This file was created to check simple FEA dynamic response models.

$Hz := \frac{2 \cdot \pi}{s}$ Mathcad's Hz unit is incorrect. Therefore, redefining it.

$mass := 1kg$ $k := 1\frac{N}{m}$ $x := 4.3m$ $w := 1m$

$E = \frac{F}{area} = \frac{k \cdot x}{area}$ Young's Modulus

$E := \frac{k \cdot x}{w^2} = 4.3\,Pa$ Young's Modulus

$\omega_n := \sqrt{\frac{k}{mass}} = 1 \cdot \frac{rad}{s}$ Natural Angular Frequency

$\omega_n = 0.159155 \cdot Hz$ Natural Frequency

$area := w^2 = 1\,m^2$ Element Area

$x1 := 1m$ $x2 := 1m$

$area1 := 1m^2$ $area2 := 1m^2$

$E1 := .5Pa$ $E2 := 5Pa$

$rho1 := .5\frac{kg}{m^3}$ $rho2 := 1\frac{kg}{m^3}$

$k1 := \frac{E1 \cdot area1}{x1} = 0.5\frac{kg}{s^2}$ $k2 := \frac{E2 \cdot area2}{x2} = 5\frac{kg}{s^2}$

$k1 = 0.5 \cdot \frac{N}{m}$ $k2 = 5 \cdot \frac{N}{m}$

$m1 := rho1 \cdot area1 \cdot x1 = 0.5\,kg$ $m2 := rho2 \cdot area2 \cdot x2 = 1\,kg$

$meq := m1 + m2 = 1.5\,kg$

Boundary Conditions:

The base of the model is fixed in x,y,z. The rest of the model is fixed in two directions. If the elements used have rotational degrees of freedom, they are all fixed. Thus, there is only linear motion in one dimension.

System response:

Depending on the elements used, there may be one or two resonance values. Only the lowest resonance value is of interest. Moreover, the forcing function frequency is set based on the first natural frequency of the model. Peak amplitude occurs when the forcing function frequency equals the natural frequency. Beating occurs if damping is low and the forcing frequency is near the natural frequency. The amplification factor depends on the damping ratio. A damping ratio of 0.01 is typical for structural models. This will cause an amplification factor of 50. The amplification factor is the peak vibratory displacement / the steady state displacement. I have a different file that computes the transient response and amplification factor, for a given set of inputs. The Mathcad state space solver is very good at solving this problem. You can even use a pulse width modulated controller. In this case, the amplification factor will be much less than with a cosine or sine wave (depending on the PWM duty cycle). The amplification factor of 50, mentioned previously, is for a cosine or sine wave.

The FEA models I made use various tricks, depending upon the limitations of the software.

One trick involves using two spring elements in series. m1 is zero, while m2 is the mass that sets the resonance frequency. m2 is located at the end of the second spring element. This method will only produce one resonance frequency. This method will also calculate the first resonance value, when m1 is very near zero. In this case, m1 is located at the end of the first spring element. However, there will be a second resonance that this method can't calculate.

In other cases, two point masses and two springs are used in series. m1 is located at the end of the first spring element. m2 is located at the end of the second spring element. This case has two resonance values. However, only the first one is of interest.

The last situation is where the FEA software has no spring elements. Thus you have two beam elements with a continous mass distribution. You need to use a continous mass matrix to model this. This case also has two resoance values. As with before, only the first one is of interest.

So, below, you have three different types of physics. The model in the last position is used for the resonance calculations near the end of the document.

Two springs in series:

$$keq := \frac{k1 \cdot k2}{k1 + k2} = 0.454545 \frac{kg}{s^2}$$

$$keq = 0.454545 \cdot \frac{N}{m}$$

$m1 = 0.5\,kg$ $\quad$ $m2 = 1\,kg$ $\quad$ $meq = 1.5\,kg$

If m1 is not equal to or near zero, this model shouldn't be used. Moreover, meq should be equal to or very close to m2.

$$\omega_{eq} := \sqrt{\frac{keq}{meq}} = 0.550482 \cdot \frac{rad}{s}$$

Note; this method only has one resonace value

$$\omega_{eq} = 0.087612 \cdot Hz$$

2 Beam Elements that use a continous mass matrix:

$$K3 := \begin{pmatrix} k1 + k2 & -k2 \\ -k2 & k2 \end{pmatrix}$$

This is for 2 beam elements, where the first element has a near zero mass. The second element's mass is split (50/50). So m1 is very close to m2.

$$M3 := \begin{pmatrix} \frac{meq}{3} & \frac{meq}{6} \\ \frac{meq}{6} & \frac{meq}{3} \end{pmatrix}$$

This will also work if the first element is a spring element (which has no mass) and the second element uses a continous mass matrix. In this case, the mass (m2) is split 50/50. Moreover, m1 exactly equals m2.

This method will have two resonance values. Only the first one (lowest frequency of the two) is of interest.

$$m1a := \frac{meq}{3} + \frac{meq}{6} = 0.75\,kg \qquad m2a := \frac{meq}{6} + \frac{meq}{3} = 0.75\,kg$$

2 Point Masses, 2 Springs:

$$K3 := \begin{pmatrix} k1 + k2 & -k2 \\ -k2 & k2 \end{pmatrix}$$

This is similar to the two springs in series method. However, this will work for any value of m1. It also produces two resonance values. Whereas, the two springs in series method will only produce one resonace value. Only the first resonance (lowest frequency of the two) is of interest.

$$M3 := \begin{pmatrix} m1 & 0 \\ 0 & m2 \end{pmatrix}$$

Start of the eigenvalue and eigenvector calculations:

$$F = k \cdot x = m \cdot a \qquad a = \frac{F}{m} = \left(\frac{k}{m}\right) \cdot x \qquad A1 = \frac{k}{m}$$

$$A1 := \frac{K3}{M3} = \begin{pmatrix} 11 & -5 \\ -10 & 5 \end{pmatrix} \frac{1}{s^2}$$

K3 is the stiffness matrix
M3 is the mass matrix

Note this is the same as a more common notation; $K3 \cdot M3^{-1} = \begin{pmatrix} 11 & -5 \\ -10 & 5 \end{pmatrix} \frac{1}{s^2}$

$$\frac{K3}{M3} - K3 \cdot M3^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{s^2}$$

This check should also equal zero

$$freq := \sqrt{eigenvals(A1)} = \begin{pmatrix} 3.959943 \\ 0.564672 \end{pmatrix} \frac{1}{s}$$

Eigenvalues (rad/s)

$$\phi := eigenvecs(A1) = \begin{pmatrix} 0.73 & 0.423962 \\ -0.683447 & 0.90568 \end{pmatrix}$$

Eigenvectors (non-dim)

Note; Mathcad's version of eigenvalues are the square of what FEA resonance values are. Therefore, you need to take the square root of the Mathcad version. Also, the units of Mathcad are erroneous. They list the units as 1/s, which is meaningless. 1 what, should be asked. Mathcad has taken the liberty of thinking this means 1 rad/s. Another problem is they seem to think Hz = 1 rad/s, when in reality it is 1 rev/s. I have corrected the the definition of Hz. So the results shown in Hz, in this document, are revolutions per second.

Sorting eigenvalues and eigenvectors (from lowest to highest):

FEA software has always reported the natural frequencies from lowest to highest. Unfortunately, Mathcad will randomly output them. Sometimes, they will be in order and other times they won't be. This section is meant to correct this, so that Mathcad will report what FEA software does.

$i1 := 0 .. (\mathrm{length}(freq) - 1)$

$order1_{i1} := \mathrm{order}(freq \cdot s)_{i1}$

$Freq_{i1} := freq_{order1_{i1}}$ Eigenvalues (rad/s) - Sorted

$eig^{\langle i1 \rangle} := \phi^{\langle order1_{i1} \rangle}$ Eigenvectors (non-dim) - Sorted

$Freq = \blacksquare \cdot \frac{rad}{s}$ $Freq = \blacksquare \cdot Hz$ $eig = \blacksquare$

$mode1 := eig^{\langle 0 \rangle} = \blacksquare$ $mode2 := eig^{\langle 1 \rangle} = \blacksquare$

$mode1a := mode1 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} j$ $mode2a := mode2 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} j$

These plots show non-dimensional node displacement. They are consistent with the plots you get from FEA software.

mode1a mode2a

Note, in my testing; genvals, genvecs, and eigenvec can fail sometimes. Whereas, eigenvals and eigenvecs always seem to give the correct result. Therefore, I recommend using eigenvals and eigenvecs.