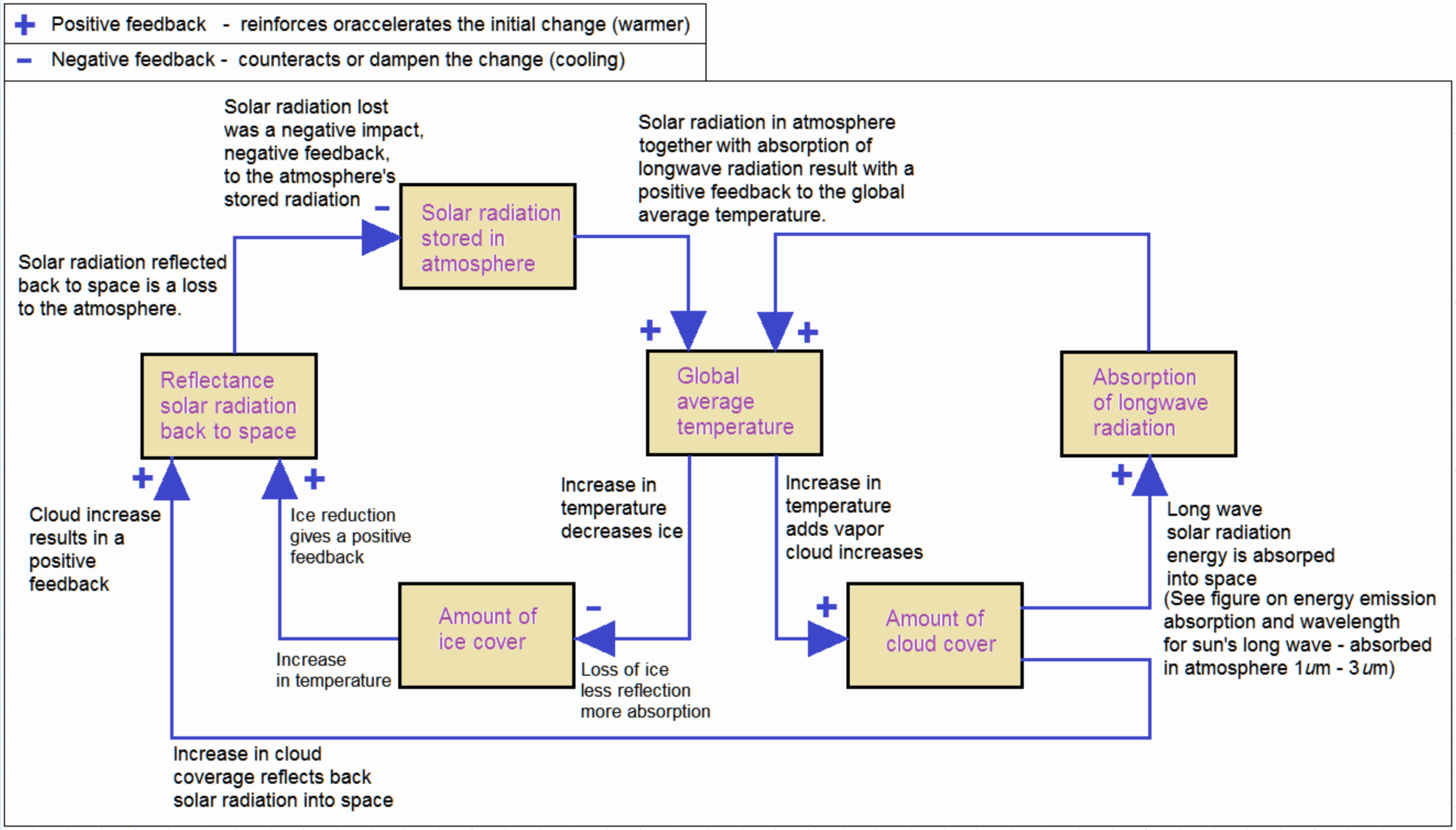
The causal (relationship) loop would be easier to view.
Positive feedback increases temperature it places the climate in the not desired direction. (+) is appropriate since its an increase in temperature. (-) is associated to lowering in temperature and providing the desired impact ie cooling.
The direction the atmosphere-earth surface is going is in the increased warming direction, positive feedback - increased temperature, which most scientist indicate is less likely to turn around.

Next another figure then Causal loop diagram linking global average temperature, cloud or ice cover, and absorption of incoming solar energy - Chapter 4 VAA. *With notes added.*

Figure below typical in solar energy resource calculations with additional identification to longwave radiation from earth's surface into atmosphere and space.
GREEN DASHED lines are longwaves from earth, and SOLID DARK AND LIGHT BLUE lines are shortwaves from sun.



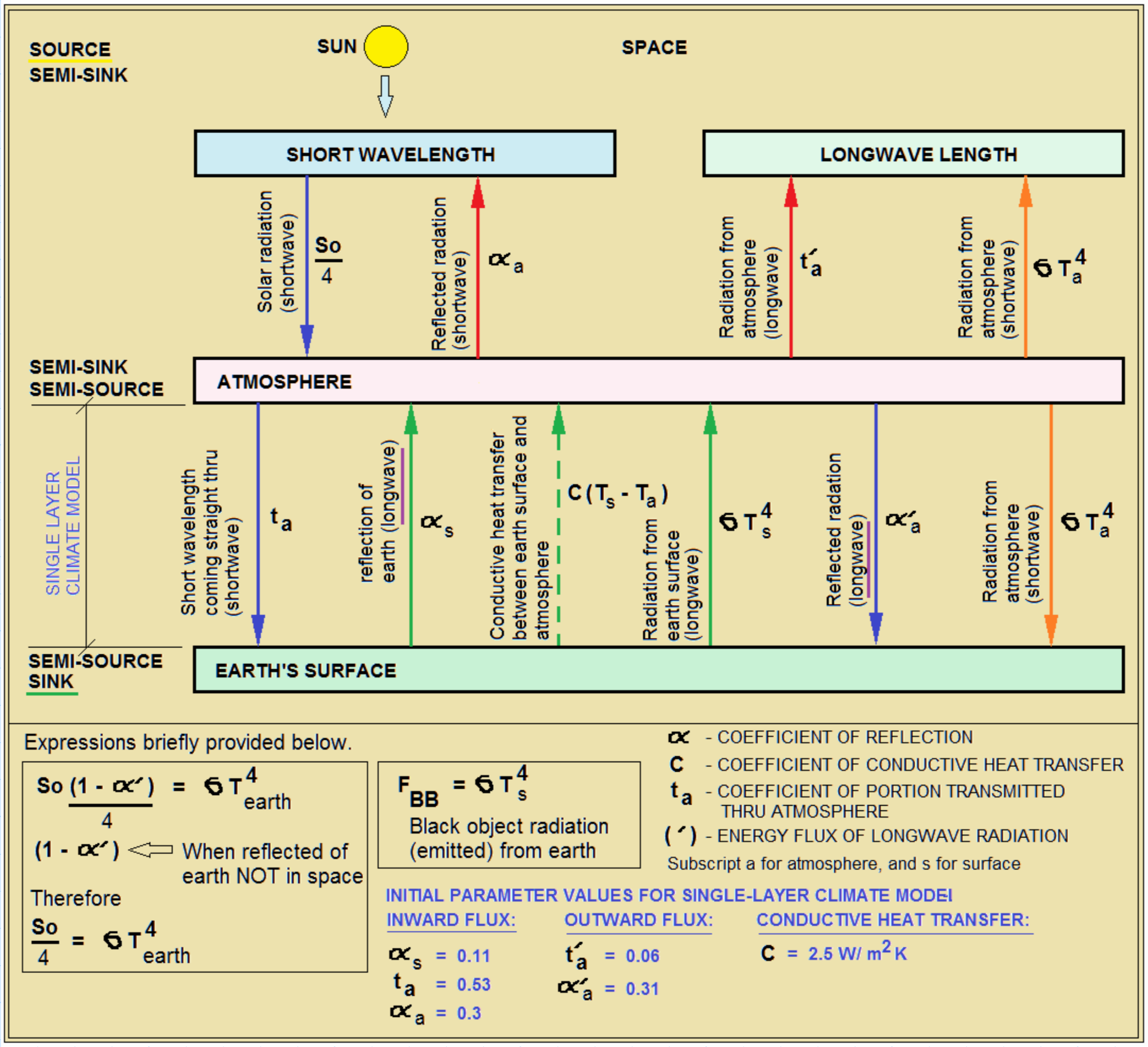
Climate Change Basic Modeling. Basic Models On Climate Change and Climate Modeling.
Textbook Resources: 1). Energy Systems Engineering (Vanek Albright & Angenent) - Primary.
2). Other references indicated in notes.
Notes by **Karl Bogha**. My Homework.



Causal loop (concept relationship loop) diagram connecting global average temperature, cloud, ice cover, and absorption of incoming solar energy. Source: Chapter 4 VAA 3rd Ed. Textbook notes added into diagram.

Note: Solar radiation stored (or retained to be released into atmosphere) and longwave radiation directly connected to the increase of global average temperature. Both providing positive input. Diagram help build the base of climate change modeling.

Energy balance starter climate model - single layer



Model diagram above found similarly in other disciplines such as petroleum reservoir engineering, chemistry-chemical engineering, thermodynamics, power plant analysis,.....

Here energy balance principles are applied to solar radiation energy flux.
Generally we start with inbound waves from sun as short wavelength, reflected waves from earth surface as long wavelength.

Check: Aim is to keep consistent on the origin of the wave, may not be consistent dependent on term being radiant or conduction or convection, and if reflected back/derived from atmosphere changes its wavelength .

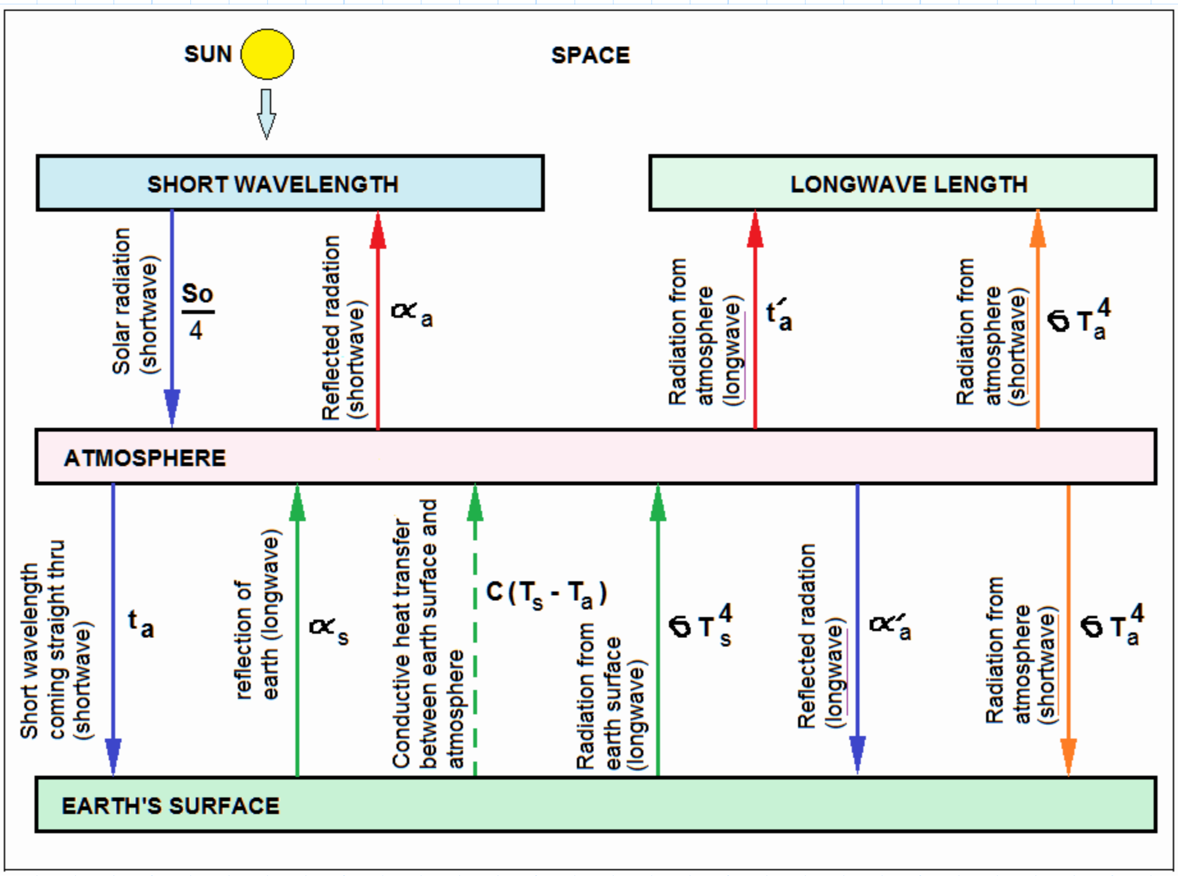
Assumptions to the single layer model:

1. One dimensional model.
   Energy is transferred along a line straight from space through atmosphere to earth surface and reflected back to space. Looking in the vertical distance up-down - straight. Not lateral - not sideways or not adjacent.
2. Atmosphere is taken for a uniform average temperature Ta, and earth's surface is taken for a uniform average temperature Ts.
3. Energy transfer is taken only for radiation and conduction.
   Convection energy transfer between earth surface and atmosphere is not included.

Energy balance equations provided for this model below. Will get into its formation after continuing with energy flow explanation related to the model.

$$t_a(1-\alpha_s)\frac{S_o}{4} - C(T_s - T_a) - \sigma T_s^4 (1-\alpha'_a) + \sigma T_a^4 = 0 \quad \text{Equation 1}$$

$$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} + C(T_s - T_a) + \sigma T_s^4 (1-t'_a-\alpha'_a) - 2\,\sigma T_a^4 = 0 \quad \text{Equation 2}$$



kqj

Figure above all solid lines are radiation and dashed line is conduction energy transfer.

Energy flux flow:

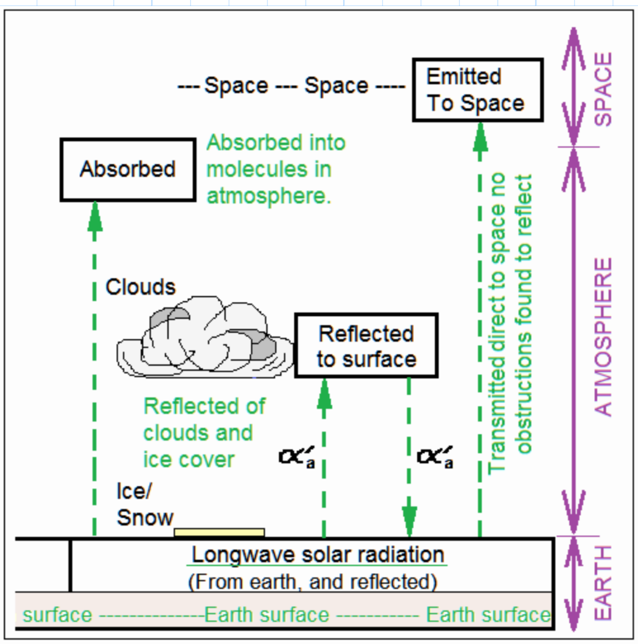
1. Radiation from sun to atmosphere
   - part of it is transmitted straight to earth by atmosphere transmission factor t_a.
   - part of it is reflected back to space in any direction outward to space
   - part of it is absorbed in the atmosphere

Note: Clouds are in the atmosphere at a higher elevation than earth's surface.

2. Straight thru transmitted flux from sun reaches the earth's surface (absorped).
   - part of this is reflected of earth's surface

The energy flux absorbed (transmitted) into earth heats the earth up to an equilibrium temperature Ts.
Does equilibrium temperature mean the same as uniform temperature?
No. In this case it means the energy balance equation where Ts is applied results in zero on the RHS. *For simplifcity the surface temperature is assumed uniform.*

3. The energy absorbed in 2 above increases in earth until a point where it starts to radiate back to space, black object-body radiation, this is the solid green arrow (sigma T^4) it is a longwave. *New wavelength? Generated by new source.*
   - This energy is transmitted straight thru atmosphere to space - long wave
   - part of this energy flux is absorbed by the atmosphere (air molecules and clouds) this is a longwave.
   - part of the enegy flux is reflected of the atmosphere back to earth surface this energy flux is completely absorbed by the earth's surface.



<--- This partial figure from a previous pages may help sort point number 3.

4. Conductive heat transfer usually needs a solid medium for the energy to transfer. We want to account for as much heat transfer between the source(s) and sinks(s), we have radiation covered thus far and conduction be included with the atmosphere taken for a mass/medium for conduction.
   - C(Ts - Ta) takes place due to the temperature difference between the two.
   - Ts at earth would be higher than Ta in the atmosphere; Ts > Ta.
   - Energy flux travles from earth (Ts) to the atmosphere (Ta).
   - C is the conductive heat transfer.

5. Similar to point 3 above, there is a build up of energy flux in the atmosphere. This energy reaches a point where it begins to radiate.
   - part black body radiation from atmosphere to space.
   - part black body radiation from atmosphere to earth surface, this part when it reaches the surface it is completely absorbed and none reflected back.

These 5 points help form the 2 energy balance equations.

There are two equations:

1. Equation 1 is for the sum of energy flux entering and leaving the earth's surface equal zero.

$$t_a(1-\alpha_s)\frac{S_o}{4} - C(T_s - T_a) - \sigma T_s^{\,4}(1-\alpha'_a) + \sigma T_a^{\,4} = 0 \quad \text{Equation 1}$$

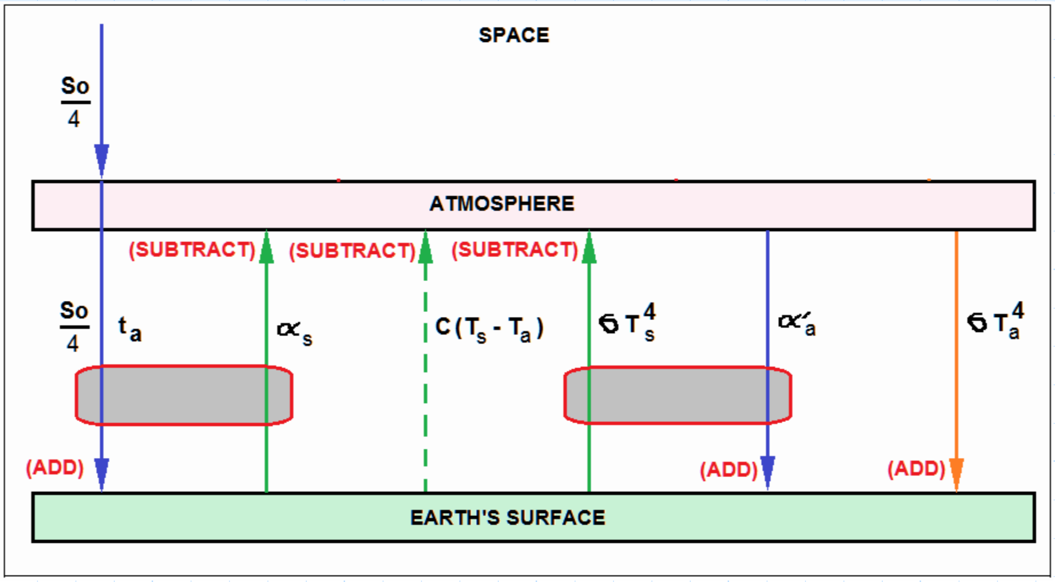
2. Equation 2 is for the sum of energy flux entering and leaving the atmosphere equal zero.

$$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} + C(T_s - T_a) + \sigma T_s^{\,4}(1 - t'_a - \alpha'_a) - 2\,\sigma T_a^{\,4} = 0 \quad \text{Equation 2}$$

Next on the reasoning of forming the two equations.

Comments:
If youre a civil engineer/student you can maybe see this in the statics-dynamics course work. Sum of forces equal zero at a point.
If a mechanical engineer/student in the thermodynamics for power plant analysis or exergy analysis.
If a electrical engineer/student in the circuits for sum of current at a node equal zero - something Kirchoff"s Law.
These three main brances would cover for all the other engineering discipline engineer/ students. A renewable energy engineer is one from any of these or a discipline of its own.

Earth surface equation:



$t_a(1-\alpha_s)\dfrac{S_o}{4}$ First two arrows to the left, blue down green up, the radiant energy coming in straight thru is So/4 and the alpha_s is that lost from reflection - incoming energy flux

$-C(T_s - T_a)$ Third arrow dashed has energy flux leaving thru conduction. Higher temperature flows from earth surface outward. Most of it could happen after sun set - conductive heat transfer

$-\sigma T_s^{\,4} + \sigma T_s^{\,4} \cdot \alpha'_a$

$= -\sigma T_s^{\,4}(1-\alpha'_a)$

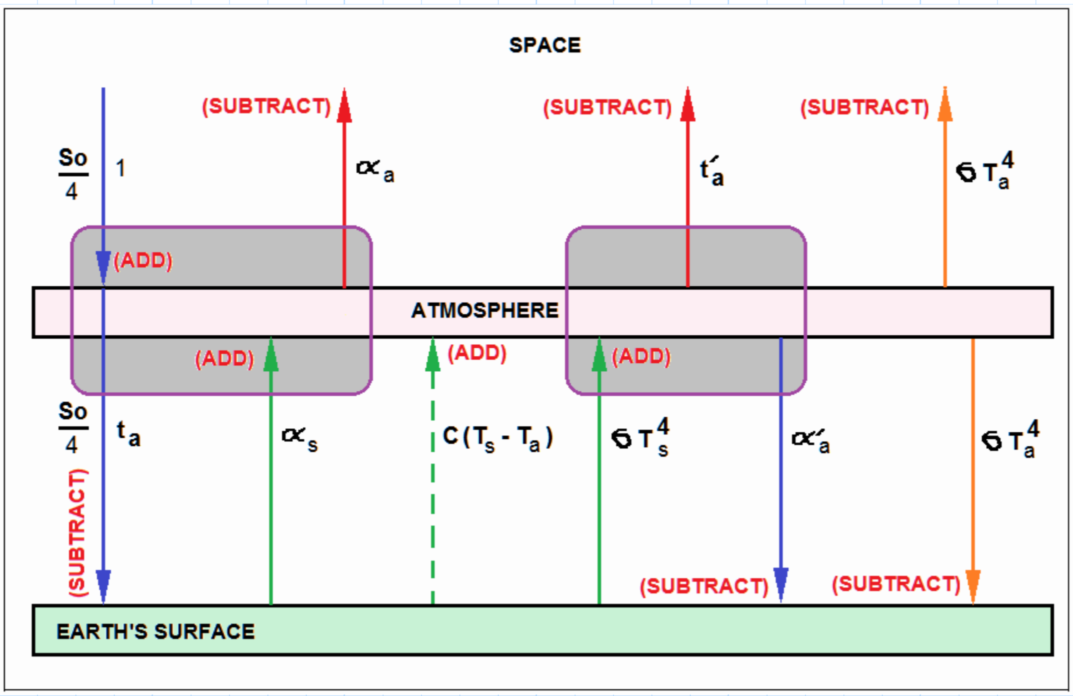
Earth is warmed up in the day, sun sets, the flux in earth escapes thru radiation outward, with portion of same (add) flux reflected back from atmosphere to surface. The outcome of which is (1 - alpha'_a). 4th and 5th arrows - radiative heat transfer outward. *Term rearranged with negative sign.*

$\sigma T_a^{\,4}$ Atmosphere has absorbed energy flux, in the daylight hours. It reaches a point where it begins to release this energy outward both to space and earth. The part to earth is added energy flux. - radiative heat transfer inward.

Equation 1 ---> Earth's surface.

$$t_a(1-\alpha_s)\frac{S_o}{4} - C(T_s - T_a) - \sigma T_s^{\,4}(1-\alpha'_a) + \sigma T_a^{\,4} = 0$$

Atmosphere equation:



$1\left(\frac{S_o}{4}\right)$ Energy flux transmitted directly-straight thru.

$-t_a\left(\frac{S_o}{4}\right)$ Portion of transmitted flux escaping atmosphere leaving for earth surface.

$-\alpha_a\left(\frac{S_o}{4}\right)$ Energy flux reflected by atmosphere back into space.

$t_a\left(\alpha_s\left(\frac{S_o}{4}\right)\right)$ Portion of radiation reaching (transmitted) to earth reflected back to atmosphere

$(1-\alpha_a-t_a(1-\alpha_s))\cdot\frac{S_o}{4}$ Quantity of energy flux held by the atmosphere: after subtracting amount reflected by atmosphere to space, and subtracting amount transmitted straigtht to earth surface adding its flux reflected back to atmosphere - incoming energy flux

$C(T_s-T_a)$ Green arrow dashed has energy flux entering atmosphere thru conduction - add. Higher temperature flows from earth surface outward. This most likely to happen after sun set - conductive heat transfer

σT_s^4 Earth surface warmed in day sends flux out at later time in evening, added to atmosphere.

$\sigma T_s^4\ (-t'_a)$ Same transmitted out of atmospere into space

$\sigma T_s^4\ (-\alpha'_a)$ Same reflected back to earth's surface.

$\sigma T_s^{\ 4}\ \left(1 - t'_a - \alpha'_a\right)$ Earth is warmed up in the day, sun sets the flux in earth escapes thru radiation - add to atmosphere, with portion of same flux reflected from atmosphere to surface - subtract, and with portion of same transimitted out to space (subtract) - radiative heat transfer inward (net) from earth's surface.

$-\sigma T_a^{\ 4} - \sigma T_a^{\ 4}$

$= -2\left(\sigma T_a^{\ 4}\right)$

Atmosphere absorbed energy flux, in the daylight hours. It reaches a point where it begins to release this energy outward to space and earth. Both directions of flux leaving for space and earth surface are subtracted - radiative heat transfer outward.

Equation 2 ---> Atmosphere.

$$\left(1 - \alpha_a - t_a\left(1 - \alpha_s\right)\right)\frac{S_o}{4} + C\left(T_s - T_a\right) + \sigma T_s^{\ 4}\ \left(1 - t'_a - \alpha'_a\right) - 2\ \sigma T_a^{\ 4}\ = 0$$

Those are the 2 equations.
What is to be solved from them?
Using them to solve Ts and Ta which are earth and atmosphere temperatures.
Use math techniques like solver-numerical methods techniques.
Some of the variables/data need to be available or given.

Discussion:

1. Both the equations do not operate as a function of time - f(t).
Its logical to say the equations can be taken for a period of 24 hours, day and night.

2. Day time energy absorbed into earth, though on white surfaces or reflective surface energy can be reflected back to atmosphere in the day time.

3. Earth was warmed in day hours, night hours earth's surface looses energy flux from inside the upper layer of earth to the atmosphere. Here Ts > Ta.

4. Atmosphere has energised flux in day hours from solar energy, it becomes a reservoir, it can release energy flux in day and same for night time. Most likely more released in day time since the molecules are more excited thus collide more compared to night time.

5. Most times electromagnetic field equations are similar no time function same for heat transfer equations.

6. This was the basic model its one layer. The atmosphere can be split into multiple layers, same the ocean depths. Layers are dependent on altitude/depth.
Hence variables that may apply are humidity, evaporation, pressure, density of air, and other variables used in meteorology. This basic model is theorectically solid, there are assumptions/limitations/simplifications in both single and multilayers.

There are initial values for variables/constants to start with
for determining the temperature of Ts and Ta - *Given in Chapter 4 by VAA.*

Initial values:

Inward Flux

$\alpha_s := 0.11$

$t_a := 0.53$

$\alpha_a := 0.3$

$S_o := 1368$ W/m^2

Outward Flux

$t'_a := 0.06$

$\alpha'_a := 0.31$

Conductive heat transfer

$C := 2.5$ W/m^2 K

Boltzman constant:

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2 K^4

Substitute the above into equations 1 and 2 :

$$t_a (1-\alpha_s)\frac{S_o}{4} - C(T_s - T_a) - \sigma T_s^{\,4}(1-\alpha'_a) + \sigma T_a^{\,4} = 0$$ Equation 1- earth surface

$$0.53\ (1-0.11)\ \frac{1369}{4} - 2.5\ (T_s - T_a) - (5.67\cdot 10^{-8})\cdot T_s^{\,4}\ (1-0.31) + (5.67\cdot 10^{-8})\ T_a^{\,4} = 0$$

$$0.53\ (1-0.11)\ \frac{1369}{4} = 161.439 \qquad (5.67\cdot 10^{-8})\cdot(1-0.31) = 3.912\cdot 10^{-8}$$

$$161.439 - 2.5\ (T_s - T_a) - (3.912\cdot 10^{-8})\cdot T_s^{\,4} + (5.67\cdot 10^{-8})\ T_a^{\,4} = 0$$

$$161.439 - 2.5\cdot T_s + 2.5\cdot T_a - (3.912\cdot 10^{-8})\cdot T_s^{\,4} + (5.67\cdot 10^{-8})\ T_a^{\,4} = 0$$

$$161.439 - 2.5\cdot T_s + 2.5\cdot T_a - (3.912\cdot 10^{-8})\cdot T_s^{\,4} + (5.67\cdot 10^{-8})\ T_a^{\,4} = 0$$

$$161.439 + 2.5\cdot T_a + 5.67\cdot 10^{-8}\cdot T_a^{\,4} - 2.5\cdot T_s - 3.912\cdot 10^{-8}\cdot T_s^{\,4} = 0$$

Above expression requires some values of Ta and Ts that would make RHS equal zero.
That is not easy! Ta and Ts are to the 4th power. Requires numerical method solver.

Next substitute for atmosphere equation.

$$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} + C(T_s - T_a) + \sigma T_s^{\,4}(1 - t'_a - \alpha'_a) - 2\ \sigma T_a^{\,4} = 0$$ Equation 2- atmosphere

$$(1-\alpha_a - t_a\cdot(1-\alpha_s))\frac{S_o}{4} = 78.079 \qquad \sigma\cdot(1 - t'_a - \alpha'_a) = 3.572\cdot 10^{-8} \qquad 2\cdot\sigma = 1.134\cdot 10^{-7}$$

$$78.08 + 2.5\cdot T_s - 2.5\cdot T_a + 3.572\cdot 10^{-8}\cdot T_s^{\,4} - 1.134\cdot 10^{-7}\cdot T_a^{\,4} = 0$$

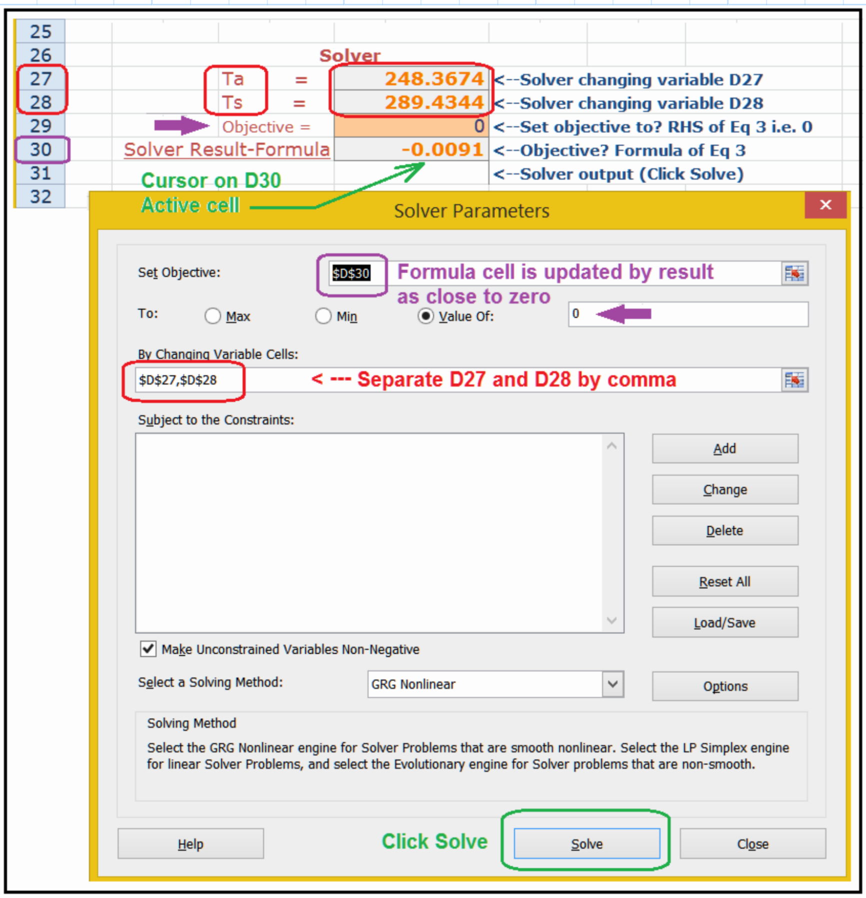
$$78.08 - 2.5\cdot T_a - 1.134\cdot 10^{-7}\cdot T_a^{\,4} + 2.5\cdot T_s + 3.572\cdot 10^{-8}\cdot T_s^{\,4} = 0$$

$$161.439 + 2.5 \cdot T_a + 5.67 \cdot 10^{-8} \cdot T_a^{\ 4} - 2.5 \cdot T_s - 3.912 \cdot 10^{-8} \cdot T_s^{\ 4} = 0 \quad \text{Equation 3}$$

$$78.08 - 2.5 \cdot T_a - 1.134 \cdot 10^{-7} \cdot T_a^{\ 4} + 2.5 \cdot T_s + 3.572 \cdot 10^{-8} \cdot T_s^{\ 4} = 0 \quad \text{Equation 4}$$

Discussion why a solver method:

Equation 3 and 4 cannot be solved using the simultaneous equations because there is an expression to the 4th power. To plot where either equation 3 or 4 crossed the x-axis equal zero would be tedious. Need both values guessed close enough to start then a tabulation then the plot. So a discipline in Mathematics called Numerical Methods is used which Excel has a Solver which can assist. Tried solver and found several pairs of Ts and Ta that met equation 3 RHS equal close enough to zero. *Some hindsight or expectation helps on what the result should be close too*. Since 255K is the somewhat typical for Ts, values close to it were acceptable. Plugged in 255 for both Ta and Ts below. The Solver returned the results given by VAA. Satisfied. College/University mathematics course like Numerical Methods has this content, and Excel's Solver is taught at lower level courses. How to use solver provided below.



Excel solver results shown to the left. Cell numbers and their locations to the solver box are shown.

The results for Ta and Ts were updated by Excel.

To turn Solver go to Options-Add-In and click Analysis, then choose Solver. Check on your Excel.

Several pairs of results may meet the equation requirement. Only one equation, 3 or 4, is needed to solve for RHS equal zero.

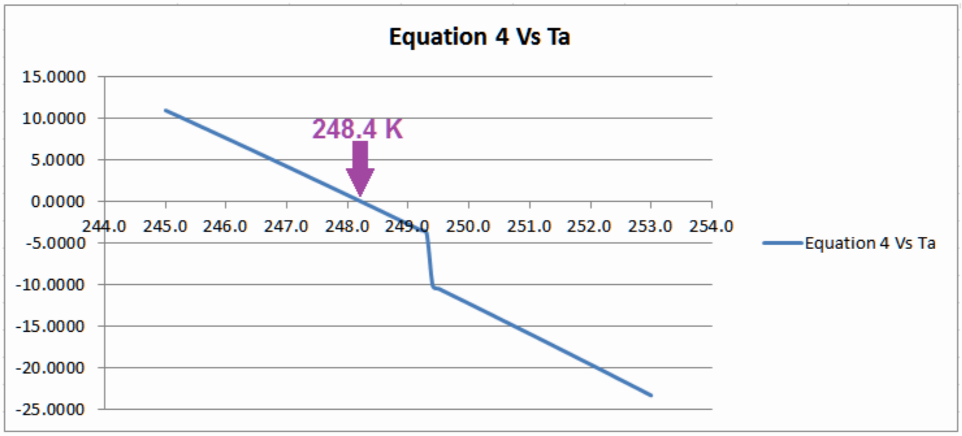
Set objective field only takes one equation. So may work both equations one after the other. Success in getting a solution may require you manually check the roots (Ts and Ta) and level of tolerance i.e. decimal places. Several pairs of solution may result, the curve may crosses twice or the tolerance limit dictates here it was set to 0.

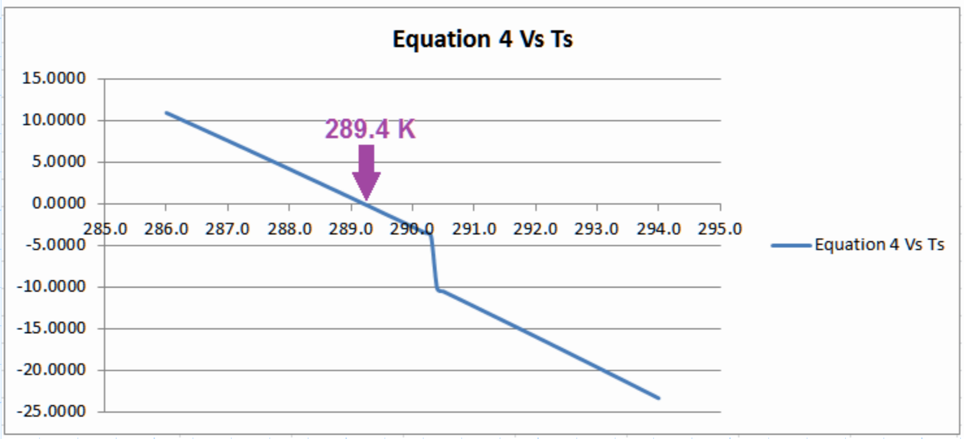
VAA provided Ts = 289.4K and Ta = 248.4K for the solution.

Discussion:

Manual check can be made setting D27 and D28 for Ta and Ts in another cell for the same equation. Compute the result. Calculate its deviation from zero.

Excel plots revealed the same values within limit.
Ta plot crosses x-axis, so did Ts using equation 4.





Ta	Ts	Eq 3	Eq 4
248.0	289.0	0.5288	0.7911
248.1	289.1	0.4971	0.4439
248.2	289.2	0.4654	0.0962
248.3	289.3	0.4337	-0.2520
248.4	**289.4**	0.4020	-0.6007
248.5	289.5	0.3703	-0.9499
248.6	289.6	0.3387	-1.2995

When the tabulations change sign, + to -, the curve has crossed the x-axis. At times its close to 0, here 0.4020, and in Excel the 'set objective' could be set to 0.4 or what is acceptably close enough (tolerance).

Tabulations narrowed down to where the values of Ta and Ts come close to zero for equation 4. Equation 3 value shown is near zero but not zero, close enough, and its not crossing the x-axis. *Though close to it. I have better results with solver for equation 3 while the plot did better for equation 4. Anyway, whatever it takes, theres no end to tabulations and taking it to the nth degree!*
Keep it light and simple.

Chapter 4 VAA has several graphs with the implication of earth's surface temperature was 288K. 'The observed average temperature in the absence of recent warming in the period 1980-2000 is 288K (earth surface)'. So this is that 288K.

Earth atmosphere temperature at preindustrial, *maybe further back to ice age*, was around 255K calculated from some Physics expression with alpha (albedo) equal 0.3. 288K can be set at 0 on a vertical scale with temperature deviations of +/- above and below it. *Some base value need be established? Right? Yes. Debatable it may be.*

The results expected were Ta = 255 K
Ts = 288 K.

The results calculated were Ta = 248.4 K
Ts = 289.4 K.

VAA - the model chosen was a basic single layer one, should a multilayer model be chosen the results could/would/may improve.

Comment:
Dependent on data, what data you use is what you can work on modeling wise to get results that preferably fit your theory for the hoped/intended outcome.

Difference from expected:

$288 - 289.4 = -1.4$ Surface temperature off by -1.4K

$255 - 248.4 = 6.6$ Atmosphere off by 6.6K

Surface temperature close and atmosphere off by under 7K.
Starter single layer model can return good results/prediction for surface temperature.

VAA - 'One possible refinement is to expand the atmosphere into several layers, which will tend to create a temperature gradient from the coolest layer furthest from the Earth, to the warmest layer adjacent to the earth, as occurs in the actual atmosphere'.

Within the atmosphere, as we travel higher up in altitude for example Mt Cook NZ the temperature drops and the ground up there is snow covered.
At the near earth surface, ground sea-level, temperature is higher. Correct.

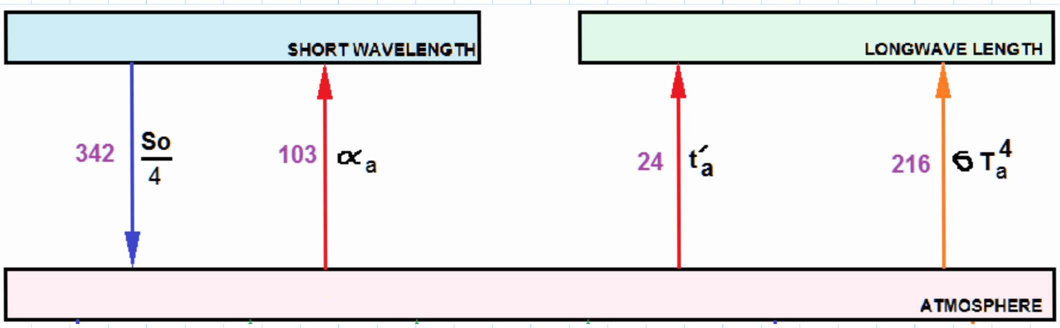
Suggestion:
Got an idea for the multilayer modeling. Suggest, if possible theory wise, to do multiple connected engine cycles, going from higher temperature to lower.
Heat engine cycle model from thermodynamics - Engineering Idea. Not touch on that here.
You may investigate further. Could be MS/ME/phd research topic. You know more about it.

Now try for an analysis of the earth atmosphere single layer model
and it can be performed using the values calculated thus far (including Ta and Ts).
Map the values of energy flows measured in W/m^2 for
the base case on the model diagram.

$\frac{S_o}{4} = 342$ $\alpha_a \cdot 342 = 103$ $T_a := 248.4$ $\sigma \cdot (T_a)^4 = 216$

Radiate heat transfer from surface to atmosphere: $T_s := 289.4$ $\sigma \cdot (T_s)^4 = 398$

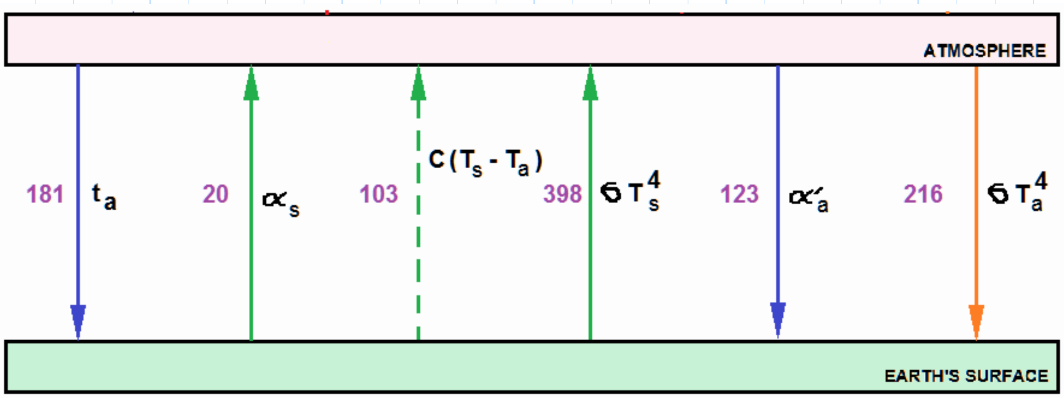
Last value between atmosphere and space : $t'_a := 398 \cdot 0.06 = 24$



Next the lower side of the diagram between atmosphere and surface :

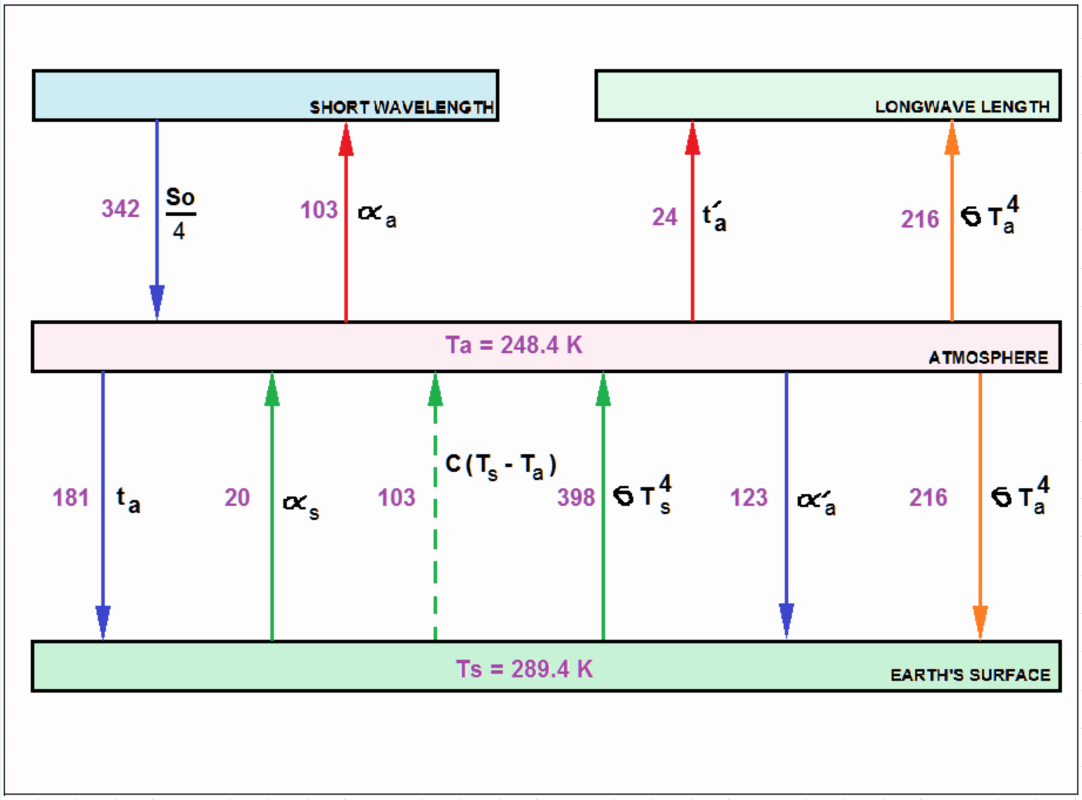
$t_a \cdot \left(\frac{S_o}{4}\right) = 181$ $\alpha_s \cdot 181 = 20$ $C \cdot (T_s - T_a) = 103$ $\sigma \cdot (T_s)^4 = 398$

$\alpha'_a \cdot 398 = 123$ $\sigma \cdot (T_a)^4 = 216$



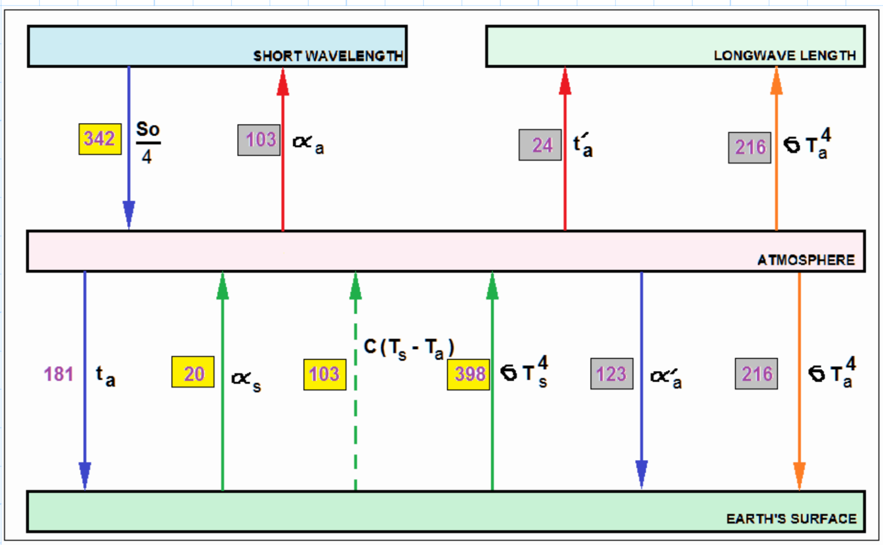
Next Putting the top and bottom together.

Climate Change Basic Modeling. Basic Models On Climate Change and Climate Modeling.
Textbook Resources: 1). Energy Systems Engineering (Vanek Albright & Angenent) - Primary.
2). Other references indicated in notes.
Notes by **Karl Bogha**. My Homework.



Single layer climate model with representative energy flows in W/m^2.
Provided in figure above.

Energy balance? Meaning the flow in and out equal the same for each region under study, here there is the earth's surface region and the atmosphere region.



Sum of yellow box flux entering atmosphere:

342
20
103
398

Total In: 863 W/m^2

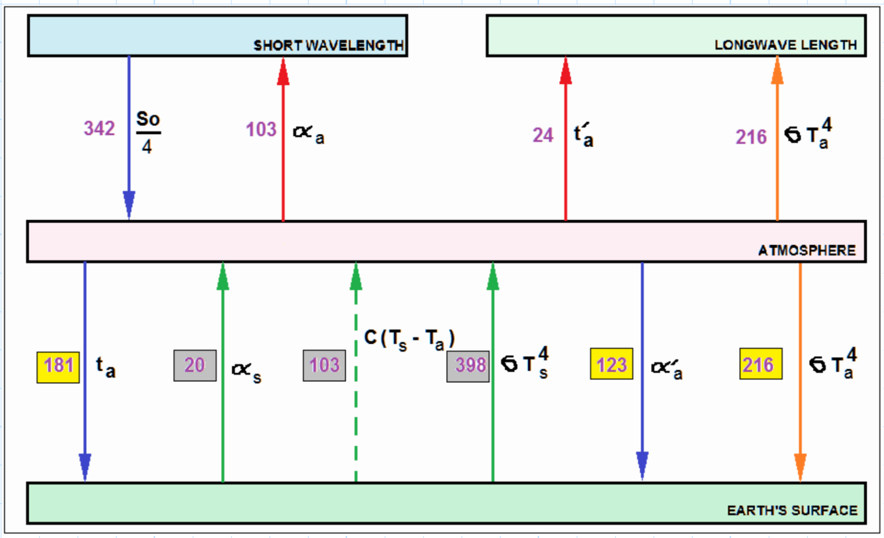
Sum of grey box flux leaving atmosphere:

103
24
216
123
216
181

Total Out: 863 W/m^2

Sum of energy flux entering and leaving the atmospheric region is equal 863W/m^2.
Its the same as Kirchoff's law on sum of currents at a node equal zero.
You got the node being the atmosphere's region under study.
Same thermodynamics or physics principle on black body radiation,....etc.

Next try for the sum of energy flux flow at the earth surface,
what goes in must equal what goes out.



Sum of yellow box flux entering earth surface:

181
123
216
Total In: 520 W/m^2

Sum of grey box flux leaving surface:

20
103
398
Total Out: 521 W/m^2

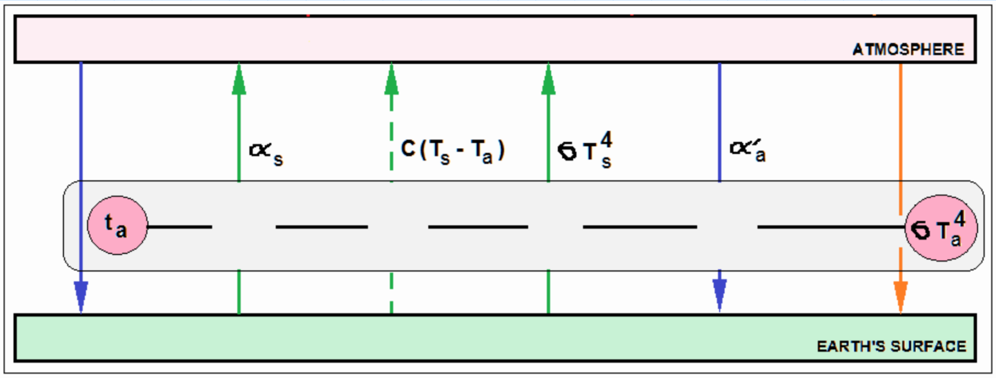
Sum of energy flux entering and leaving earth surface region is equal to 520 W/m^2.

The sum in is 520 the sum out is 521. The difference of 1 between them can be over looked, since its a large region under analysis/study.
Results were kept to nearest whole number instead of to decimal because it was known minor differences would exist. As in most scientific energy studies-analysis.

In the atmosphere during the travel of energy flux t_a to the earth's surface some of this energy flux is used up in the absorption process in the atmosphere.

How is that absorped portion of energy flux evaluated?

By use of the energy balance expressions already attained, with some maybe sorting out, and the use of black body radiation equations already used.



For the analysis use of ta and sigmaTa^4 shown in above figure.

A review of an earlier equation shown below.

$$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} + C(T_s - T_a) + \sigma T_s^4 (1 - t'_a - \alpha'_a) - 2\sigma T_a^4 = 0$$ Equation 2-atmosphere

The first term has parts of the energy flux expression, that makes the black object radiation, for the LHS and the RHS being sigma Ta^4.

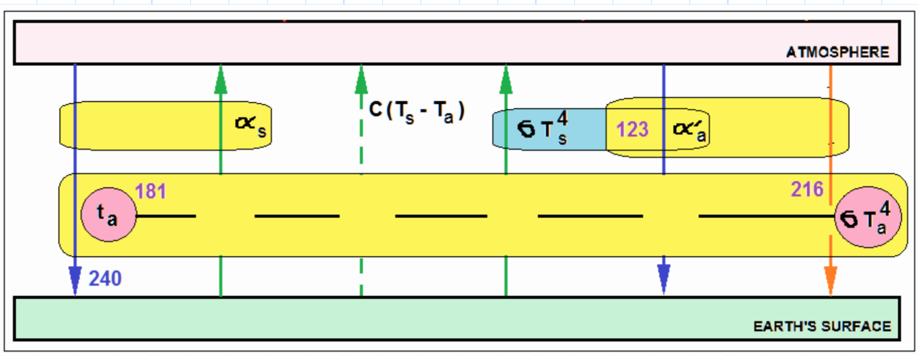
$$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} \quad \text{---> Reduces to --->} \quad (1-\alpha_a)\frac{S_o}{4}$$

The expression above to the right is related to the atmosphere, the surface reflection -t_a(1 - alpha_s) to the left excluded since it was accounted for in energy balance prior.
The black object radiation expression used prior in calculation shown below, its used for the continuing analysis.

$$\frac{S_o}{4}\cdot(1-\alpha_a) = \sigma \cdot T_a^4$$

$$\frac{S_o}{4}\cdot(1-\alpha_a) = 239.4 = 240 \qquad \sigma \cdot T_a^4 = 216$$

Rounded to the nearest 240.



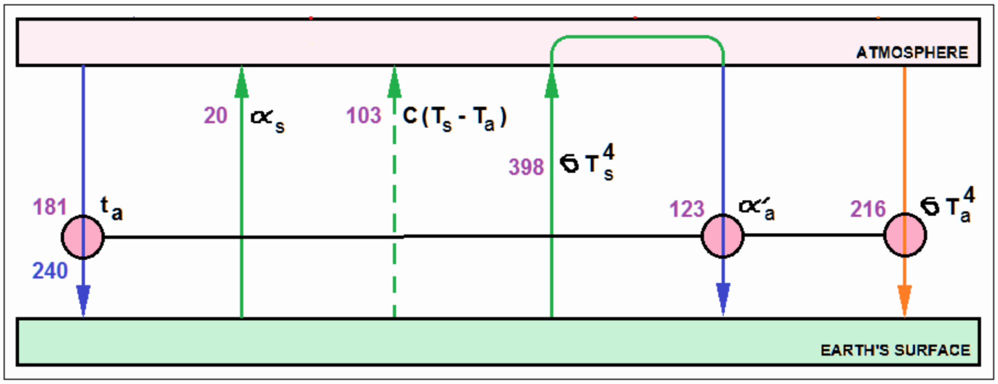
Relationships shown thru the yellow and light blue highlighting.

Absorption in the atmospheric layer caused short wave flux from the sun to be reduced from 240 to 181 W/m^2.

Difference: $240 - 181 = 59$ W/m^2.

$\sigma \cdot T_s^{\ 4} = 398$ **<---** Reflected back 123 W/m^2.

$\alpha'_a \cdot 398 = 123$ **<---** Reflected back.



Of the 240 sent 181 delivered with a loss to atmospheric absorption of 59 W/m^2.

Of the 398 W/m^2 long wave radiation sent out to atmosphere
123 W/m^2 returned thru reflection.

Then there is the atmosphere's radiation sent to earth surface of 216 W/m^2.

The loss due to absorption 59 W/m^2 is more than made-up for by the gain of 216 and 123 W/m^2. Resulting with an increase of the overall earth's surface temperature.

Thanks to VAA this much was accomplished with respect to the single layer modeling.

Next to example 4-3 the final example in Chapter 4.

Continued on next page.

Example 4-3 - Changing coefficients in energy blance equation(s):

Use an appropriate change in coefficients, on both energy balance equations (atmosphere and earth surface), from those given in the previous table to model the effect of changing characteristics of the atmosphere due to increasing GHGs on predicted Ts and Ta.

Solution:

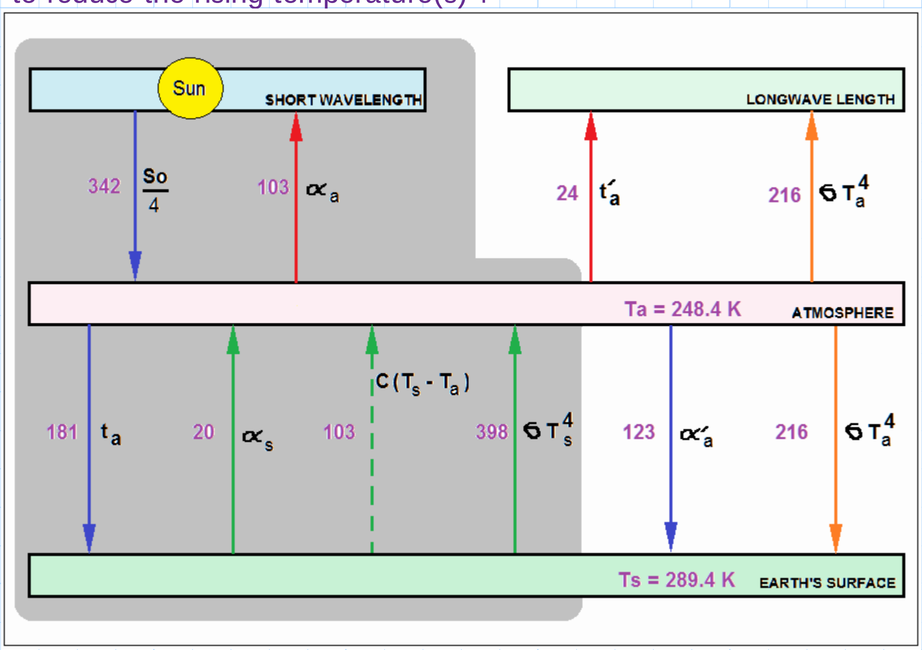
Previous table values provided below.

Initial values:

Inward Flux	Outward Flux	Conductive heat transfer
$\alpha_s := 0.11$	$t'_a := 0.06$	$C := 2.5$ W/m^2 K
$t_a := 0.53$	$\alpha'_a := 0.31$	
$\alpha_a := 0.3$		

Applicable constant not related to table above for the purpose of calculations:
$S_o := 1368$ W/m^2 $\sigma := 5.67 \cdot 10^{-8}$ W/m^2 K^4 Boltzman constant

Another way to simplify the question's requirement is to state it like this
'What characteristic (variable) could be changed in the energy balance equation(s) to reduce the rising temperature(s) ?'



Sun a resource provides solar energy flux this reaches earth's surface and is not likely to be reduced by human intervention. Shaded area parameters. Thus, solar energy flux stored in the atmosphere which is transmitted to space and earth's surface maybe the area to look at.

VAA - approach to the solution is to look at the long wave radiation.

The unshaded area of figure on previous page has the following terms:

Atmosphere: $t'_a := 398 \cdot 0.06 = 24$ $\sigma \cdot (T_a)^4 = 216$

Earth surface: $\alpha'_a \cdot 398 = 123$ $\sigma \cdot (T_a)^4 = 216$

$\alpha'_a \cdot 398 = 123$ **<---** This term is a reflection of sigma Ts^4.
Sigma Ts^4 is derived from the Sun's
solar energy flux (stored in earth's surface).

$\sigma \cdot (T_a)^4 = 216$ **<---** This term is from the atmosphere.
However it cancels off since its the same
value in two opposite directions.

Leaving only **t'a** to work with and its such a small value 24 W/m^2
in comparison to other values.

The temparatures for reference used thus far lets name them here the classic values
T-earth = 255K (earth surface) at a time in the past.
Lets say the recent decade studies place T-earth at 288 K (1980-2000).
Classic because they reference to a past 255K and a recent reference 288K to where GHG goes over 30 plus years to early 1980. 288K impact is considered a climate impact.
I am calling them classic numbers otherwise how do I reference to them since there are so many possible temperatures from calculations, and these 2 numbers keep coming up.
The difference between 288-255= 33K is another classic number. ***255K, 288K, and 33K.***

VAA - makes a strong case of what a coefficient change in t'a could do
to impact the temperatures of Ts and Ta in this example-theory.

$t'_a := 0.06$ reduce this to **--->** $t'_a := 0.04$ outward flux transmission
coefficient of a longwave.

Next the process need to be repeated to calculate Ts and Ta.
Set up the equations with the new ta' value then apply solver.

Initial values with only t'a updated:

Inward Flux

$\alpha_s := 0.11$
$t_a := 0.53$
$\alpha_a := 0.3$
$S_o := 1368$ W/m^2

Outward Flux

$t'_a := 0.04$
$\alpha'_a := 0.31$

Conductive heat transfer

$C := 2.5$ W/m^2 K

Boltzman constant:

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2 K^4

Substitute the above values into equations 1 and 2 :

$t_a(1-\alpha_s)\frac{S_o}{4} - C(T_s - T_a) - \sigma T_s^{\,4}\,(1-\alpha'_a) + \sigma T_a^{\,4} = 0$ Equation 1- earth surface

$161.439 + 2.5 \cdot T_a + 5.67 \cdot 10^{-8} \cdot T_a^{\,4} - 2.5 \cdot T_s - 3.912 \cdot 10^{-8} \cdot T_s^{\,4} = 0$

Equation 1 remains the same t'a term does not exist here.

$(1-\alpha_a - t_a(1-\alpha_s))\frac{S_o}{4} + C(T_s - T_a) + \sigma T_s^{\,4}\,(1 - t'_a - \alpha'_a) - 2\,\sigma T_a^{\,4} = 0$ Equation 2- atmosphere

$\sigma \cdot (1 - t'_a - \alpha'_a) = 3.686 \cdot 10^{-8}$ <--- New value.

$3.572 \cdot 10^{-8}$ <--- Previous value when t'a = 0.06.

Decrease from 0.06 to 0.04 results in the coefficient reduction of :

$3.680 \cdot 10^{-8} - 3.572 \cdot 10^{-8} = 1.08 \cdot 10^{-9}$

$78.08 - 2.5 \cdot T_a - 1.134 \cdot 10^{-7} \cdot T_a^{\,4} + 2.5 \cdot T_s + 3.686 \cdot 10^{-8} \cdot T_s^{\,4} = 0$ Updated equation.

Seting the same equation numbers to the theory side of things prior to Example 4-3 the new equation 3 shown below.

$161.439 + 2.5 \cdot T_a + 5.67 \cdot 10^{-8} \cdot T_a^{\,4} - 2.5 \cdot T_s - 3.912 \cdot 10^{-8} \cdot T_s^{\,4} = 0$ Equation 3 same here.

$78.08 - 2.5 \cdot T_a - 1.134 \cdot 10^{-7} \cdot T_a^{\,4} + 2.5 \cdot T_s + 3.686 \cdot 10^{-8} \cdot T_s^{\,4} = 0$ Equation 4 Updated.

Next to Excel Solver - same procedure.
Only work equation 4 here since one term changed here.
The RHS can be set to 0.05 that is the tolerance instead of 0.
When set to 0.0 the results is finer/closer.

Results Excel provided: $T_a := 249.4981$ $T_s := 291.2463$

$T_a := 249.5$ $T_s := 291.3$

VAA - results: $T_a := 250.5$ $T_s := 291.4$

Close enough, Excel results are satisfactory. Use VAA results to continue.

Change in earth surface temperature: $291.4 - 289.4 = 2$ K Increase in temperature.

In the updated energy balance NOT just one term is changed. Some of the other terms are also changed due to the new value of Ts and Ta.

Upper side of the diagram between space and atmosphere :

$\frac{S_o}{4} = 342$ $\alpha_a \cdot 342 = 103$ $T_a := 250.5$ $\sigma \cdot (T_a)^4 = 223$

Radiative heat transfer from surface to atmosphere : $T_s := 291.4$ $\sigma \cdot (T_s)^4 = 409$

New value for t'a : $t'_a := 409 \cdot 0.04 = 16$

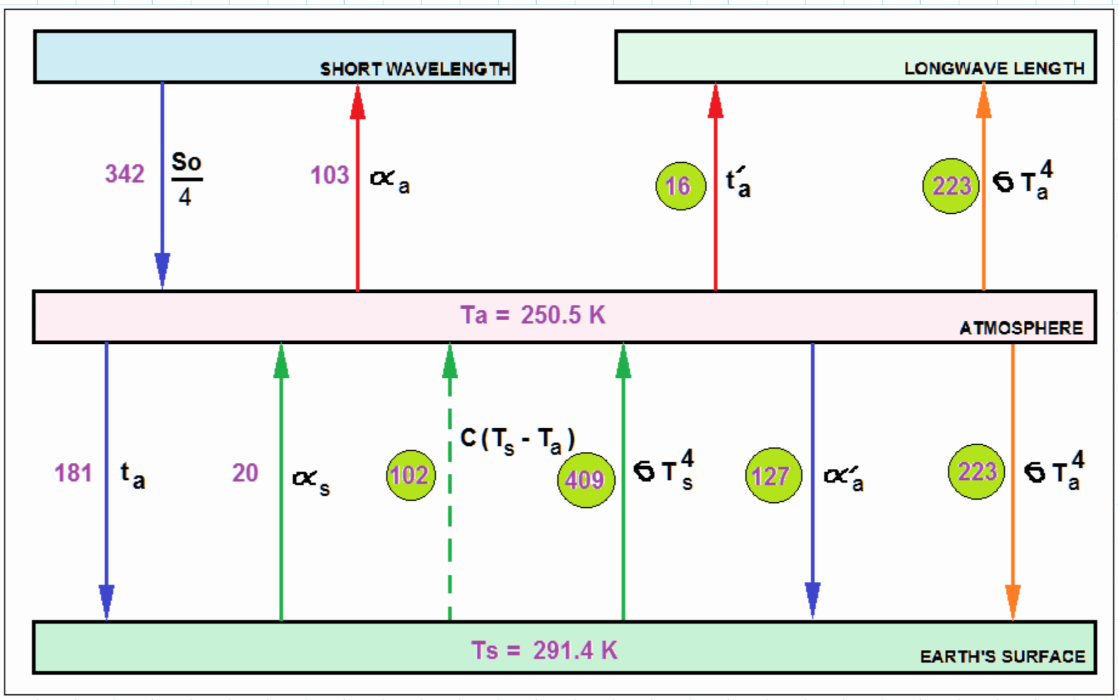
Change of t'a reduced by : $24 - 16 = 8$

Next the lower side of the diagram between atmosphere and surface :

$t_a \cdot \left(\frac{S_o}{4}\right) = 181$ $\alpha_s \cdot 181 = 20$ $C \cdot (T_s - T_a) = 102$ $\sigma \cdot (T_s)^4 = 409$

$\alpha'_a \cdot 409 = 127$ $\sigma \cdot (T_a)^4 = 223$

Next the updated energy balance diagram.



Energy balance diagram for t'a = 0.04 above with new/changed values highlighted in a green filled circle.

This change in Ts from 289.4 to 291.4 is temperature increase of 2K.
2K increase in earth surface temperature.

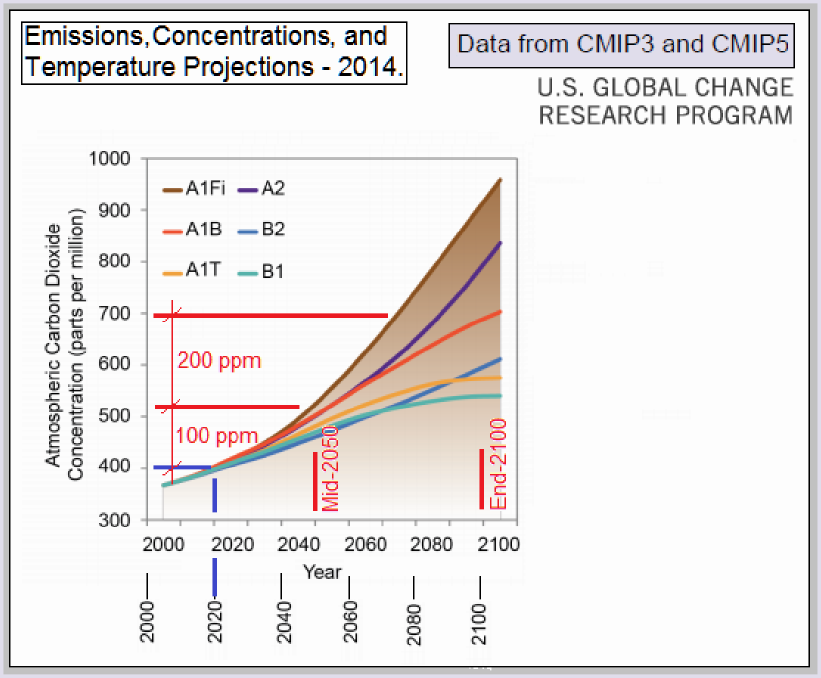
A decrease from t'a = 0.06 to 0.04, approximately a 33% decrease, gave a 2K rise in earth's surface temperature.

VAA page 126 - Thus a 33% decrease in the value of t'a in the model is equivalent to the predicted effect of an increase of 200 to 300 ppm in the concentration of CO_2 in the atmosphere, which might happen between the middle and end of the twenty-first century - Conclusion.

The parts per million increase of 200 to 300 is obtained from scientific predictions which have been plotted/graphed. Figure 4-5 in VAA serves to provide that resource for that CO_2 ppm. There are current graphs attainable elsewhere today on government agency websites. VAA figure 4-5 maybe prefered since they present it in a change of an established base 288K which is identified at 0 deg C. Re-sketched provided below.

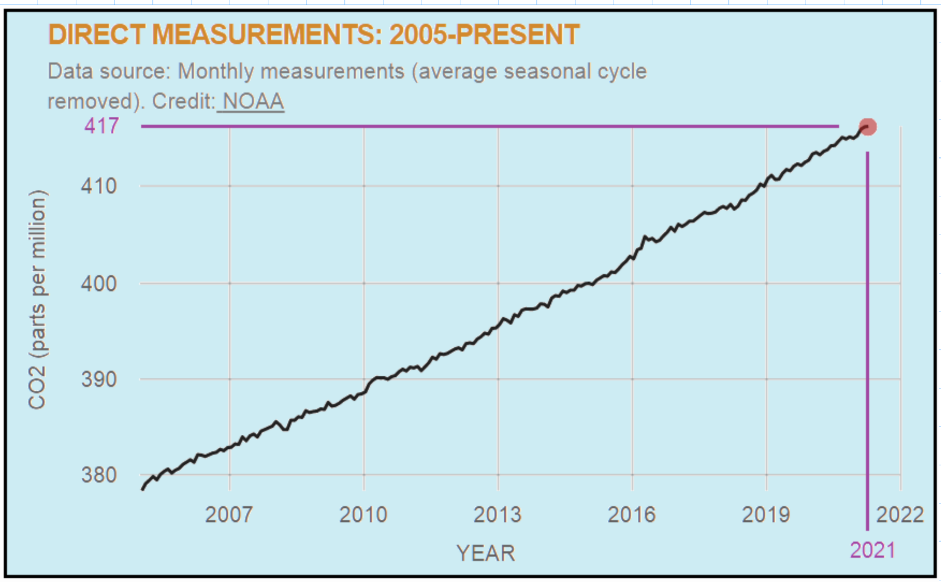
Rough sketch of Figure 4-5 VAA (source NOAA, and CDIAC)

The observed average temperature in the absence of recent warming in the period 1980-2000 is 288 K (i.e. the baseline temperature, *shown in red with 288 in the figure above*, where temperature anamoly equals 0 deg C) so the difference can be attributed to the effect of GHGs.



The SRES scenarios are named by family (A1, A2, B1, and B2), where each family is designed around a set of consistent assumptions: for example, a world that is more integrated or more divided.

The figure above is roughly one way to determine the 200 and 300 ppmv.
This figure was not intended for that though something like it may assist.
VAA has something similar to the above in figure 4-16.
However the temperature was not included as in the graph like in figure 4-5.
Figure below gives a monthly average thru measurement on a website.



There maybe more supporting information you may find elsewhere to provide additional results and conclusions. With that the solution to 4-3 comes to end.

General Circulation Models (GCMs) for global climate system.

VAA - General circulation models (GCMs) are sophisticated computer models of the global climate system that are used to predict the effect of changes in GHG concentration on climate in the future.

GCMs require multi-disciplines skills and knowledge working together toward a mathematical model that reaches to an acceptable level of confidence to predict future climate. Its a software algorithm with a relatively large database together with climate instrumentation for collecting data. Senior engineering students and practicing engineers are familair with these types of software, requirements for data capture, data base, and analysis.

The numerical model must resemble or act in the same manner as the physical processes in the climate's connected environment. The physical processs include atmosphere, land surface, ocean, cryosphere - *all snow and ice surfaces permafrost*, etc.

Areas of discipline include:
1. biology
2. chemistry
3. thermodynamics - heat transfer
4. fluid dynamics
5. meteorology
6. other related fields

Much can be said about GCMs in words however with a pictorial figure on a GCM things may become a little easier to understand.

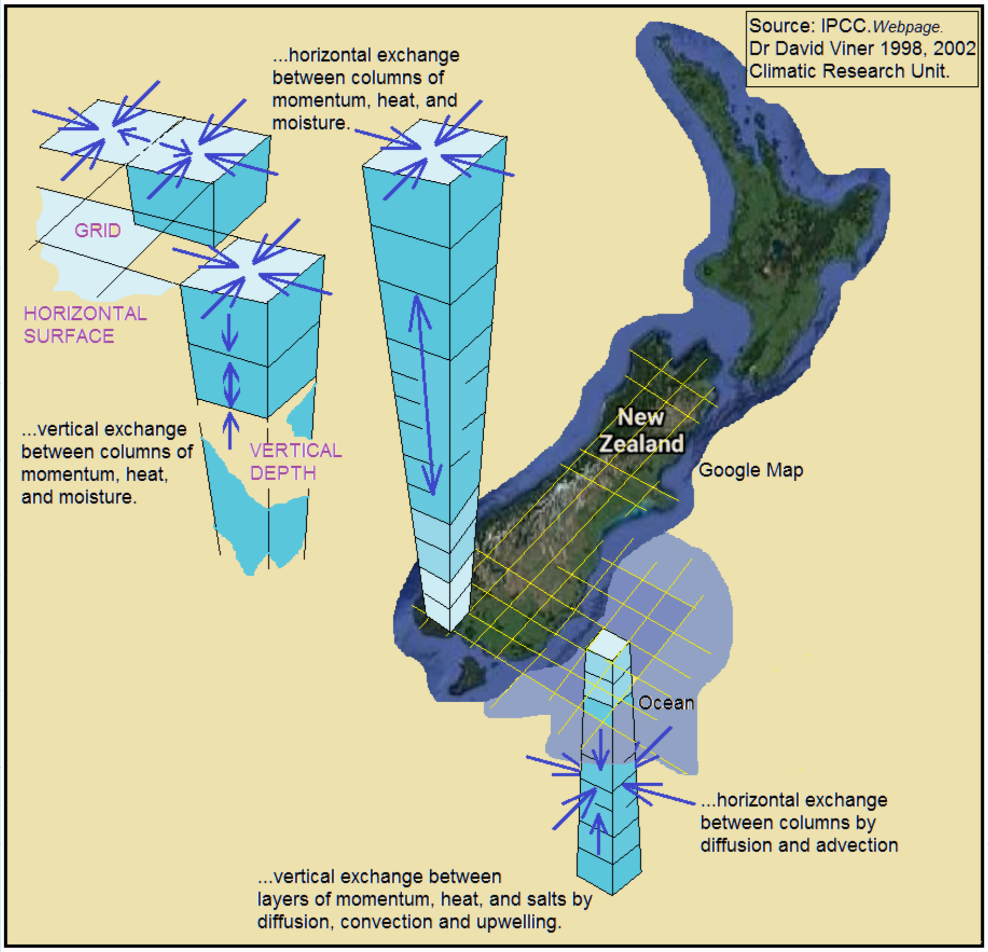
The figure on the next page comes from IPCC and it is creditted to Dr David Vilner.

This can be found on an IPCC, or similar multilayer model pictures on other related webpages.

Re-sketched it with most same notes and instead of England has New Zealand for the landscape.

The basic idea is a 3D grid with each cube being acted on vertically and horizontally.

Multilayer energy balance system shown below.



The skills from these fields generate system equations that model changes of state for a three dimensional system of grid points over multiple time periods. Typical time increment is 1 day (24 hours).

GCM-3D grid expands on the simple concepts laid out using the single layer model which was explained in the energy balance for climate change model by including the following additional criteria:

1. Dynamic modeling:

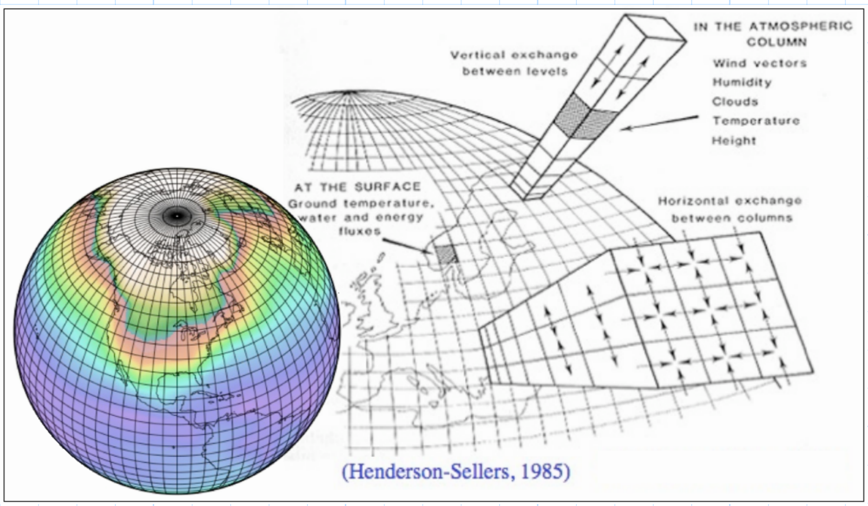
The basic, or starter, climate model based on energy blance did not have a time element. The values applied were static not changing with time OR they were not updated with changing data in a time interval. However GCMs are modeled with respect to changing time. Temperatures in multilayers changing with time so the model is a dynamic model.

2. Multiple vertical layers:

Energy balance concept-laws can be applied in multilayers together with atmospheric science and other related principles. Layers can be as high as 30 from ground surface into the atmosphere, and similarly if not more going down into the ocean depth. From the energy balance method we know the critical need is to solve for temperatures Ts and Ta. Multilayers permit the capture of temperature measurements at various layers using air baloons aircrafts, deep sea vessels...... to form a temperature gradient in the analysis. Also capture CO2 concentrations in the multilayers, and all other pretinent data for the succcess of the model. The outcome of which is to produce predictions of the effects in climate change. Example higher temperature recorded does that show higher concentrations of CO2? Same the other way round. Same for other parameters like nitrogen, methane, ...

3. Horizontal matrix grid points:

The grid is horizontal as far as you can see, curvature of the earth may or may not be applied. Valuable data is that collected by a region's major agency or a country's agency. In deveopled countires a few could work tgether to build a grid. Data need to be uploaded periodically to monitor the changes. Grid resolution impacts the accuracy of the result. Too small a grid is not practical, it has to be meaningful and where data collected represents somewhat uniformly. A 1985 GCM model is shown below. Data collected is wind vectors, humidity, clouds, temperature, and height with respect to layer.



(Henderson-Sellers, 1985)

Source:
SERC website.
Global grid maybe a reality for who is able to provide the resources to capture the data. It would certainly be more meaningful. *USA usually is capable of providing the finances.*

IPCC has a grid size of 250 km x 600 km.
This is the horizontal plane of the grid.
You may also see grid resolution of 2.5 degrees latitude x 3.75 degrees longitude.
Canadian GCM 3.75 x 3.75 degs, France has 2.8 x 2.8 degs,.... relative to country's need.
Vertical depth can be 1 km each layer for 20 layers that is 20 km high a column.
Depth dimension dependent on the resources available for data capture.

4. Convection:

In the simple model only conductive and radiative heat transer were included, in the GCM type national or commercial model convective heat transfer would be included. Comes down to amount of financial resources spent in capturing accurate data which then can be run thru computer simulations. There is real time data transmitted from land based weather stations with some at higher elevations, and here too GCM of a specific purpose can be running in real time with outputs over an interval of time.

5. More sophisticated temperature forcing:

In a single layer model the forcing function is the solar energy input, assumed constant. In addition historic fluctuations of solar energy may be applied for future simulations, together with impact from volcanic activity in grid and surrounding girds, and other similar energy deviations.

6. Biological processes:

GCM's take into consideration the forest cover which takes in CO_2, and earth reflective surfaces that reflect sunlight. Plants take in CO_2 reduces GHG, and high reflection increases radiative energy sent into the atmosphere.

7. Changes in ice cover:

Similar to biological processes, ice cover in the artic, antartic, and winter snow reflects sunlight back into the atmosphere. Though the earth's surface temperature at these locations are low during winter. So changing temperature impacts ice coverage - artic icebergs breaking away is an effect of global warming.

Many scientific groups around the world have their GCM based on their own thinking operating in various parts of the world. There are national GCMs run by a country's agency these provide climate change predictions, GHG emissions, suggest corrective actions, and may also assist the agricultural industry.

Closing Comments:

There is much more to say however not a single exercise file as this can cover the work of volumes of journals and textbooks on this subject. Resources are available for you to find online, book shops, and technical college libraries. It is obvious from some of the calculations, and its true, this filed is also moving from science to engineering. When theories become more solid, instrumentation device appear, data capturing begin, simulations run,......the work becomes more engineering like. *Thanks to VAA.*

Climate Change Basic Modeling. Basic Models On Climate Change and Climate Modeling.
Textbook Resources: 1). Energy Systems Engineering (Vanek Albright & Angenent) - Primary.
2). Other references indicated in notes.
Notes by **Karl Bogha**. My Homework.

Informative figure below well known to the public where
the consequences are concerned.
Its on the news all to often.

Human Influences Apparent in Many Aspects of the Changing Climate

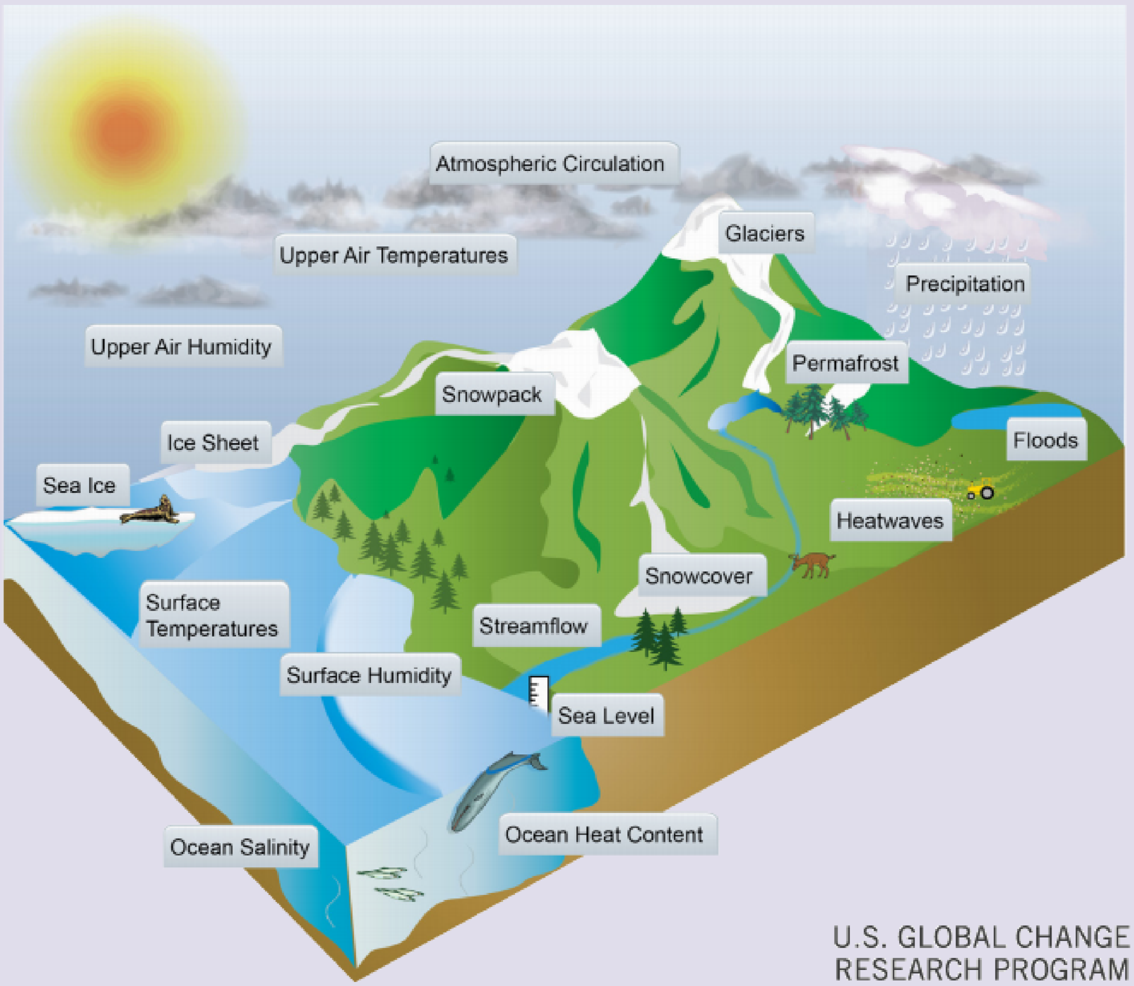


Figure shows examples of the many aspects of the climate system in which changes have been formally attributed to human emissions of heat-trapping gases and particles by studies published in peer-reviewed science literature. For example, observed changes in surface air temperature at both the global and continental levels, particularly over the past 50 years or so, cannot be explained without including the effects of human activities. While there are undoubtedly many natural factors that have affected climate in the past and continue to do so today, human activities are the dominant contributor to recently observed climate changes. (Figure source: NOAA NCDC).