**Retrofitted slab**

Existing slab

$l_n := 2500\ mm$ $\quad f'_c := 27\ MPa$ $\quad A_s := 355\ mm^2$

$h := 150\ mm$ $\quad \varepsilon_{cu} := 0.003$ $\quad d := 120\ mm$

$b := 900\ mm$ $\quad \gamma_c := 24\ \dfrac{kN}{m^3}$ $\quad A_{sc} := 355\ mm^2$

$w_c := \gamma_c \cdot b \cdot h = 3.24\ \dfrac{kN}{m}$ $\quad \alpha_1 := 0.85$ $\quad d_c := 30\ mm$

$E_c := 4700 \cdot \sqrt{f'_c \cdot (MPa)} = \left(2.44 \cdot 10^4\right)\ MPa$ $\quad E_s := 200000\ MPa$

$$f\beta_1(f') := \text{if}\left(f' \le 28\ MPa, 0.85, \text{if}\left(f' \ge 56\ MPa, 0.65, 1.05 - 0.05 \cdot \frac{f' \cdot MPa^{-1}}{6.9}\right)\right)$$

$\beta_1 := f\beta_1(f'_c) = 0.85$ $\quad n := \dfrac{E_s}{E_c} = 8.2$ $\quad f_y := 400\ MPa$

$\phi_v := 0.75$ $\quad \phi_f := 0.9$ $\quad \psi_f := 0.85$

Overlay $\quad t_H := 30\ mm$ $\quad f'_H := 70\ MPa$ $\quad \beta_{1H} := f\beta_1(f'_H) = 0.65$

CFRP $\quad t_F := 0.36\ mm$ $\quad f'_{fu} := 1170\ MPa$ $\quad \gamma_F := 1200\ \dfrac{kg}{m^3}$ $\quad \gamma_a := 1200\ \dfrac{kg}{m^3}$

$b_F := b = 900\ mm$ $\quad CE := 1$ $\quad n_F := 1$ $\quad t_a := 1\ mm$

$E_F := 72600\ MPa$ $\quad f_{fu} := f'_{fu} \cdot CE = 1170\ MPa$ $\quad \varepsilon_{fu} := \dfrac{f_{fu}}{E_F} = 0.016$

$A_F := t_F \cdot b_F = 324\ mm^2$

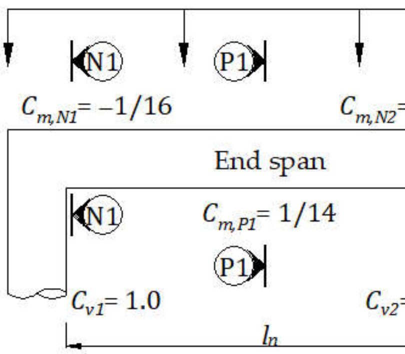
Moment and shear coefficients

$C_{mN1} := \dfrac{1}{16}$ $\quad C_{mN2} := \dfrac{1}{10}$ $\quad C_{mN3} := \dfrac{1}{11}$ $\quad C_{v1} := 1$ $\quad C_{v2} := 1.15$ $\quad C_{v3} := 1$

$C_{mP1} := \dfrac{1}{14}$ $\quad C_{mP2} := \dfrac{1}{16}$

$$C_m := \begin{bmatrix} -\frac{1}{16} \\ \frac{1}{14} \\ -\frac{1}{10} \\ -\frac{1}{11} \\ \frac{1}{16} \\ -\frac{1}{11} \end{bmatrix}$$

$$M_u := C_m \cdot w_c \cdot (l_n)^2 = \begin{bmatrix} -1.266 \\ 1.446 \\ -2.025 \\ -1.841 \\ 1.266 \\ -1.841 \end{bmatrix} kN \cdot m$$



$$C_v := \begin{bmatrix} 1 \\ 1.15 \\ 1 \\ 1 \end{bmatrix}$$

$$V_u := \frac{C_v \cdot w_c \cdot (l_n)}{2} = \begin{bmatrix} 4.05 \\ 4.658 \\ 4.05 \\ 4.05 \end{bmatrix} \; \boldsymbol{kN}$$

Maximum

$$M_{u.P} := \left|\max\left(M_u\right)\right| = 1.446 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$M_{u.N} := \left|\min\left(M_u\right)\right| = 2.025 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$V_{u.max} := \max\left(\left|\max\left(V_u\right)\right|, \left|\min\left(V_u\right)\right|\right) = 4.658 \; \boldsymbol{kN}$$

Compute Icr at support

$$\rho := \frac{A_s}{b \cdot d} = 0.003$$

$$k_N := \sqrt{2 \cdot \rho \cdot n + (\rho \cdot n)^2} - \rho \cdot n = 0.207$$

$$x_N := k_N \cdot d = 24.8 \; \boldsymbol{mm}$$

$$I_{crN} := \frac{b \cdot x_N^{\,3}}{3} + n \cdot A_s \cdot \left(d - x_N\right)^2 = \left(3.09 \cdot 10^7\right) \; \boldsymbol{mm}^4$$

Control slab

- At support

$$c_{cN} := \frac{A_s \cdot f_y}{\alpha_1 \cdot \beta_1 \cdot f'_c \cdot b} = 8.09 \; \boldsymbol{mm}$$

$$M_{nN} := \left(d - \frac{c_{cN} \cdot \beta_1}{2}\right) \cdot A_s \cdot f_y = 16.55 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$M_{rNc} := \phi_f \cdot M_{nN} = 14.9 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$V_{rc} := \phi_v \cdot \left(d \cdot \sqrt{f'_c \cdot \boldsymbol{MPa}}\right) \cdot \frac{b}{6} = 70.1 \; \boldsymbol{kN}$$

- At mid-span

$$c_{cP} := \frac{A_s \cdot f_y}{\alpha_1 \cdot \beta_1 \cdot f'_c \cdot b} = 8.09 \; \boldsymbol{mm}$$

$$M_{nP} := \left(d - \frac{c_{cP} \cdot \beta_1}{2}\right) \cdot A_s \cdot f_y = 16.55 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$M_{rPc} := \phi_f \cdot M_{nP} = 14.9 \; \boldsymbol{kN} \cdot \boldsymbol{m}$$

Control slab + Retrofit system

- At support

$$J_{1N} := \alpha_1 \cdot f'_c \cdot b$$

$$J_{2N} := t_F \cdot b_F \cdot E_F \cdot \varepsilon_{cu} - A_s \cdot f_y$$

$$J_{3N} := -\beta_1 \cdot t_F \cdot b_F \cdot E_F \cdot \varepsilon_{cu} \cdot \left(h + \frac{t_F}{2}\right)$$

$$a_{sN} := \frac{-J_{2N} + \sqrt{{J_{2N}}^2 - 4 \cdot J_{1N} \cdot J_{3N}}}{2 \cdot J_{1N}} = 22.68\ mm$$

$$c_{sN} := \frac{a_{sN}}{\beta_1} = 26.69\ mm$$

- At mid-span

$$J_{1P} := \alpha_1 \cdot f'_H \cdot b$$

$$J_{2P} := t_F \cdot b_F \cdot E_F \cdot \varepsilon_{cu} - A_s \cdot f_y$$

$$J_{3P} := -\beta_{1H} \cdot t_F \cdot b_F \cdot E_F \cdot \varepsilon_{cu} \cdot \left(t_H + \frac{t_F}{2}\right)$$

$$a_{sP} := \frac{-J_{2P} + \sqrt{{J_{2P}}^2 - 4 \cdot J_{1P} \cdot J_{3P}}}{2 \cdot J_{1P}} = 5.79\ mm$$

$$c_{sP} := \frac{a_{sP}}{\beta_{1H}} = 8.92\ mm$$

2. Check Overlay strength is large enough to generate tension in CFRP at mid-span section

$$f'_{H1} := \frac{\varepsilon_{cu} \cdot E_F}{1.445} \left(\frac{t_F}{t_H}\right)^2 + f_y \cdot \frac{A_s}{b} \frac{1}{0.7225 \cdot t_H} = 7.3\ MPa$$

$$f'_{H2} := 0.15 \cdot f'_c + \frac{\varepsilon_{cu} \cdot E_F}{1.7} \left(\frac{t_F}{t_H}\right)^2 + f_y \cdot \frac{A_s}{b} \cdot \frac{1}{0.85 \cdot t_H} = 10.26\ MPa$$

$$\text{if}\left(f'_H \ge \max\left(f'_{H1}, f'_{H2}\right), \text{“OK”}, \text{“N.G”}\right) = \text{“OK”}$$

3. Existing state of strain at support

$$M_{DN2} := C_{mN2} \cdot w_c \cdot \left(l_n\right)^2 = 2.03\ kN \cdot m$$

$$\varepsilon_{bi} := M_{DN2} \cdot \frac{\left(h - c_{cN}\right)}{I_{crN} \cdot E_c} = 3.81 \cdot 10^{-4}$$

4. Design strain of CFRP

$$\varepsilon_{fd} := min\left(0.41 \cdot \sqrt{\frac{f'_c}{n_F \cdot E_F \cdot t_F \left(\frac{1}{mm}\right)}}, 0.9 \cdot \varepsilon_{fu}\right) = 0.0132$$

5. Neutral axis depth for

6. Check FRP strain

$$\varepsilon_{feN}\left(c_N\right) := min\left(\varepsilon_{cu} \cdot \left(\frac{h - c_N}{c_N}\right), \varepsilon_{fd}\right)$$

$$\varepsilon_{feP}\left(c_P\right) := min\left(\varepsilon_{cu} \cdot \left(\frac{t_H - c_P}{c_P}\right), \varepsilon_{fd}\right)$$

$$\varepsilon_{cN}\left(c_N\right) := min\left(\left(\varepsilon_{feN}\left(c_N\right) + \varepsilon_{bi}\right) \frac{c_N}{h - c_N}, \varepsilon_{cu}\right)$$

$$\varepsilon_{cP}\left(c_P\right) := min\left(\varepsilon_{feP}\left(c_P\right) \frac{t_H + \frac{t_F}{2} - c_P}{c_P}, \varepsilon_{cu}\right)$$

7,8. Check stress level in the steel and FRP

$$\varepsilon_{sN1}(c_N) := (\varepsilon_{feN}(c_N) + \varepsilon_{bi}) \cdot \frac{(d - c_N)}{h - c_N}$$

$$\varepsilon_{sP1}(c_P) := \varepsilon_{cP}(c_P) \cdot \frac{(d + t_H + t_F - c_P)}{c_P}$$

$$f_{sN}(c_N) := min(E_s \cdot \varepsilon_{sN1}(c_N), f_y)$$

$$f_{sP}(c_P) := min(E_s \cdot \varepsilon_{sP1}(c_P), f_y)$$

$$f_{feN}(c_N) := min(E_F \cdot \varepsilon_{feN}(c_N), f_{fu})$$

$$f_{feP}(c_P) := min(E_F \cdot \varepsilon_{feP}(c_P), f_{fu})$$

9. Force equilibrium is verified be checking the initial estimate of c with Eq. (10-12).

$$\varepsilon'_{cN} := 1.7 \cdot \frac{f'_c}{E_c} = 0.002$$

$$\varepsilon'_{cP} := 1.7 \cdot \frac{f'_H}{E_c} = 0.005$$

$$\beta_{1N}(c_N) := \frac{4 \cdot \varepsilon'_{cN} - \varepsilon_{cN}(c_N)}{6 \cdot \varepsilon'_{cN} - 2 \cdot \varepsilon_{cN}(c_N)}$$

$$\beta_{1P}(c_P) := \frac{4 \cdot \varepsilon'_{cP} - \varepsilon_{cP}(c_P)}{6 \cdot \varepsilon'_{cP} - 2 \cdot \varepsilon_{cP}(c_P)}$$

$$\alpha_{1N}(c_N) := \frac{3 \cdot \varepsilon'_{cN} \cdot \varepsilon_{cN}(c_N) - (\varepsilon_{cN}(c_N))^2}{3 \cdot \beta_{1N}(c_N) \cdot (\varepsilon'_{cN})^2}$$

$$\alpha_{1P}(c_P) := \frac{3 \cdot \varepsilon'_{cP} \cdot \varepsilon_{cP}(c_P) - (\varepsilon_{cP}(c_P))^2}{3 \cdot \beta_{1P}(c_P) \cdot (\varepsilon'_{cP})^2}$$

$$\alpha\beta_{1N}(c_N) := \text{if}\left(\varepsilon_{cN}(c_N) < \varepsilon_{cu}, \begin{bmatrix} \alpha_{1N}(c_N) \\ \beta_{1N}(c_N) \end{bmatrix}, \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}\right)$$

$$\alpha\beta_{1P}(c_P) := \text{if}\left(\varepsilon_{cP}(c_P) < \varepsilon_{cu}, \begin{bmatrix} \alpha_{1P}(c_P) \\ \beta_{1P}(c_P) \end{bmatrix}, \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}\right)$$

$$U(c_N) := A_s \cdot f_{sN}(c_N) + A_F \cdot f_{feN}(c_N)$$

$$U_p(c_P) := A_s \cdot f_{sP}(c_P) + A_F \cdot f_{feP}(c_P)$$

$$\phi S(c_N) := \prod \alpha\beta_{1N}(c_N) \cdot f'_c \cdot b \cdot c_N$$

$$\phi S_p(c_P) := \prod \alpha\beta_{1P}(c_P) \cdot f'_H \cdot b \cdot c_P$$

$$Q(c_N) := \phi S(c_N) - U(c_N)$$

$$Q_p(c_P) := \phi S_p(c_P) - U_p(c_P)$$

**Option 1: Using solve block**

Guess Values

$c_N := 30\ mm$

Constraints

$-U(c_N) + \phi S(c_N) = 0$

Solver

$c_N := \text{find}(c_N)$

Guess Values

$c_P := 1\ mm$

Constraints

$-U_p(c_P) + \phi S_p(c_P) = 0$

Solver

$c_P := \text{find}(c_P)$

$c_N = 24.85\ mm$

$c_P = 7.649\ mm$

$U_p(c_P) = 348.185\ kN$

$\phi S_p(c_P) = 348.185\ kN$

$$Q_p\left(c_P\right) = \left(5.821 \cdot 10^{-11}\right)\ N$$

**Option 2: Using function**

$c_N := \mathrm{root}\left(Q(c), c, 0.1\ mm, t_H\right) = 24.85\ mm$ $c_P := \mathrm{root}\left(Q_p(c), c, 0.1\ mm, t_H\right) = 30\ mm$