The soil pressure resultant and corresponding $\bar{y}$ are shown on Fig. E11-5*b* (and *this calculation does not include water*).

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11-7 ACTIVE AND PASSIVE EARTH PRESSURE USING THEORY OF PLASTICITY

The Coulomb and Rankine passive earth pressure methods consistently overestimate the passive pressure developed in field and model tests for ϕ much over 35°. This estimate may or may not be conservative, depending on the need for the passive pressure value. Because of the problem of overestimation, Caquot and Kerisel (1948) produced tables of earth pressure based on nonplane-failure surfaces; later Janbu (1957) and then Shields and Toluany (1973) proposed an approach to the earth pressure problem similar to the method of slices used in slope-stability analyses. Sokolovski (1960) presented a finite-difference solution using a highly mathematical method. All these methods give smaller values for the passive earth pressure coefficient. None of these methods significantly improves on the Coulomb or Rankine active earth pressure coefficients.

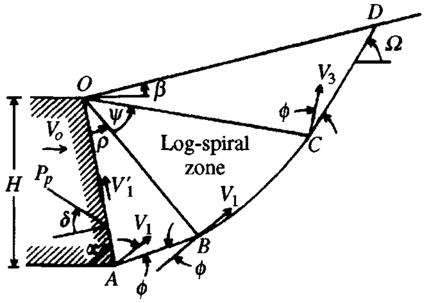
Rosenfarb and Chen (1972) developed a closed-form earth pressure solution using plasticity theory that can be used for both active and passive earth pressure computations. The closed-form solution requires a computer program with an iteration routine, which is not particularly difficult. This method is included here because of the greater clarity over the alternative methods.

Rosenfarb and Chen considered several failure surfaces, and the combination of a so-called *log-sandwich* mechanism gave results that compared most favorably with the Sokolovski solution, which has been accepted as correct by many persons. Figure 11-10 illustrates the passive log-sandwich mechanism. From this mechanism and appropriate consideration of its velocity components the following equations are obtained.

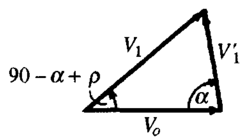
Cohesionless Soil

For a smooth wall ($\delta < \phi$):

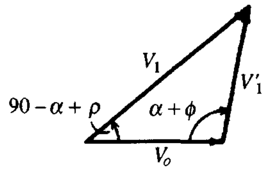
$$
\begin{aligned}
\left\{ \begin{matrix} K_{a\gamma} \\ K_{p\gamma} \end{matrix} \right\} &= \frac{\mp \sec\delta}{\mp \sin\alpha + \tan\delta\cos\alpha - [\tan\delta\cos(\alpha-\rho)/\cos\rho]} \\
&\quad \times \left(\frac{\tan\rho\cos(\rho\pm\phi)\cos(\alpha-\rho)}{\sin\alpha\cos\phi} + \frac{\cos^2(\rho\pm\phi)}{\cos\rho\sin\alpha\cos^2\phi(1+9\tan^2\phi)} \right. \\
&\quad \times \left\{ \cos(\alpha-\rho)[\pm 3\tan\phi + (\mp 3\tan\phi\cos\psi + \sin\psi) \right. \\
&\quad \times \exp(\mp 3\psi\tan\phi)] \\
&\quad \left. + \sin(\alpha-\rho)[1 + (\mp 3\tan\phi\sin\psi - \cos\psi)\times\exp(\mp 3\psi\tan\phi)] \right\} \\
&\quad \left. + \frac{\cos^2(\rho\pm\phi)\sin(\alpha-\rho-\psi+\beta)\cos(\alpha-\rho-\psi)\exp(\mp 3\psi\tan\phi)}{\cos\phi\sin\alpha\cos(\alpha-\rho-\psi\mp\phi+\beta)\cos\rho} \right)
\end{aligned}
\qquad (11\text{-}10)
$$



(a) Passive log-sandwich mechanism with $V_3 = V_1 \exp(\psi \tan\phi)$.



(b) Velocity diagram for a smooth wall $\delta < \phi$.



(c) Velocity diagram for a rough wall $\delta = \phi$.

Figure 11-10 Plastic stress fields for earth pressure using the theory of plasticity. [*Rosenfarb and Chen (1972).*]

For a rough wall ($\delta = \phi$):

$$\begin{Bmatrix} K_{a\gamma} \\ K_{p\gamma} \end{Bmatrix} = \frac{\mp \sec\delta}{\mp \sin\alpha + \tan\delta\cos\alpha}\left(\frac{\sin^2\rho\cos(\rho\pm\phi)\cos(\alpha-\rho)\sin(\alpha\mp\phi)}{\sin^2\alpha\cos\phi\cos(\rho\mp\phi)}\right.$$
$$\mp \frac{\cos^2(\rho\pm\phi)\sin(\alpha\mp\phi)}{\sin^2\alpha\cos^2\phi(1+9\tan^2\phi)\cos(\rho\mp\phi)}$$
$$\times\left\{\cos(\alpha-\rho)[\pm 3\tan\phi + (\mp 3\tan\phi\cos\psi + \sin\psi)\exp(\mp 3\psi\tan\phi)]\right.$$
$$\left. + \sin(\alpha-\rho)[1 + (\mp 3\tan\phi\sin\psi - \cos\psi)\exp(\mp 3\psi\tan\phi)]\right\}$$
$$\left. + \frac{\cos^2(\rho\pm\phi)\sin(\alpha-\rho-\psi+\beta)\cos(\alpha-\rho-\psi)\sin(\alpha\mp\phi)\exp(\mp 3\psi\tan\phi)}{\sin^2\alpha\cos\phi\cos(\alpha-\rho-\psi+\beta\mp\phi)\cos(\rho\mp\phi)}\right) \tag{11-11}$$

Cohesive Soil

For a smooth wall ($\delta < \phi$):

$$\begin{Bmatrix} K_{ac} \\ K_{pc} \end{Bmatrix} = \frac{\sec\delta}{\mp\sin\alpha + \tan\delta\cos\alpha - [\tan\delta\cos(\alpha-\rho)/\cos\rho]}$$
$$\times\left\{\tan\rho + \frac{\cos(\rho\pm\phi)\sin(\alpha-\rho-\psi+\beta)\exp(\mp\psi\tan\phi)}{\cos\rho\cos(\alpha-\rho-\psi\mp\phi+\beta)}\right.$$
$$\left.\mp \frac{\cos(\rho\pm\phi)[\exp(\mp 2\psi\tan\phi)-1]}{\sin\phi\cos\rho}\right\} \tag{11-12}$$

For a rough wall ($\delta = \phi$):

$$\begin{Bmatrix} K_{ac} \\ K_{pc} \end{Bmatrix} = \frac{\sec\delta}{\mp\sin\alpha + \tan\delta\cos\alpha}\left\{\frac{\cos\phi\cos(\alpha-\rho)}{\sin\alpha\cos(\rho\mp\phi)} + \frac{\sin\rho\sin(\alpha\mp\phi)}{\sin\alpha\cos(\rho\mp\phi)} + \frac{\cos(\rho\pm\phi)\sin(\alpha-\rho-\psi+\beta)\sin(\alpha\mp\phi)\exp(\mp\psi\tan\phi)}{\sin\alpha\cos(\alpha-\rho-\psi\mp\phi+\beta)\cos(\rho\mp\phi)} \mp \frac{\cos(\rho\pm\phi)\sin(\alpha\mp\phi)[\exp(\mp 2\psi\tan\phi)-1]}{\sin\phi\sin\alpha\cos(\rho\mp\phi)}\right\} \tag{11-13}$$

In solving Eqs. (11-10) through (11-13), it is necessary to solve for the maximum value of K_p or K_a. The maximizing of these equations depends on the two variables ρ and ψ. This requires a search routine in computer program B-23. The values of the two dependent variables are initialized to approximately

$$\rho \cong 0.5(\alpha + \beta)$$
$$\psi \cong 0.2(\alpha + \beta)$$

With these initial values, the search routine is used to revise the values until convergence is obtained. In most cases values from which K_p is computed are found after not more than 20 iterations. A computer program should shut off after 46 to 50 iterations. In a few cases the program may not find a solution using the above initial values because of the programming search routine. For these cases, one must change the initial values and retry as necessary to obtain the solution. Table 11-5 gives selected values of K_p for cohesionless soils. Note that these equations correctly give K_p increasing with β. Values of $\beta = \delta = 0$ are not shown, as they are identical to the Coulomb or Rankine solution.

The "smooth" wall solution is used for wall friction $\delta < \phi$; when $\delta = \phi$ the "rough" wall equation is used. Equations (11-12) and (11-13) can readily be programmed, using the same routines to solve an equation for minimum or maximum with two dependent variables, to obtain passive pressure coefficients for cohesive soil. This solution does not give greatly different values from the Coulomb passive pressure theory until the ϕ angle becomes larger than 35° and with δ on the order of $\phi/2$ or more and $\beta \neq 0°$ (since the back slope can have $\pm\beta$).

11-8 EARTH PRESSURE ON WALLS, SOIL-TENSION EFFECTS, RUPTURE ZONE

The Rankine or Coulomb earth pressure equations can be used to obtain the force and its approximate point of application acting on the wall for design. Soil-tension concepts can also be investigated. These will be taken up in the following discussion.

11-8.1 Earth Forces on Walls

From Eq. (2-55) and temporarily considering a soil with $c = 0$, γ constant with depth z and referring to Fig. 11-9a, we can compute the wall force as

$$P_a = \int_0^H \sigma_3 K_a\,dz = \int_0^H \gamma z K_a = \frac{\gamma z^2 K_a}{2}\Bigg|_0^H = \frac{\gamma H^2}{2}K_a \tag{a}$$

from which it is evident that the soil pressure diagram is hydrostatic (linearly increases with depth) as shown in the figure. If there is a surcharge q on the backfill as shown in Fig. 11-9c (other surcharges will be considered in Sec. 11-13), the wall force can be computed as