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1 restart
with(Units) : with(ThermophysicalData) :
Automatically loading the Units[Simple] subpackage

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## Variables

### physical parameters

$$d := 0.038 \text{ cm}$$

$$d := 0.038 \text{ cm} \quad (1.1.1)$$

$$\phi := 0.620$$

$$\phi := 0.620 \quad (1.1.2)$$

$$\varphi := 0.789$$

$$\varphi := 0.789 \quad (1.1.3)$$

$$Void_f := .41241$$

$$Void_f := 0.41241 \quad (1.1.4)$$

$$Bloodpath := 10.2 \text{ cm}$$

$$Bloodpath := 10.2 \text{ cm} \quad (1.1.5)$$

$$A_f := 21.55 \text{ cm}^2$$

$$A_f := 21.55 \text{ cm}^2 \quad (1.1.6)$$

### biological variables

$$P_{baro} := 765 \text{ mmHg}$$

$$P_{baro} := 765 \text{ mmHg} \quad (1.2.1)$$

$$FiO2 := .45$$

$$FiO2 := 0.45 \quad (1.2.2)$$

$$Q_{blood} := 4990 \frac{\text{mL}}{\text{min}}$$

$$Q_{blood} := 4990 \frac{\text{mL}}{\text{min}} \quad (1.2.3)$$

$$T_{art} := 32.5 \text{ Celsius}$$

$$T_{art} := 32.5 \text{ } ^\circ\text{C} \quad (1.2.4)$$

$$Hgb := 101 \frac{\text{g}}{\text{L}}$$

$$Hgb := 101 \frac{\text{g}}{\text{L}} \quad (1.2.5)$$

$$Hct := \frac{\text{convert}(Hgb, \text{unit\_free}) \cdot 3}{1000.0} \quad Hct := 0.3030000000 \quad (1.2.6)$$

$$PvO2 := 45.2 \text{ mmHg} \quad PvO2 := 45.2 \text{ mmHg} \quad (1.2.7)$$

## Calculate water vapor

The formula used below comes from Huang 2018 "A Simple Accurate Formula for Calculating Saturation Vapor Pressure of Water and Ice"

$$T_b := \text{convert}(T_{art}, \text{unit\_free}) \quad T_b := 32.5 \quad (2.1)$$

$$P_s(T) := \frac{\exp\left(34.494 - \frac{4924.99}{(T + 237.1)}\right)}{(T + 105)^{1.57}} \quad P_s := T \mapsto e^{34.494 + \left(-\frac{4924.99}{T + 237.1}\right)} \left(\gamma((T + 105)^{1.57})\right) \quad (2.2)$$

$$P_s(T_b) \text{ Pa} \quad 36.72190236 \text{ mmHg} \quad (2.3)$$

$$P_{baroCor} := (P_{baro} - P_s(T_b) \text{ Pa}) \cdot FiO2 \quad 327.7251438 \text{ mmHg} \quad (2.4)$$

## Temperature conversion PaO2

$$pH_a := 7.37 \quad pH_a := 7.37 \quad (3.1)$$

$$pH_v := 7.342 \quad pH_v := 7.342 \quad (3.2)$$

$$n := 2.7 \quad n := 2.7 \quad (3.3)$$

$$T_{bloodgasMachine} := 37 \text{ Celsius} \quad T_{bloodgasMachine} := 37 \text{ }^{\circ}\text{C} \quad (3.4)$$

$$P_{50}(pH) := (26.6 \cdot 10^{(0.48 \cdot (7.4 - pH))}) \quad P_{50} := pH \mapsto 26.6 \cdot 10^{0.48 \cdot (7.4 + (-pH))} \quad (3.5)$$

$$P50(pH, T) := (P_{50}(pH) \cdot 10^{-(0.024 \cdot (37 - \text{convert}(T, \text{unit\_free})))}) \text{ mmHg} \quad P50 := (pH, T) \mapsto P_{50}(pH) \cdot 10^{-0.024 \cdot (37 + (-\text{convert}(T, \text{unit\_free})))} \text{ 'mmHg'} \quad (3.6)$$

$$P50_a := P50(pH_a, T_{bloodgasMachine}) \\ P50_a := 27.49676718 \text{ mmHg} \quad (3.7)$$

$$P50_v := P50(pH_v, T_{bloodgasMachine}) \\ P50_v := 28.36100632 \text{ mmHg} \quad (3.8)$$

$$S(PO2, P50) := \left( \frac{\left( \frac{PO2}{P50} \right)^n}{1 + \left( \frac{PO2}{P50} \right)^n} \right) \\ S := (PO2, P50) \mapsto (PO2 (\vee(P50)))^n \left( \vee(1 + (PO2 (\vee(P50)))^n) \right) \quad (3.9)$$

$$S_v := S(PvO2, P50_v) \\ S_v := 0.7787541952 \quad (3.10)$$

$$P50_{ac} := P50(pH_a, T_{art}) \\ P50_{ac} := 21.44280699 \text{ mmHg} \quad (3.11)$$

$$P50_{vc} := P50(pH_v, T_{art}) \\ P50_{vc} := 22.11676669 \text{ mmHg} \quad (3.12)$$

$$P50_c := \frac{P50_{ac} + P50_{vc}}{2} \\ P50_c := 21.77978684 \text{ mmHg} \quad (3.13)$$

$$PO2_c(S, P50) := \left( \exp \left( \frac{\ln \left( \frac{-S}{(S-1)} \right)}{n} \right) \cdot P50 \right) \\ PO2_c := (S, P50) \mapsto e^{\ln((-S)(\vee(S-1)))(\vee(n))} P50 \quad (3.14)$$

$$PvO2_c := PO2_c(S_v, P50_{vc}) \\ PvO2_c := 35.24832098 \text{ mmHg} \quad (3.15)$$

$$S_{vc} := S(PvO2_c, P50_{vc}) \\ S_{vc} := 0.7787541952 \quad (3.16)$$

## Calculation Mass Transfer

### Solubility of Oxygen

$$k_c := 4.658 \cdot 10^{-5} \cdot (1.01)^{(37 - T_b)} \frac{\text{mL}}{\text{mmHg}}$$

$$k_c := 0.00004871308857 \frac{1}{\text{mmHg}} \quad (4.1.1)$$

$$k_p := 2.855 \cdot 10^{-5} \cdot 1.01^{(37 - T_b)} \frac{\frac{\text{mL}}{\text{mmHg}}}{\text{mL}}$$

$$k_p := 0.00002985742118 \frac{1}{\text{mmHg}} \quad (4.1.2)$$

$$k := (k_c \cdot Hct + k_p \cdot (1 - Hct))$$

$$3.56 \times 10^{-5} \frac{1}{\text{mmHg}} \quad (4.1.3)$$

$$\lambda(PvO2_c) := \frac{1}{1000} \left( \frac{1.34}{convert(k, unit\_free)} \cdot convert(Hgb, unit\_free) \cdot \left( \frac{n}{convert(P50_c, unit\_free)} \right) \cdot \left( \frac{convert(PvO2_c, unit\_free)}{convert(P50_c, unit\_free)} \right)^{(n-1)} \cdot \left( \frac{1}{\left( 1 + \left( \frac{convert(PvO2_c, unit\_free)}{convert(P50_c, unit\_free)} \right)^n \right)^2} \right) \right)$$

$$\lambda := PvO2_c \mapsto \frac{1.34}{convert(k, unit\_free)} convert(Hgb, unit\_free) (n (\wedge (convert(P50_c, unit\_free)))) (convert(PvO2_c, unit\_free) (\wedge (convert(P50_c, unit\_free))))^n$$

$$-1 (\wedge ((1 + (convert(PvO2_c, unit\_free) (\wedge (convert(P50_c, unit\_free))))^n)^2)) (\wedge (1000)))$$

$$\lambda(PvO2_c) = 49.05348866 \quad (4.1.5)$$

## Diffusivity of Oxygen

$$D_c := 0.76 \cdot 10^{-5} \cdot 1.025^{(T_b - 25)}$$

$$D_c := 9.146239783 \cdot 10^{-6} \quad (4.2.1)$$

$$D_p := 1.62 \cdot 10^{-5} \cdot 1.025^{(T_b - 25)}$$

$$D_p := 0.00001949593217 \quad (4.2.2)$$

$$N := \frac{k_c}{k_p}$$

$$N := 1.631523643 \quad (4.2.3)$$

$$\beta := \left( \frac{1}{3} \cdot \left( \left( \left( \frac{2}{1 + \left( \left( N \cdot \frac{D_c}{D_p} - 1 \right) \cdot \left( \frac{0.283}{2} \right) \right)} + \frac{1}{1 + \left( \left( N \cdot \frac{D_c}{D_p} - 1 \right) \cdot (1 - 0.283) \right)} \right) \cdot \left( N \cdot \frac{D_c}{D_p} \right) - 1 \right) \right) \right) \right)$$

$$\beta := -0.2557766615 \quad (4.2.4)$$

$$\kappa := -\frac{\left( N \cdot \frac{D_c}{D_p} - 1 \right) - \left( N \cdot \frac{D_c}{D_p} \right) \cdot \beta}{\left( N \cdot \frac{D_c}{D_p} - 1 \right) - \beta}$$

$$\kappa := 1.832656150 \quad (4.2.5)$$

$$R := \left( Hct \cdot \frac{\left( N \cdot \frac{D_c}{D_p} - 1 \right)}{\left( N \cdot \frac{D_c}{D_p} + \kappa \right)} \right)$$

$$R := -0.02735959621 \quad (4.2.6)$$

$$D_m := \left( D_p \cdot \left( \frac{k_p}{k} \right) \cdot \frac{1 + \kappa \cdot R}{1 - R} \right) \frac{\text{cm}^2}{\text{s}}$$

$$1.513006565 \cdot 10^{-9} \frac{\text{m}^2}{\text{s}} \quad (4.2.7)$$

$$D_{eff} := \frac{D_m}{1 + \lambda(PvO_2_c)}$$

$$3.022779442 \cdot 10^{-11} \frac{\text{m}^2}{\text{s}} \quad (4.2.8)$$

## Density

$$\rho_c := 1.090 \frac{\text{kg}}{\text{L}}$$

$$\rho_c := 1.090 \frac{\text{kg}}{\text{L}} \quad (4.3.1)$$

$$\rho_p := 1.035 \frac{\text{kg}}{\text{L}}$$

$$\rho_p := 1.035 \frac{\text{kg}}{\text{L}} \quad (4.3.2)$$

$$\rho := (\rho_c \cdot Hct + \rho_p \cdot (1 - Hct))$$

$$1051.66 \frac{\text{kg}}{\text{m}^3} \quad (4.3.3)$$

## Viscosity

$$T_{visc} := convert(T_b (\text{Unit}(Celsius, preserve)), \text{temperature}, K)$$

$$T_{visc} := 305.6500000 \text{ K} \quad (4.4.1)$$

$$\eta_p := \exp\left(-5.64 + \left(\frac{1800}{convert(T_{visc}, \text{unit\_free})}\right)\right) \text{ mPa}\cdot\text{s}$$

$$\eta_p := 1.282855980 \text{ mPa s} \quad (4.4.2)$$

$$\text{eta} := (\eta_p \cdot \exp(2.31 \cdot Hct))$$

$$2.583173875 \text{ mPa s} \quad (4.4.3)$$

$$v := \frac{\text{eta}}{\rho}$$

$$v := 2.456270652 \cdot 10^{-6} \frac{\text{m}^2}{\text{s}} \quad (4.4.4)$$

## Dimensionless Numbers

$$N_{re} := \frac{Q_{blood} \cdot d}{(1 - Void_f) \cdot A_f \cdot v}$$

$$N_{re} := 10.16096609 \quad (4.5.1)$$

$$N_{sc} := \frac{\text{eta}}{\rho \cdot D_m}$$

$$N_{sc} := 1623.436877 \quad (4.5.2)$$

## Differential Equation

Convert flow from mL/min to mL/s

$$Q_{bc} := convert(Q_{blood}, \text{'units'}, \frac{\text{'mL'}}{\text{'s'}})$$

$$\frac{499}{6} \frac{\text{mL}}{\text{s}} \quad (4.6.1)$$

Convert  $\eta$  from mPa\*s to g/cm\*s

$$\eta_c := convert(\text{eta}, \text{'units'}, \frac{\text{'g'}}{\text{'cm'} \cdot \text{'s'}})$$

$$\eta_c := 0.02583173875 \frac{\text{g}}{\text{cm s}} \quad (4.6.2)$$

$$a := \left( \frac{4}{Void_f} \cdot \left( \frac{(1 - Void_f)}{convert(d, unit\_free)} \right)^{(1 + \phi)} \cdot \left( \frac{convert(A_f, unit\_free) \cdot convert(\eta_c, unit\_free)}{convert(Q_{bc}, unit\_free) \cdot convert(rho, unit\_free)} \right)^\phi \cdot \frac{\phi}{\left( \frac{v}{D_m} \right)^{\frac{2}{3}}} \right) \cdot 0.1080566016 \quad (4.6.3)$$

$$P_b := convert(convert(P_{baroCor}, 'units', 'mmHg'), unit\_free) \\ P_b := 327.7251438 \quad (4.6.4)$$

$$BP := convert(Bloodpath, unit\_free) \\ BP := 10.2 \quad (4.6.5)$$

$$ODE := diff(P(x), x) = a \cdot \frac{(P_b - P(x))}{\left(1 + lambda(P(x))\right)^{\frac{2}{3}}} \\ ODE := \frac{d}{dx} P(x) = \frac{0.1080566016 (327.7251438 - P(x))}{1 + 1.844881967 \left( \frac{P(x)^{1.7}}{(1 + 0.0002439243816 P(x)^{2.7})^2} \right)^{2/3}} \quad (4.6.6)$$

$$IC := P(0) = convert(PvO2_c, unit\_free) \\ IC := P(0) = 35.24832098 \quad (4.6.7)$$

$$h := 1 \# stepsize \\ h := 1 \quad (4.6.8)$$

$$sol := dsolve(\{ODE, IC\}, numeric) \\ sol := \text{proc}(x\_rkf45) \dots \text{end proc} \quad (4.6.9)$$

$$eval(sol(BP)) \\ [x = 10.2, P(x) = 78.3119963266407] \quad (4.6.10)$$

plots:-odeplot(sol, [x, P(x)], x = 0 .. BP, gridlines, labels = ["Blood path length [cm]", "Increase in PO2"], labeldirections = [horizontal, vertical])

