When saving or printing, disable Automatic Calculation.

$t0\_ := 0 \quad \tau_{init\_} := time(t0\_) \rightarrow time(0) \quad \tau_{init\_} = 1.618 \times 10^{9}$

WAVEFORM SPECTRA

Francesco Mezzanino

This worksheet is a collection of some common (and not), signals used in electronics. It deals with the harmonic analysis of periodic signals, satisfying the Dirichlet conditions, determined without any particular artifice to speed up the calculation but using the definition formulas. For each one first, it is plotted a graph, then is calculated its bandwidth in order to do a correct sampling of it. The Fourier harmonics and phase, are plotted in two graphs. Then, the sampled signal is rebuilt with the Shannon interpolation formula. Given the sampled signal, the fft function is applied and plotted to compare the result (first 18 functions only). The previous procedure is repeated for each signal (41).

DATA

DATA

FOURIER

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Signal's Bandwidth Calculation

1) Bandwidth() — Bandwidth calculation
2) FurSr — Computation of the polynomial coefficients
3) BCSA — Bandwidth Calculation and Signal Analysis

The signal bandwidth is constituted by a number of harmonics such that it is possible to reconstruct it by the Fourier series with sufficient accuracy.
A criterion for determining the number of harmonics necessary to reconstruct the signal, with negligible error is to neglect all harmonics whose amplitude is lower than a certain percentage, established a priori, of the fundamental harmonic. It is what is done in the following program:
The program returns a vector containing the following four variables:

percent: is the percentage chosen,
j: is the harmonic's number,
B_w: is the bandwidth in MHz,
Temp1: is a temporary variable.

Function's parameters description:
Bandwidth(signal frequency, Fourier coefficients vector a1, Fourier coefficients vector b1, the percentage chosen, polynomial degree).

$\text{Bandwidth}(f_{ptfs\_}, a1\_, b1\_, rt_{fs\_}, N1\_) :=$

- return "T" if $f_{ptfs\_} \le 0.0$
- otherwise
  - return "The percentage chosen is less than or = 0" if $rt_{fs\_} \le 0.0$
  - otherwise
    - return "The polinomial degree N1_ is less than or = 0" if N1_ :
    - otherwise
      - $percent \leftarrow rt_{fs\_}$
      - $B_w \leftarrow 0.0$
      - $Temp1 \leftarrow 0.0$
      - $mx \leftarrow 0.0$
      - $U1\_ \leftarrow \begin{pmatrix} percent \\ 0.0 \\ 0 \\ 0.0 \end{pmatrix}$
      - $mx \leftarrow percent \cdot \dfrac{\sum_{k=1}^{N1\_-1} \sqrt{(a1\_{k})^2 + (b1\_{k})^2}}{(N1\_ - 1)}$
      - for $j \in 1 .. N1\_ - 1$
        - if $\sqrt{(a1\_{j})^2 + (b1\_{j})^2} \ne 0.0$
          - $Temp1 \leftarrow \sqrt{(a1\_{j})^2 + (b1\_{j})^2}$
          - $Temp2 \leftarrow \sum_{k=j}^{N1\_-1} \left[\dfrac{\sqrt{(a1\_{k})^2 + (b1\_{k})^2}}{N1\_ - j} \right]$ if $j \le (N1$
          - $U1\_{2} \leftarrow j$
          - $U1\_{1} \leftarrow (j - 1) \cdot f_{ptfs\_}$ if $j > 1$
          - $U1\_{1} \leftarrow f_{ptfs\_}$ otherwise
          - $U1\_{3} \leftarrow Temp1$
          - break if $Temp2 < Temp1 \wedge Temp1 \le mx \wedge Temp1$
        - continue otherwise
      - return U1_

Computation of the polynomial coefficients

Function's parameters description:
FurSr(Dimensonless signal name, polynomial degree, start time, signal period)

$\text{FurSr}(s\_, n_{fs\_}, t_{\_0fs}, T_{fs\_}) :=$

- $res\_ \leftarrow$
  - $msg \leftarrow 0$
  - $msg \leftarrow$ "The Fourier_polinomial_degree n.fs_ less than or = 0" if $n_{fs\_} \le$
  - otherwise
    - $msg \leftarrow$ "The period T.fs_ less than or = 0" if $T_{fs\_} \le 0$
    - otherwise
      - $Coeff_{ab}^{\langle 0 \rangle} \leftarrow \dfrac{2}{T_{fs\_}} \cdot \displaystyle\int_{t\_0fs}^{t\_0fs + T_{fs\_}} s\_(t\_)\, dt\_$
      - for $k \in 1 .. n_{fs\_}$
        - $Coeff_{ab}^{\langle k \rangle} \leftarrow \begin{pmatrix} \dfrac{2}{T_{fs\_}} \cdot \displaystyle\int_{t\_0fs}^{t\_0fs + T_{fs\_}} s\_(t\_) \cdot \cos\left(\dfrac{2 \cdot \pi}{T_{fs\_}} \cdot k \cdot t\_\right) dt\_ \\ \dfrac{2}{T_{fs\_}} \cdot \displaystyle\int_{t\_0fs}^{t\_0fs + T_{fs\_}} s\_(t\_) \cdot \sin\left(\dfrac{2 \cdot \pi}{T_{fs\_}} \cdot k \cdot t\_\right) dt\_ \end{pmatrix}$
  - $Coeff_{ab}^{T}$ if $msg = 0$
  - msg otherwise
- return res_ if IsString(res_)
- otherwise

$xx\_ \leftarrow res\_$

$\text{return } xx\_$

Bandwidth Calculation and Signal Analysis

Function parameters description:

FurSr(*Dimensonless signal name, relative error, polynomial degree, start time, signal period*) .

Function parameters description:

fsr(*time cosine coefficient, sine coefficient, signal period, polynomial degree*).

The function parameters are described below:

Bandwidth(*signal frequency, Fourier coefficients vector a1, Fourier coefficients vector b1, percentage chosen, polynomial degree*).

Function parameters description:

BCSA(*Dimensonless signal name, relative error, polynomial degree, start time, signal period*)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

The function returns a matrix made of three columns.

The first column contains:

pos. 0: relative error,
pos. 1: bandwidth (dimensionless),
pos. 2: the nth. harmonic number corresponding tp the give relative error,
pos. 3: temporary variable,
pos. 4: Parseval,
pos. 5: signal average,
pos. 6: signal RMS.

The second column contains the coefficients a_k of the Fourier series,
the third column contains the coefficients b_k of the Fourier series.

$\text{BCSA}(Vi\_, rtfs\_, N1\_, T_{0\_}, T_{fs\_}) :=$

$\quad \text{return "The percentage chosen is less than or = 0"} \quad \text{if } rtfs\_ \le 0.0$

$\quad \text{otherwise}$

$\quad\quad \text{return "N1\_ less than or equal 0"} \quad \text{if } N1\_ \le 0$

$\quad\quad \text{otherwise}$

$\quad\quad\quad \text{return "T.fsless than or equal T.0\_"} \quad \text{if } T_{fs\_} \le T_{0\_}$

$\quad\quad\quad \text{otherwise}$

$\quad\quad\quad\quad \text{for } \xi \in 0..N1\_ - 1$

$\quad\quad\quad\quad\quad \text{for } \zeta \in 0..3$

$\quad\quad\quad\quad\quad\quad U2_{\xi,\zeta} \leftarrow 0.0$

$\quad\quad\quad\quad polycoeff \leftarrow \text{FurSr}(Vi\_, N1\_, T_{0\_}, T_{fs\_})$

$\quad\quad\quad\quad \text{return } polycoeff \quad \text{if IsString}(polycoeff)$

$\quad\quad\quad\quad coeffa1 \leftarrow polycoeff^{\langle 0 \rangle}$

$\quad\quad\quad\quad coeffb1 \leftarrow polycoeff^{\langle 1 \rangle}$

$$Pars\_ \leftarrow \frac{(coeffa1_0)^2}{2} + \sum_{k=1}^{N1\_} \left[(coeffa1_k)^2 + (coeffb1_k)^2\right]$$

$$UB \leftarrow \text{Bandwidth}\left(\frac{1}{T_{fs\_}} \cdot sec, coeffa1, coeffb1, rtfs\_, N1\_\right)$$

$\quad\quad\quad\quad \text{return } UB \quad \text{if IsString}(UB)$

$\quad\quad\quad\quad (U2^{\langle 0 \rangle})_0 \leftarrow UB_0$

$\quad\quad\quad\quad (U2^{\langle 0 \rangle})_1 \leftarrow UB_1$

$\quad\quad\quad\quad (U2^{\langle 0 \rangle})_2 \leftarrow UB_2$

$\quad\quad\quad\quad (U2^{\langle 0 \rangle})_3 \leftarrow UB_3$

$\quad\quad\quad\quad (U2^{\langle 0 \rangle})_4 \leftarrow Pars\_$

$$(U2^{\langle 0 \rangle})_5 \leftarrow \frac{1}{T_{fs\_}} \cdot \int_0^{T_{fs\_}} Vi\_(t)\, dt$$

$$(U2^{\langle 0 \rangle})_6 \leftarrow \sqrt{\frac{1}{T_{fs\_}} \cdot \int_0^{T_{fs\_}} Vi\_(t)^2\, dt}$$

$\quad\quad\quad\quad U2^{\langle 1 \rangle} \leftarrow coeffa1$

$\quad\quad\quad\quad U2^{\langle 2 \rangle} \leftarrow coeffb1$

$\quad\quad\quad\quad \text{return } U2$

```
SPCT(Vi_, rt_gd, N1_, T_0, T_fs_) := | UBCSA ← BCSA(Vi_, rt_gd, N1_, T_0, T_fs_)
                                     | ""
                                     | cfa2 ← UBCSA^⟨1⟩
                                     | cfb2 ← UBCSA^⟨2⟩
                                     | Bw ← (UBCSA^⟨0⟩)_1 / sec
                                     | for j2 ∈ 0 .. rows(cfa2) − 1
                                     |   φgs1_j2 ← | 0.0  if cfb2_j2 = 0.0
                                     |             | otherwise
                                     |             |   | −atan(cfb2_j2 / cfa2_j2)  if cfa2_j2 ≠ 0
                                     |             |   | otherwise
                                     |             |   |   | −π/2  if −atan(cfb2_(j2−1) / cfa2_(j2−1)) < 0.0 ∧ (0 < j2 < rows(cfa2) -
                                     |             |   |   | π/2  if −atan(cfb2_(j2−1) / cfa2_(j2−1)) > 0.0 ∧ (0 < j2 < rows(cfa2) −
                                     | X2 ← max[√((cfa2)^2 + (cfb2)^2)]
                                     | for j2 ∈ 0 .. rows(cfa2) − 1
                                     |   AmplitudeSpectrum_j2 ← √((cfa2_j2)^2 + (cfb2_j2)^2) / X2
                                     | posj2 ← (UBCSA^⟨0⟩)_2
                                     | mx2 ← √((cfa2_posj2)^2 + (cfb2_posj2)^2) / X2
                                     | Bw ← (UBCSA^⟨0⟩)_1
                                     | Average2 ← (UBCSA^⟨0⟩)_5
                                     | RMS2 ← (UBCSA^⟨0⟩)_6
                                     | Parseval ← (UBCSA^⟨0⟩)_4
                                     | Temp2 ← (UBCSA^⟨0⟩)_3
                                     | relerr2 ← (UBCSA^⟨0⟩)_0
                                     | ( AmplitudeSpectrum )
                                     | ( φgs1              )
                                     | ( mx2               )
                                     | ( Bw                )
                                     | ( Average2          )
```



```
                                     | ( RMS2              )
                                     | ( Parseval          )
                                     | ( relerr2           )
                                     | ( Temp2             )
                                     | ( cfa2              )
                                     | ( cfb2              )
```

$$fs(t, a_{fs}, b_{fs}, T_{fs}, n_{fs}) := \left[\frac{a_{fs_0}}{2} + \sum_{k=1}^{n_{fs}} \left(a_{fs_k} \cos\left(\frac{2\cdot\pi}{T_{fs}}\cdot k\cdot t\right) + b_{fs_k}\cdot \sin\left(\frac{2\cdot\pi}{T_{fs}}\cdot k\cdot t\right)\right)\right]$$

FOURIER

Pulses and Priodic Formulae Only

When saving or printing, disable Automatic Calculation.

PULSES AND WAVEFORMS FORMULAE DEFINITION

Francesco Mezzanino

The subscript gd is the acronym of general Data.xmcd
The subscript fs is the acronym of Fourier series.xmcd
The subscript sl is the acronym of Signal List.xmcd
The subscript dp is the acronym of Dirac Pulse - formulas.xmcd

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INTRODUCTION

Pulse Definitions

Periodic Waveforms Definitions

Pulses

-1) Dirac Pulse and its Derivatives - Definition and Approximation

Dirac pulse definition: $\Delta(t) = \left(\begin{array}{l} \infty \ \text{ if } t = 0.0 \\ 0.0 \ \text{ otherwise} \end{array} \right)$

Some text of electrical engineering, use the symbol: $\mathbf{u_0}(t) := \begin{array}{|l} \infty \ \text{ if } t = 0.0 \\ 0.0 \ \text{ otherwise} \end{array}$

$$\int_{-\infty}^{\infty} \Delta(t)\, dt = 1 \qquad \int_{-\infty}^{\infty} \mathbf{u_0}(t)\, dt = 1$$

Let's now approximate the Dirac Pulse in a way that it can be drawn, namely define a time interval as small as desired, for example:

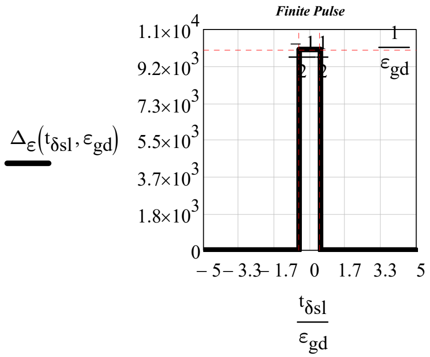
approximation. $\Delta_\varepsilon(t, \varepsilon_{gd}) := \begin{array}{|l} \dfrac{1}{\varepsilon_{gd}} \ \text{ if } \dfrac{-\varepsilon_{gd}}{2} \le t \le \dfrac{\varepsilon_{gd}}{2} \\ 0 \ \text{ otherwise} \end{array}$

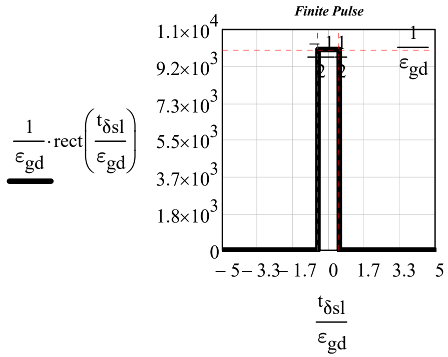
Dirac Pulse property: $\displaystyle\int_{-\infty}^{\infty} \Delta_\varepsilon(t, \varepsilon_{gd})\, dt = \int_{-\frac{\varepsilon_{gd}}{2}}^{\frac{\varepsilon_{gd}}{2}} \Delta_\varepsilon(t, \varepsilon_{gd})\, dt = 1$

$$\lim_{\varepsilon \to 0} \int_{-\frac{\varepsilon_{gd}}{2}}^{\frac{\varepsilon_{gd}}{2}} \Delta_\varepsilon(t, \varepsilon_{gd})\, dt = 1 \qquad \int_{-\infty}^{\infty} \lim_{\varepsilon \to 0} \Delta_\varepsilon(t, \varepsilon_{gd})\, dt = 0$$

(Physics) $\text{rect}(t) := \begin{array}{|l} 1 \ \text{ if } |t| < \dfrac{1}{2} \\ \dfrac{1}{2} \ \text{ if } |t| = \dfrac{1}{2} \\ 0 \ \text{ if } |t| > \dfrac{1}{2} \end{array}$

$\varepsilon_{gd} = 1 \times 10^5 \dfrac{1}{s} \cdot ns \qquad t_{\delta sl} := -5\cdot\varepsilon_{gd}, -5\cdot\varepsilon_{gd} + \dfrac{10\cdot\varepsilon_{gd}}{2000} .. 5\cdot\varepsilon_{gd}$





Pulses

-2) Voltage step

Some text in electrical engineering indicate the unitary step with the symbol: $\mathbf{u}_{\_1}(t) = \int_{-\infty}^{t} \mathbf{u_0}(\xi)\,d\xi = \Phi(t)$,

therefore: $\mathbf{u_0}(t) = \frac{d}{dt}\mathbf{u}_{\_1}(t)$.

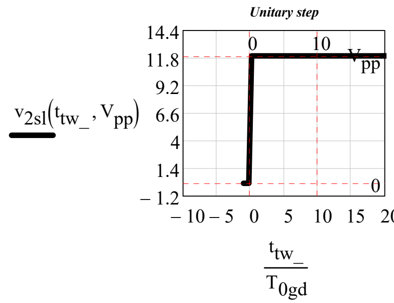
Other definition are $\Phi(t) = \lim_{\varepsilon_{gd} \to 0} \int_{-\infty}^{t} \frac{1}{\varepsilon_{gd}} \cdot \Pi\left(\frac{\xi}{\varepsilon_{gd}}\right) d\xi = \int_{-\infty}^{t} \mathbf{u_0}(\xi)\,d\xi = \int_{-\infty}^{t} \Delta(\xi)\,d\xi$

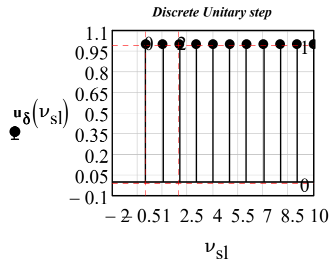
Voltage step $V_{stpsl}(t, V_{pp}) := V_{pp} \cdot \Phi(t)$

Discrete time Unitary step (Unitary pulse: $\delta(\nu, k)$):

$$v_{2sl}(t, V_{pp}) := \frac{V_{stpsl}(t, V_{pp})}{V} \qquad \mathbf{u}_\delta(\nu) := \begin{cases} \sum_{k=0}^{\nu} \delta(\nu, k) & \text{if } \nu \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$V_{pp} = 1.2 \times 10^4 \cdot mV$ $\qquad t_{tw\_} := -1 \cdot T_{0gd}, -1 \cdot T_{0gd} + \frac{201 \cdot T_{0gd}}{500} .. 200 \cdot T_{0gd}$ $\qquad \nu_{sl} := 0..20$





-3) Ramp with slope V_1/T

Some text of electrical engineering indicate the ramp function, use the symbol: $u_2(t) := t \cdot \Phi(t)$

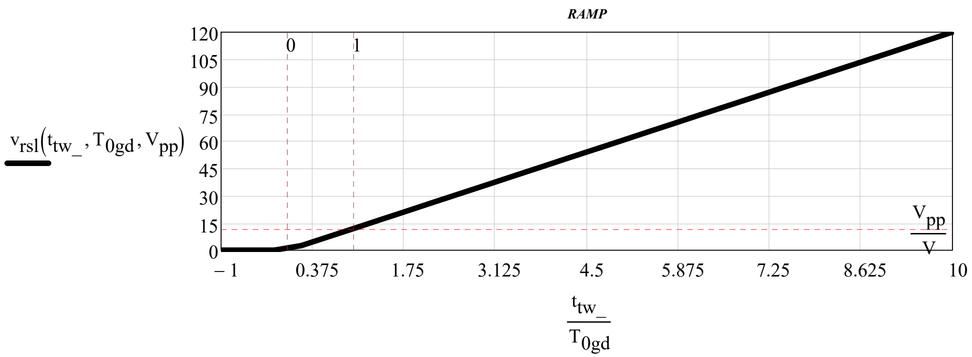
T_{0gd} and V_{pp} are defined in "global data.xmcd "

$$\mathbf{u_2}(t) = t \cdot \Phi(t) = \int_{-\infty}^{t} \Phi(\xi)\, d\xi$$

Voltage ramp:

$$\mathbf{u_2}(t) := \int_{0}^{t} \Phi(\tau_{tw})\, d\tau_{tw} \rightarrow t$$

$$\boxed{v_{rsl}(t, T_{0gd}, V_{pp}) := \frac{V_{pp}}{T_{0gd}} \cdot \frac{t \cdot \Phi(t)}{V}}$$



-4) Voltage Pulse

Description of the Function's parameters:

$V_4(t, \tau_\delta, \tau_{ptd}, V_{pp}) = \text{Adimensional\_amplitude} \cdot \text{rect1}(\text{time}, \text{risingedge}, \text{width})$

Data file " pulse train data.xmcd"

Pulse width: τ_{ptd}, ,

Amplitude: V_{pp}

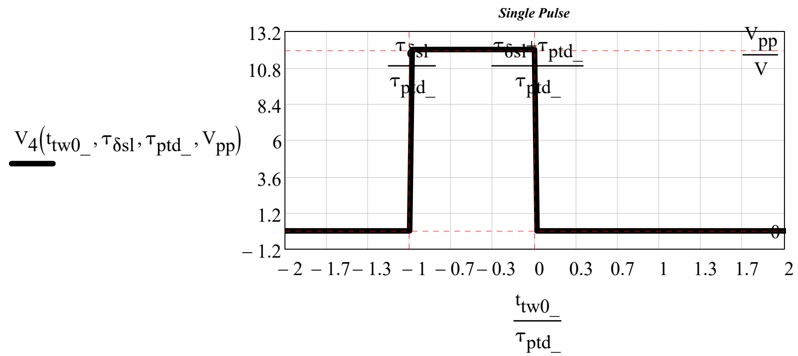
Pulse displacement from the origin: $\xi_{tw} := 0 \cdot s$, $\tau_{ptd\_} = \tau_{ptd\_} \cdot (1 - \xi_{tw}) + \xi_{tw} \cdot \tau_{ptd\_}$,

Time delay from the origin: $\tau_{\delta sl} := -\tau_{ptd\_} \cdot (1 - \xi_{tw})$, risingedge $= \tau_{\delta sl}$, width $= \tau_{ptd\_}$.

$$\boxed{\text{rect1}(t\_, \text{risingedge}, \text{width}) := [\Phi(t\_ - \text{risingedge}) - \Phi[t\_ - (\text{width} + \text{risingedge})]]}$$

$$V_4(t, \tau_\delta, \tau_{ptd}, V_{pp}) := \frac{V_{pp}}{V} \cdot \text{rect1}(t, \tau_\delta, \tau_{ptd})$$

$$t_{tw0\_} := -2 \cdot \tau_{ptd\_}, -2 \cdot \tau_{ptd\_} + \frac{102 \cdot \tau_{ptd\_}}{5000} .. 100 \cdot \tau_{ptd\_}$$



Pulses

-5 Doublet Voltage Pulse

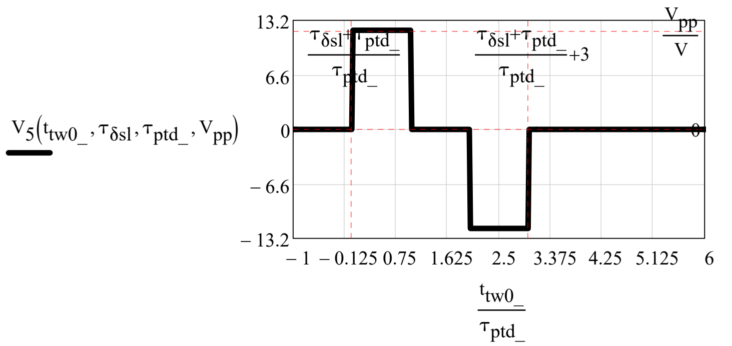
Description of the Function's parameters: $V_4(t, risingedge, width, pulse\_amplitude)$;

$V_5(t, risingedge, width, pulse\_amplitude)$

Data file " pulse train data.xmcd"

$\tau_{\delta sl} = -250\cdot\mu s$ $\tau_{ptd\_} = 250\cdot V_4(t, \tau_\delta, \tau_{ptd}, V_{pp}) = \frac{V_{pp}}{V}\cdot rect1(t, \tau_\delta, \tau_{ptd})$

$$V_5(t, \tau_\delta, \tau_{ptd}, V_{pp}) := V_4(t - \tau_{ptd}, \tau_\delta, \tau_{ptd}, V_{pp}) - V_4(t - 3\cdot\tau_{ptd}, \tau_\delta, \tau_{ptd}, V_{pp})$$



Pulses

-6 Staircase 1 Voltage Pulse

Data file "staircase pulse data.xmcd"

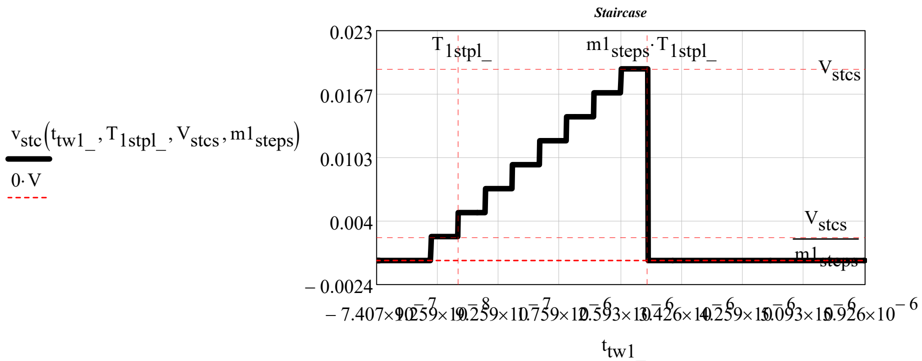
Description of the Function's parameters: $v_{stc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

Test signal:

$$v_{stc}(t, T_{1stpl\_}, V_{stcs}, m1_{steps}) := \frac{V_{stcs}}{m1_{steps}}\cdot\left[\sum_{k=0}^{m1_{steps}-1} \left(\Phi(t - k\cdot T_{1stpl\_})\right) - m1_{steps}\cdot\Phi(t - m1_{steps}\cdot T_{1stpl\_})\right]$$

Area under the staircase:

$$A_{stc} = T_{1stpl\_}\cdot\frac{V_{stcs}}{m1_{steps}}\cdot\sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1)$$



Pulses

- 7 Staircase 2 Voltage Pulse

Data: staircase 2 pulse data   shift := 6

Description of the Function's parameters: $v_{stcc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

$$v_{stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps}) := \frac{V_{stc}}{m2_{steps}}\cdot\left[\sum_{k=1}^{m2_{steps}} \left(\Phi(t - k\cdot T_{2stpl\_})\right) - \sum_{k=1}^{m2_{steps}} \Phi\left[t - T_{2stpl\_}\cdot(k + m2_{steps})\right]\right]$$

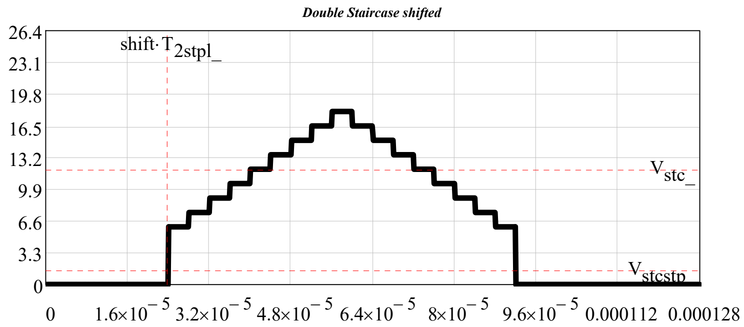
Area under the staircase $A_{stcc} = 2\cdot T_{2stpl\_}\cdot\frac{V_{stc}}{m2_{steps}}\cdot\sum_{k=1}^{m2_{steps}} \left((m2_{steps} - k + 1)\right) - V_{stc}\cdot T_{2stpl\_}$

$$v_{2stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps}, shift) := v_{stcc}(t - shift\cdot T_{2stpl\_}, T_{2stpl\_}, V_{stc}, m2_{steps}) \ldots$$
$$+ V_{stc}\cdot rect1\left[t - shift\cdot T_{2stpl\_}, 0\cdot T_{2stpl\_}, (2\cdot m2_{steps}+1)\cdot T_{2stpl\_}\right] \ldots$$
$$+ \left[v_{stcc}\left[t - shift\cdot T_{2stpl\_} - (2\cdot m2_{steps}+1)\cdot T_{2stpl\_}, T_{2stpl\_}, V_{stc}, m2_{step}\right.\right.$$
$$\left.+ 2\cdot V_{stc}\cdot rect1\left[t - shift\cdot T_{2stpl\_} - (2\cdot m2_{steps}+1)\cdot T_{2stpl\_}, 0\cdot T_{2stpl\_}, (\right.\right.$$

$$v_H(t, T_{2stpl\_}, V_{stc\_}, m2_{steps\_}, shift) := v_{2stcc}(t, T_{2stpl\_}, V_{stc\_}, m2_{steps\_}, shift) \ldots$$
$$+ v_{stcc}\left[t - T_{2stpl\_}\cdot(4\cdot m2_{steps\_} + 2 + shift), T_{2stpl\_}, V_{stc\_}, m2_{steps\_}\right] \ldots$$
$$+ V_{stc\_}\cdot rect1\left[t - T_{2stpl\_}\cdot(4\cdot m2_{steps\_} + 2 + shift), 0\cdot T_{2stpl\_}, (2\cdot m2_{steps\_}\right.$$

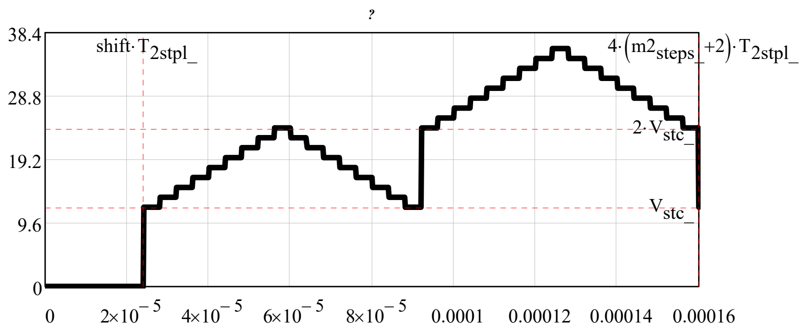
$$T_T = 2\cdot T_{2stpl\_}\cdot(3\cdot m2_{steps} + 4) + shift\cdot T_{2stpl\_}$$

$$v_{HDoor}(t, T_{2stpl\_}, V_{stc\_}, m2_{steps\_}, shift) := V_{stc\_}\cdot rect1\left[t + \frac{T_{2stpl\_}\cdot(1 - shift)}{2}, 1\cdot\frac{(6\cdot m2_{steps\_} + shift + 3)\cdot T_{2stpl\_}}{2}\right.$$

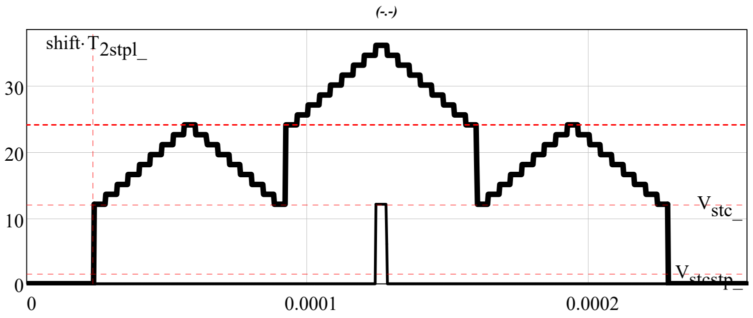


$V_{stc\_} = 12\,V$

$T_{2stpl\_} = 4\cdot\mu s$



$V_{stc\_} = 12\,V$ $\quad m2_{steps\_} = 8$ $\quad shift = 6$



Pulses

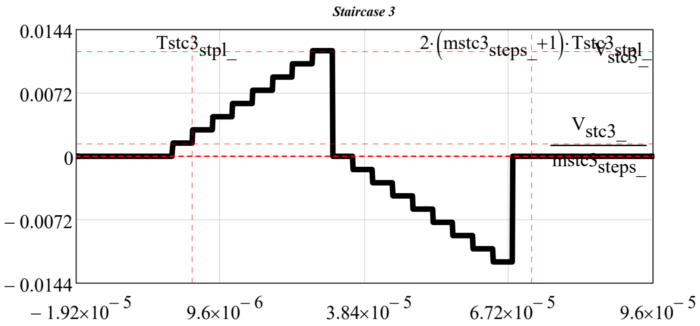
-8 Staircase 3 Voltage Pulse

Data file "staircase 3 pulse data.xmcd"

Description of the Function's parameters: vstc1(t, step_length, signal_amplitude, number_of_steps)

$$vstc1\left(t, T3_{stpl\_}, V_{stc3}, m3_{steps}\right) := v_{stc}\left(t, T3_{stpl\_}, V_{stc3}, m3_{steps}\right) \ldots$$
$$+ (-1)\cdot v_{stc}\left[t - \left(m3_{steps} + 1\right)\cdot T3_{stpl\_}, T3_{stpl\_}, V_{stc3}, m3_{steps}\right]$$

$$v_{12}\left(t, T3_{stpl\_}, V_{stc3}, m3_{steps}\right) := \frac{vstc1\left(t, T3_{stpl\_}, V_{stc3}, m3_{steps}\right)}{V}$$



Pulses

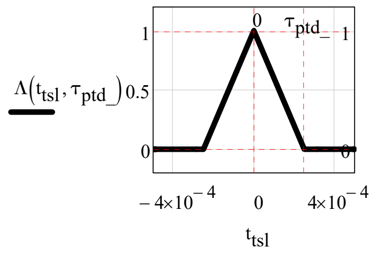
-9 Triangular Voltage Pulse

Data file " general data.xmcd"

Definition of the Triangle function:
$$\Lambda\left(t, \tau_{ptd}\right) := \begin{cases} 1 - \left|\frac{t}{\tau_{ptd}}\right| & \text{if } \left|\frac{t}{\tau_{ptd}}\right| < 1 \\ 0 & \text{if } \left|\frac{t}{\tau_{ptd}}\right| > 1 \end{cases}$$

Alternative definition using the function rect(t) or ($\Pi\left(\tau_{ptd}\right)$) :

$$\Lambda\left(t, \tau_{ptd}\right) := \left(1 - \left|\frac{t}{\tau_{ptd}}\right|\right)\cdot\left(\Phi\left(t + \tau_{ptd}\right) - \Phi\left(t - \tau_{ptd}\right)\right) \qquad \tau_{ptd\_} = 250\cdot\mu s$$

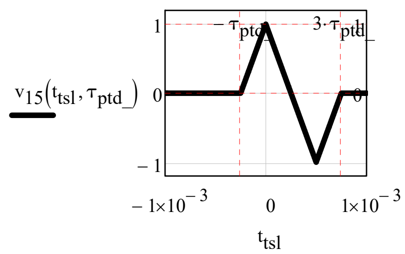


Pulses

-10 Bipolar Triangular Voltage Pulse

Data file " general data.xmcd"

$$v_{15}\left(t, \tau_{ptd}\right) := \Lambda\left(t, \tau_{ptd}\right) - \Lambda\left(t - 2\cdot\tau_{ptd}, \tau_{ptd}\right)$$



Pulses

-11 Sawtooth Voltage Pulse with positive slope

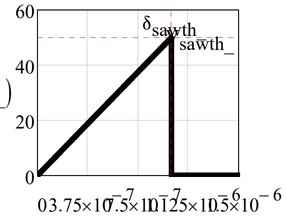
Data file " sawtooth pulse data.xmcd"

Signal amplitude: V_{sawth}

Slope: sp_{sawth}

$$f_{sw}(t, \delta_{sawth}, V_{sawth}) := \frac{V_{sawth}}{\delta_{sawth}} \cdot t \cdot rect1(t, 0.0 \cdot sec, \delta_{sawth})$$

$$t_{tsw} := -\delta_{sawth\_} \cdot 0, -\delta_{sawth\_} \cdot 0 + \frac{5 \cdot \delta_{sawth\_} + \delta_{sawth\_} \cdot 0}{10000} .. 5 \cdot \delta_{sawth\_}$$



$f_{sw}(t_{tsw}, \delta_{sawth\_}, V_{sawth\_})$

t_{tsw}

Pulses

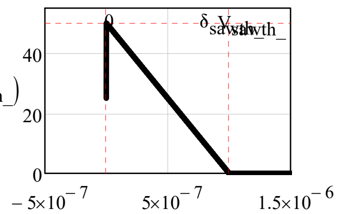
-12 Sawtooth Voltage Pulse with negative slope

Data file " sawtooth pulse data.xmcd"

Signal amplitude: V_{sawth}

Slope: sp_{nsawth}

$$f_{sl}(t, \delta_{sawth}, V_{sawth}) := V_{sawth} \cdot \left(\frac{-t}{\delta_{sawth}} + 1\right) \cdot (\Phi(t) - \Phi(t - \delta_{sawth}))$$



$f_{sl}(t_{tsw}, \delta_{sawth\_}, V_{sawth\_})$

t_{tsw}

Pulses

-13 Bipolar Single Sawtooth with adjustable rising and falling edges Pulse Train

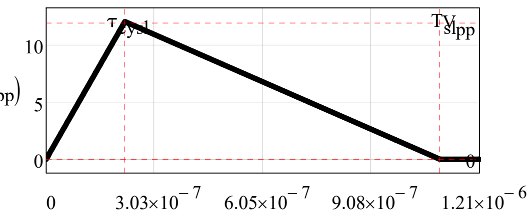
Data file " sawtooth pulse data.xmcd"

$$s1s2(t, T_{sl}, \delta_{cycl}, Ampl) := \left[\frac{t \cdot (\Phi(t - T_{sl} \cdot \delta_{cycl}) - \Phi(t) + \delta_{cycl} \cdot \Phi(t) - \delta_{cycl} \cdot \Phi(t - T_{sl}))}{T_{sl} \cdot \delta_{cycl}} \cdots + \Phi(t - T_{sl}) - \Phi(t - T_{sl} \cdot \delta_{cycl})\right] \cdot \frac{Ampl}{(\delta_{cycl} - 1)}$$

$V_{pp} = 12\,V$

$T_{sl} := 1.1 \cdot \mu s$ $\quad \omega_{0sl} := \frac{2 \cdot \pi}{T_{sl}}$ $\quad \omega_{0sl} = 5.712 \cdot \frac{Mrads}{sec}$ $\quad \delta_{cycl} = 0.2$

$\tau_{cysl} := \delta_{cycl} \cdot T_{sl}$



$s1s2(t_{sw}, T_{sl}, \delta_{cycl}, V_{pp})$

t_{sw}

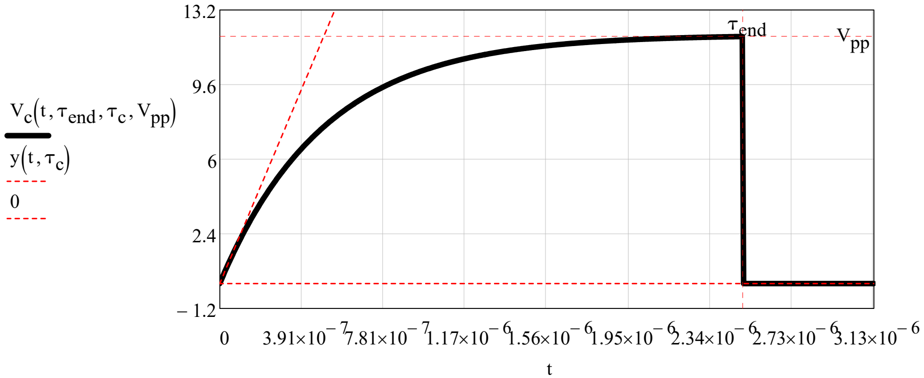
Pulses

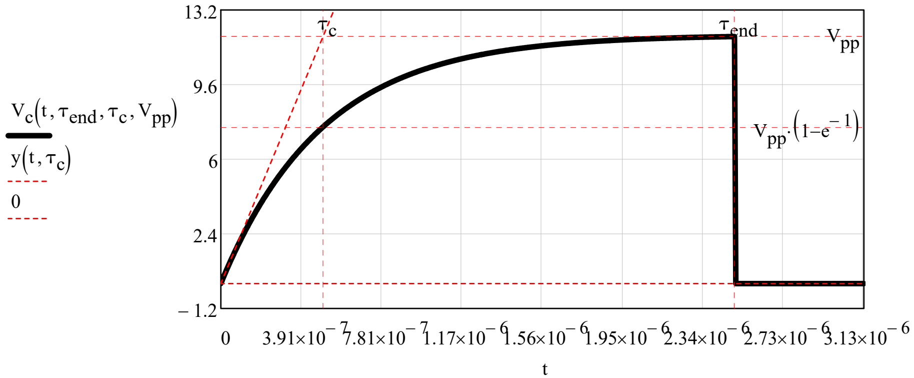
-14 Voltage Pulse Exponentially Rising

τ_c = time constant

$$V_c(t,\tau_{end},\tau_c,V_{pp}) := \left(1 - e^{\frac{-t}{\tau_c}}\right) \cdot V_{pp} \cdot \left(\Phi(t) - \Phi(t-\tau_{end})\right)$$

$$y(t,\tau_c) := \frac{V_{pp}}{\tau_c} \cdot t \qquad V_{end} := \left(1 - e^{\frac{-\tau_{end}}{\tau_c}}\right) \cdot V_{pp}$$





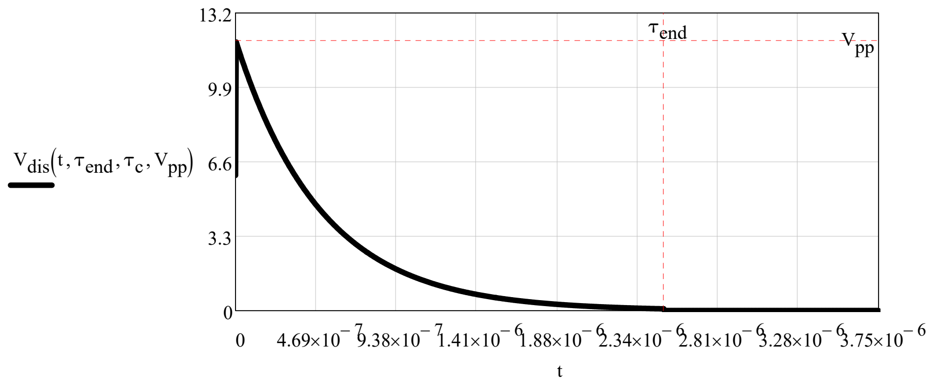
Dimensionless function: $V_{cd}(t,\tau_{end},\tau_c,V_{pp}) := \dfrac{V_c(t,\tau_{end},\tau_c,V_{pp})}{V}$

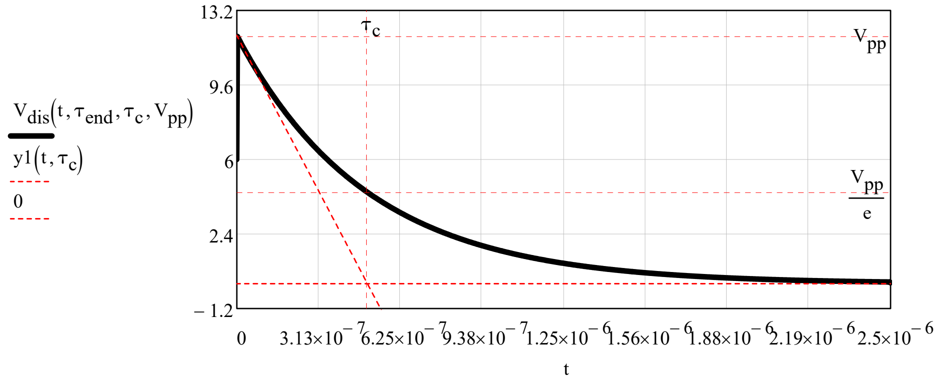
Pulses

-15 Voltage Pulse Exponentially Decaying

$$V_{dis}(t,\tau_{end},\tau_c,V_{pp}) := V_{pp} \cdot e^{\frac{-t}{\tau_c}} \cdot \left(\Phi(t) - \Phi(t-\tau_{end})\right)$$

$$y1(t,\tau_c) := V_{pp} \cdot \left(-\frac{t}{\tau_c} + 1\right) \qquad \tau_c = 0.5 \cdot \mu s$$





Dimensionless function: $V_{disad}(t,\tau_{end},\tau_c,V_{pp}) := \dfrac{V_{dis}(t,\tau_{end},\tau_c,V_{pp})}{V}$

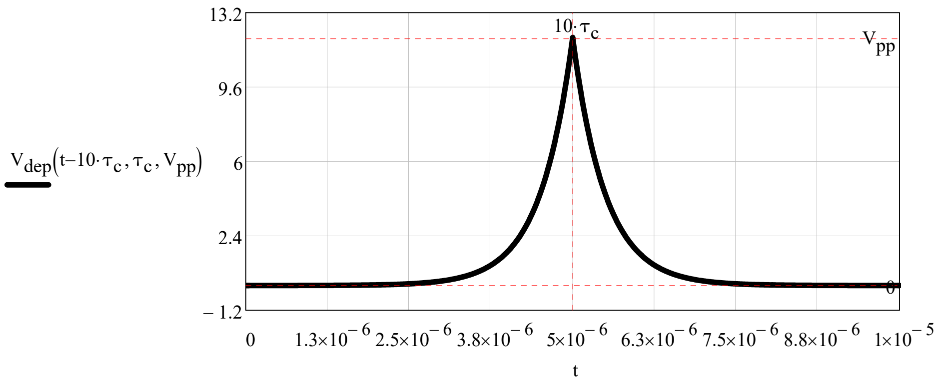
TEST Waveforms

Pulses

- 16 Double Exponential Pulse

$$V_{dep}(t,\tau_c,V_{pp}) := \begin{array}{|l} \text{return "τ.ptd less or =0.0"} \quad \text{if } \tau_c \le 0 \\ \text{return "Vpp=0.0"} \quad \text{if } V_{pp} = 0.0\cdot V \quad \text{otherwise} \\ V_{pp}\cdot e^{\frac{-|t|}{\tau_c}} \end{array}$$

Dimensionless function: $V_{depad}(t,\tau_c,V_{pp}) := \dfrac{V_{dep}(t,\tau_c,V_{pp})}{V}$



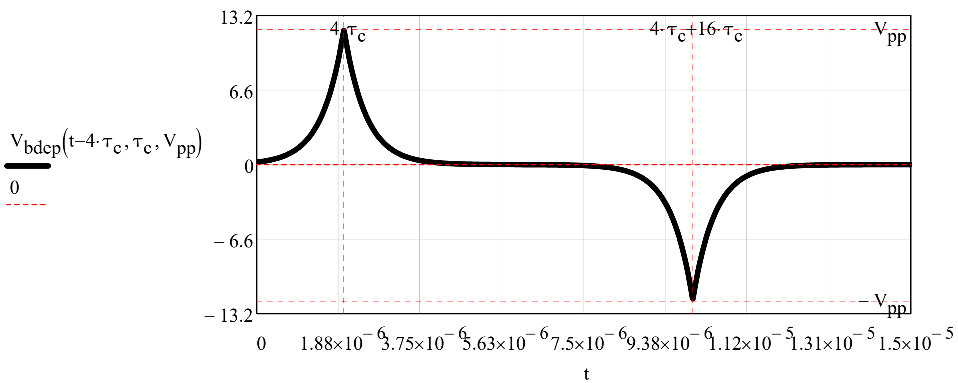
TEST Waveforms

Pulses

- 17 Bipolar Double Exponential Pulse

$$V_{bdep}(t,\tau_{tw},V_{pp}) := V_{dep}(t,\tau_{tw},V_{pp})\cdot \text{rect1}(t,-8\cdot\tau_{tw},16\cdot\tau_{tw}) \ldots$$
$$+(-1)\cdot V_{dep}(t-16\cdot\tau_{tw},\tau_{tw},V_{pp})\cdot\text{rect1}(t,8\cdot\tau_{tw},16\cdot\tau_{tw})$$

Dimensionless function: $V_{bdepad}(t,\tau_{tw},V_{pp}) := \dfrac{V_{bdep}(t,\tau_{tw},V_{pp})}{V}$



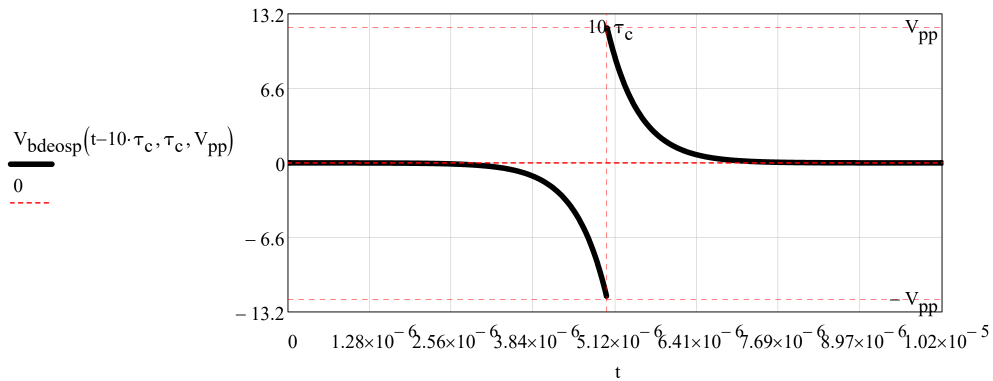
TEST Waveforms

Pulses

- 18 Bipolar Double Exponential Odd symmetric Pulse

$$V_{bdeosp}(t,\tau_{tw},V_{pp}) := \begin{array}{|l} \text{return "τ.tw less or =0.0"} \quad \text{if } \tau_{tw} \le 0 \\ \text{return "Vpp=0.0"} \quad \text{if } V_{pp} = 0.0\cdot V \quad \text{otherwise} \\ e^{\frac{-t}{\tau_{tw}}}\cdot V_{pp} \quad \text{if } t > 0.0 \\ 0\cdot V_{pp} \quad \text{if } t = 0.0 \\ -e^{\frac{t}{\tau_{tw}}}\cdot V_{pp} \quad \text{if } t < 0.0 \end{array}$$

Dimensionless function: $V_{bdeospad}(t,\tau_{tw},V_{pp}) := \dfrac{V_{bdeosp}(t,\tau_{tw},V_{pp})}{V}$



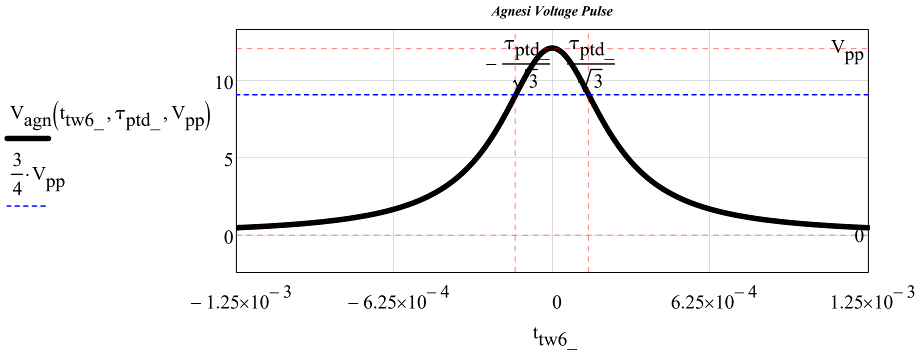
TEST Waveforms

Pulses

- 19 Agnesi Profile Voltage Pulse

$$V_{agn}(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}} \cdot \frac{\tau_{tw}^3}{t^2 + \tau_{tw}^2}$$

Dimensionless function: $V_{agnad}(t, \tau_{tw}, V_{pp}) := \dfrac{V_{agn}(t, \tau_{tw}, V_{pp})}{V}$ $\quad \tau_{ptd\_} = 250 \cdot \mu s$



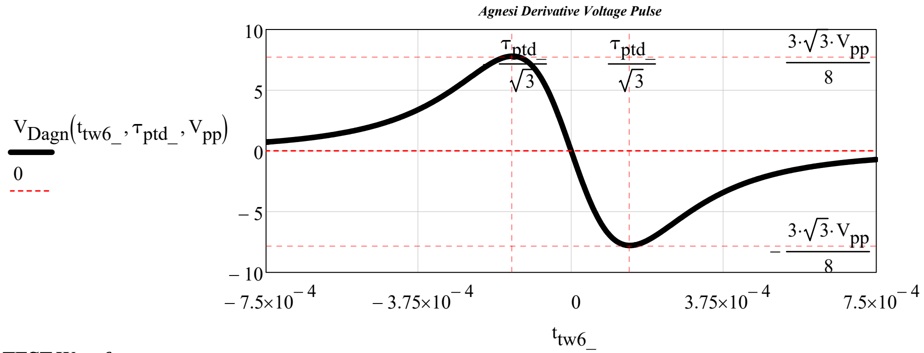
TEST Waveforms

Pulses

- 20 Agnesi Derivative Voltage Pulse

$$V_{Dagn}(t, \tau_{tw}, V_{pp}) := -\tau_{tw} \cdot \frac{2 \cdot V_{pp} \cdot t \cdot \tau_{tw}^2}{\left(t^2 + \tau_{tw}^2\right)^2}$$

Dimensionless function: $V_{Dagnad}(t, \tau_{tw}, V_{pp}) := \dfrac{V_{Dagn}(t, \tau_{tw}, V_{pp})}{V}$ $\quad \tau_{ptd\_} = 250 \cdot \mu s$



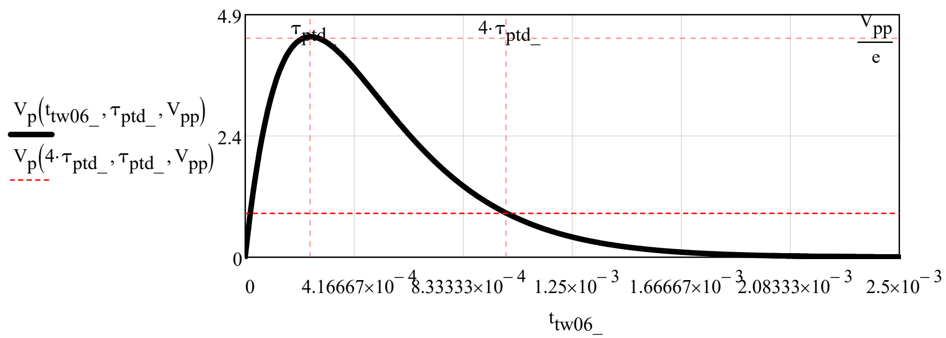
TEST Waveforms

Pulses

- 21 Poisson Profile Voltage Pulse

$$V_p(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}} \cdot t \cdot e^{\frac{-t}{\tau_{tw}}}$$

Dimensionless function: $V_{pad}(t, \tau_{tw}, V_{pp}) := \dfrac{V_p(t, \tau_{tw}, V_{pp})}{V}$ $\quad \tau_{ptd\_} = 250 \cdot \mu s$



TEST Waveforms

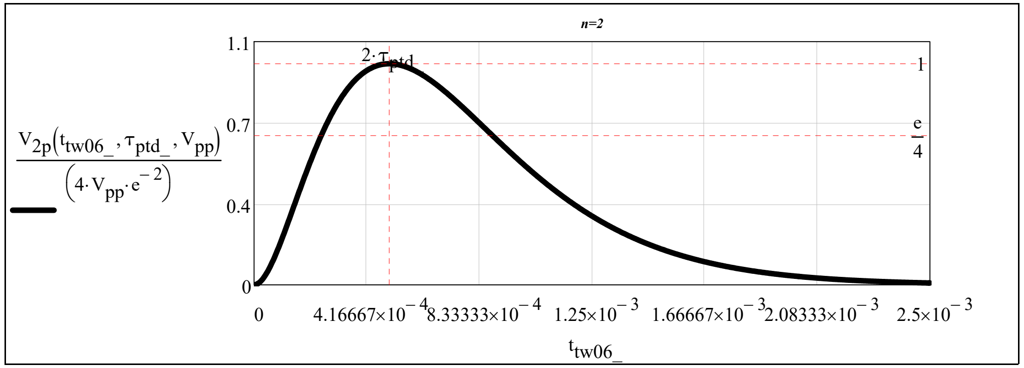
Pulses

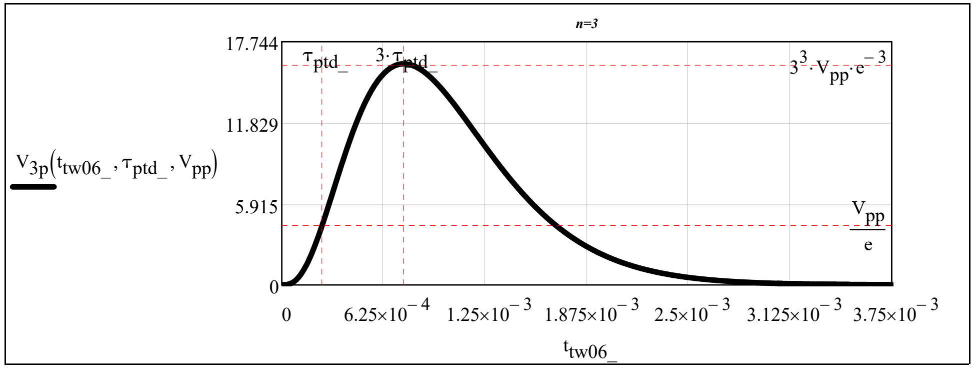
- 22 n^th Poisson Profile Voltage Pulse

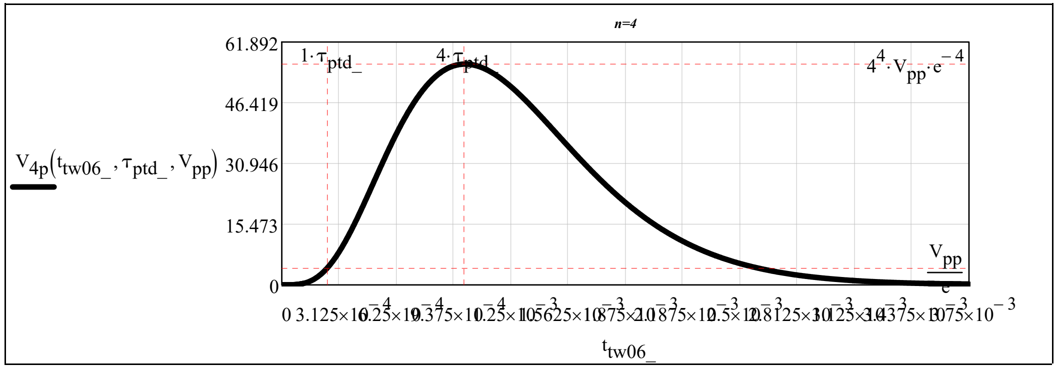
$maxy = V_{pp} \cdot e^{-n} \cdot n^n$ $\qquad maxx = n \cdot \tau_{ptd}$

$$V_{2p}(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}^2} \cdot t^2 \cdot e^{\frac{-t}{\tau_{tw}}} \qquad V_{3p}(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}^3} \cdot t^3 \cdot e^{\frac{-t}{\tau_{tw}}} \qquad V_{4p}(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}^4} \cdot t^4 \cdot e^{\frac{-t}{\tau_{tw}}}$$

Dimensionless function: $V_{2pad}(t, \tau_{ptd}, V_{pp}) := \dfrac{V_{2p}(t, \tau_{ptd}, V_{pp})}{V}$ $\quad \tau_{ptd\_} = 250 \cdot \mu s$







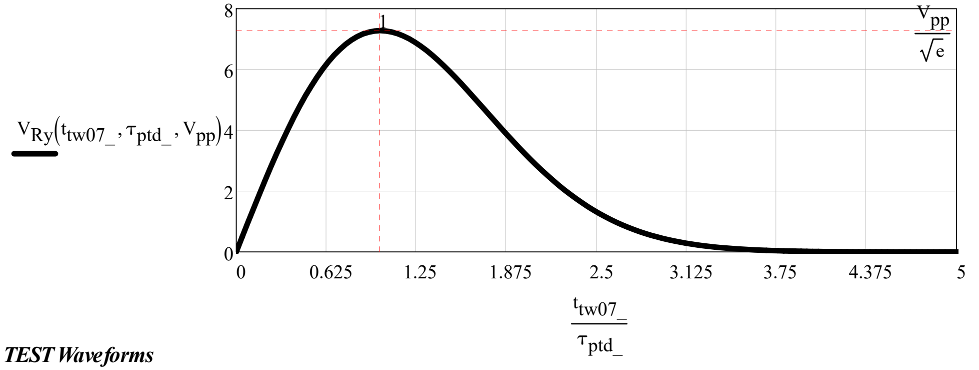
TEST Waveforms

Pulses

- 23 Rayleigh Profile Voltage Pulse

$$V_{Ry}(t, \tau_{tw}, V_{pp}) := \frac{V_{pp}}{\tau_{tw}} \cdot t \cdot e^{\frac{-t^2}{2 \cdot \tau_{tw}^2}}$$

$$t_{tw07\_} := 0 \cdot \tau_{ptd\_}, 0 \cdot \tau_{ptd\_} + \frac{200 \cdot \tau_{ptd\_}}{5000} .. 200 \cdot \tau_{ptd\_} \qquad \tau_{ptd\_} = 250 \cdot \mu s$$



TEST Waveforms

Pulses

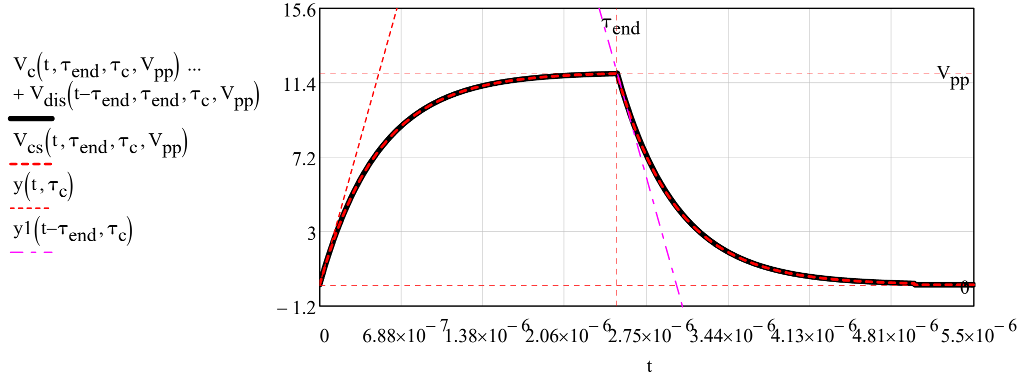
- 24 Cap. Charge and Discharge Voltage Pulse

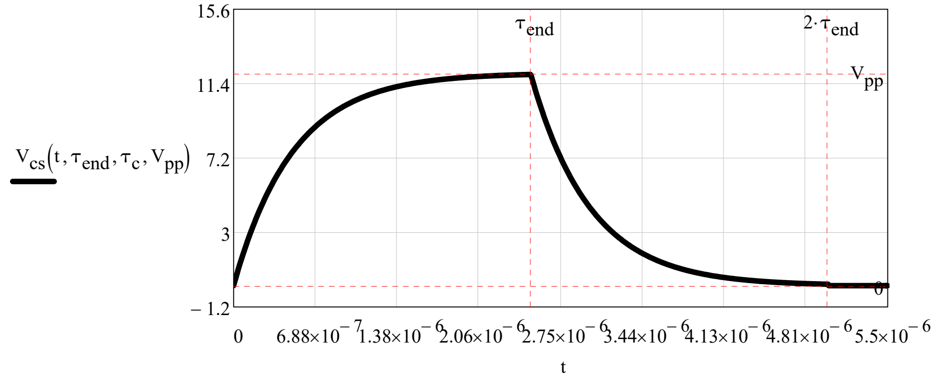
$V_c(t, \tau_{init}, \tau_{end}, \tau_c, V_{pp})$ τ_c = time constant

Parameters description:
V_{cs}(time, time constant, pulse width, supply voltage)

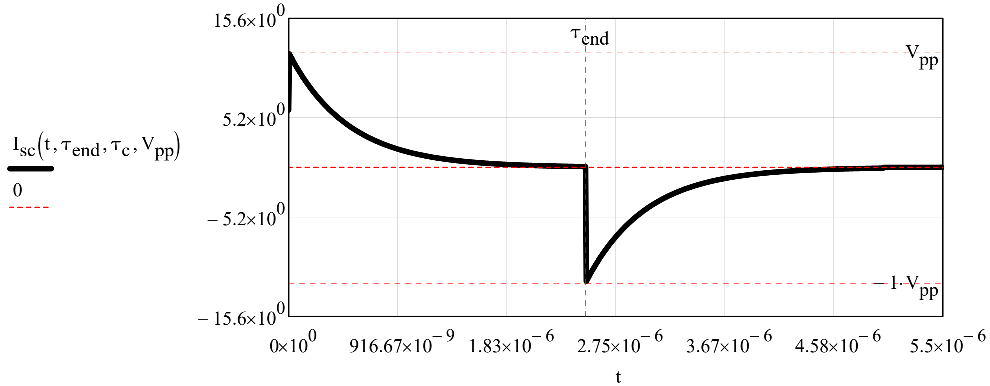
$$V_{cs}(t, \tau_{end}, \tau_c, V_{pp}) := V_c(t, \tau_{end}, \tau_c, V_{pp}) \dots + V_{dis}(t - \tau_{end}, \tau_{end}, \tau_c, V_{end})$$

$\tau_{end} = 2.5 \cdot \mu s$ $\quad \tau_c = 0.5 \cdot \mu s$ $\quad \tau_{end} = 2.5 \cdot \mu s$ $\quad V_{pp} = 12\,V$





$$I_{sc}(t_{tw}, \tau_{end}, \tau_c, V_{pp}) := V_{dis}(t_{tw}, \tau_{end}, \tau_c, V_{pp}) \cdots + (-1) \cdot V_{dis}(t_{tw} - \tau_{end}, \tau_{end}, \tau_c, V_{pp})$$

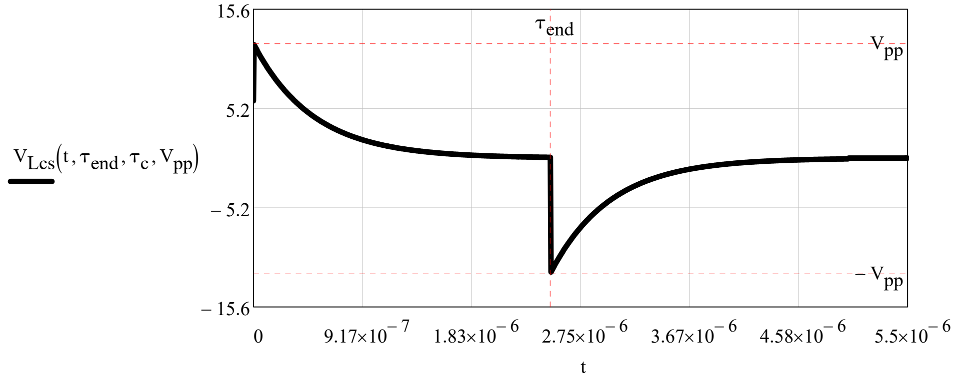


TEST Waveforms

Pulses

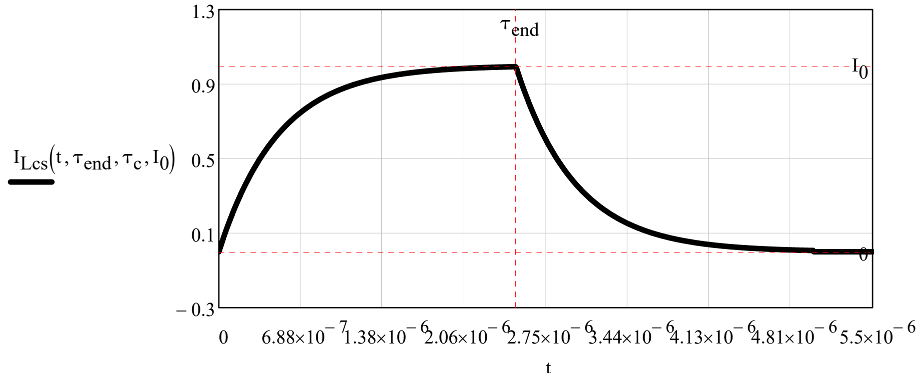
- 25 Induct. Charge and Discharge Pulse

$$V_{Lcs}(t_{tw}, \tau_{end}, \tau_c, V_{pp}) := V_{dis}(t_{tw}, \tau_{end}, \tau_c, V_{pp}) \cdots + (-1) \cdot V_{dis}(t_{tw} - \tau_{end}, \tau_{end}, \tau_c, V_{pp})$$



$I_0 := 1 \cdot A$ $\qquad$ $I_{end} := \left(1 - e^{\frac{-\tau_{end}}{\tau_c}}\right) \cdot I_0$

$$I_{Lcs}(t, \tau_{end}, \tau_c, I_0) := V_c(t, \tau_{end}, \tau_c, I_0) \cdots + V_{dis}(t - \tau_{end}, \tau_{end}, \tau_c, I_{end})$$



Pulses

- 26 Triangular Cusp Pulse

Signal amplitude: V_{pp}

Pulse width: $p_w = \tau_{ptd}$

Max pulse amplitude and cusp ratio: $a_p = \frac{1}{4}$ $\quad a_p < 1$

Cusp slope $c_s = V_{pp}\cdot\frac{2\cdot(1-a_p)}{p_w}$

$$\mathrm{cusp0}(t,p_w,a_p,V_{pp}) := V_{pp}\cdot\left[\left[1+t\cdot\frac{2\cdot(1-a_p)}{p_w}\right]\cdot\left(\Phi\left(t+\frac{p_w}{2}\right)-\Phi(t)\right)\ \ldots \atop +\left[1-t\cdot\frac{2\cdot(1-a_p)}{p_w}\right]\cdot\left(\Phi(t)-\Phi\left(t-\frac{p_w}{2}\right)\right)\right]$$

Signal amplitude: $V_{pp} = 12\cdot V$

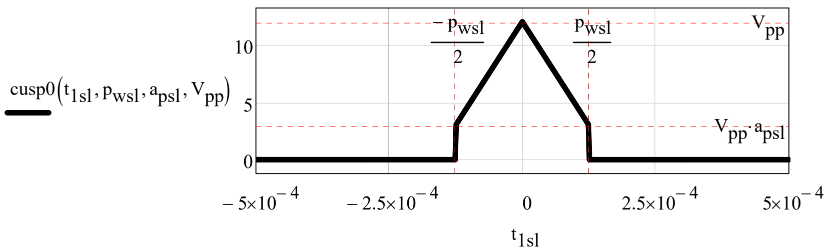
Pulse width: $p_{wsl} := \tau_{ptd\_}$ $\quad p_{wsl} = 2.5\times10^{-4}\,s$

$p_{wsl} = 250\cdot\mu s$

Max pulse amplitude and cusp ratio: $a_{psl} := \frac{1}{4}$ $\quad a_{psl} < 1$

Cusp slope $c_{ssl} := V_{pp}\cdot\frac{2\cdot(1-a_{psl})}{p_{wsl}}$ $\quad c_{ssl} = 0.072\cdot\frac{V}{\mu s}$

One Cusp Pulse



Pulses

- 27 Parabolic Cusp Pulse

Signal amplitude: V_{pp}

Pulse width: p_w

Max pulse amplitude and cusp ratio: $a_{p1} = 0.61$ $\quad a_{p1} < 1$

$$y_{parab}(t,p_w,a_{p1},V_{pp}) := V_{pp}\cdot\left[\frac{4\cdot t^2\cdot(a_{p1}-1)}{p_w^{\,2}}+1\right]$$

$$\mathrm{cusp1}(t,p_w,a_{p1},V_{pp}) := \left[\frac{4\cdot t^2\cdot(a_{p1}-1)}{p_w^{\,2}}+1\right]\cdot\left(\Phi\left(t+\frac{p_w}{2}\right)-\Phi\left(t-\frac{p_w}{2}\right)\right)\cdot V_{pp}$$

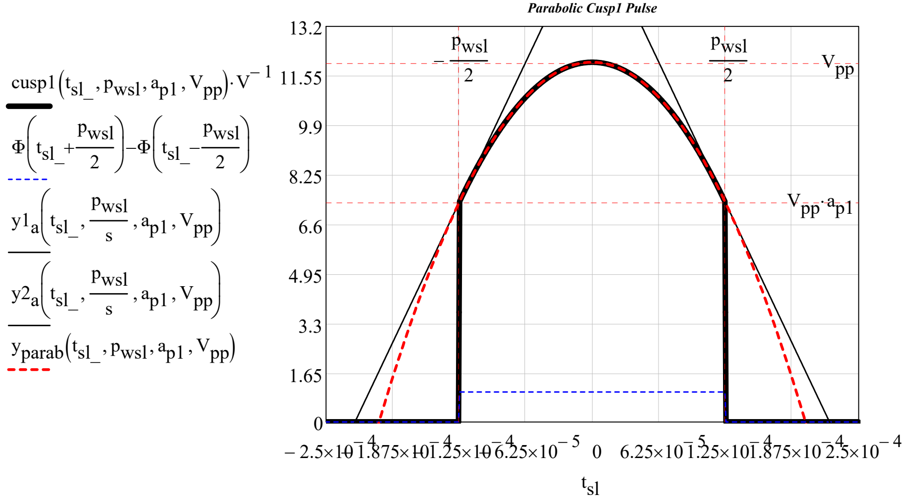
Geometric tangens in $\pm\frac{p_w}{2}$:

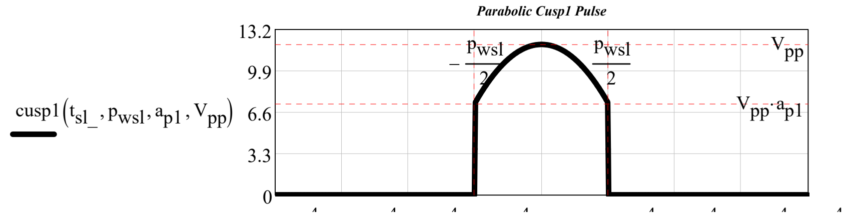
$$y1_a(t,p_w,a_{p1},V_{pp}) := \frac{4\cdot V_{pp}\cdot(a_{p1}-1)}{p_w}\cdot t+(2-a_{p1})\cdot V_{pp}$$

$$y2_a(t,p_w,a_{p1},V_{pp}) := -\frac{4\cdot V_{pp}\cdot(a_{p1}-1)}{p_w}\cdot t+(2-a_{p1})\cdot V_{pp}$$

Max pulse amplitude and cusp ratio: $a_{p1} := 0.61$ $\quad a_{p1} < 1$

Parabolic Cusp1 Pulse





-5×10^{-4} -3.75×10^{-4} -2.5×10^{-4} -1.25×10^{-4} 0 1.25×10^{-4} 2.5×10^{-4} 3.75×10^{-4} 5×10^{-4}

$t_{sl\_}$

TEST Waveforms

Pulses

- 28 Elliptic Cusp Pulse

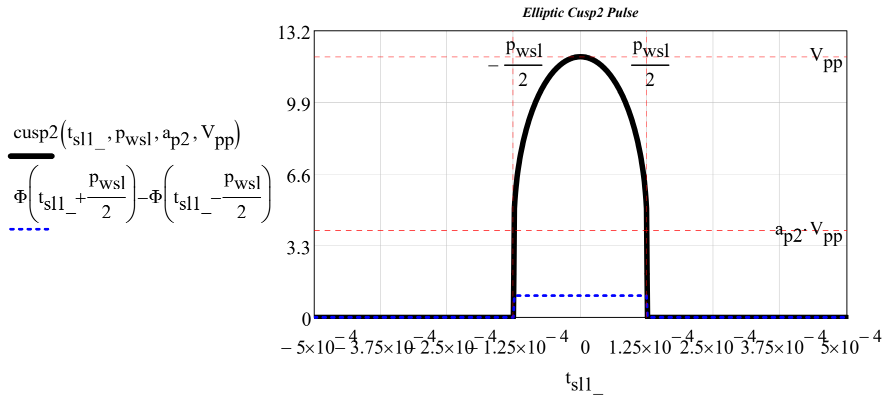
Signal amplitude: V_{pp}

Pulse width: p_w

Max pulse amplitude and cusp ratio: $a_{p2} = \frac{1}{3}$ $a_p < 1$

$$cusp2(t, p_w, a_{p2}, V_{pp}) := V_{pp} \cdot \left[\frac{2 \cdot (1 - a_{p2})}{p_w} \cdot \sqrt{\left(\frac{p_w}{2}\right)^2 - t^2} + a_{p2} \right] \cdot \left(\Phi\left(t + \frac{p_w}{2}\right) - \Phi\left(t - \frac{p_w}{2}\right) \right)$$

Max pulse amplitude and cusp ratio: $a_{p2} := \frac{1}{3}$ $a_{psl} < 1$



Pulses and Priodic Formulae Only

Periodic Waveforms *Periodic Waveforms* *Periodic Waveforms*

Periodic Waveforms Formulae Only

TEST Waveforms

Periodic Waveforms

1) Half wave

Data file " general data.xmcd"

Amplitude: $B1_{hw}$

Period: T_{hw} Angular frequency: $\omega_{hw} = \frac{2\cdot\pi}{T_{0gd}}$

$$g1hw(t, T_{hw}, B1_{hw}, N0_{gd}) := \frac{B1_{hw}}{V}\cdot\sum_{k=0}^{N0_{gd}}\left(rect1\left(t-k\cdot T_{hw}, -1\cdot T_{hw}, \frac{T_{hw}}{2}\right)\cdot\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\right)$$

TEST Waveforms

Periodic Waveforms

2 Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{hw}$,

Amplitude of the decreasing exponential for t=0: V_{pp},

Exponential Time constant: $\tau_{hw1} = 2\cdot T_{0gd}$,

Period: $T_{hw} = T_{0gd}$,

Pulsation: ω_{hw},

Intersection abscissa between half wave and exponential: ζ (scalar),
Tangent points abscissas between half wave and exponential: τ_{tpa} (vector)

$$Z01(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{tpv}) := \begin{array}{|l} \xi \leftarrow \frac{2\cdot\pi}{\omega_{hw}\cdot 1} \\ f_{sl}(\xi) \leftarrow B1_{hw}\cdot\sin(\omega_{hw}\cdot\xi) \\ g(\xi) \leftarrow e^{\frac{-\xi}{\tau_{hw1}}}\cdot V_{tpv} \\ S \leftarrow root(f_{sl}(\xi) - g(\xi), \xi) \\ return\ \ S \end{array}$$

$$g02hw(t_{sl}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}) := \begin{array}{|l} Y \leftarrow \frac{B1_{hw}}{V}\cdot\sin(\omega_{hw}\cdot t_{sl}) \text{ if } 0 \le t_{sl} \le \tau_{tpa_1} \\ \text{otherwise} \\ \quad \begin{array}{|l} Y \leftarrow e^{\frac{-t_{sl}}{\tau_{hw1}}}\cdot\frac{V_{tpv}}{V} \text{ if } \tau_{tpa_1} < t_{sl} < \zeta_{hw} \\ Y \leftarrow \frac{B1_{hw}}{V}\cdot\sin(\omega_{hw}\cdot\zeta_{hw}) \text{ if } t_{sl} = \zeta_{hw} \text{ otherwise} \end{array} \\ return\ \ Y \end{array}$$

$$g03hw(t_{sl}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}) := \begin{array}{|l} X \leftarrow \frac{B1_{hw}}{V}\cdot\sin(\omega_{hw}\cdot t_{sl}) \text{ if } \zeta_{hw} \le t_{sl} \le \tau_{tpa_3} \\ X \leftarrow e^{\frac{-\left(t_{sl}-\frac{2\cdot\pi}{\omega_{hw}}\right)}{\tau_{hw1}}}\cdot\frac{V_{tpv}}{V} \text{ if } \tau_{tpa_3} < t_{sl} \le \zeta_{hw} + \frac{2\cdot\pi}{\omega_{hw}} \text{ oth} \\ return\ \ X \end{array}$$

$$g2hw(t_{sl}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}, N0_{gd}) := g02hw(t_{sl}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv})\cdot rect1\Big(t_{sl}\cdot$$
$$+ \sum_{k=0}^{N0_{gd}}\left(g03hw\left(t_{sl} - k\cdot\frac{2\cdot\pi}{\omega_{hw}}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{h}\right.\right.$$

Periodic Waveforms

3 Double Half wave

$$g3\left(t, T_{hw}, B1_{hw}\right) := \frac{B1_{hw}}{V} \cdot \left|\sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t\right)\right|$$

TEST Waveforms

Periodic Waveforms

4 Double Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{hw}$,

Amplitude of the decreasing exponential for t=0: V_{pp},

Exponential Time constant: τ_{hw1},

Period: $\frac{T_{hw}}{2}$,

Pulsation: ω_{hw},

Intersections between half wave and exponential: τ_{tpa} (vector),

Tangent points between half wave and exponential: θ_{dhw} (scalar)

Intersection

$$Z1\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{ppt}\right) := \left| \begin{array}{l} k \leftarrow 1 \\ \tau_{tpa_k} \leftarrow \dfrac{\operatorname{atan}\left(-\omega_{hw} \cdot \tau_{hw1}\right) + k \cdot \pi}{\omega_{hw}} \\ V_{ppt} \leftarrow B1_{hw} \cdot \sin\left(\omega_{hw} \cdot \tau_{tpa_1}\right) \cdot e^{\frac{\tau_{tpa_1}}{\tau_{hw1}}} \\ \xi \leftarrow \dfrac{\pi}{\omega_{hw}} \\ f(\xi) \leftarrow B1_{hw} \cdot \left|\sin\left(\omega_{hw} \cdot \xi\right)\right| \\ g(\xi) \leftarrow e^{\frac{-\xi}{\tau_{hw1}}} \cdot V_{ppt} \\ S \leftarrow \operatorname{root}\left(f(\xi) - g(\xi), \xi\right) \\ \text{return } S \end{array} \right.$$

$$\theta_{dhw} = Z1\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{ppt}\right)$$

$$h02\_\left(t, \tau_{tw}, \tau_{tpa}, \theta, \omega_{hw}, B1_{hw}, V_{ppt}\right) := \left| \begin{array}{l} Y \leftarrow \dfrac{B1_{hw}}{V} \cdot \left|\sin\left(\omega_{hw} \cdot t\right)\right| \quad \text{if } 0 \le t \le \tau_{tpa_1} \\ \text{otherwise} \\ \quad \left| \begin{array}{l} Y \leftarrow e^{\frac{-t}{\tau_{tw}}} \cdot \dfrac{V_{ppt}}{V} \quad \text{if } \tau_{tpa_1} < t < \theta \\ Y \leftarrow \dfrac{B1_{hw}}{V} \cdot \left|\sin\left(\omega_{hw} \cdot \theta\right)\right| \quad \text{if } t = \theta \quad \text{otherwise} \end{array} \right. \\ \text{return } Y \end{array} \right.$$

$$h03\_\left(t, \tau_{tw}, \tau_{tpa}, \theta, \omega_{hw}, B1_{hw}, V_{ppt}\right) := \left| \begin{array}{l} X \leftarrow \dfrac{B1_{hw}}{V} \cdot \left|\sin\left(\omega_{hw} \cdot t\right)\right| \quad \text{if } \theta \le t \le \tau_{tpa_2} \\ X \leftarrow e^{\frac{-\left(t - \frac{\pi}{\omega_{hw}}\right)}{\tau_{tw}}} \cdot \dfrac{V_{ppt}}{V} \quad \text{if } \tau_{tpa_2} < t \le \theta + \dfrac{\pi}{\omega_{hw}} \quad \text{otherwise} \\ \text{return } X \end{array} \right.$$

$$g4\left(t, \tau_{hw1}, \tau_{tpa}, \theta, \omega_{hw}, B1_{hw}, V_{ppt}, N0_{gd}\right) := h02\_\left(t, \tau_{hw1}, \tau_{tpa}, \theta, \omega_{hw}, B1_{hw}, V_{ppt}\right) \cdot \operatorname{rect1}\left(t, 0 \cdot \frac{\pi}{\omega_{hw}}, \theta\right) \dots$$
$$+ \sum_{k=0}^{N0_{gd}} \left(h03\_\left(t - k \cdot \frac{\pi}{\omega_{hw}}, \tau_{hw1}, \tau_{tpa}, \theta, \omega_{hw}, B1_{hw}, V_{ppt}\right) \cdot \operatorname{rect}\right.$$

Periodic Waveforms

5 Voltage Pulse Train

Data " pulse train data"

Pulse period:	T_{ptd}
Pulse Cadence:	f_{ptd}
Pulse width:	τ_{ptd}
Duty Cycle:	δ_{ptd}
Amplitude:	B_{ptd}
Pulse delay from the origin:	$\tau_{ptd\delta 0}$
Average value:	$v_{ptm} = B_{ptd} \cdot \delta_{ptd}$

$$\tau_{ptd} = T_{ptd} \cdot \delta_{ptd} \qquad T_{ptd} = \frac{\tau_{ptd}}{\delta_{ptd}}$$

Generic pulse definition: rect1 (t, risingedge, width)

$$v_{ptsl}\left(t, T_{ptd}, \tau_{\delta 0}, \delta_{ptd}, B_{ptd}, N0_{gd}\right) := B_{ptd} \cdot \sum_{k=0}^{N0_{gd}} \mathrm{rect1}\left(t - k \cdot T_{ptd}, \tau_{\delta 0}, T_{ptd} \cdot \delta_{ptd}\right)$$

$$\mathrm{Vip1}\left(t, \tau_{ptd}, \tau_{\delta 0}, \delta_{tw}, B2_{tw}, N0_{gd}\right) := \frac{v_{ptsl}\left(t, \tau_{ptd}, \tau_{\delta 0}, \delta_{tw}, B2_{tw}, N0_{gd}\right)}{\mathrm{volt}}$$

Function definition: Vip1(time, Period, rising edge delay, Duty Cycle, pulse amplitude)

Periodic Waveforms

6 RF Pulse Train

Data " rf pulse data"

Generic pulse definition: $\mathrm{rect1}(t, \mathrm{risingedge}, \mathrm{width}) = \mathrm{rect1}\left(t, \tau_{\delta rf}, \tau_{ptd}\right)$

Period:	T_{ptd}
Pulse Cadence:	f_{ptd}
time constant:	τ_{rfpd}
Rising edge delay:	$\tau_{\delta rfpd}$
Signal period:	T_{rfpd}
Signal angular frequency:	ω_{rfpd}
Pulse width:	τ_{ptd}
Duty Cycle:	δ_{ptd}
Amplitude:	B_{ptd}
Pulse delay from the origin:	$\tau_{ptd\delta 0}$
Average value:	$v_{rfptm} = B_{ptd} \cdot \delta_{ptd}$

$$\tau_{ptd} = T_{ptd} \cdot \delta_{ptd}$$

$$f_{rfpt}\left(t, \tau_{\delta rf}, \tau_{ptd}, \omega_{rfpt}\right) := \mathrm{rect1}\left(t, \tau_{\delta rf}, \tau_{ptd}\right) \cdot \cos\left(\omega_{rfpt} \cdot t\right)$$

(v_{ptrfs} parameters: v_{pt}(time, period, pulse_width, duty_cycle, pulse_amplitude))

$$v_{ptrf}\left(t, T_{ptd}, \tau_{\delta rfpd}, \delta_{ptd}, \omega_{rfpd}, V_{rf}, N0_{gd}\right) := V_{rf} \cdot \sum_{k=0}^{N0_{gd}} f_{rfpt}\left(t - k \cdot T_{ptd}, \tau_{\delta rfpd}, T_{ptd} \cdot \delta_{ptd}, \omega_{rfp}\right.$$

Average value: $v_{ptmrf} = B_{ptd} \cdot \delta_{ptd}$

$$Vi_{pt}\left(t, T_{ptd}, \tau_{\delta rf}, \delta_{ptd}, \omega_{rfpt}, V_{rf}, N0_{gd}\right) := \frac{v_{ptrf}\left(t, T_{ptd}, \tau_{\delta rf}, \delta_{ptd}, \omega_{rfpt}, V_{rf}, N0_{gd}\right)}{\mathrm{volt}}$$

(v_{ptrfs} parameters: Vi_{pt}(time, period, pulse_width, duty_cycle, pulse_amplitude))

Periodic Waveforms

7 Bipolar Square Wave

Data file " pulse train data.xmcd"

Signal amplitude: V_{pp}

Square wave period: T_{0gd}

ω_{ptd}

Test signal: $$v_{sqw}(t, T_{0gd}, V_{pp}, N0_{gd}) := V_{pp} \cdot \sum_{k=0}^{N0_{gd}} \left[\left[\Phi(t - k \cdot T_{0gd}) - 2 \cdot \Phi\left[t - \left(\frac{2 \cdot k + 1}{2}\right) \cdot T_{0gd}\right]\right] \ldots \\ + \Phi[t - (k+1) \cdot T_{0gd}] \right]$$

$$Vsqw(t, T_{0gd}, V_{pp}, N0_{gd}) := \frac{v_{sqw}(t, T_{0gd}, V_{pp}, N0_{gd})}{volt}$$

Periodic Waveforms

8 Bipolar Square Wave 1

Data file " pulse train data.xmcd"

Period: T_{ptd} $T_{ptd} := \tau_{\delta} + 4 \cdot \tau_{ptd}$

Pulse Cadence: f_{ptd}

Pulse width: τ_{ptd}

Amplitude: V_{pp}

Pulse delay from the origin: τ_{δ}

$$V_6(t, \tau_{\delta}, \tau_{ptd}, T_6, V_{pp}, N0_{gd}) := \sum_{k=0}^{N0_{gd}} V_5(t - k \cdot T_6, \tau_{\delta}, \tau_{ptd}, V_{pp})$$

$V_5(\ldots)$ is defined in ***-5 Doublet Voltage Pulse***

Periodic Waveforms

9 Staircase 1 Voltage Pulse Train

Description of the Function's parameters: $v_{stcp}(t, period, signal\_amplitude, number\_of\_steps)$,

: $v_{stc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

For data, see *staircase pulse data*

Period:	T_{stcpt}	$T_{stcpt} = (m1_{steps} + 1) \cdot T_{1stpl\_} \cdot 2$
Step length:	$T_{1stpl\_}$	$T_{1stpl\_} = \frac{T_{stcpt}}{2 \cdot (m1_{steps} + 1)}$
Number of steps:	$m1_{steps}$	
Duty Cycle:	δ_{ptd}	$\tau_{ptd} = T_{ptd} \cdot \delta_{ptd}$
Step Amplitude:	$V_{stcstp0}$	
Amplitude:	V_{stcs}	
Pulse delay from the origin:	$\tau_{ptd\delta 0}$	

Average value: $$v_{stcpta} = \frac{V_{stcs}}{2 \cdot m1_{steps} \cdot (m1_{steps} + 1)} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1)$$

$$\frac{1}{T_{stcpt}} \cdot \int_{T_{1stpl}}^{(m1_{steps}+1) \cdot T_{1stpl\_}} v_{stc}(t, T_{1stpl\_}, V_{stcs}, m1_{steps})\, dt$$

Test signal: $$v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd}) := \sum_{k=0}^{N0_{gd}} v_{stc}\left[t - k \cdot T_{stcpt}, \frac{T_{stcpt}}{2 \cdot (m1_{steps} + 1)}, V_{stcs}, m1_{steps}\right]$$

Area under the staircase $$A_{stcp} = T_{1stpl\_} \cdot \frac{V_{stcs}}{m1_{steps}} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1)$$

Dimensionless function:

$$Vistcp(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd}) := \frac{v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd})}{volt}$$

Description of the Function's parameters:
$Vistcp(t, period, signal\_amplitude, number\_of\_steps, max\_number\_of\_periods)$

Periodic Waveforms

10 Staircase 2 Voltage Pulse Train

Description of the Function's parameters: $v_{stct}(time, period, max\_amplitude, number\_of\_steps)$,

$v_{stcc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

For data, see "staircase 2 pulse data"

max amplitude:	V_{stc}	Period:	$T2_{stp\_}$
Number of steps:	$m2_{steps}$	Step amplitude:	V_{stcstp}
Step length:	$T_{2stpl\_}$		

$$v_{stct}(t, T2_{stp\_}, V_{stc}, m2_{steps}, N0_{gd}) := \sum_{k=0}^{N0_{gd}} v_{stcc}\left(t - k \cdot T2_{stp\_}, \frac{T2_{stp\_}}{2 \cdot m2_{steps}}, V_{stc}, m2_{steps}\right)$$

Dimensionless function:

$$Vistct(t, T2_{stp\_}, V_{stc}, m2_{steps}, N0_{gd}) := \frac{v_{stct}(t, T2_{stp\_}, V_{stc}, m2_{steps}, N0_{gd})}{V}$$

Area under the staircase

$$A_{stccp} = 2 \cdot T_{2stpl\_} \cdot \frac{V_{stc}}{m2_{steps}} \cdot \sum_{k=1}^{m2_{steps}} (m2_{steps} - k + 1)$$

$$A_{stccp} = \int_{T_{2stpl\_}}^{T2_{stp\_}} v_{stct}(t, T2_{stp\_}, V_{stc}, m2_{steps} - 2)\, dt$$

Description of the Function's parameters: $Vistct(t, step\_length, max\_amplitude, number\_of\_steps, period)$

Periodic Waveforms

11 Staircase 2 Voltage Pulse Train + sinus

Description of the Function's parameters: Vstcsin(t, period, max_amplitude, number_of_steps)
For data, see the worksheet *"staircase 2 pulse data.xmcd"*

max amplitude:	V_{stc}	Period:	$T2_{stp\_}$
Number of steps:	$m2_{steps}$	Step amplitude:	V_{stcstp}
Step length:	$T_{2stpl\_}$		

$$\mathrm{Vstcsin}\left(t, T2_{stp\_}, V_{stc}, m2_{steps}, N0_{gd}\right) := \mathrm{Vistct}\left(t, T2_{stp\_}, V_{stc}, m2_{steps}, N0_{gd}\right) + \frac{V_{stc}}{4 \cdot m2_{steps} \cdot V} \cdot \sin\left(\frac{2 \cdot \pi \cdot m2_{steps}}{T2_{stp\_}} \cdot \right.$$

Periodic Waveforms

12 Staircase 3 Voltage Pulse Train

Description of the Function's parameters: v_{stct}(t, period, step_amplitude, number_of_steps),

: v_{stctA0}[t, (period, step_amplitude, number_of_steps)]

You can find the data in "*staircase 3 pulse data*"

$$v_{stctA0}\left(t, T3, V_{stc3}, m3_{steps}, N0_{gd}\right) := v_{stct}\left(t, T3, V_{stc3}, m3_{steps}, N0_{gd}\right) - \frac{V_{stc3}}{2}$$

Dimensionless function:

$$\mathrm{VistctA0}\left(t, T3, V_{pbds}, m3_{steps}, N0_{gd}\right) := \frac{v_{stctA0}\left(t, T3, V_{pbds}, m3_{steps}, N0_{gd}\right)}{V}$$

Periodic Waveforms

13 Staircase 3 Voltage Pulse Train + sinus

$$\mathrm{VistctA0sin}\left(t, T3, V_{pbds}, m3_{steps}, N0_{gd}\right) := \mathrm{VistctA0}\left(t, T3, V_{pbds}, m3_{steps}, N0_{gd}\right) \ldots$$
$$+ \frac{V_{pbds}}{2 \cdot m3_{steps} \cdot V} \cdot \sin\left(\frac{2 \cdot \pi}{T3} \cdot 8 \cdot m3_{steps} \cdot t\right)$$

Periodic Waveforms

14 Staircase 4 Voltage Pulse Train

Description of the Function's parameters : vstc1p(time, step length, step amplitude, number of steps)

To modify data, see " *staircase 4 pulse data*"

$$vstc1p\left(t, T_{0gd}, V_{stc4}, m4_{steps}, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} vstc1\left[t - k\cdot 2\cdot\left(m4_{steps}+1\right)\cdot T_{0gd}, T_{0gd}, V_{stc4}, m4_{steps}\right]$$

Dimensionless function:

$$vstc1p1\left(t, T_{0gd}, V_{stc4}, m8_{steps}, N0_{gd}\right) := \frac{vstc1p\left(t, T_{0gd}, V_{stc4}, m8_{steps}, N0_{gd}\right)}{V}$$

Periodic Waveforms

15 Bipolar Triangular Voltage Wave

Description of the Function's parameters : Λ_V(time, triangle half base, triangle amplitude)

Signal amplitude: V_{pp}

Time constant: τ_{twt}

Period: T_9 $\qquad f_9 = \frac{1}{T_9}$

$$\Lambda_V\left(t, \tau_{twt}, V_{pp}, N0_{gd}\right) := V_{pp}\cdot \sum_{k=-N0_{gd}}^{N0_{gd}} \left[(-1)^k\cdot\Lambda\left(t - 2\cdot k\cdot\tau_{twt}, \tau_{twt}\right)\right]$$

Bipolar Triangular Voltage Wave Built using the Step Function

Signal amplitude: V_{pp}

Time constant: τ_{twt}

Period: T_9

$$\omega_9 = 2\cdot\pi\cdot f_9$$

$$v_{tri0}(t, T_9, V_{pp}, N0_{gd}) := \left[\frac{4\cdot V_{pp}}{T_9}\cdot\sum_{k=0}^{N0_{gd}}\left[\begin{array}{l}(t - k\cdot T_9)\cdot\Phi(t - k\cdot T_9)\ ... \\ +(-1)\cdot\left[2\cdot\left[t - \left(k + \frac{1}{2}\right)\cdot T_9\right]\cdot\Phi\left[t - \left(k + \frac{1}{2}\right)\cdot T_9\right]\right]\ ... \\ +\left[t - (k+1)\cdot T_9\right]\cdot\Phi\left[t - (k+1)\cdot T_9\right]\end{array}\right]\right] - V_{pp}$$

Dimensionless function: $$Vi3(t, T_9, V_{pp}, N0_{gd}) := \frac{v_{tri0}(t, T_9, V_{pp}, N0_{gd})}{V}$$

Periodic Waveforms

16 Triangular Cusps Voltage Pulse Train

Signal amplitude: V_{pp}

Pulse width: p_w

Period: T_{0csp}

Max pulse amplitude and cusp ratio: a_p $a_p < 1$

Cusp slope $c_s = V_{pp}\cdot\frac{2\cdot(1 - a_p)}{p_w}$

$$cusp0(t, p_w, a_p, V_{pp}) = V_{pp}\cdot\left[\begin{array}{l}\left[1 + t\cdot\frac{2\cdot(1 - a_p)}{p_w}\right]\cdot\left(\Phi\left(t + \frac{p_w}{2}\right) - \Phi(t)\right)\ ... \\ +\left[1 - t\cdot\frac{2\cdot(1 - a_p)}{p_w}\right]\cdot\left(\Phi(t) - \Phi\left(t - \frac{p_w}{2}\right)\right)\end{array}\right]$$

$$csp01(t, p_w, a_p, T_{0csp}, V_{pp}, N0_{gd}) := \sum_{k=0}^{N0_{gd}}\left(cusp0\left(t - k\cdot T_{0csp} - \frac{p_w}{2}, p_w, a_p, V_{pp}\right)\right)$$

Dimensionless function: $$fc5(t, p_w, a_p, T_{0gd}, V_{pp}, N0_{gd}) := \frac{csp01(t, p_w, a_p, T_{0gd}, V_{pp}, N0_{gd})}{V}$$

Periodic Waveforms

17 Bipolar Sawtooth with positive slope Pulse Train

Amplitude: V_{sawth}

Sawtooth length: δ_{sawth}

Slope: sp_{sawth}

Period: T_{sawth}

f_{sawth}

$$f_{sw}(t, T_{sawth}, V_{sawth}) = \frac{V_{sawth}}{\delta_{sawth}} \cdot t \cdot rect1(t, 0.0 \cdot sec, \delta_{sawth})$$

Defined in -4) Voltage Pulse

$$\alpha_{saw} = atan\left(sp_{sawth} \cdot \frac{sec}{volt}\right)$$

$$v1_{sw}(t, T_{sawth}, V_{sawth}, N0_{gd}) := \sum_{k=0}^{N0_{gd}} \left(f_{sw}(t - k \cdot T_{sawth}, T_{sawth}, 2 \cdot V_{sawth})\right) - V_{sawth}$$

Dimensionless function: $v1sw(t, T_{sawth}, V_{sawth}, N0_{gd}) := \dfrac{v1_{sw}(t, T_{sawth}, V_{sawth}, N0_{gd})}{V}$

Periodic Waveforms

18 Bipolar Sawtooth with negative slope Pulse Train

Amplitude: V_{sawth}

Sawtooth length: δ_{sawth}

Slope: sp_{sawth}

Period: T_{sawth}

Frequency: f_{sawth}

$$f(t, T_{sawth}, V_{sawth}) := V_{sawth} \cdot \left(\frac{-t}{T_{sawth}} + 1\right) \cdot \left(\Phi(t) - \Phi(t - T_{sawth})\right)$$

Defined in -12 Sawtooth Voltage Pulse with negative slope

$$v2_{sw}(t, \delta_{sawth}, V_{sawth}, N0_{gd}) := \sum_{k=-N0_{gd}}^{N0_{gd}} f_{sl}(t - k \cdot \delta_{sawth}, \delta_{sawth}, 2 \cdot V_{sawth}) - V_{sawth}$$

Dimensionless function: $fc7(t, T_{sawth}, V_{sawth}, N0_{gd}) := \dfrac{v2_{sw}(t, T_{sawth}, V_{sawth}, N0_{gd})}{V}$

Periodic Waveforms

19 Bipolar Sawtooth with adjustable rising and falling edges Pulse Train

$$n = 11 \qquad \tau_{cy} = \frac{T}{n}$$

Use the following definition to facilitates the signal's Laplace transformation, instead of the recurrence relation whose Laplace transformation can't be so immediate.

$$V_s(t, T_{sl}, \delta_{cycl}, V_{pp}, N_{gd}) := \sum_{k=0}^{N_{gd}} s1s2\left[t-(k-1)\cdot T_{sl}, T_{sl}, \delta_{cycl}, V_{pp}\right] - \frac{V_{pp}}{2}$$

TEST Waveforms

Periodic Waveforms

20 AM test signal (single tone)

Carrier Amplitude: A1

Modulating signal's amplitude: B1

$$\omega 1_c = \frac{\omega_{0gd}}{2} \qquad T1_c = \frac{2\cdot\pi}{\omega 1_c} \qquad \omega 1_m = \frac{\omega_{0gd}}{10} \qquad T1_m = \frac{2\cdot\pi}{\omega 1_m} \qquad f1_m = \frac{\omega 1_m}{2\cdot\pi} \qquad f_{15} = \frac{\omega 1_c}{2\cdot\pi}$$

$$v_{ammax} = A1 + B1 \qquad v_{ammin} = A1 - B1 \qquad A1 = v_{ammax} + v_{ammin} \qquad B1 = v_{ammax} - v_{ammin}$$

$$m_{am} = \frac{v_{ammax} - v_{ammin}}{v_{ammax} + v_{ammin}}$$

$$v2_i(t, \omega 1_m, \omega 1_c, A1, B1) := A1\cdot\cos(\omega 1_c\cdot t) \ldots + \frac{B1}{2}\cdot\cos\left[(\omega 1_c + \omega 1_m)\cdot t\right] \ldots + \frac{B1}{2}\cdot\cos\left[(\omega 1_c - \omega 1_m)\cdot t\right]$$

Dimensionless function: $V2am(t, \omega 1_m, \omega 1_c, A1, B1) := \frac{v2_i(t, \omega 1_m, \omega 1_c, A1, B1)}{V}$

TEST Waveforms

Periodic Waveforms

21 AM test signal (triangular wave)

$$\omega_{2m} = \frac{\omega 1_c}{10} \qquad T2_m = \frac{2\cdot\pi}{\omega_{2m}} \qquad f_{16} = \frac{\omega 1_c}{2\cdot\pi}$$

$$v_{am}(t, \omega_{2m}, \omega 1_c, m_{am}, A1, B1, N0_{gd}) := A1\cdot\left[\left(1 + \frac{m_{am}}{B1}\cdot v_{tri0}\left(t, \frac{2\cdot\pi}{\omega_{2m}}, B1, N0_{gd}\right)\right)\cdot\cos(\omega 1_c\cdot t)\right]$$

Dimensionless function: $V3am(t, \omega 1_m, \omega 1_c, m_{am}, A1, B1, N0_{gd}) := \frac{v_{am}(t, \omega 1_m, \omega 1_c, m_{am}, A1, B1, N0_{gd})}{V}$

Periodic Waveforms

22 AM DSBSC test signal (single tone)

$$\omega2_m = \frac{\omega1_c}{10} \qquad T2_{mdsb} = \frac{2\cdot\pi}{\omega2_m} \qquad \omega2_m = \frac{2\cdot\pi}{T2_{mdsb}}$$

$$f2_m = \frac{1}{T2_m} \qquad f_{1c} = \frac{\omega1_c}{2\cdot\pi}$$

$$T_{1c} = \frac{1}{f_{1c}} \qquad V_{dsbsc}(t, f1_c, f2_m, A1) = A1\cdot\cos(2\cdot\pi\cdot f1_c\cdot t)\cdot v_m(t)$$

$$V_m(t, f2_m) = B1\cdot\cos(2\cdot\pi\cdot f2_m\cdot t)$$

$$V_{dsbsc}(t, f1_c, f2_m, A1, B1) = A1\cdot\cos(2\cdot\pi\cdot f1_c\cdot t)\cdot B1\cdot\cos(2\cdot\pi\cdot f2_m\cdot t)$$

$$V_{dsbsc}(t, f1_c, f2_m, A1, B1) := \frac{A1\cdot B1}{2}\cdot\cos\left[\left(2\cdot\pi\cdot f1_c + 2\cdot\pi\cdot f2_m\right)\cdot t\right] + \frac{A1\cdot B1}{2}\cdot\cos\left[2\cdot\pi\cdot\left(f1_c - f2_m\right)\cdot t\right]$$

Dimensionless function: $V4dsbsc(t, f1_c, f2_m, A1, B1) := \dfrac{V_{dsbsc}(t, f1_c, f2_m, A1, B1)}{V^2}$

Periodic Waveforms

23 AM DSBSC test signal (triangular wave)

$$T_{18} = T2_{mdsb} \qquad V3_{dsbsc}(t, T2, f1_c, f2_m, A1, B1, N0_{gd}) := A1\cdot\cos(\pi\cdot 2\cdot f1_c\cdot t)\cdot v_{tri0}(t, T2\cdot 2, B1, N0_{gd})$$

Dimensionless function: $V5_{dsbsc}(t, T2, f1_c, f2_m, A1, B1, N0_{gd}) := \dfrac{V3_{dsbsc}(t, T2, f1_c, f2_m, A1, B1, N0_{gd})}{volt^2}$

Periodic Waveforms

24 AM SSBSC test signal (single tone)

$$V_{ssbsc}(t, f1_c, f2_m, A1, B1) := \frac{A1 \cdot B1}{2} \cdot \cos\left[2 \cdot \pi \cdot (f1_c + f2_m) \cdot t\right]$$

Dimensionless function: $$V6_{ssbsc}(t, f1_c, f2_m, A1, B1) := \frac{V_{ssbsc}(t, f1_c, f2_m, A1, B1)}{V^2}$$

Periodic Waveforms

25 AM SSBSC test signal (triangular wave)

$$V4_{ssbsc}(t, f1_c, f2_m, A1, B1, N0_{gd}) := A1 \cdot \cos(f1_c \cdot t) \cdot v_{tri0}\left(t, \frac{2}{f2_m}, B1, N0_{gd}\right)$$

Dimensionless function: $$V7_{ssbsc}(t, f1_c, f2_m, A1, B1, N0_{gd}) := \frac{V4_{ssbsc}(t, f1_c, f2_m, A1, B1, N0_{gd})}{V^2}$$

Periodic Waveforms

26 FM test signal (single tone) (change data in FM data.xmcd)

Carrier Frequency..: f_{cfm}

Carrier period..: $T_{cfm} = \frac{1.0}{f_{cfm}}$

Angular frequency of the carrier....................................: $\omega_{cfm} = 2.0\cdot\pi\cdot f_{cfm}$

Amplitude of the single tone modulating signal............: B_{fmm}

Period of the modulating signal....................: T_{fmm} $\quad f_{fmm} = \frac{1}{T_{fmm}}$

Frequency of the single tone modulating signal.............: f_{fmm}

Angular frequency of the single tone modulating signal: $\omega_{fmm} = 2.0\cdot\pi\cdot f_{fmm}$

$$v_{fmsl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd}) := \mathrm{Re}\left[A_{fm}\cdot e^{j\cdot 2\cdot\pi\cdot f_{cfm}\cdot t}\cdot\sum_{k=-N_{gd}}^{N_{gd}}\left(Jn(k, m_{fm})\cdot e^{j\cdot k\cdot 2\cdot\pi\cdot f_{fmm}\cdot t}\right)\right]$$

Dimensionless function: $V7_{fm}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd}) := \frac{v_{fmsl}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd})}{V}$

Periodic Waveforms

27 FM test signal (triangular wave)

Modulating triangular voltage wave: $vtri_m(t) = v_{tri0}(t, T_{fmm}, B_{fmm}, N0_{gd})$

Generic FM signal: $v_{fmsl}(t) = A\cdot\cos(\varphi(t))$

where: $\varphi(t) = \omega_{cfm}\cdot t + Kst_{fm}\cdot\int vtri_m(t)\,dt$

results:

$$\int vtri_m(t)\,dt = \frac{4\cdot B_{fmm}}{T_{fmm}}\cdot\sum_{k=0}^{N0_{gd}}\left[\frac{\Phi(t - T_{fmm}\cdot k)\cdot(t - T_{fmm}\cdot k)^2}{2} \ldots + (-1)\cdot\frac{\Phi\left[t - T_{fmm}\cdot\left(k+\frac{1}{2}\right)\right]\cdot\left[2\cdot t - 2\cdot T_{fmm}\cdot\left(k+\frac{1}{2}\right)\right]^2}{4} \ldots + \frac{\Phi[t - T_{fmm}\cdot(k+1)]\cdot[t - T_{fmm}\cdot(k+1)]^2}{2}\right] - B_{fmm}\cdot t$$

or written as a function of t.

$$I_{vtri}(t, T_{fmm}, B_{fmm}, N0_{gd}) := \frac{4\cdot B_{fmm}}{T_{fmm}}\cdot\sum_{k=0}^{N0_{gd}}\left[\frac{\Phi(t - T_{fmm}\cdot k)\cdot(t - T_{fmm}\cdot k)^2}{2} \ldots + (-1)\cdot\Phi\left[t - T_{fmm}\cdot\left(k+\frac{1}{2}\right)\right]\cdot\left[t - T_{fmm}\cdot\left(k+\frac{1}{2}\right)\right]^2 \ldots + \frac{\Phi[t - T_{fmm}\cdot(k+1)]\cdot[t - T_{fmm}\cdot(k+1)]^2}{2}\right] - B_{fmm}\cdot t$$

$I_{vtrimax} = B_{fmm}\cdot\frac{T_{fmm}}{8}$ $\quad T_{fmm} = \frac{2\cdot\pi}{\omega_{fmm}}$

FM signal:

$$v_{fm3}(t_{fm}, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, k_{fm}, N0_{gd}) := A_{fm}\cdot\cos\left(2\cdot\pi\cdot f_{cfm}\cdot t_{fm} + k_{fm}\cdot I_{vtri}\left(t_{fm}, \frac{1}{f_{fmm}}, B_{fmm}, N0\right.\right.$$

Dimensionless function:

$$V8_{fm}(t_{fm}, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, k_{fm}, N0_{gd}) := \frac{v_{fm3}(t_{fm}, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, k_{fm}, N0_{gd})}{V}$$

Periodic Waveforms

28 PM test signal (single tone)

Carrier Amplitude ....: A_{pm}

Carrier frequency ....: $f_{cpm} = j_{pm} \cdot f_{0gd}$

Carrier period ....: $T_{cpm} = \frac{1.0}{f_c}$,

Angular frequency of the carrier ....: $\omega_{cpm} = 2.0 \cdot \pi \cdot f_c$,

Modulating Signal Amplitude ....: B_{pm}

Modulating Signal period ....: T_{pmm}

One Tone Modulating Signal frequency ....: $f_{pm} = \frac{1}{T_{pmm}}$

Angular frequency of the modulating signal ....: $\omega_{pmm} = 2.0 \cdot \pi \cdot f_{pmm}$.

$$m_{pm} = \frac{j_{pm}}{100} \qquad k_{pm} = \frac{m_{pm}}{B_{pm}}$$

Phase modulartion index ....: B_{pm}

Phase-sensitivity factor ....: k_{pm}

For any modulating signal $v_m(t)$, results: $v_{pm}(t) = A_{pm} \cdot \cos\left(\omega_{cpm} \cdot t + k_{pm} \cdot v_m(t)\right)$

$$v_{pm}\left(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N0_{gd}\right) = A_{pm} \cdot \sum_{k=-N0_{gd}}^{N0_{gd}} \left[Jn\left(k, m_{pm}\right) \cdot \cos\left[\left[2 \cdot \pi \cdot \left(f_{cpm} + k \cdot f_{pmm}\right)\right] \cdot t - k \cdot f_m(t)\right.\right.$$

while for a cosinusoidal test signal, the modulated carrier is:

$$v_{pm}(t) = A_{pm} \cdot \cos\left(\omega_{cpm} \cdot t + m_{pm} \cdot \cos\left(\omega_{pmm} \cdot t\right)\right)$$

$$v_{pm}\left(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}\right) = A_{pm} \cdot \cos\left(2 \cdot \pi \cdot f_{cpm} \cdot t + m_{pm} \cdot \cos\left(2 \cdot \pi \cdot f_{pmm} \cdot t\right)\right)$$

$$v_{pm}\left(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N0_{gd}\right) := \operatorname{Re}\left[A_{pm} \cdot e^{j \cdot 2 \cdot \pi \cdot f_{cpm} \cdot t} \cdot \sum_{k=-N0_{gd}}^{N0_{gd}} \left(e^{j \cdot \frac{k \cdot \pi}{2}} \cdot Jn\left(k, m_{pm}\right) \cdot \cos\left(k \cdot 2 \cdot \pi \cdot f_{pmm}\right.\right.\right.$$

Dimensionless function: $V9_{pm}\left(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N0_{gd}\right) := \frac{v_{pm}\left(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N0_{gd}\right)}{V}$

TEST Waveforms

Periodic Waveforms

29 PM test signal (triangular wave)

$$k_{pm} = \frac{m_{pm}}{B_{pm}}$$

$$v_{pmtri}\left(t, T_{pmm}, f_{cpm}, k_{pm}, A_{pm}, B_{pm}, N0_{gd}\right) = A_{pm} \cdot \cos\left(2 \cdot \pi \cdot f_{cpm} \cdot t + k_{pm} \cdot v_{tri0}\left(t, T_{pmm}, B_{pm}, N0_{gd}\right)\right)$$

$$v_{pmtri}\left(t, T_{pmm}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd}\right) := A_{pm} \cdot \cos\left(2 \cdot \pi \cdot f_{cpm} \cdot t + \frac{m_{pm}}{B_{pm}} \cdot v_{tri0}\left(t, m_{pm}, B_{pm}, N0_{gd}\right)\right)$$

Dimensionless function:

$$V10_{pm}\left(t, T_{pmm}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd}\right) := \frac{A_{pm} \cdot \cos\left(2 \cdot \pi \cdot f_{cpm} \cdot \frac{t}{s} + \frac{m_{pm}}{B_{pm}} \cdot v_{tri0}\left(\frac{t}{s}, m_{pm}, B_{pm}, N0_{gd}\right)\right)}{V}$$

Periodic Waveforms

30 Staircase based test signal

$$T_{Hsl} := \left(6\cdot m2_{steps\_} + 13\right)\cdot T_{2stpl\_}$$

$$v_{HD}\left(t, T_{2stpl\_}, m2_{steps\_}, V_{stc\_}, shift, N0_{gd}\right) := V_{stc\_}\cdot\sum_{k=1}^{N0_{gd}} rect1\left[\left[t-(2\cdot k-1)\cdot\left[\frac{\left(6\cdot m2_{steps\_}+shift+3\right)\cdot T_{2stpl\_}}{2} + \frac{\left(6\cdot m2_{steps\_}+shift+3\right)\cdot T_{2stpl\_}}{2}\right] + (1-k)\cdot 2\,\frac{(shift+1)\cdot T_{2stpl\_}}{2} - T_{2stpl\_}\cdot shift\right.\right.$$

$$V_{HD}\left(t, T_{2stpl\_}, m2_{steps\_}, V_{stc\_}, shift, N0_{gd}\right) := \frac{v_{HD}\left(t, T_{2stpl\_}, m2_{steps\_}, V_{stc\_}, shift, N0_{gd}\right)}{V}$$

$$v_{HH}\left(t, T_T, T_{2stpl\_}, V_{stc\_}, m2_{steps\_}, shift, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} v_H\left(t-k\cdot T_T, T_{2stpl\_}, V_{stc\_}, m2_{steps\_}, shift\right)$$

$$V_H\left(t, T_T, T_{2stpl\_}, V_{stc}, m2_{steps}, shift, N0_{gd}\right) := \frac{v_{HH}\left(t, T_T, T_{2stpl\_}, V_{stc}, m2_{steps}, shift, N0_{gd}\right)}{V}$$

$m2_{steps\_} = ∎$ $\qquad T_{2stpl\_} = ∎$ $\qquad mstc3_{steps\_} = ∎$

Periodic Waveforms

31 Bipolar Double Exponential Pulse Train

$$V_{bdept}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} V_{bdep}\left(t-k\cdot T, \tau_{ptd}, V_{pp}\right)$$

$$V_{bdepta}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right) := \frac{V_{bdept}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right)}{V}$$

TEST Waveforms

Periodic Waveforms

32 Bipolar Double Exponential Odd symmetric Pulse Train

$$V_{bdeospp}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} V_{bdeosp}\left(t-k\cdot T, \tau_{ptd}, V_{pp}\right)$$

TEST Waveforms

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33 Agnesi Profile Voltage Pulse Train

$$V_{agnp}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} V_{agn}\left(t-k\cdot T, \tau_{ptd}, V_{pp}\right)$$

TEST Waveforms

Periodic Waveforms

34 Agnesi Derivative Profile Voltage Pulse Train

$$V_{Dagnp}\left(t, \tau_{ptd}, T, V_{pp}, N0_{gd}\right) := \sum_{k=0}^{N0_{gd}} V_{Dagn}\left(t-k\cdot T, \tau_{ptd}, V_{pp}\right)$$

TEST Waveforms

Periodic Waveforms

35 Poisson Profile Voltage Pulse Train

$$T_{pp} = 10\cdot\tau_{ptd}$$

$$V_{p2p}(t,\tau_{ptd},T,V_{pp},N_{gd}) := \sum_{k=0}^{N_{gd}} \left(V_p(t-k\cdot T,\tau_{ptd},V_{pp})\cdot \mathrm{rect1}(t-k\cdot T,0\cdot T,T)\right)$$

TEST Waveforms

Periodic Waveforms

36 Poisson Derivative Profile Voltage Pulse Train

$$V_{2pDp}(t,\tau_{ptd},T,V_{pp},N_{gd}) := \sum_{k=0}^{N_{gd}} \left(V_{2p}(t-k\cdot T,\tau_{ptd},V_{pp})\cdot \mathrm{rect1}(t-k\cdot T,0\cdot T,T)\right)$$

TEST Waveforms

Periodic Waveforms

37 Rayleigh Profile Voltage Pulse Train

$$V_{Ryp}(t,\tau_{ptd},T,V_{pp},N0_{gd}) := \sum_{k=0}^{N0_{gd}} \left(V_{Ry}(t-k\cdot T,\tau_{ptd},V_{pp})\cdot \mathrm{rect1}(t-k\cdot T,0\cdot T,T)\right)$$

TEST Waveforms

Periodic Waveforms

38 Cap. Charge and Discharge Pulse Train

pulse width: $p_w = \tau_{end} - \tau_{init}$

time constant $\tau_c = \frac{p_w}{20}$

Period: $T_{cdsc} = \tau_{end} - \tau_{init}$

Function's parameters description:
V_{cs}(*time, time constant, pulse width, supply voltage*)

Function's parameters description:
V_{Ccd}(*time, time constant, pulse width,period, supply voltage*)

$$V_{Ccd}(t,T_{cdsc},\tau_{end},\tau_c,V_{pp},N0_{gd}) := \sum_{k=0}^{N0_{gd}-1} V_{cs}(t-k\cdot T_{cdsc},\tau_{end},\tau_c,V_{pp})$$

TEST Waveforms

Periodic Waveforms

39 Inductance Charge and Discharge Voltage Pulse Train

$$Ind_{sc}(t_{tw},\tau_c,\tau_{end},T_{ind},V_{pp},N_{gd}) := \sum_{k=0}^{N_{gd}-1} V_{Lcs}(t_{tw}-k\cdot T_{ind},\tau_{end},\tau_c,V_{pp})$$

TEST Waveforms

Periodic Waveforms

40 Parbolic Cusps Pulse Train

Signal amplitude: V_{pp}

Pulse width: p_w

Duty cycle: δ_{cv}

Period: $T_{pcsp} = \frac{p_w}{\delta_{cy}}$

Max pulse amplitude and cusp ratio: $a_{pp1} = \frac{4}{9}$ $a_p < 1$

$$q = (2 - a_{pp1})\cdot V_{pp}$$

asymptotes: $y1_a(t,a_{pp1},p_w,V_{pp}) := \frac{4\cdot V_{pp}\cdot(a_{pp1}-1)}{p_w}\cdot t + (2-a_{pp1})\cdot V_{pp}$

$$y2_a(t,a_{pp1},p_w,V_{pp}) := -\frac{4\cdot V_{pp}\cdot(a_{pp1}-1)}{p_w}\cdot t + (2-a_{pp1})\cdot V_{pp}$$

$$\mathrm{cusp1p}(t,p_w,a_p,V_{pp}) := \left[\frac{4\cdot t^2\cdot(a_p-1)}{p_w^{\ 2}} + 1\right]\cdot\left(\Phi\left(t+\frac{p_w}{2}\right) - \Phi\left(t-\frac{p_w}{2}\right)\right)\cdot V_{pp}$$

$$\mathrm{csp11}(t,p_w,a_{p1},T_{pcsp},V_{pp},N0_{gd}) := \sum_{k=0}^{N0_{gd}} \left(\mathrm{cusp1p}\left(t-k\cdot T_{pcsp}-\frac{p_w}{2},p_w,a_{p1},V_{pp}\right)\right)$$

Dimensionless function: $\mathrm{fcsp11}(t,p_w,a_{p1},T_{0gd},V_{pp},N0_{gd}) := \frac{\mathrm{csp11}(t,p_w,a_{p1},T_{0gd},V_{pp},N0_{gd})}{V}$

TEST Waveforms

Periodic Waveforms

41 Elliptic Cusps Pulse Train

Signal amplitude: V_{pp}

Pulse width: p_w

Duty cycle: δ_{cv}

Period: $T_{0csp2} = \frac{p_w}{\delta_{cy}}$

Max pulse amplitude and cusp ratio: $a_{pe} = \frac{2}{10}$ $a_{pe} < 1$

$$cusp2(t, p_w, a_{pe}, V_{pp}) := V_{pp} \cdot \left[\frac{2 \cdot (1 - a_{pe})}{p_w} \cdot \sqrt{\left(\frac{p_w}{2}\right)^2 - t^2} + a_{pe} \right] \cdot \left(\Phi\left(t + \frac{p_w}{2}\right) - \Phi\left(t - \frac{p_w}{2}\right) \right)$$

$$csp22(t, p_w, a_{pe}, T, V_{pp}, N0_{gd}) := \sum_{k=0}^{N0_{gd}} \left(cusp2\left(t - k \cdot T - \frac{p_w}{2}, p_w, a_{pe}, V_{pp}\right) \right)$$

Dimensionless function: $$fcsp22(t, p_w, a_p, T, V_{pp}, N0_{gd}) := \frac{csp22(t, p_w, a_p, T, V_{pp}, N0_{gd})}{V}$$

Periodic Waveforms

Periodic Waveforms Formulae Only

When saving or printing, disable Automatic Calculation.

WAVEFORM SPECTRA

Francesco Mezzanino

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INTRODUCTION

The subscript "gd" is the acronym of "Global Data.xmcd"
The subscript "fs" is the acronym of "Fourier series.xmcd"
The subscript "sl" is the acronym of "Signal List.xmcd"
The subscript "dp" is the acronym of "Dirac Pulse - formulas.xmcd"

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Periodic Waveforms Frequency Spectra

PERIODIC WAVEFORMS' FREQUENCY SPECTRA

Function parameters description:

BCSA(*Adimensional signal name, relative error, polinomial degree, start time, signal period*)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

The function returns a matrix made of three columns.

The first column contains:

pos. 0: relative error,
pos. 1: bandwidth (adimensional),
pos. 2: the nth. harmonic number corresponding tp the give relative error,
pos. 3: temporary variable,
pos. 4: Parseval,
pos. 5: signal average,
pos. 6: signal r.m.s..

The ***second column contains the coefficients*** a_k ***of the Fourier series***,
the ***third column contains the coefficients*** b_k ***of the Fourier series***.

TEST Waveforms

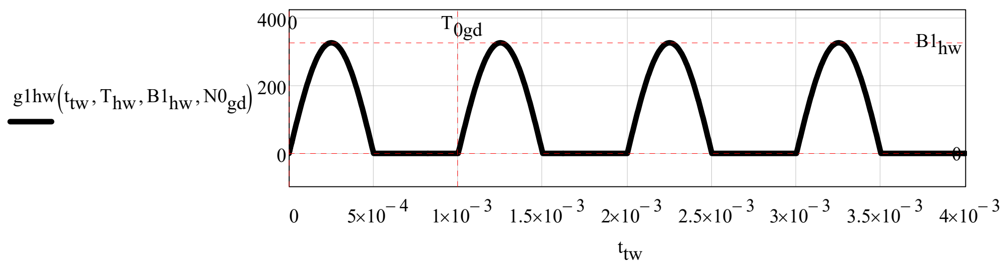
Periodic Waveforms   Periodic Waveforms   Periodic Waveforms

1) Half wave

Amplitude: $B1_{hw} := 230\cdot\sqrt{2}\cdot V$

$f_{hw} := \frac{1}{T0_{gd}}$ $T_{hw} := T0_{gd}$ Angular frequency: $\omega_{hw} := \frac{2\cdot\pi}{T0_{gd}}$ $T_{hw} = 1\cdot ms$

Half wave

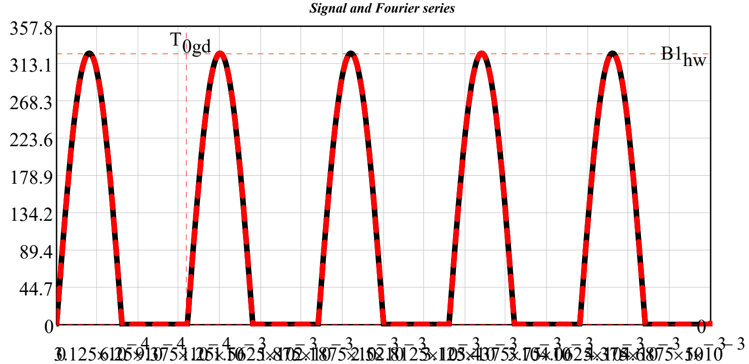


$V_{hw}(t) := g1hw(t, T_{hw}, B1_{hw}, N0_{gd})$ $B1_{hw} = 325.269\,V$

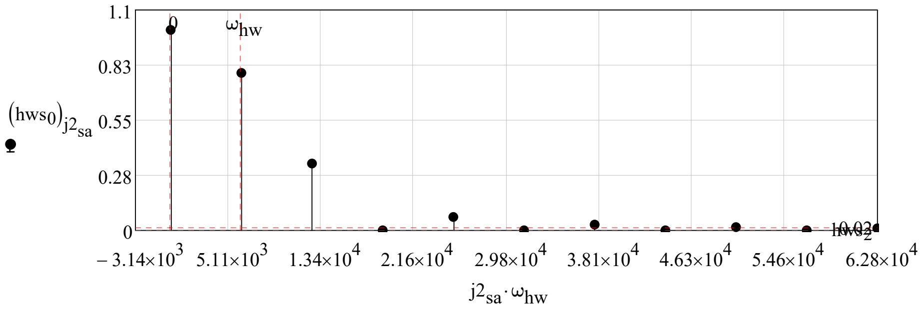
$hws := SPCT(V_{hw}, rt_{gd}, N1\_, 0\cdot sec, T_{hw})$ $N1\_ = 50$

$N_{gd} = 40$ $j2_{sa} := 0..rows(hws_0) - 1$ $(relerr) = 0.1$

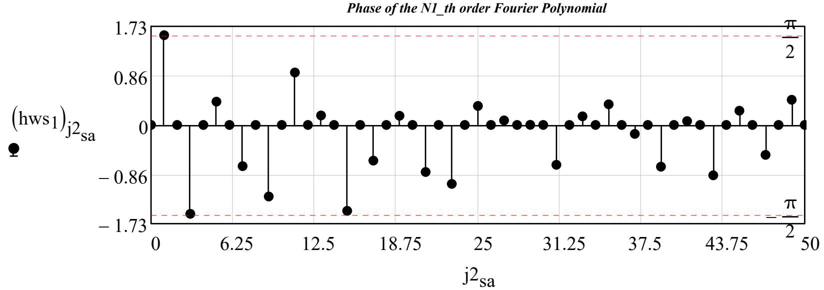
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := hws_3\cdot Hz$ $Bw_{sa} = 0.019\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $fpt_{so} = 0.038\cdot MHz$

$relerr := hws_7$ $relerr = 10\cdot\%$

$k := 0..2^8 - 1$ $npt_k := \frac{k}{fpt_{so}}$

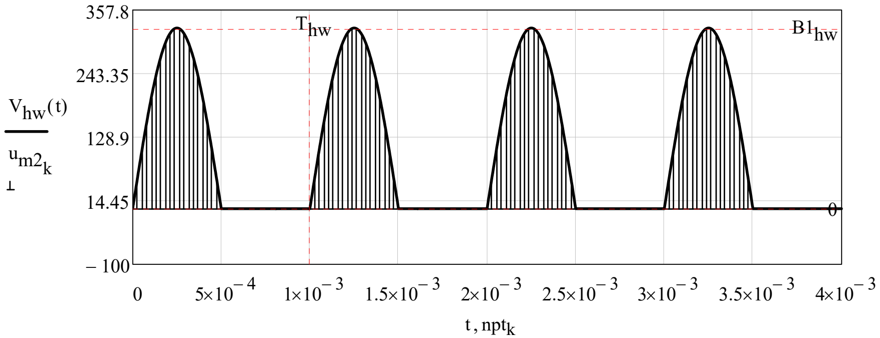
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{hw}} = 6.737$

Signal sampling: $u_{m2_k} := V_{hw}(npt_k)$

$u_{m2}^T =$

	0	1	2	3	4
0	0	53.538	105.615	154.811	...

Sampling



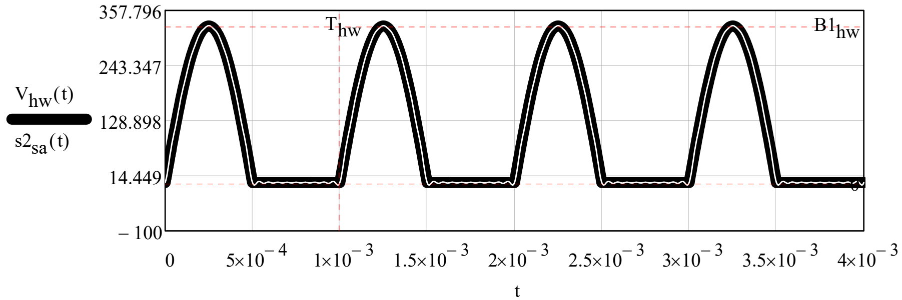
$relerr = 10\cdot\%$ $\omega_{bw} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bw} = 0.119\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bw}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s2_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m2_n}\cdot sinc\left(\omega_{bw}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ $u_{m2_{12}} = 297.873$

$rows\left(u_{m2}\right) = 256$ $relerr = 10\cdot\%$

Signal Rebuilding from samples

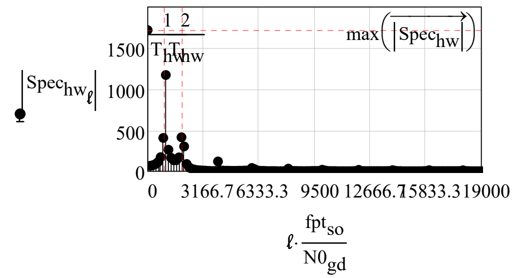


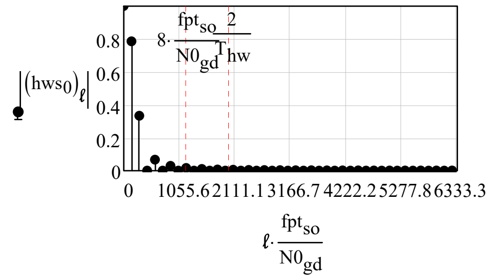
$length\left(u_{m2}\right) = 256$

$fpt_{so} = 38\cdot kHz$

$Spec_{hw} := fft\left(u_{m2}\right)$ $length\left(Spec_{hw}\right) = 129$

$\ell := 0..\frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





Periodic Waveforms

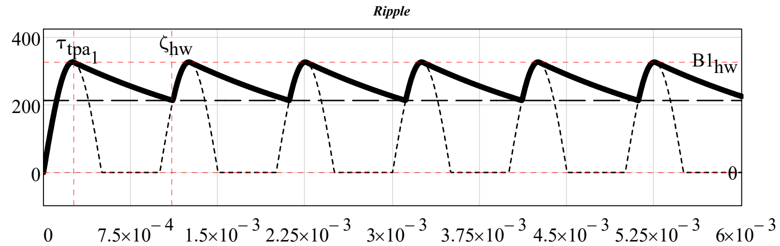
2 Half wave filtered (Capacitive)

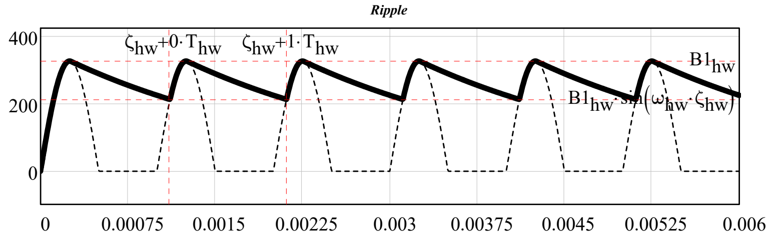
Max half wave amplitude: $B1_{hw} = 325.269 \cdot V$,
Amplitude of the decreasing exponential for t=0: V_{pp},
Exponential Time constant: $\tau_{hw1} := 2 \cdot T0_{gd}$
Period: , $T_{hw} = 1 \times 10^{3} \cdot \mu s$,
Pulsation: $\omega_{hw} := \dfrac{2 \cdot \pi}{T_{hw}} = 6.283 \cdot \dfrac{krads}{sec}$,
Intersection abscissa between half wave and exponential: ζ (scalar),
Tangent points abscissas between half wave and exponential: τ_{tpa} (vector)

$$\tau_{tpa_{k_{sl}}} := \frac{\operatorname{atan}\left(-\omega_{hw} \cdot \tau_{hw1}\right) + k_{sl} \cdot \pi}{\omega_{hw}}$$

$$V_{tpv} := B1_{hw} \cdot \sin\left(\omega_{hw} \cdot \tau_{tpa_1}\right) \cdot e^{\frac{\tau_{tpa_1}}{\tau_{hw1}}} \qquad V_{tpv} = 369.746\,V$$

$$\zeta_{hw} := Z01\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{tpv}\right)$$

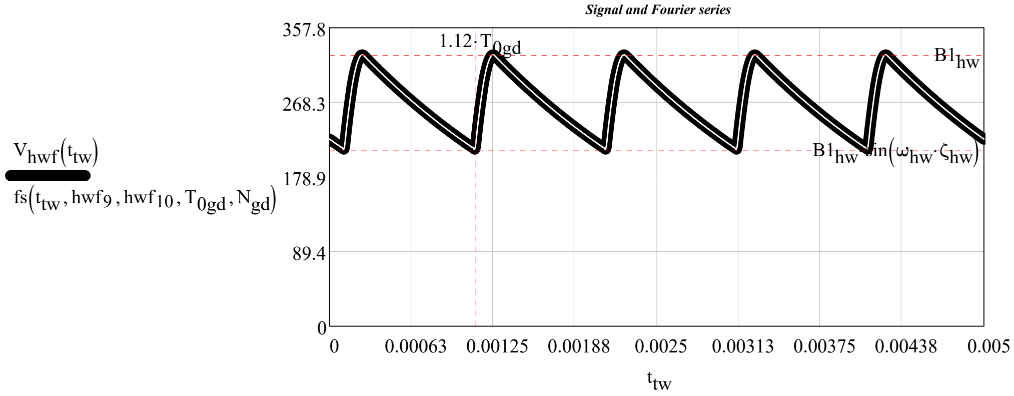


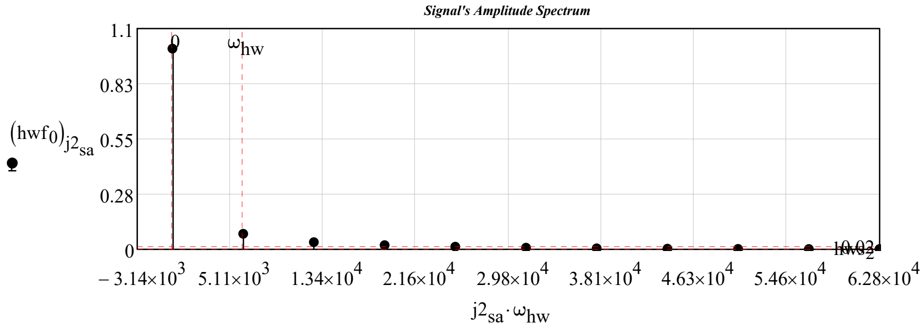


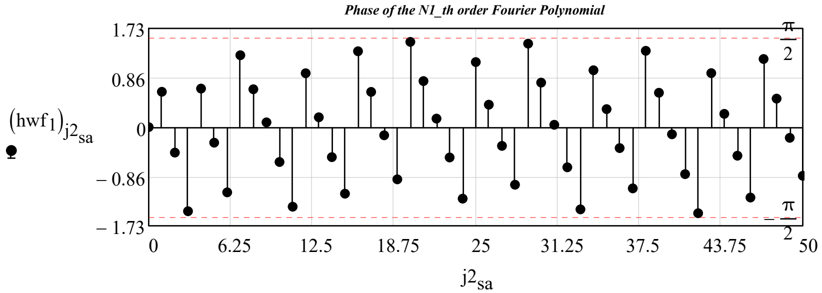
$B1_{hw} = 325.269\,V$ $\qquad V_{hwf}(t) := g2hw\left(t + \tau_{hw1}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}, N0_{gd}\right)$

$hwf := SPCT\left(V_{hwf}, rt_{gd}, N1\_, 0 \cdot sec, T0_{gd}\right) \qquad N1\_ = 50$

$j2_{sa} := 0 .. rows\left(hws_0\right) - 1$







$Bw_{sa} := hwf_3 \cdot Hz$

$Bw_{sa} = 0.02 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.04 \cdot MHz$

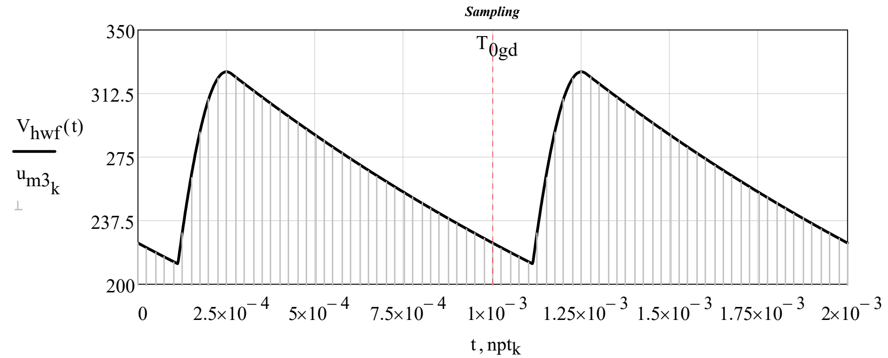
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 6.4$

$$u_{m3_k} := V_{hwf}(npt_k)$$

$u_{m3}^T =$

	0	1	2	3	4	5	6
0	224.262	221.476	218.725	216.008	213.325	230	...

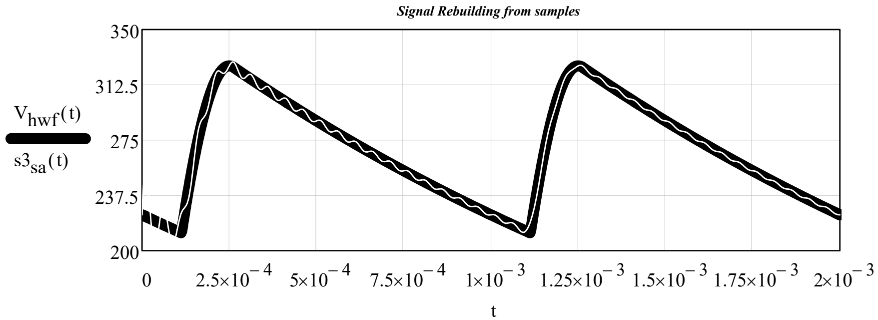


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.126 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s3_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m3_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$ relerr =

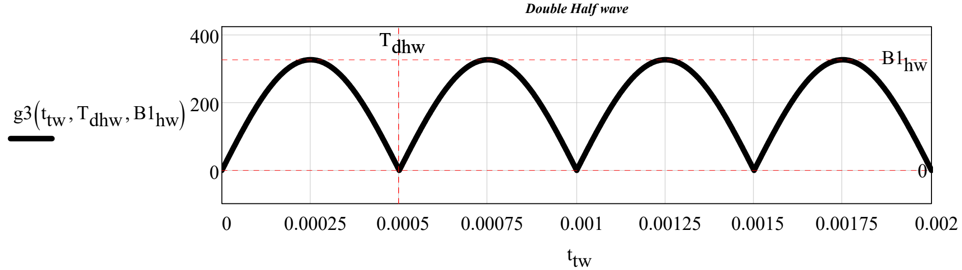


Symbol frequency:

Periodic Waveforms

3 Double Half wave

$T_{dhw} := \frac{T_{hw}}{2}$ $\omega_{dhw} := \frac{\pi}{T_{dhw}}$ $g3(t_{sl}, T_{dhw}, B1_{hw}) := \frac{B1_{hw}}{V} \cdot \left| \sin\left(\frac{2 \cdot \pi}{T_{hw}} \cdot t_{sl} \right) \right|$



Dirichlet conditions

A periodic function s(t)=s(t+T), can be expressed by the Fourier series provided that (Dirichlet conditions):
(1) it is discontinuous and presents a finite number of discontinuities in the period T;
(2) has a limited average value in the period T;
(3) it has a finite number of maximum positive or negative.
If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

1) signals of energy for which holds: $\int_{-\infty}^{\infty} \left(\left| s_{fs}(t) \right| \right)^2 dt < \infty$,

2) power signals for which holds: $\lim_{T \to \infty} \left[\frac{1}{T} \cdot \int_{-T}^{T} \left(\left| s_{fs}(t) \right| \right)^2 dt \right] < \infty$

Fourier series definition $s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(\omega \cdot k \cdot t) + b_k \cdot \sin(\omega \cdot k \cdot t) \right)$

The coefficients are defined as follows:

$$\frac{a_0}{2} = A_{fs} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t)\, dt$$

$$a_k = \frac{2}{T}\cdot\int_{t_0}^{t_0+T} s_{fs}(t)\cdot\cos(\omega\cdot k\cdot t)\,dt$$

$$b_k = \frac{2}{T}\cdot\int_{t_0}^{t_0+T} s_{fs}(t)\cdot\sin(\omega\cdot k\cdot t)\,dt$$

$B1_{hw} = 325.269\,V$ $\quad s_{fs}(t) := \frac{B1_{hw}}{V}\cdot\left|\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\right|$

$T_{dhw} := T_{dhw}$ $\quad t := t$ $\quad T_{hw} := T_{hw}$

$$\frac{a_0}{2} = A_{fs} = \frac{2}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right) dt = \frac{2\cdot B1_{hw}}{\pi\cdot V}$$

$$a_k = \frac{4}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot k\cdot t\right) dt = \frac{2\cdot B1_{hw}\cdot(\cos(\pi\cdot k)+1)}{-\pi\cdot V\cdot\left(k^2-1\right)}$$

$$b_k = \frac{4}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot k\cdot t\right) dt = -\frac{2\cdot B1_{hw}\cdot\sin(\pi\cdot k)}{\pi\cdot V\cdot\left(k^2-1\right)}$$

$$s_{fs}(t) = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\sum_{k=1}^{\infty}\left[\frac{(\cos(\pi\cdot k)+1)}{-\left(k^2-1\right)}\cdot\cos(\omega\cdot k\cdot t)+\frac{\sin(\pi\cdot k)}{\left(k^2-1\right)}\cdot\sin(\omega\cdot k\cdot t)\right]\right] \qquad \cos[k\cdot(\pi+\omega\cdot t)] = (-1)^k\cdot\cos(k\cdot$$

$$\frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t)+\cos[k\cdot(\pi+\omega\cdot t)]}{1-k^2}\right] = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t)+(-1)^k\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right]$$

$$\frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t)+(-1)^k\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right] = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2}+\sum_{k=2}^{\infty}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right]$$

$$\lim_{k\to1^+}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2} \to \frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2} \qquad \lim_{k\to1^-}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2} \to \frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2}$$

$$s_{dhw}(t) = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\frac{\pi\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot i}{2}+\sum_{k=2}^{\infty}\frac{\left[1+(-1)^k\right]\cdot\cos\left(k\cdot\frac{2\cdot\pi}{T_{hw}}\cdot t\right)}{1-k^2}\right]$$

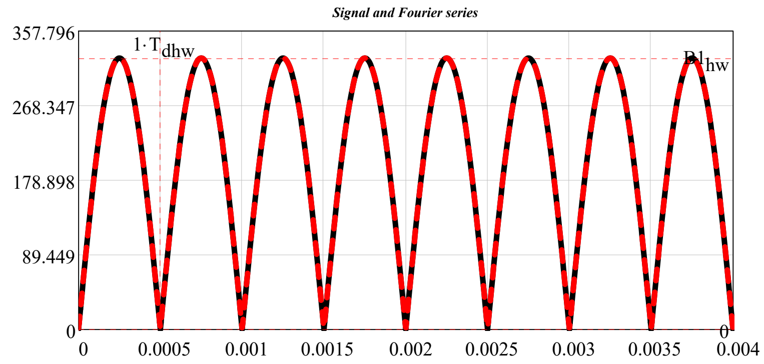
$$s_{dhw}(t) := \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\frac{\pi\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot i}{2}+\sum_{k=2}^{100}\frac{\left[1+(-1)^k\right]\cdot\cos\left(k\cdot\frac{2\cdot\pi}{T_{hw}}\cdot t\right)}{1-k^2}\right]$$

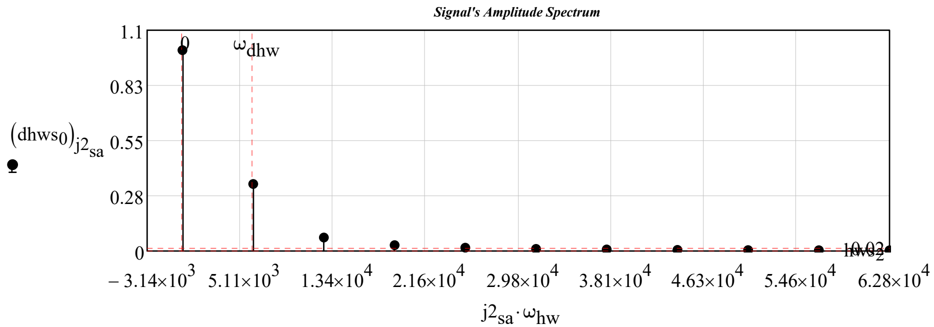
$s_{dhw}(0) = 2.05 + 325.269i$ $\quad \left|s_{dhw}(0)\right| = 325.276$ $\quad s_{dhw}\left(\frac{T_{dhw}}{2}\right) = 325.249$

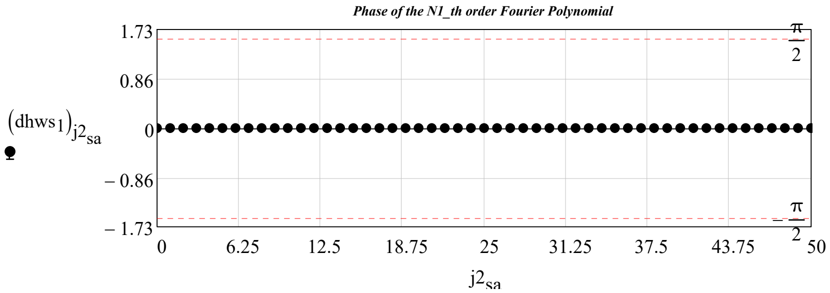
$B1_{hw} = 325.269\,V$ $\quad V_{dhw}(t) := g3\left(t, T_{dhw}, B1_{hw}\right)$ $\quad \omega_{sa} := \omega_{dhw}$ $\quad 2\cdot\frac{B1_{hw}}{\pi\cdot V} = 207.07$

$dhws := SPCT\left(V_{dhw}, rt_{gd}, N1\_, 0\cdot sec, T_{dhw}\right)$ $\quad N1\_ = 50$

$j2_{sa} := 0\,..\,rows\left(hws_0\right) - 1$







$Bw_{sa} := dhws_3 \cdot Hz$

$Bw_{sa} = 0.03 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.06 \cdot MHz$

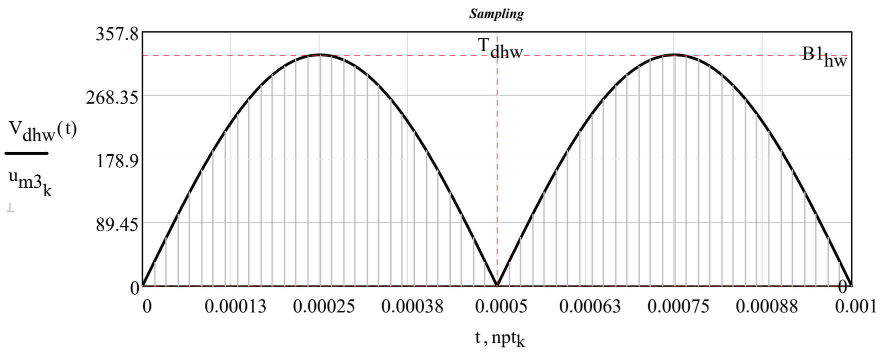
$k := 0 .. 2^8 - 1 \qquad npt_k := \dfrac{k}{fpt_{so}}$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{dhw}} = 8.533$

$u_{m3_k} := V_{dhw}(npt_k)$

$u_{m3}^T =$

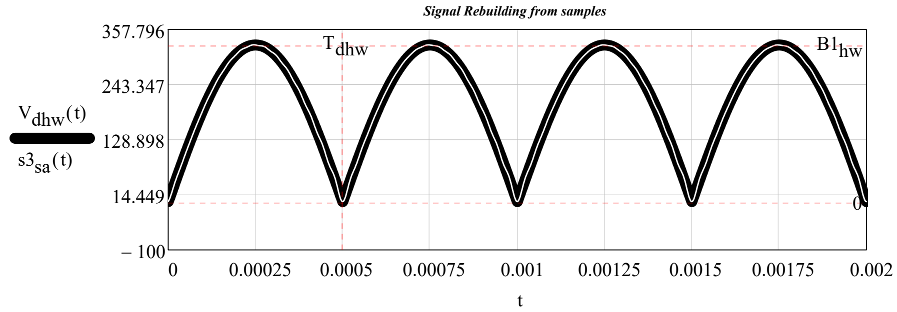
	0	1	2	3	4
0	0	34	67.627	100.514	...

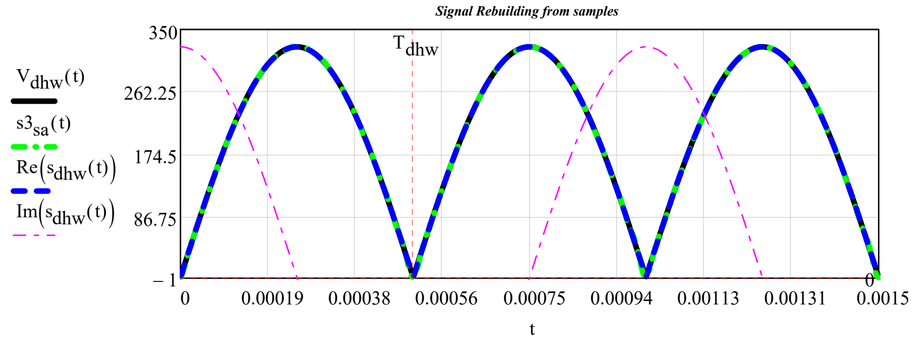


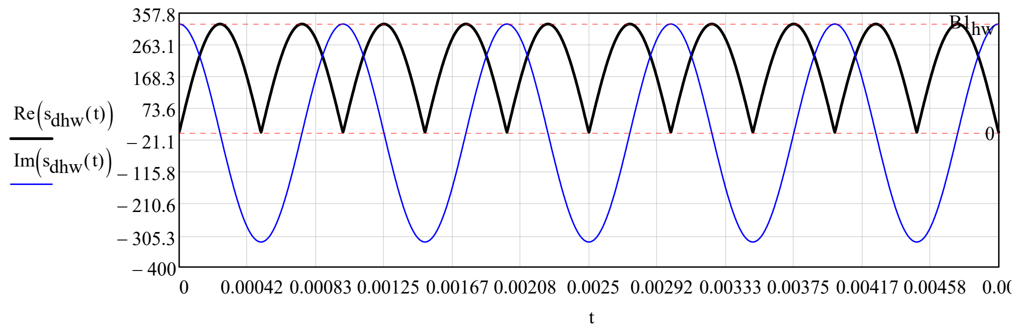
relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.188 \cdot \dfrac{Mrads}{sec} \qquad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula:: $s3_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m3_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\qquad N0_{gd} - 1 = 255 \qquad$ relerr =





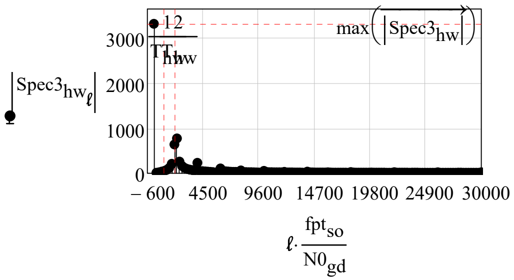


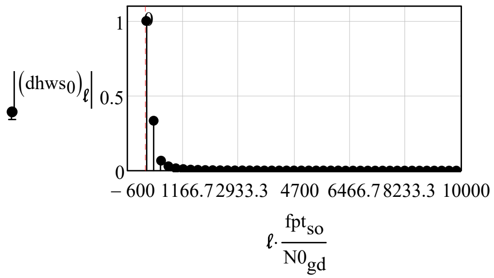
$\mathrm{length}(u_{m2}) = 256$

$fpt_{so} = 60 \cdot kHz$

$Spec3_{hw} := \mathrm{fft}(u_{m3}) \qquad \mathrm{length}(Spec3_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$





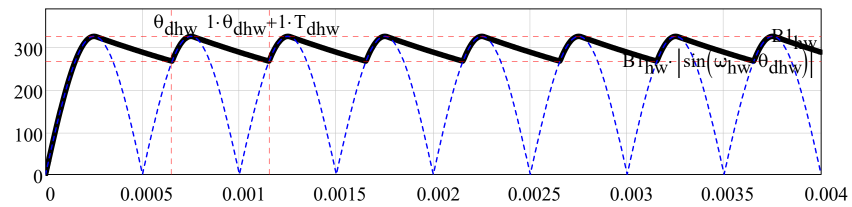
Periodic Waveforms

4 Double Half wave filtered

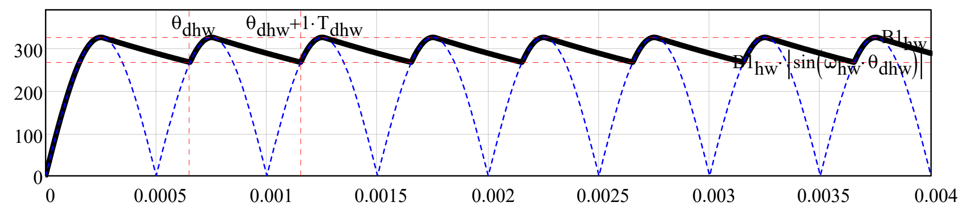
$$V_{ppt} := B1_{hw} \cdot \sin\left(\omega_{hw} \cdot \tau_{tpa_1}\right) \cdot e^{\frac{\tau_{tpa_1}}{\tau_{hw1}}} \qquad \theta_{dhw} := Z1\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{ppt}\right)$$

$$V_{ppt} = 369.746\,V \qquad rip1 = \frac{\frac{B1_{hw}}{V} - \frac{B1_{hw}}{V} \cdot \left|\sin\left(\omega_{hw} \cdot \theta_{dhw}\right)\right|}{\frac{B1_{hw}}{V}}$$

Ripple



Ripple

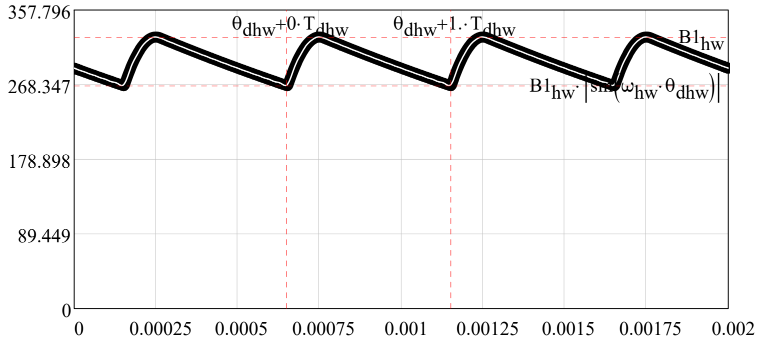


$$B1_{hw} = 325.269\,V \qquad V_{dhwf}(t) := g4\left(t + \tau_{hw1}, \tau_{hw1}, \tau_{tpa}, \theta_{dhw}, \omega_{hw}, B1_{hw}, V_{ppt}, N0_{gd}\right)$$

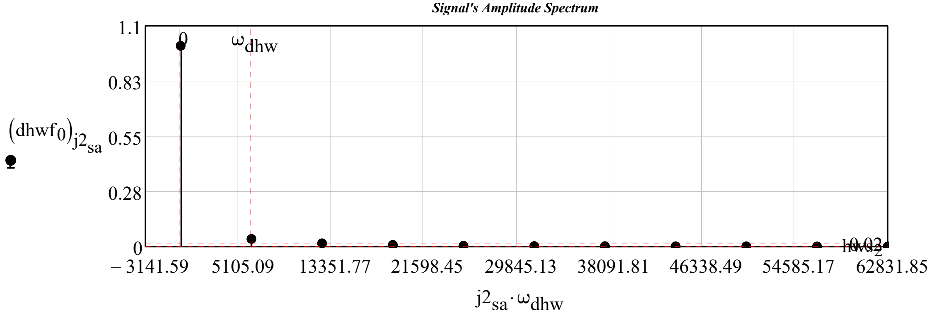
$$\tau_{hw1} = 2 \times 10^{-3}\,s \qquad dhwf := SPCT\left(V_{dhwf}, rt_{gd}, 50, 0 \cdot sec, T_{dhw}\right)$$

$$j2_{sa} := 0 .. rows\left(dhws_0\right) - 1$$

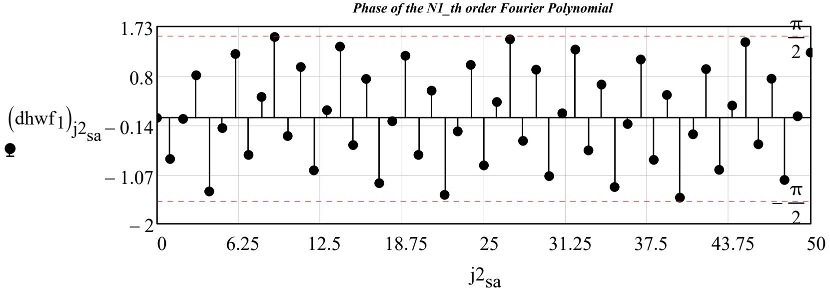
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := dhwf_3 \cdot Hz$

$Bw_{sa} = 0.036 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.072 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

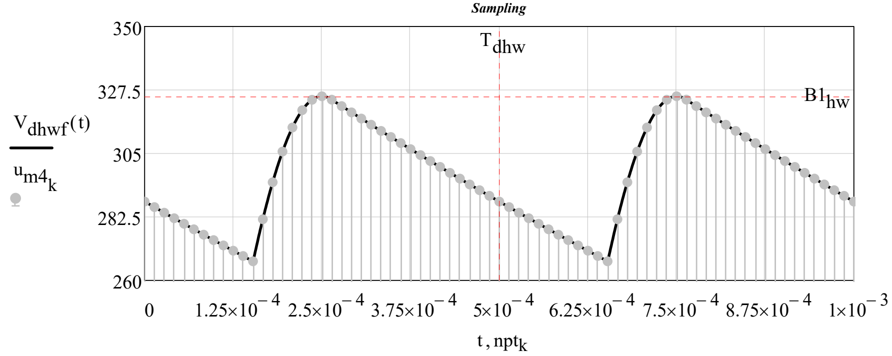
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{dhw}} = 7.111$

$u_{m4_k} := V_{dhwf}(npt_k)$

$u_{m4}^T =$

	0	1	2	3	4	5	6
0	287.958	285.966	283.987	282.021	280.07	278.131	...

Sampling

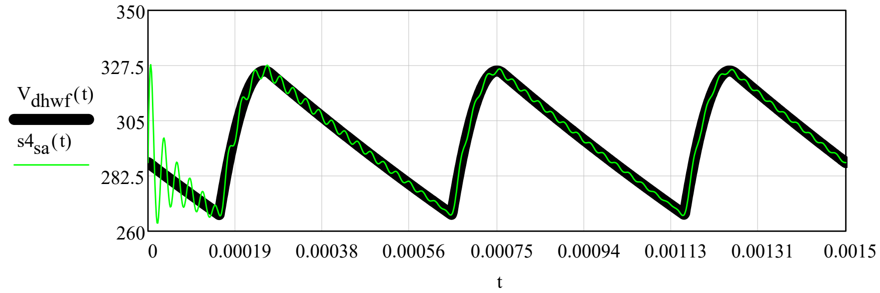


relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.226 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s4_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m4_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi \right) \right) \right]$ $\quad N0_{gd} - 1 = 255 \quad$ relerr = 10·%

Signal Rebuilding from samples

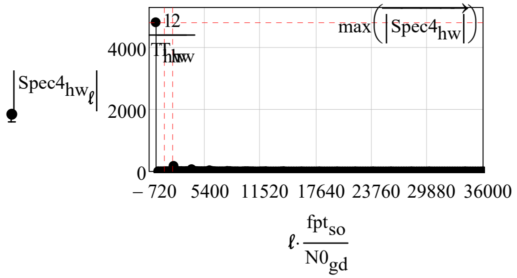


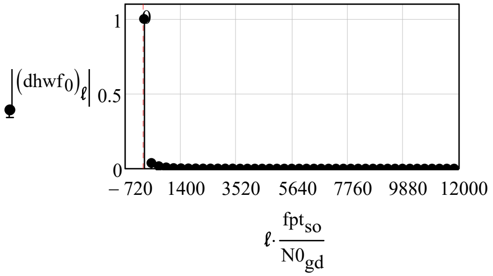
$length(u_{m4}) = 256$

$fpt_{so} = 72 \cdot kHz$

$Spec4_{hw} := fft(u_{m4})$ $\quad length(Spec4_{hw}) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





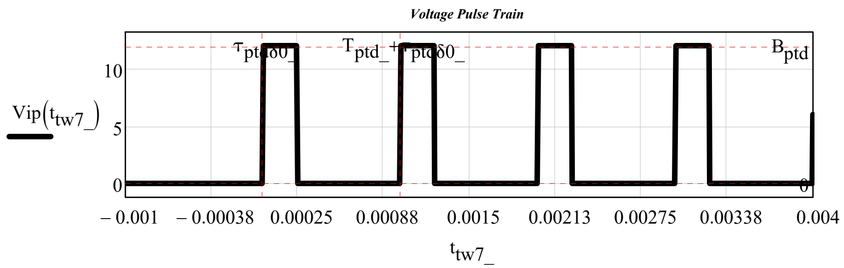
TEST Waveforms

Periodic Waveforms

5 Voltage Pulse Train

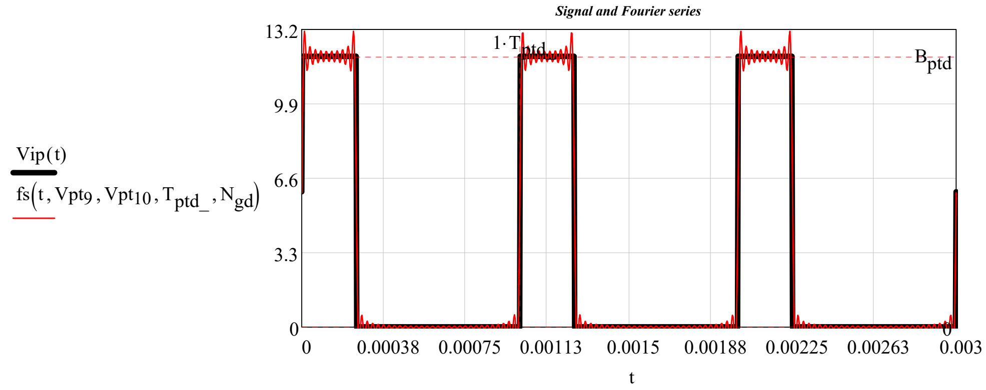
Data " pulse train data"

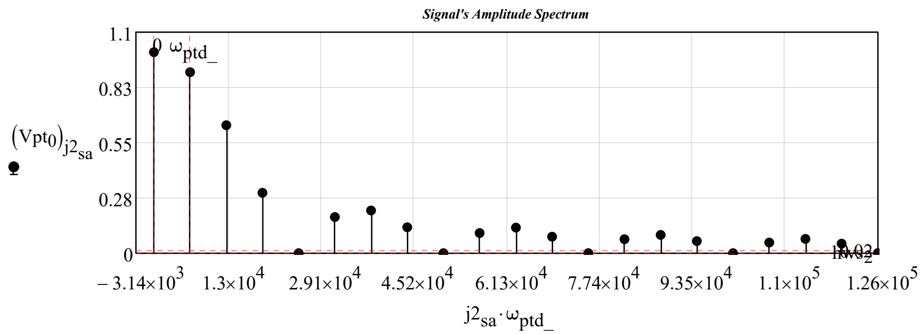
$$Vip(t) := Vip1\left(t, T_{ptd\_}, \tau_{ptd\delta0\_}, \delta_{ptd\_}, B_{ptd}, N0_{gd}\right)$$

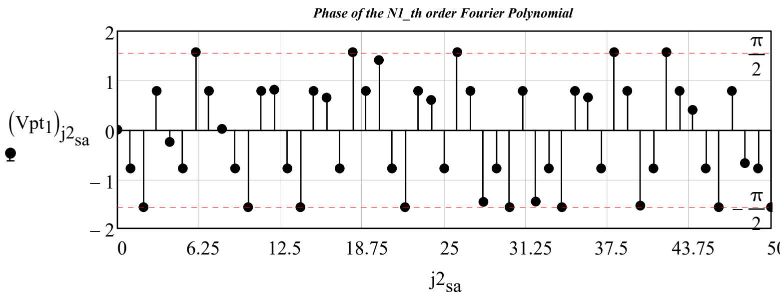


$$Vpt := SPCT\left(Vip, rt_{gd}, N1\_, 0 \cdot sec, T_{ptd\_}\right) \qquad N1\_ = 50$$

$$j2_{sa} := 0 .. rows\left(dhws_0\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$







$$Bw_{sa} := Vpt_3 \cdot Hz$$

$$Bw_{sa} = 0.048 \cdot MHz$$

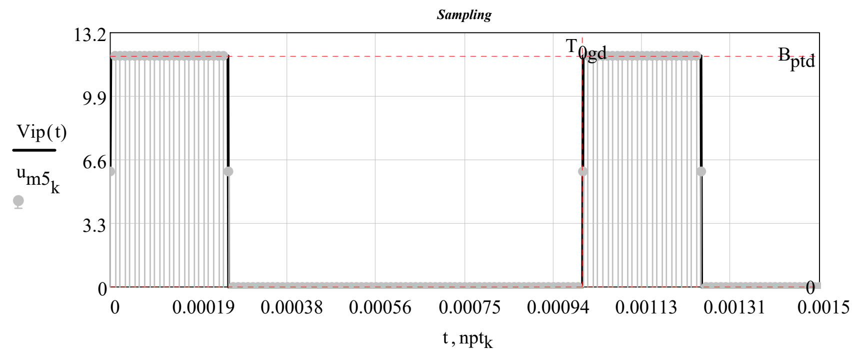
sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.096 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

$u_{m5_k} := Vip(npt_k)$

$u_{m5}^T =$	0	1	2	3	4	5	6	7	8	9
0	6	12	12	12	12	12	12	12	12	...

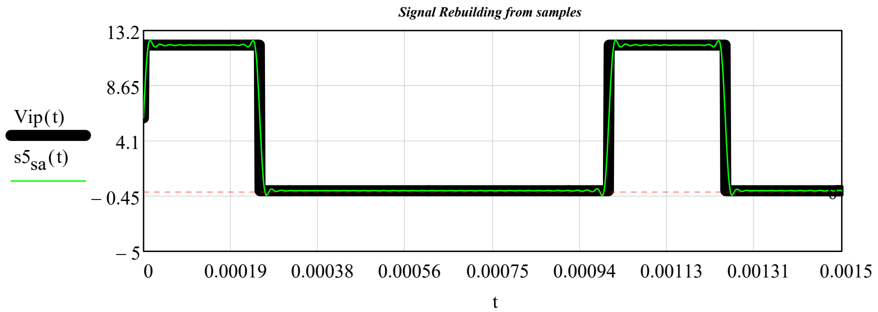


relerr = 10·%

$\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.302\cdot\frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s5_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m5_n}\cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%

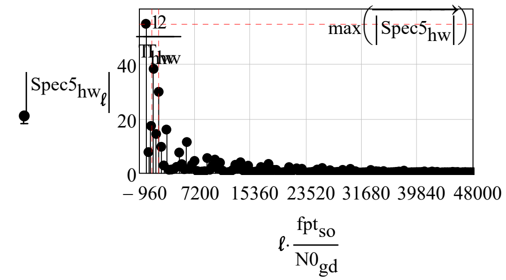


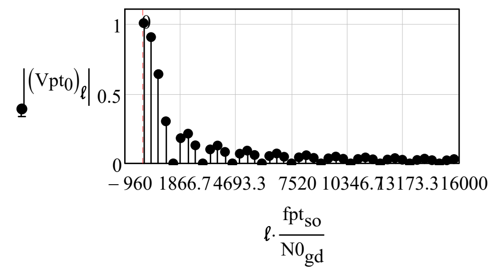
$length(u_{m5}) = 256$

$fpt_{so} = 96\cdot kHz$

$Spec5_{hw} := fft(u_{m5})$ $\quad length(Spec5_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\quad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

6 RF Pulse Train

Data " rf pulse data"

Step amplitude.............................: $V_{rfpd} := B_{ptd}$, $V_{rfpd} = 12\cdot V$

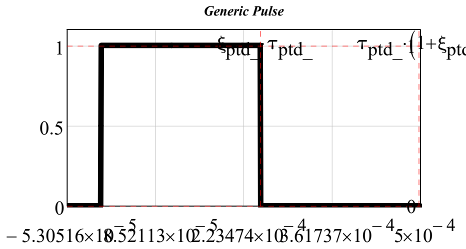
Signal frequency..........................: $f_{rfpd} := 30\cdot f_{ptd\_}$, $\quad f_{ptd\_} = 1\times 10^3\,\frac{1}{s}$

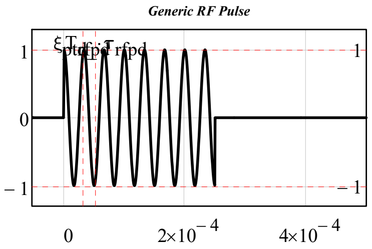
Signal period...............................: $T_{rfpd} := \frac{1}{f_{rfpd}}$, $\quad T_{ptd\_} = 1\times 10^{-3}\,s$

Signal angular frequency...............: $\omega_{rfpd} := 2\cdot\pi\cdot f_{rfpd}$ $\quad \omega_{rfpd} = 0.188\cdot\frac{Mrads}{sec}$,

time constant...............................: $\tau_{rfpd} := \frac{10}{\omega_{rfpd}}$, $\tau_{rfpd} = 53.052\cdot\mu s$

Rising edge delay ....: $\tau_{\delta rfpd} := 0\cdot ns$, $\quad \xi_{ptd\_}\cdot\tau_{ptd\_} = 250\cdot\mu s$, $\xi_{ptd\_} = 1$, $\tau_{ptd\_} = 250\cdot\mu s$

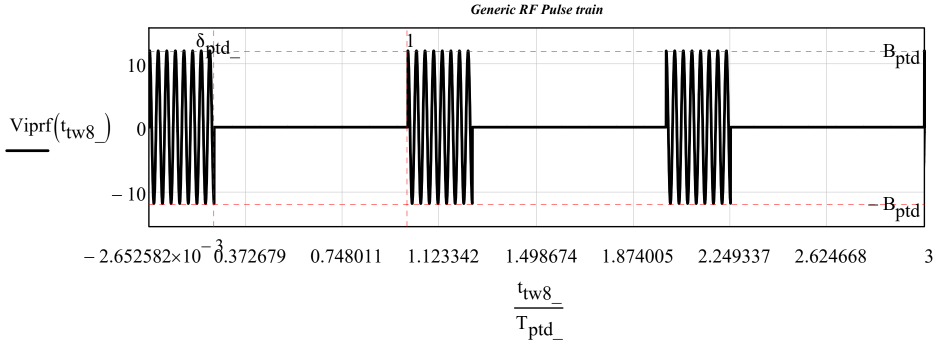




Average value: $v_{ptmrfsl} := B_{ptd\_} \cdot \delta_{ptd\_}$

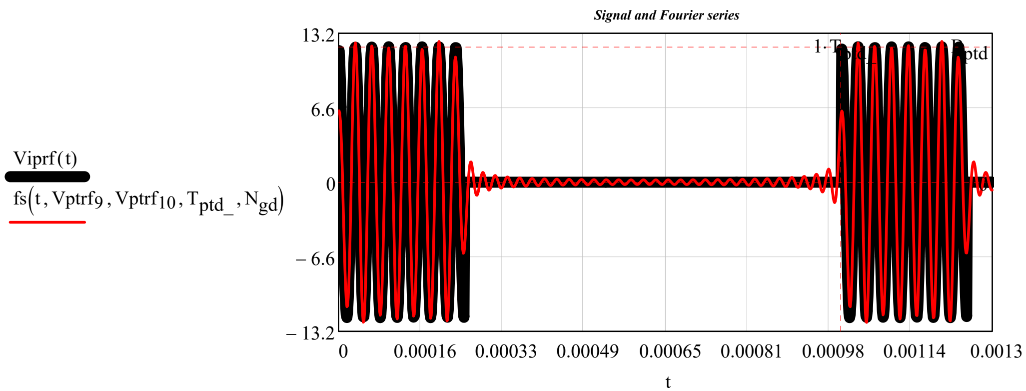
$$t_{tw8\_} := -1 \cdot \tau_{ptd\_}, -1 \cdot \tau_{ptd\_} + \frac{4 \cdot T_{ptd\_} + \tau_{ptd\_}}{8000} .. 4 \cdot T_{ptd\_}$$

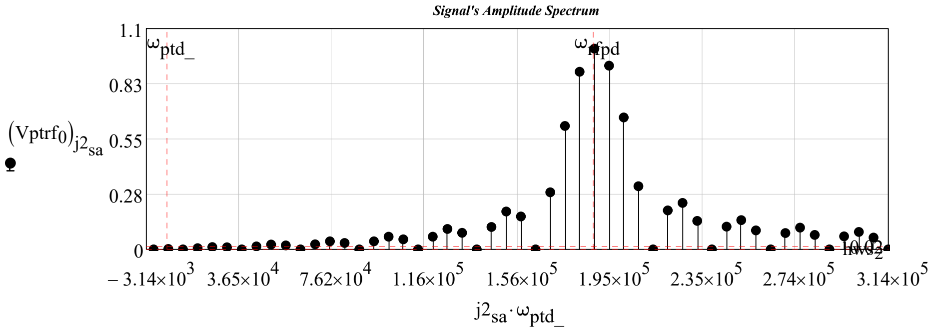
$$Viprf(t) := v_{ptrf}\left(t, T_{ptd\_}, \tau_{\delta rfpd}, \delta_{ptd\_}, \omega_{rfpd}, \frac{V_{rfpd}}{V}, N0_{gd}\right)$$

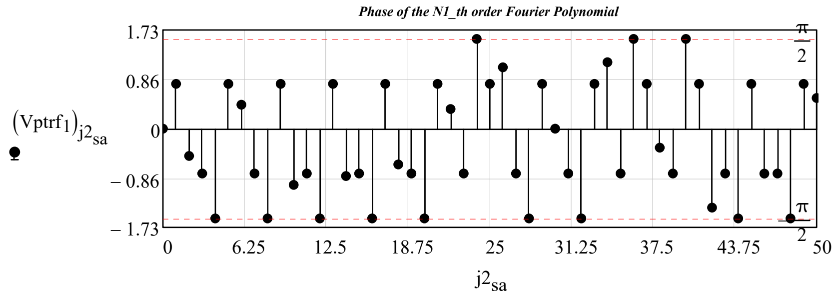


$$Vptrf := SPCT\left(Viprf, rt_{gd}, N1\_, 0 \cdot sec, T_{ptd\_}\right) \qquad N1\_ = 50$$

$N_{gd} = 40$ $\qquad \omega_{ptd\_} = 6.283 \cdot \frac{krads}{s}$ $\qquad j2_{sa} := 0 .. rows(dhws_0) - 1$ $\qquad \omega_{rfpd} = 188.496 \cdot \frac{krads}{s}$







$$Bw_{sa} := Vptrf_3 \cdot Hz$$

$$Bw_{sa} = 0.048 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.096 \cdot MHz$

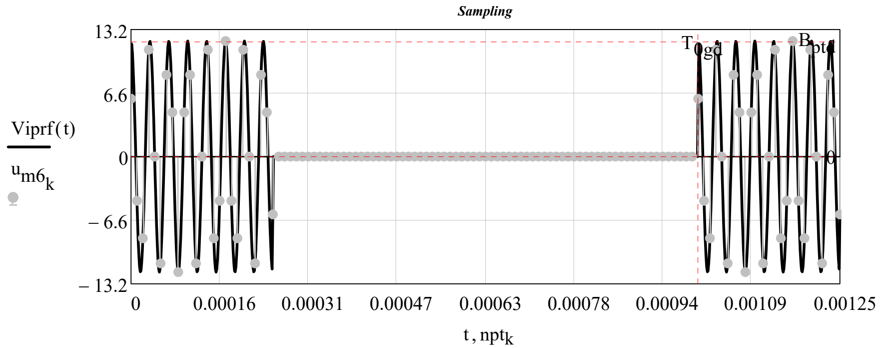
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

$$u_{m6_k} := Viprf\left(npt_k\right)$$

$u_{m6}^T =$

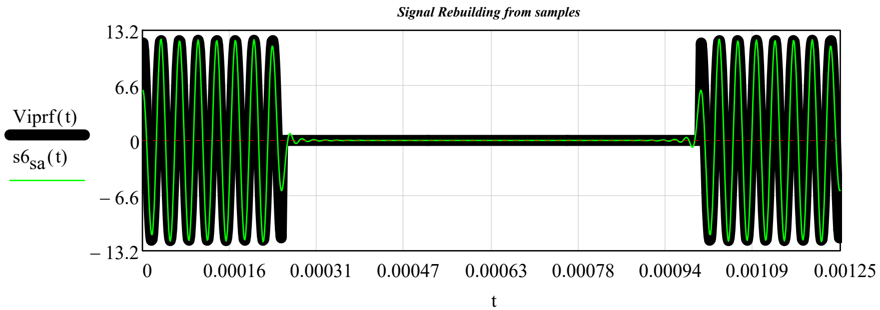
	0	1	2	3	4
0	6	-4.592	-8.485	11.087	...



relerr = 10·% $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.302\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s6_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m6_n}\cdot \text{sinc}\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

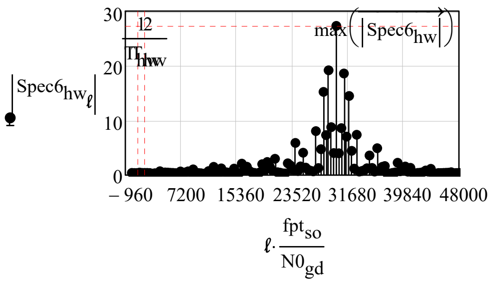


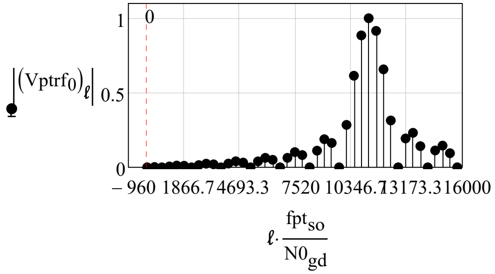
$\text{length}\left(u_{m6}\right) = 256$

$fpt_{so} = 96\cdot kHz$

$Spec6_{hw} := \text{fft}\left(u_{m6}\right)$ $\text{length}\left(Spec6_{hw}\right) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

7 Bipolar Square Wave

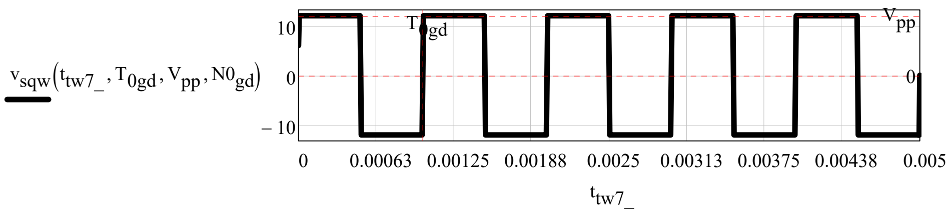
Data file " pulse train data.xmcd"

Signal amplitude: $V_{pp} = 12 \cdot V$

Square wave period: $T_{0gd} = 1 \times 10^{6} \cdot ns$

$\omega_{ptd\_} = 6.283 \times 10^{-6} \cdot \frac{Grads}{sec}$

Bipolar Square Wave

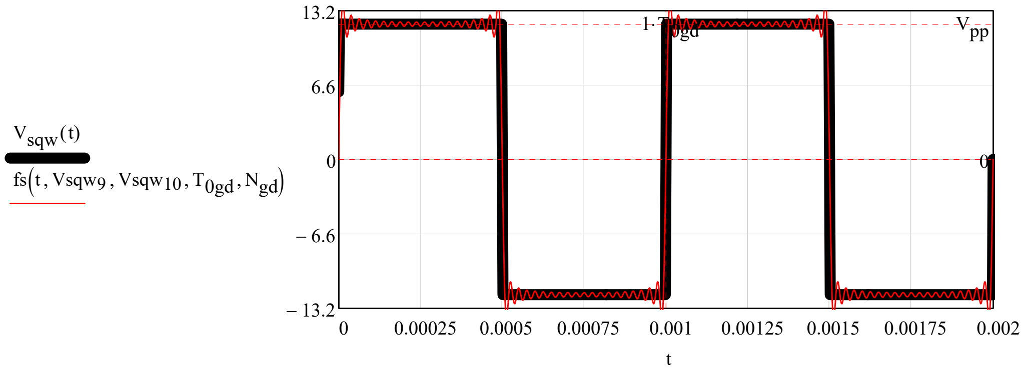


$V_{sqw}(t) := \frac{v_{sqw}(t, T_{0gd}, V_{pp}, N0_{gd})}{V}$

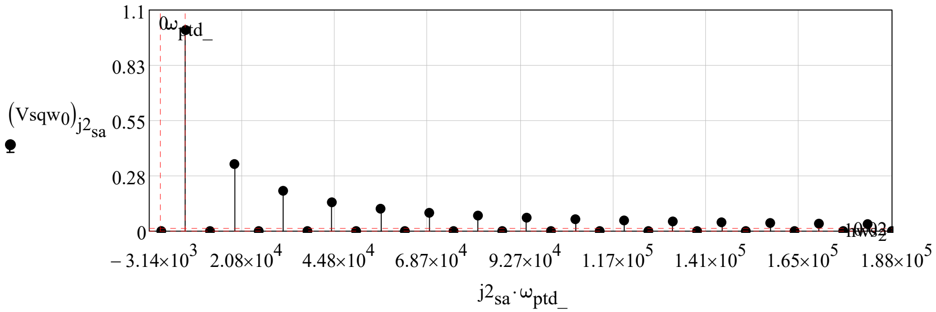
$Vsqw := SPCT(V_{sqw}, rt_{gd}, N1\_, 0 \cdot sec, T_{0gd})$ $N1\_ = 50$

$j2_{sa} := 0 .. rows(dhws_0) - 1$ $\omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

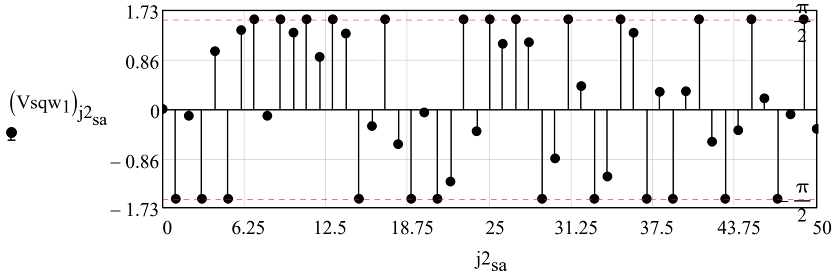
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Vsqw_3 \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.096 \cdot MHz$

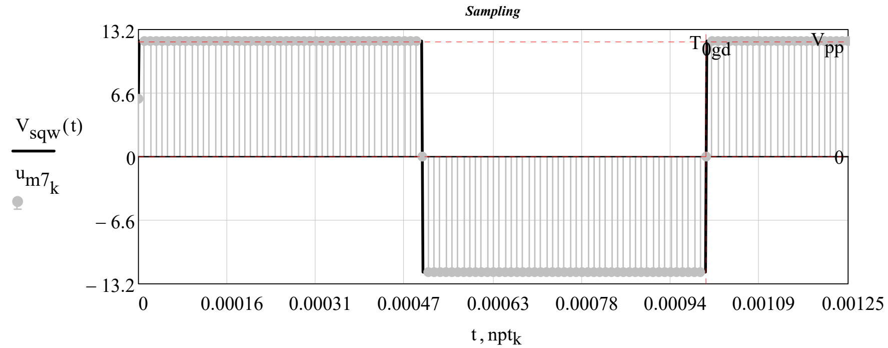
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

$u_{m7_k} := V_{sqw}(npt_k)$

$u_{m6}^T =$

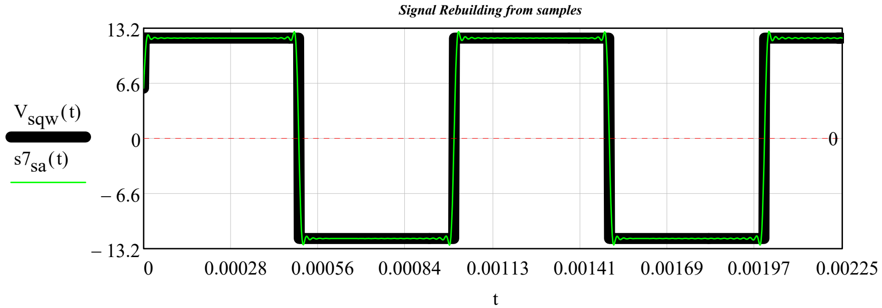
	0	1	2	3	4
0	6	-4.592	-8.485	11.087	...



$relerr = 10\cdot\%$ $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.302\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s7_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m7_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ $relerr = 10\cdot\%$

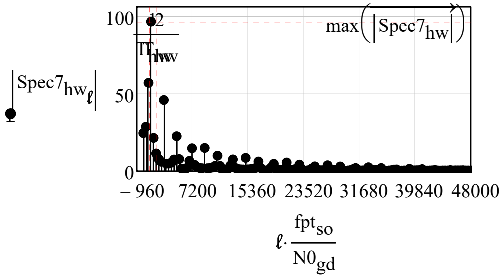


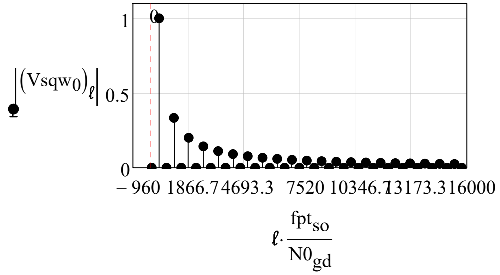
$length\left(u_{m7}\right) = 256$

$fpt_{so} = 96\cdot kHz$

$Spec7_{hw} := fft\left(u_{m7}\right)$ $length\left(Spec7_{hw}\right) = 129$

$\ell := 0..\frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

8 Bipolar Square Wave 1

Pulse train data

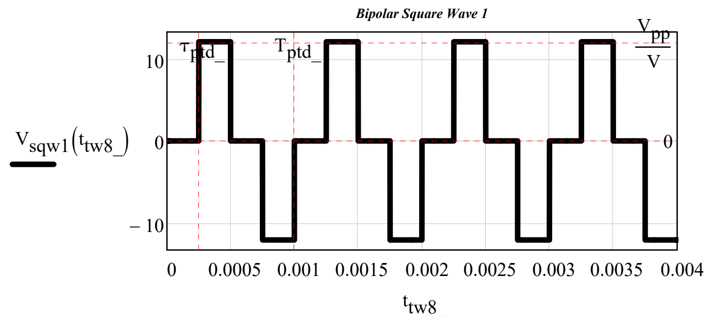
Signal amplitude: $V_{pp} = 12\,V$

Square wave period: $T_{0gd} = 1\times10^{6}\cdot ns$ $\xi_{twsl} := \xi_{ptd\_}$

$\omega_{ptd\_} = 6.283\times10^{-6}\cdot\frac{Grads}{sec}$ $\tau_{\delta sl} := -\tau_{ptd\_}\cdot\left(1-\xi_{twsl}\right)$

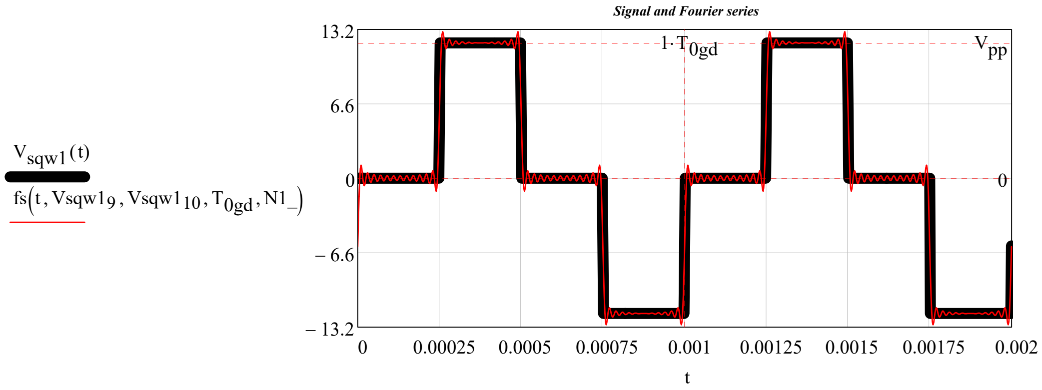
$\tau_{ptd\_} = 250\cdot\mu s$

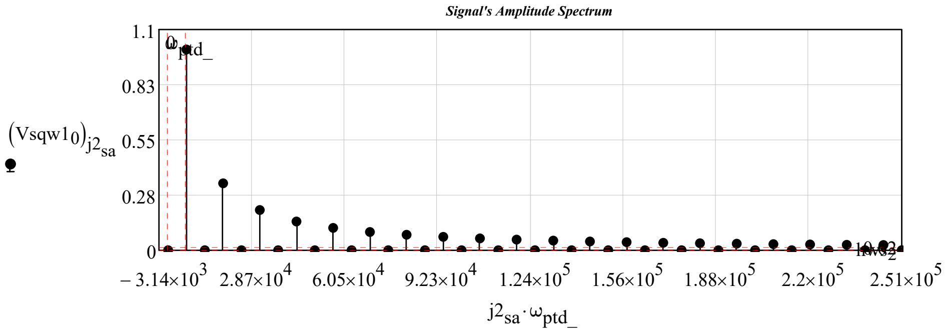
$V_{sqw1}(t) := V_6\left(t, \tau_{\delta sl}, \tau_{ptd\_}, T_{0gd}, V_{pp}, N0_{gd}\right)$

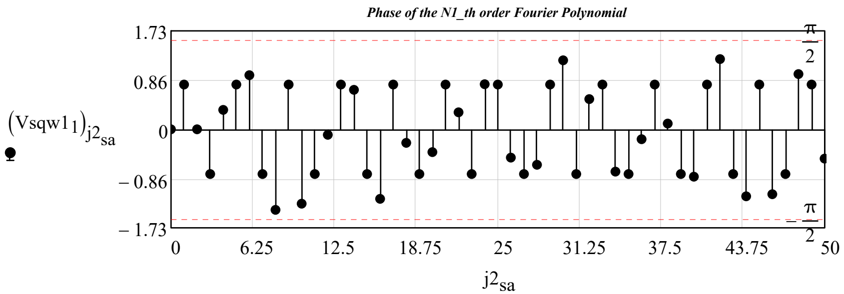


$Vsqw1 := SPCT\left(V_{sqw1}, rt_{gd}, N1\_, \tau_{ptd\_}, T_{ptd\_}\right)$ $N1\_ = 50$

$j2_{sa} := 0..rows\left(Vsqw1_0\right) - 1$ $\omega_{ptd\_} = 6.283\cdot\frac{krads}{s}$







$$Bw_{sa} := Vsqw1_3 \cdot Hz$$

$$Bw_{sa} = 0.048 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.096 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

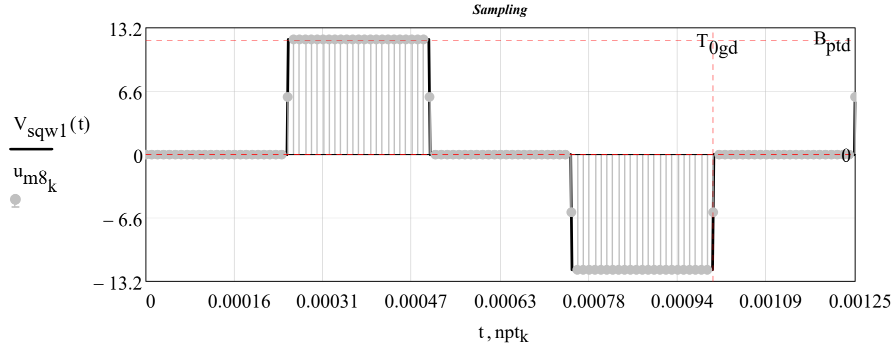
$$u_{m8_k} := V_{sqw1}(npt_k)$$

$u_{m8}^T =$

	0	1	2	3	4	5	6	7	8	9

$u_{m8} =$

0	0	0	0	0	0	0	0	0	0	...

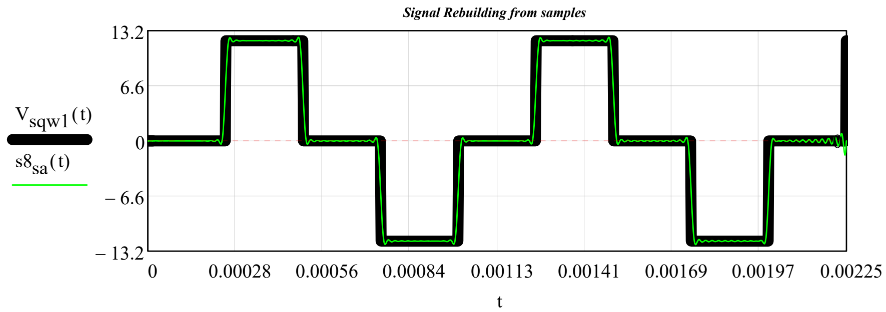


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.302 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s8_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m8_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%

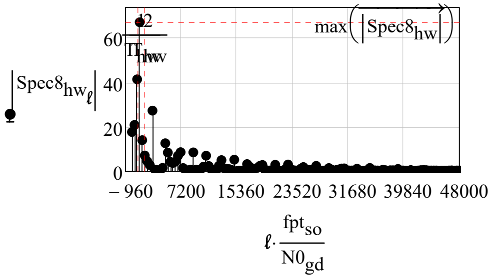


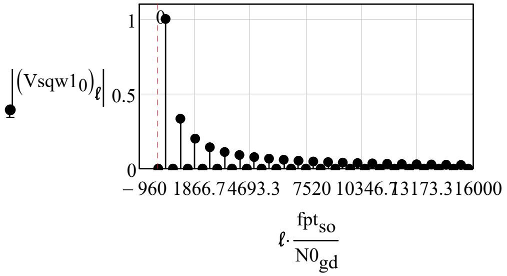
$$length(u_{m8}) = 256$$

$$fpt_{so} = 96 \cdot kHz$$

$Spec8_{hw} := fft(u_{m8})$ $\quad length(Spec8_{hw}) = 129$

$$\ell := 0..\frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

9 Staircase 1 Voltage Pulse Train

Description of the Function's parameters:

$v_{stcp}(t_{sl}, \text{period}, \text{signal\_amplitude}, \text{number\_of\_steps}, \text{max\_number\_of\_periods})$

$v_{stc}(t_{sl}, \text{step\_length}, \text{signal\_amplitude}, \text{number\_of\_steps}, \text{max\_number\_of\_periods})$

Period: $T_{stcpt} := (m1_{steps} + 1) \cdot T_{1stpl\_} \cdot 2$

Duty Cycle: $\delta_{stcpt} := \frac{m1_{steps} \cdot T_{1stpl\_}}{T_{stcpt}}$

Staircase frequency: $f_{stcpt} := \frac{1}{T_{stcpt}}$

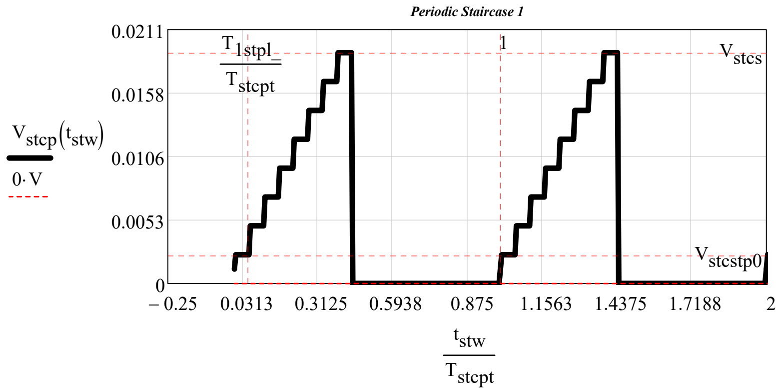
$\omega_{stcpt} := 2 \cdot \pi \cdot f_{stcpt} \qquad \omega_{1stpl} := \frac{2 \cdot \pi}{T_{1stpl\_}}$

Number of periods shown: $n_p := 20$

$$v_{stcptasl} := \frac{V_{stcs}}{2 \cdot m1_{steps} \cdot (m1_{steps} + 1)} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1) = 4.8 \cdot mV$$

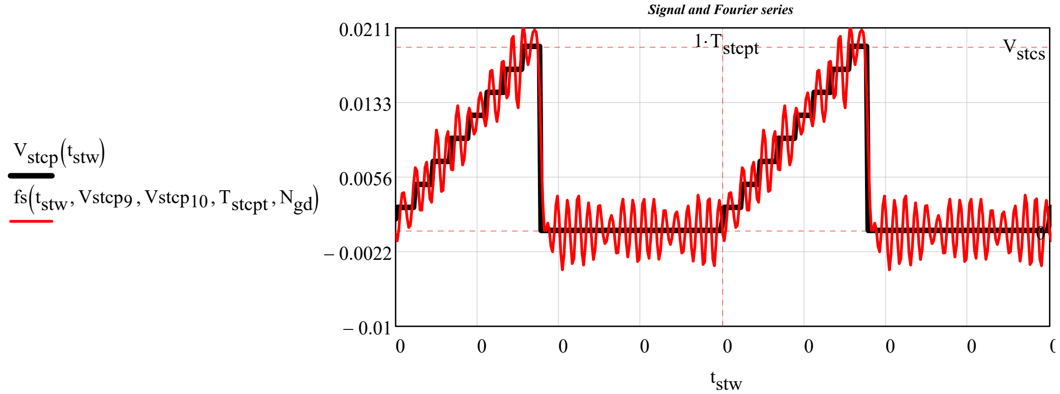
$$t_{stw} := 0 \cdot T_{stcpt}, 0 \cdot T_{stcpt} + \frac{10 \cdot T_{stcpt}}{2000} .. 10 \cdot T_{stcpt}$$

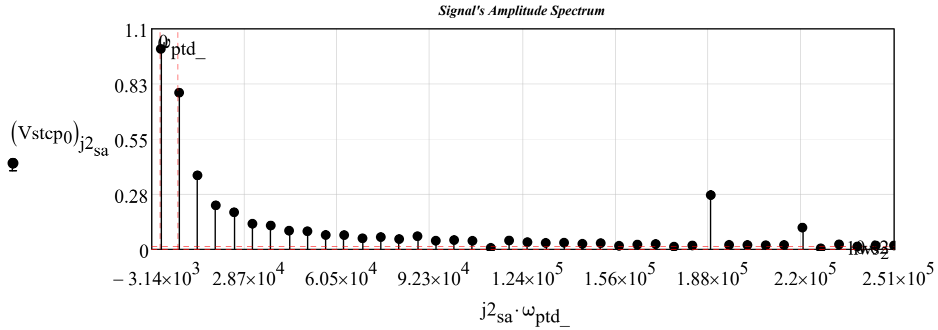
Dimensionless function: $V_{stcp}(t) := \frac{v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd})}{V}$

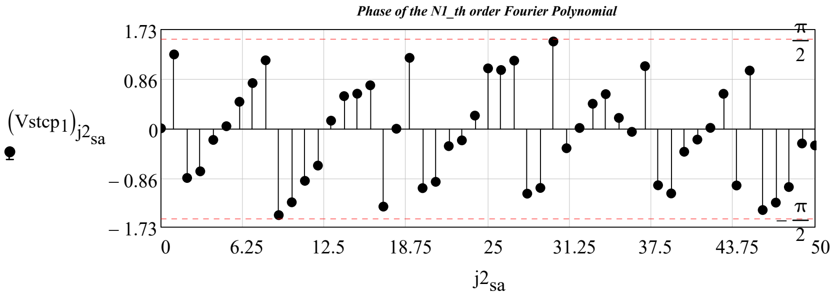


$$Vstcp := SPCT(V_{stcp}, rt_{gd}, N1\_, 0 \cdot s, T_{stcpt})$$

$$j2_{sa} := 0..rows(Vstcp_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$







$\text{Bw}_{sa} := \text{Vstcp}_3 \cdot \text{Hz}$

$\text{Bw}_{sa} = 7.2 \cdot \text{MHz}$

sampling frequency: $\text{fpt}_{so} := 2 \cdot \text{Bw}_{sa}$ $\qquad \text{fpt}_{so} = 14.4 \cdot \text{MHz}$

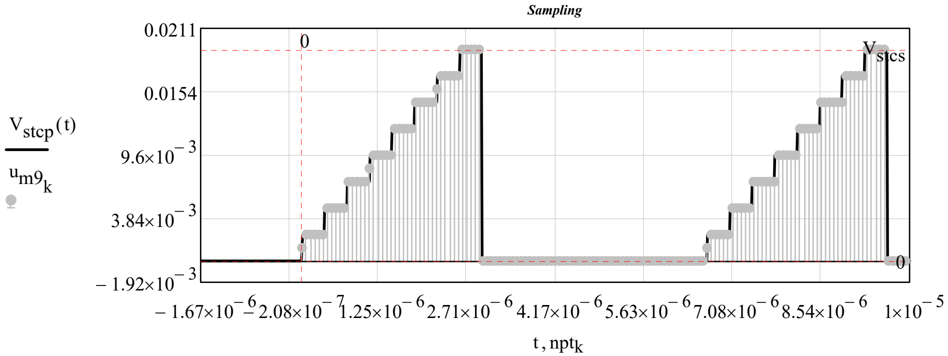
$$\text{npt}_k := \frac{k}{\text{fpt}_{so}}$$

Frequency resolution: $\dfrac{\text{N0}_{gd}}{\text{fpt}_{so}} \cdot \dfrac{1}{\text{T}_{stcpt}} = 2.667$

$$\text{u}_{m9_k} := \text{V}_{stcp}\left(\text{npt}_k\right)$$

$\text{u}_{m9}{}^{T} =$

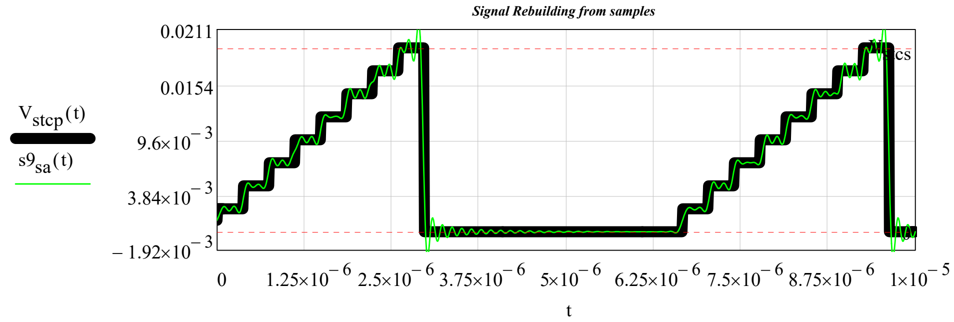
	0	1	2	3	4	5	6
0	1.2·10-3	2.4·10-3	2.4·10-3	2.4·10-3	2.4·10-3	2.4·10-3	...



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot \text{Bw}_{sa} \qquad \omega_{bwr} = 45.239 \cdot \dfrac{\text{Mrads}}{\text{sec}} \qquad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot \text{Bw}_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\text{s9}_{sa}(t) := \left[\displaystyle\sum_{n=0}^{\text{N0}_{gd}-1} \left(\text{u}_{m9_n} \cdot \text{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right] \qquad \text{N0}_{gd} - 1 = 255 \qquad \text{relerr} = 10 \cdot \%$

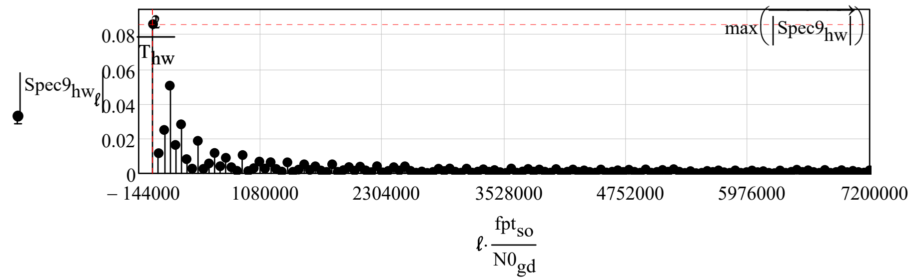


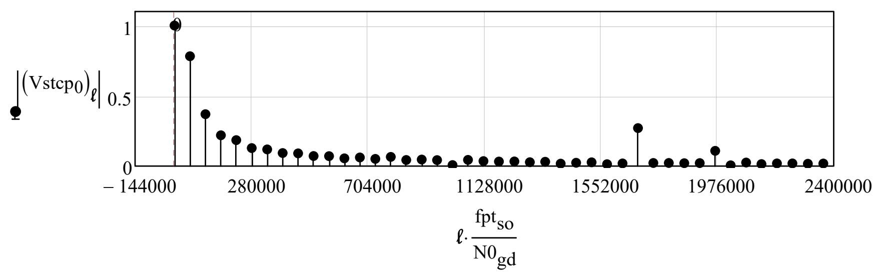
$\text{length}\left(\text{u}_{m9}\right) = 256$

$\text{fpt}_{so} = 1.44 \times 10^{4} \cdot \text{kHz}$

$\text{Spec9}_{hw} := \text{fft}\left(\text{u}_{m9}\right) \qquad \text{length}\left(\text{Spec9}_{hw}\right) = 129$

$\ell := 0 .. \dfrac{\text{N0}_{gd}}{2} \qquad \dfrac{\text{N0}_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

10 Staircase 2 Voltage Pulse Train

Description of the Function's parameters:

v_{stct}(time, period, max_amplitude, number_of_steps, max_number_of_periods)

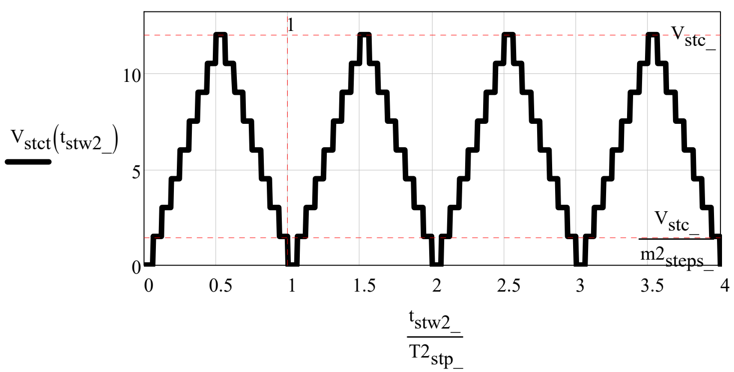
$v_{stcc}(t_{sl}$, step_length, signal_amplitude, number_of_steps, number_max_of_periods)

For data, see *"staircase 2 pulse data"*

$$t_{stw2\_} := 0\cdot T2_{stp\_}, 0\cdot T2_{stp\_} + \frac{10\cdot T2_{stp\_}}{2000} .. 10\cdot T2_{stp\_}$$

$$V_{stct}(t) := \frac{v_{stct}(t, T2_{stp\_}, V_{stc\_}, m2_{steps\_}, N0_{gd})}{V}$$

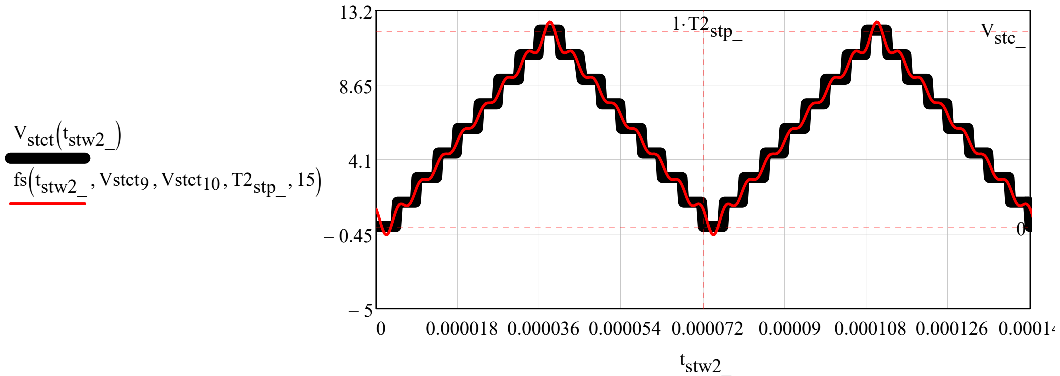
Dimensionless Staircase 2 Wave



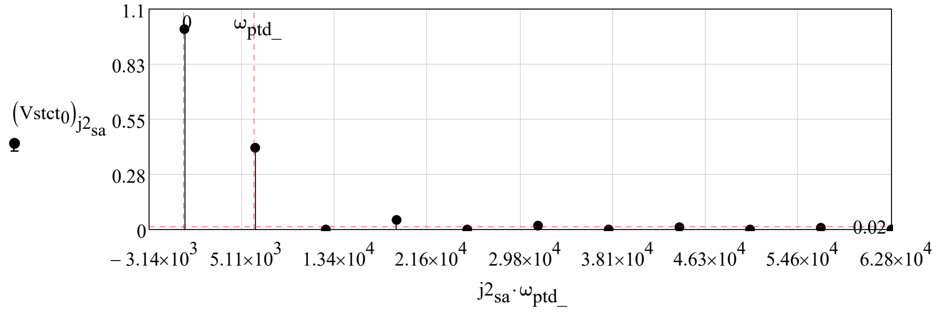
$$Vstct := SPCT(V_{stct}, rt_{gd}, N1\_, 0\cdot s, T2_{stp\_}) \qquad N1\_ = 50$$

$$j2_{sa} := 0 .. rows(Vstct_0) - 1 \qquad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$$

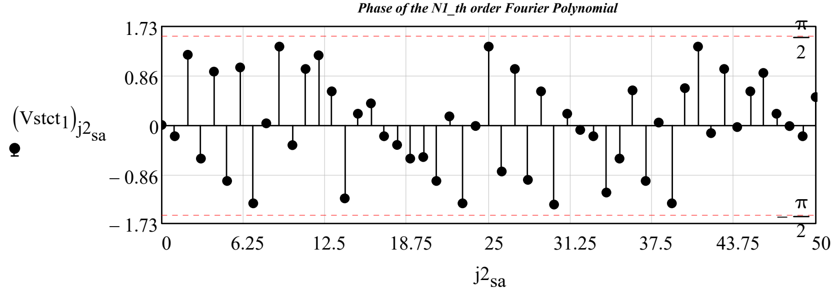
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := Vstct_3\cdot Hz$$

$$Bw_{sa} = 0.667\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 1.333\cdot MHz$

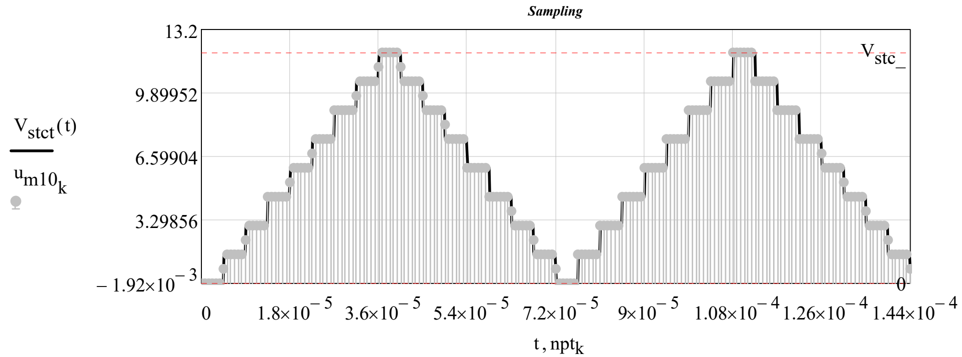
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T2_{stp\_}} = 2.667$

$$u_{m10_k} := V_{stct}(npt_k)$$

$u_{m10}^T =$

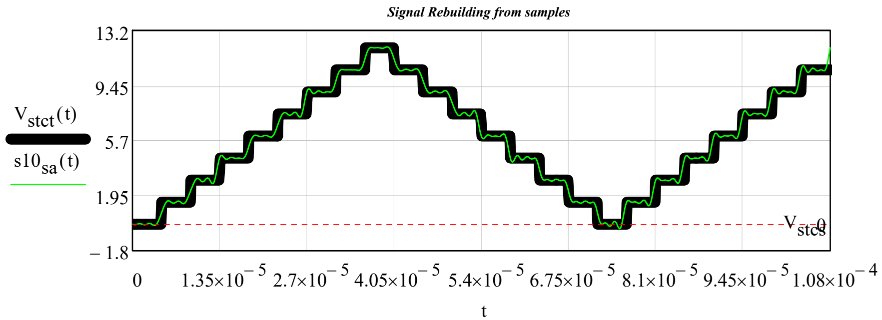
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0.75	1.5	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 4.189\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s10_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m10_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $\quad N0_{gd} - 1 = 255 \quad relerr = 10\cdot\%$

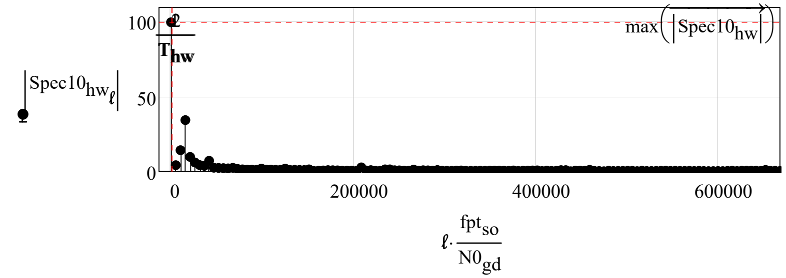


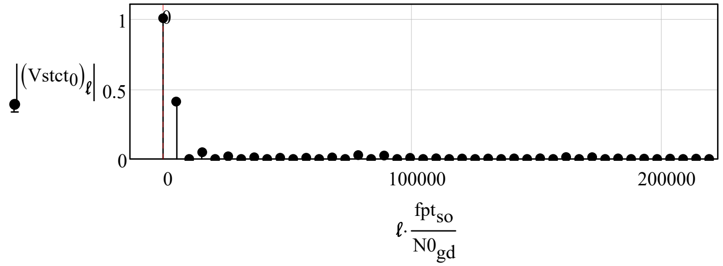
$length\left(u_{m10}\right) = 256$

$fpt_{so} = 1.333\times10^{3}\cdot kHz$

$Spec10_{hw} := fft\left(u_{m10}\right) \quad length\left(Spec10_{hw}\right) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

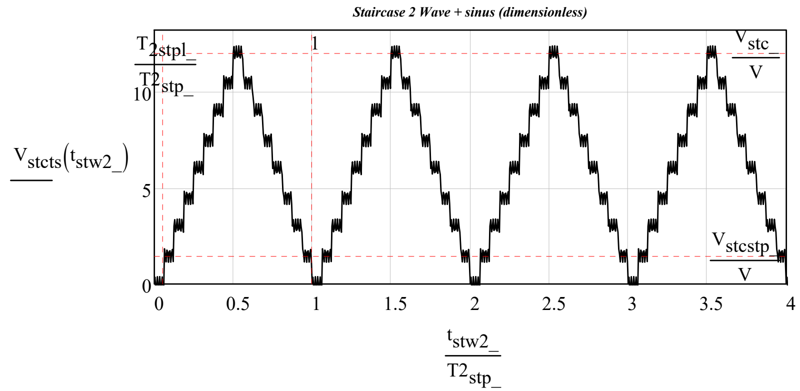
11 Staircase 2 Voltage Pulse Train + sinus

Description of the Function's parameters:

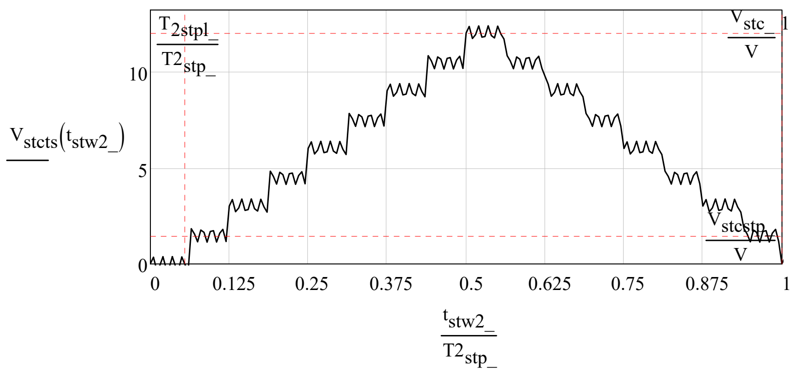
$Vstcsin\left(t_{s1}, period, max\_amplitude, number\_of\_steps, max\_number\_of\_periods, Number\_of\_periods\right)$

For data, see *" staircase 2 pulse data"*

$V_{stcts}(t) := Vstcsin\left(t, T2_{stp\_}, V_{stc\_}, m2_{steps\_}, N0_{gd}\right)$

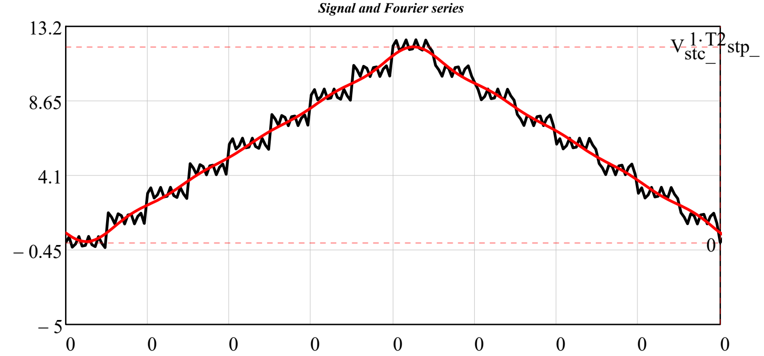


Staircase 2 Wave + sinus (dimensionless)



$N1\_ := 10$ $\qquad$ $Vstcts := SPCT\left(V_{stcts}, rt_{gd}, N1\_, 0\cdot s, T2_{stp\_}\right)$

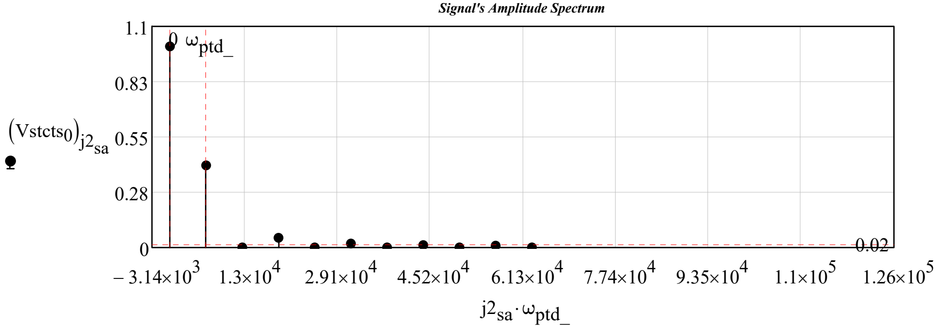
Signal and Fourier series



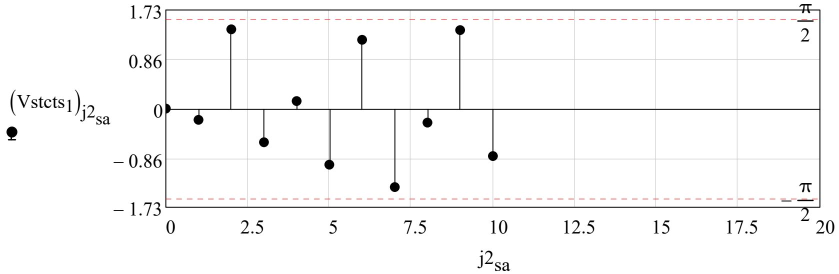
$N1\_ = 10$

$j2_{sa} := 0 .. rows\left(Vstcts_0\right) - 1 \qquad \omega_{ptd\_} = 6.283\cdot\frac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Vstcts_3\cdot Hz$

$Bw_{sa} = 0.111\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad$ $fpt_{so} = 0.222\cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

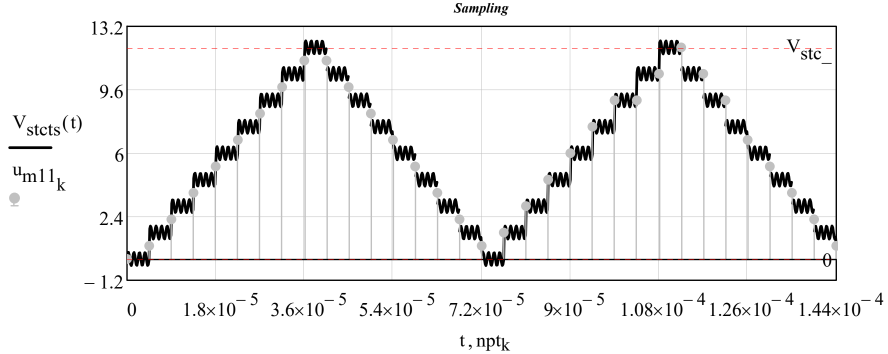
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T2_{stp\_}} = 16$

$u_{m11_k} := V_{stcts}\left(npt_k\right)$

$u_{m11}^T =$

	0	1	2	3	4
0	0	0.75	2.25	3.75	...

Sampling

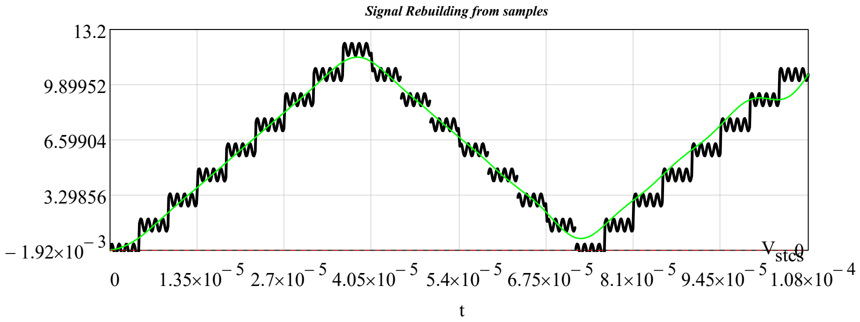


$relerr = 10\cdot\%$ $\qquad$ $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad$ $\omega_{bwr} = 0.698\cdot\frac{Mrads}{sec}$ $\qquad$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s11_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m11_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $\qquad$ $N0_{gd} - 1 = 255$ $\qquad$ $relerr = 10\cdot\%$

$N1\_ = 10$

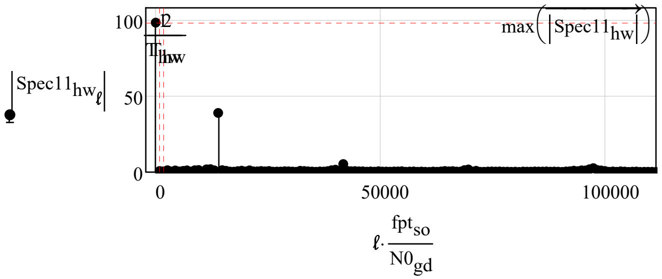


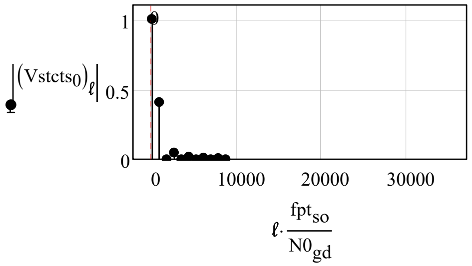
$length(u_{m11}) = 256$

$fpt_{so} = 222.222 \cdot kHz$

$Spec11_{hw} := fft(u_{m11}) \quad length(Spec11_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

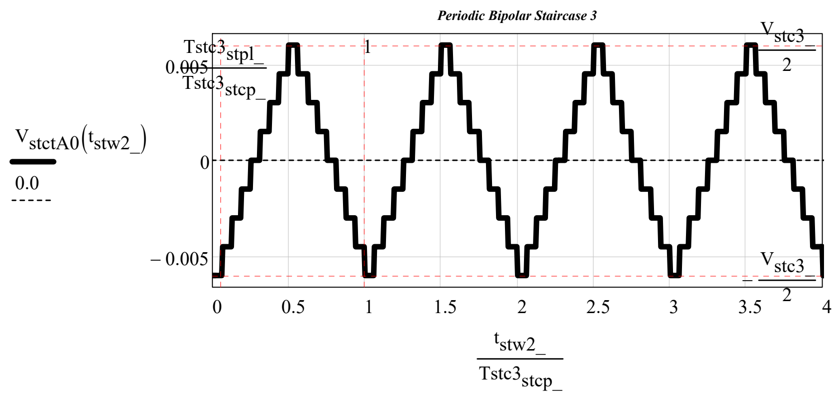
12 Staircase 3 Voltage Pulse Train

Description of the Function's parameters: $v_{stct}(t_{sl}, period, step\_amplitude, number\_of\_steps, max\_number\_of\_periods)$

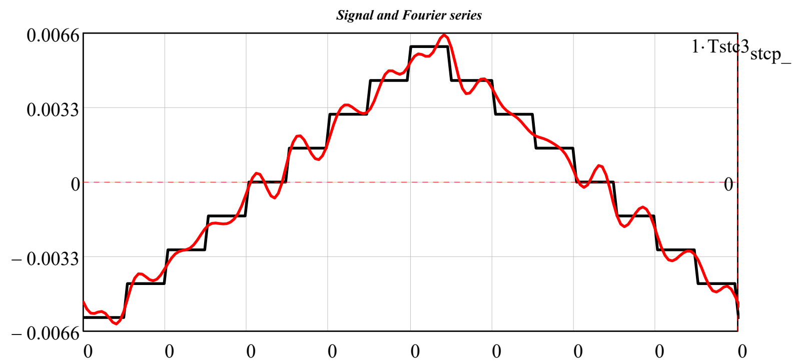
: $v_{stctA0}[t_{sl}, (period, step\_amplitude, number\_of\_steps, max\_number\_of\_periods)]$

You can find the data in "*staircase 3 pulse data*"

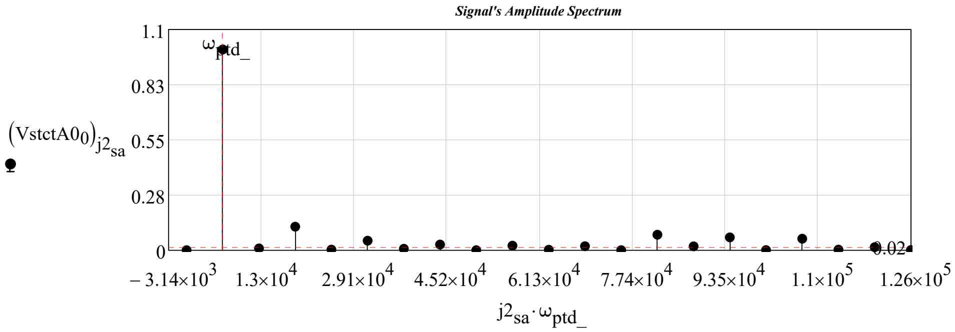
$$V_{stctA0}(t) := \frac{v_{stctA0}(t, Tstc3_{stcp\_}, V_{stc3\_}, mstc3_{steps\_}, N0_{gd})}{V} \qquad N1\_ := 25$$

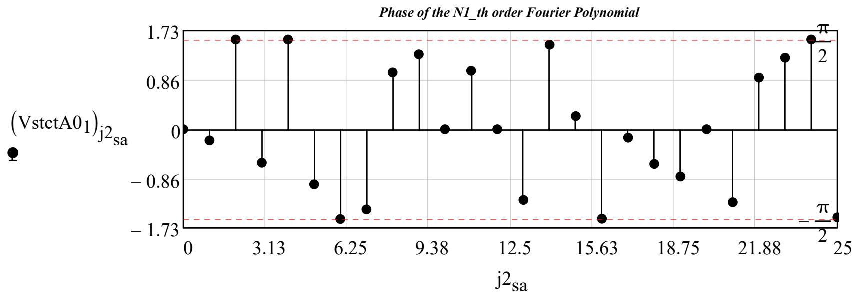


$$VstctA0 := SPCT(V_{stctA0}, rt_{gd}, N1\_, 0 \cdot s, Tstc3_{stcp\_}) \quad N1\_ = 25$$



$j2_{sa} := 0 .. rows(VstctA0_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := VstctA0_3 \cdot Hz$

$Bw_{sa} = 0.328 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.656 \cdot MHz$

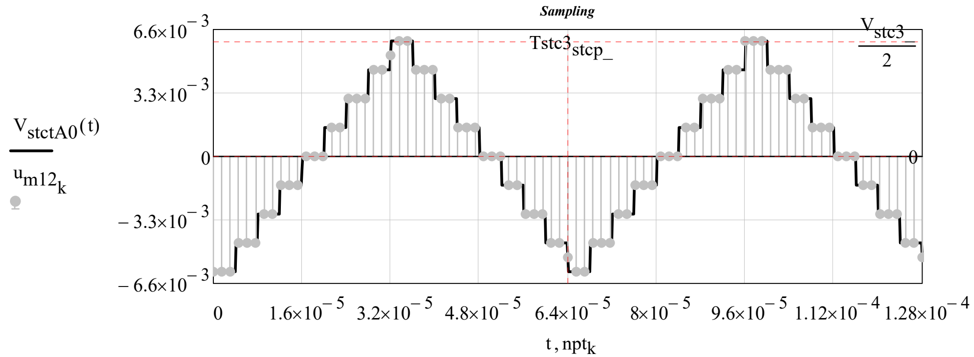
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp\_}} = 5.418$

$u_{m12_k} := V_{stctA0}(npt_k)$

$u_{m12}^T =$

	0	1	2	3	4	5
0	$-6\cdot10^{-3}$	$-6\cdot10^{-3}$	$-6\cdot10^{-3}$	$-4.5\cdot10^{-3}$	$-4.5\cdot10^{-3}$	...

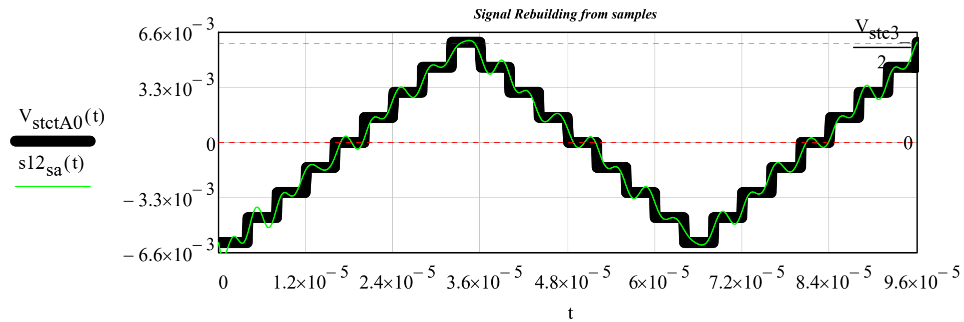


$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 2.062 \cdot \frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s12_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m12_n} \cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr = 10\cdot\%$

$N1\_ = 25$

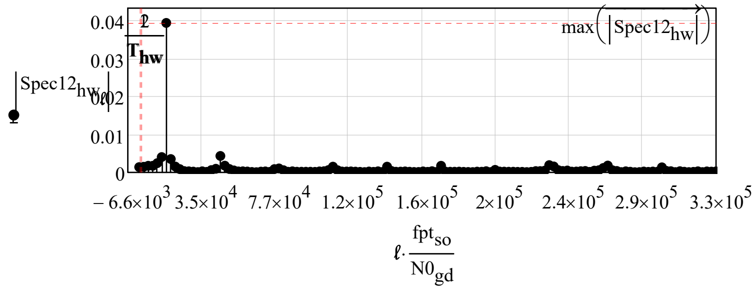


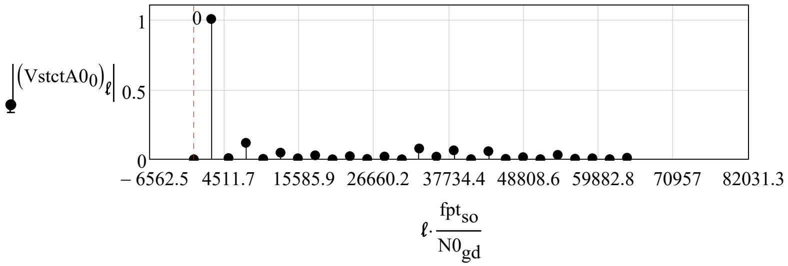
$length(u_{m12}) = 256$

$fpt_{so} = 656.25 \cdot kHz$

$Spec12_{hw} := fft(u_{m12})$ $\quad length(Spec12_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\quad \frac{N0_{gd}}{2} = 128$



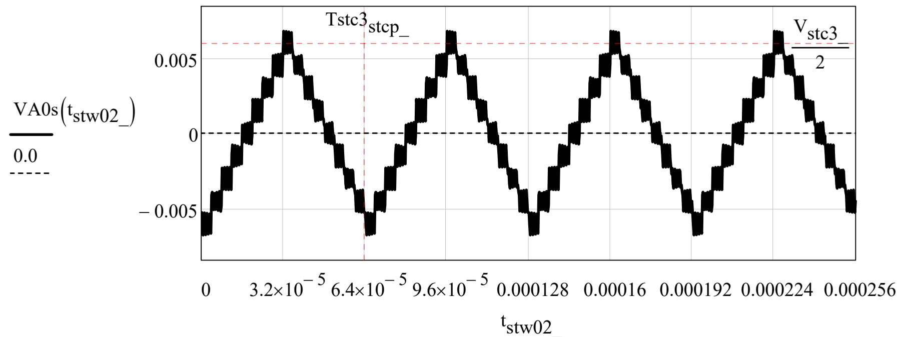


Periodic Waveforms

13 Staircase 3 Voltage Pulse Train + sinus

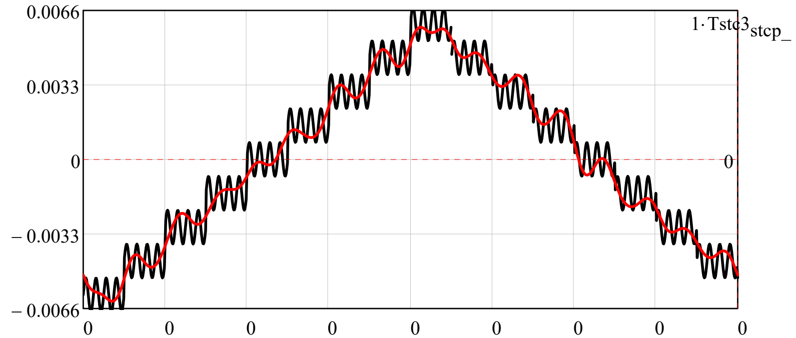
$$VA0s(t) := VistctA0sin\left(t, Tstc3_{stcp\_}, V_{stc3\_}, mstc3_{steps\_}, N0_{gd}\right)$$

Periodic Bipolar Staircase 3 + sinus

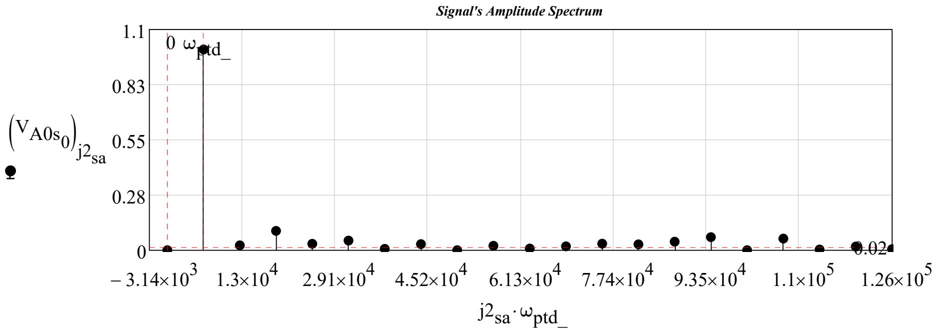


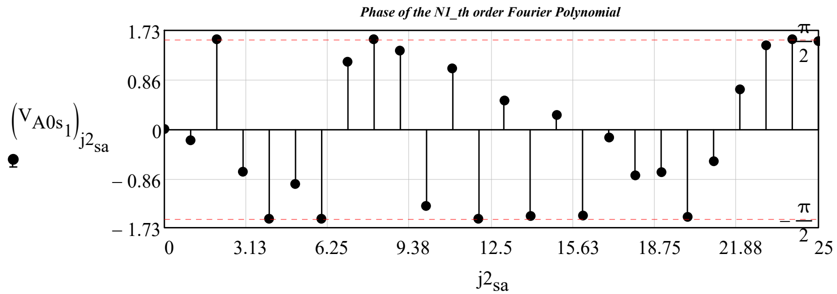
$$V_{A0s} := SPCT\left(VA0s, rt_{gd}, N1\_, 0\cdot s, Tstc3_{stcp\_}\right) \qquad N1\_ = 25$$

Signal and Fourier series



$$j2_{sa} := 0 .. rows\left(V_{A0s_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$





$\underline{Bw_{sa}} := V_{A0s_3} \cdot Hz$

$Bw_{sa} = 0.344 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.688 \cdot MHz$

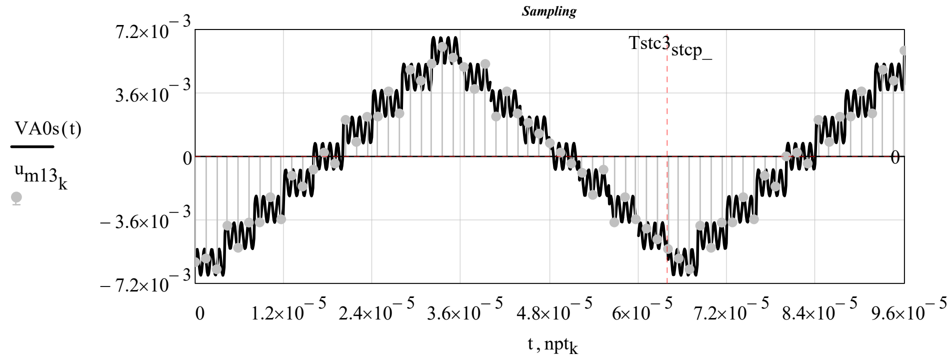
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T2_{stp\_}} = 5.172$

$u_{m13_k} := VA0s(npt_k)$

$u_{m13}^T =$

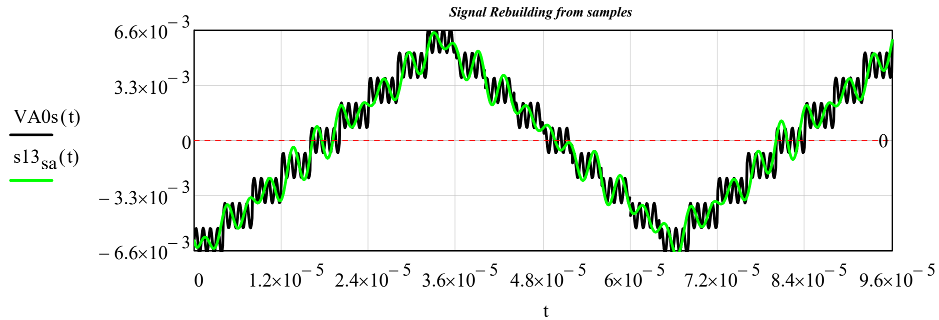
	0	1	2	3	4
0	$-6 \cdot 10^{-3}$	$-5.789 \cdot 10^{-3}$	$-6.405 \cdot 10^{-3}$	$-3.933 \cdot 10^{-3}$	...



$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 2.16 \cdot \dfrac{Mrads}{sec}$ $\quad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s13_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m13_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr = 10 \cdot \%$

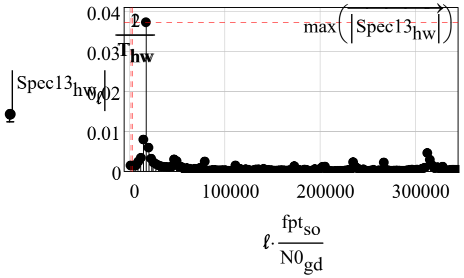


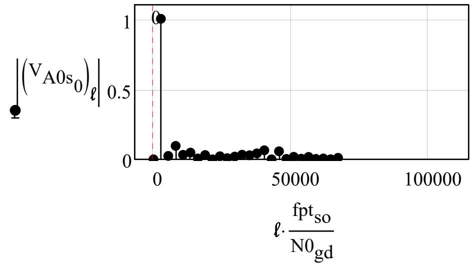
$length(u_{m13}) = 256$

$fpt_{so} = 687.5 \cdot kHz$

$Spec13_{hw} := fft(u_{m13})$ $\quad length(Spec13_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

14 Staircase 4 Voltage Pulse Train

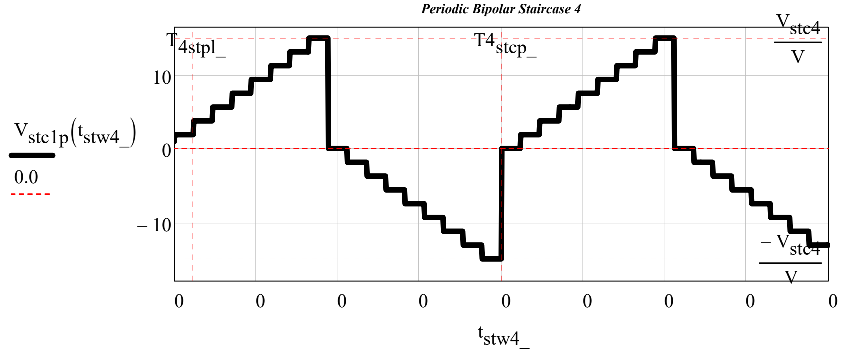
Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps in half period, max number of periods)

To modify data, see the worksheet *"staircase 4 pulse data.xmcd"*

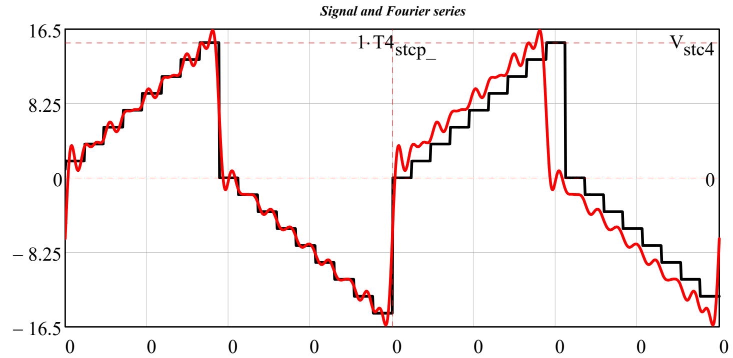
Step Amplitude:	$V_{stc4} = 15\,V$
Step length:	$T_{4stpl\_} = 1.481 \cdot \mu s$
Number of steps:	$2 \cdot m4_{steps} + 1 = 17$
Time constant:	$\tau 4\_ = 74.074 \cdot ns$
Period:	$T4_{stcp\_} = 0.025 \cdot ms$
Frequency:	$f_{44stcp\_} = 39.706 \cdot kHz$
	$\omega_{44stcp\_} = 249.479 \cdot \frac{krads}{sec}$

Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps)

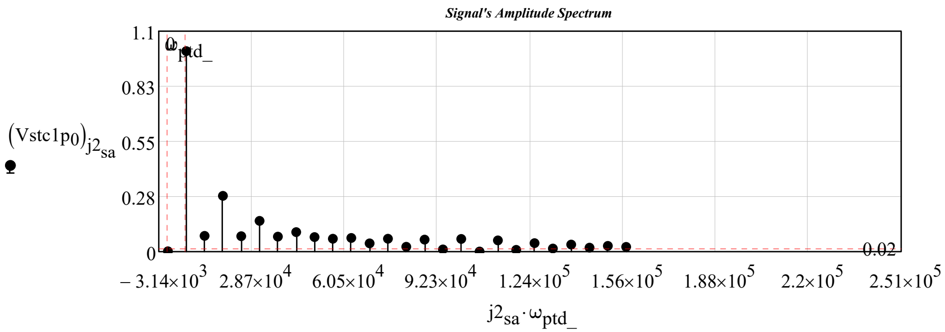
$$V_{stc1p}(t) := \frac{vstc1p\left(t, T_{4stpl\_}, V_{stc4}, m4_{steps}, N1\_\right)}{V}$$

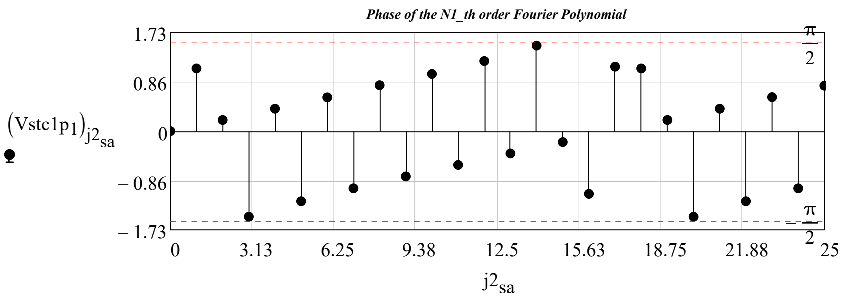


$$Vstc1p := SPCT\left(V_{stc1p}, rt_{gd}, N1\_, 0 \cdot s, T4_{stcp\_}\right) \qquad N1\_ = 25$$



$$j2_{sa} := 0 .. rows\left(VA0s_0\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$





$Bw_{sa} := Vstc1p_3 \cdot Hz$

$Bw_{sa} = 0.913 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 1.826 \cdot MHz$

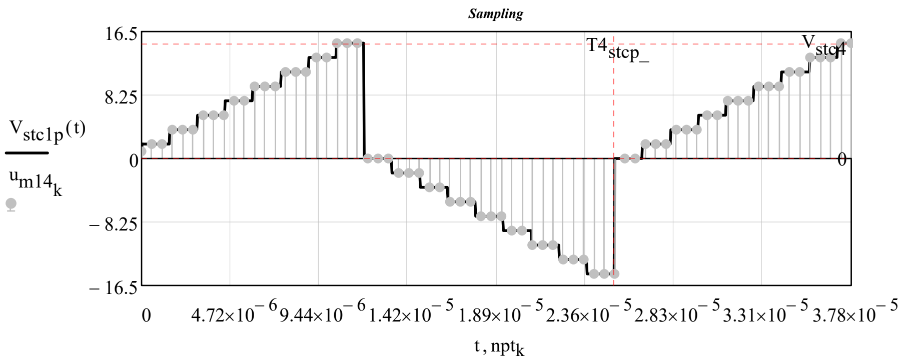
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T4_{stcp\_}} = 5.565$

$$u_{m14_k} := V_{stc1p}(npt_k)$$

$u_{m14}{}^T =$

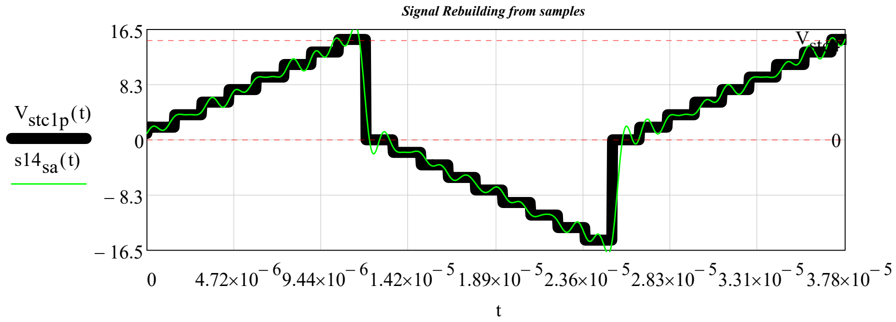
	0	1	2	3	4	5	6
0	0.938	1.875	1.875	3.75	3.75	3.75	...



$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 5.738 \cdot \dfrac{Mrads}{sec}$ $\quad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula:: $s14_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m14_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr = 10 \cdot \%$

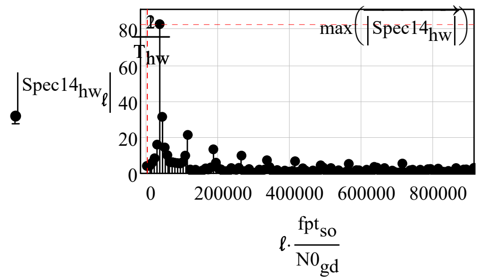


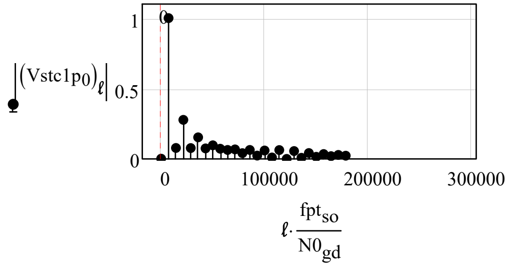
$length(u_{m14}) = 256$

$fpt_{so} = 1.826 \times 10^3 \cdot kHz$

$Spec14_{hw} := fft(u_{m14})$ $\quad length(Spec14_{hw}) = 129$

$\ell := 0 .. \dfrac{N0_{gd}}{2}$ $\quad \dfrac{N0_{gd}}{2} = 128$





Periodic Waveforms

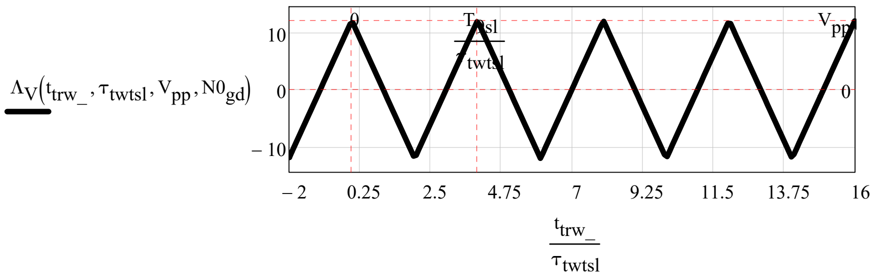
15 Bipolar Triangular Voltage Wave

Description of the Function's parameters : Λ_V(time, triangle half base, triangle amplitude, max number of periods)

Time constant: $\tau_{twtsl} := 1 \cdot \mu s$

Period: $T_{9sl} := 4 \cdot \tau_{twtsl}$ $\qquad f_{9sl} := \frac{1}{T_{9sl}}$

$$t_{trw\_} := -1 \cdot T_{9sl}, -1 \cdot T_{9sl} + \frac{20 \cdot T_{9sl} + 1 \cdot T_{9sl}}{1000} .. 20 \cdot T_{9sl}$$



Bipolar Triangular Voltage Wave Built using the Step Function

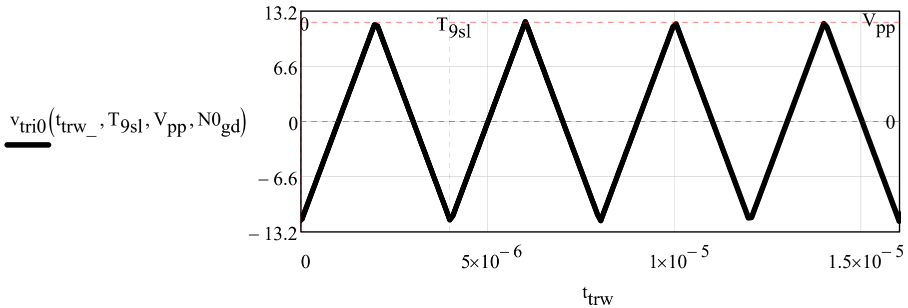
Signal amplitude: $V_{pp} = 12 \cdot V$

Time constant: $\tau_{twtsl} = 1 \cdot \mu s$

Period: $T_{9sl} = 4 \cdot \mu s$

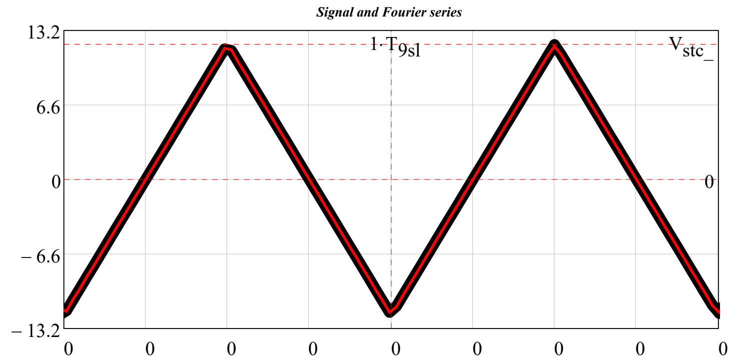
$\omega_{9sl} := 2 \cdot \pi \cdot f_{9sl}$ $\qquad \omega_{9sl} = 1.571 \times 10^{6} \cdot \frac{rad}{sec}$

$N0_{gd} = 256$

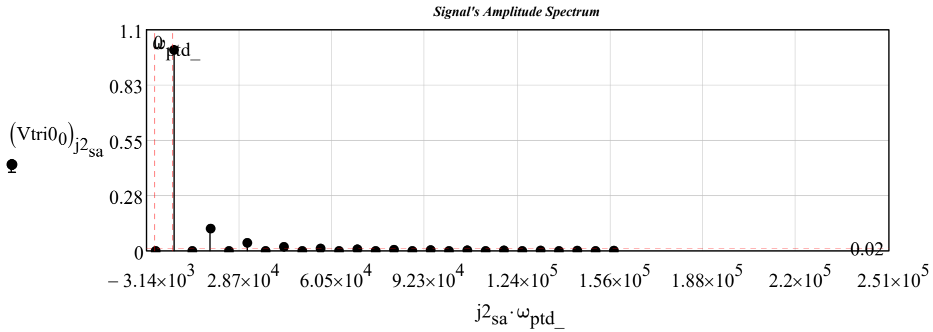


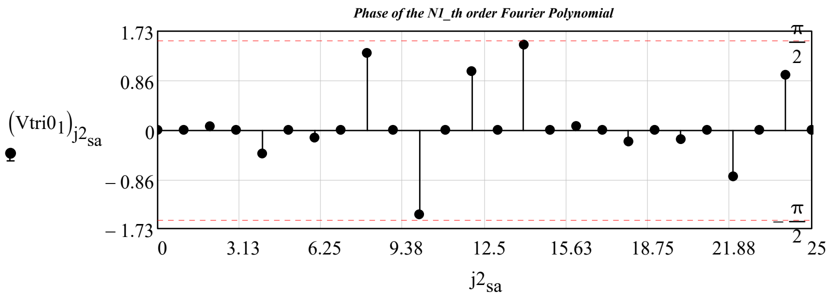
$$V_{tri0}(t) := \frac{v_{tri0}(t, T_{9sl}, V_{pp}, N0_{gd})}{V}$$

$Vtri0 := SPCT(V_{tri0}, rt_{gd}, N1\_, 0 \cdot s, T_{9sl})$ $\qquad N1\_ = 25$



$j2_{sa} := 0 .. rows(Vtri0_0) - 1$ $\qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot$





$Bw_{sa} := Vtri0_3 \cdot Hz$

$Bw_{sa} = 3.5 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 7 \cdot MHz$

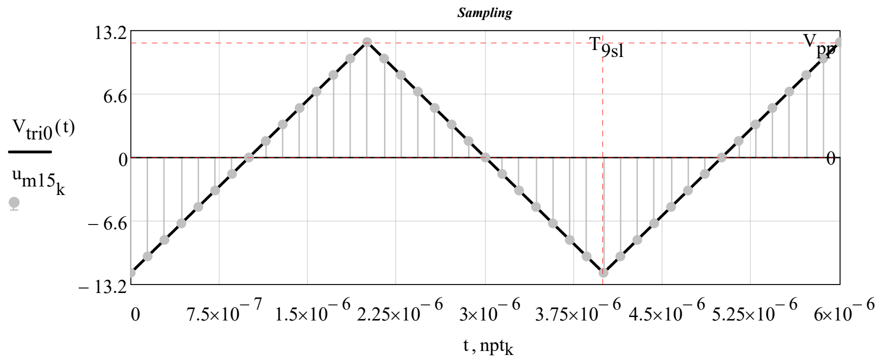
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T4_{stcp\_}} = 1.452$

$u_{m15_k} := V_{tri0}(npt_k)$

$u_{m15}{}^T =$

	0	1	2	3	4
0	-12	-10.286	-8.571	-6.857	...

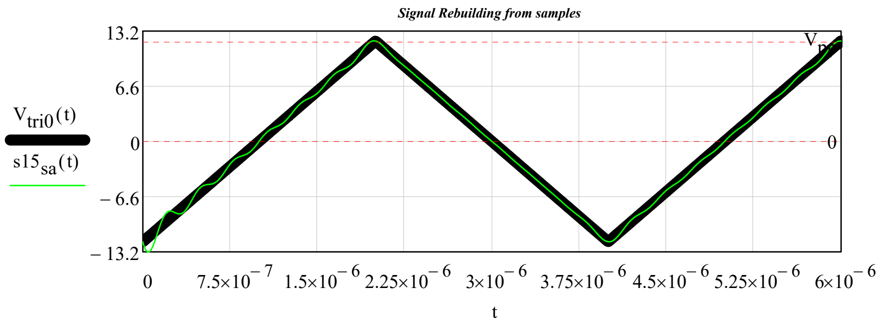


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 21.991 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s15_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m15_n} \cdot \mathrm{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi \right) \right) \right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%

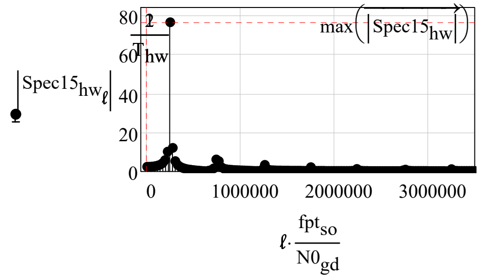


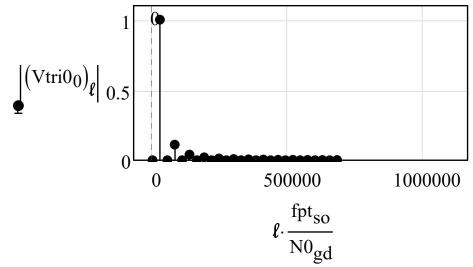
$\mathrm{length}(u_{m15}) = 256$

$fpt_{so} = 7 \times 10^3 \cdot kHz$

$Spec15_{hw} := \mathrm{fft}(u_{m15})$ $\quad \mathrm{length}(Spec15_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\quad \frac{N0_{gd}}{2} = 128$





Periodic Waveforms

16 Triangular Cusps Voltage Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$

Pulse width: $p_{wsl} := \tau_{ptd\_}$ $\quad p_{wsl} = 250 \cdot \mu s$

Max pulse amplitude and cusp ratio: $a_{psl} := \frac{1}{4}$ $\quad a_{psl} < 1$

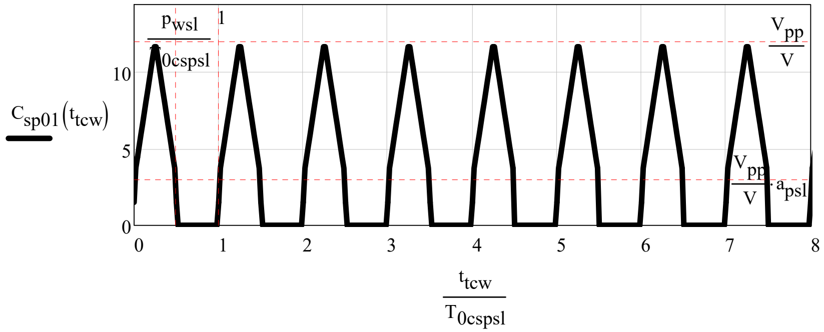
Cusp slope $c_{ssl} := V_{pp} \cdot \frac{2 \cdot (1 - a_{psl})}{p_{wsl}}$ $\quad c_{ssl} = 0.$

Period: $T_{0cspsl} := 2 \cdot p_{wsl}$

$$t_{tcw} := 0 \cdot T_{0cspsl}, 0 \cdot T_{0cspsl} + \frac{10 \cdot T_{0cspsl} - 0 \cdot T_{0cspsl}}{500} .. 10 \cdot T_{0cspsl}$$

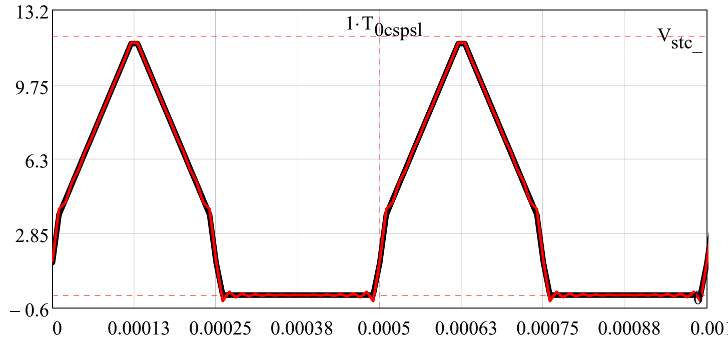
$$C_{sp01}(t) := \frac{csp01(t, p_{wsl}, a_{psl}, T_{0cspsl}, V_{pp}, N0_{gd})}{V}$$

Cusp Pulses Train



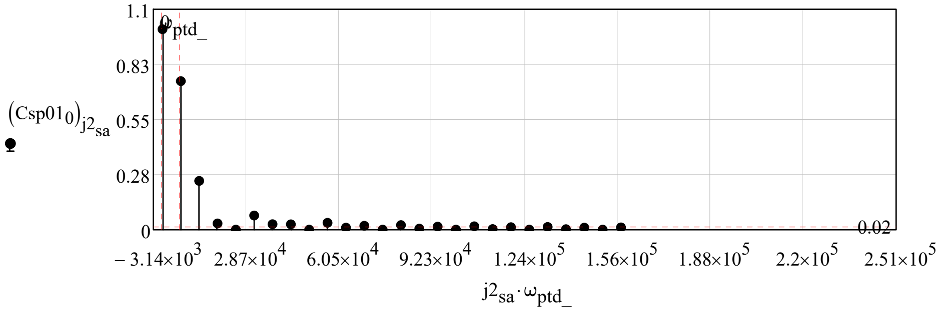
$$Csp01 := SPCT(C_{sp01}, rt_{gd}, N1\_, 0 \cdot s, T_{0cspsl}) \qquad N1\_ = 25$$

Signal and Fourier series

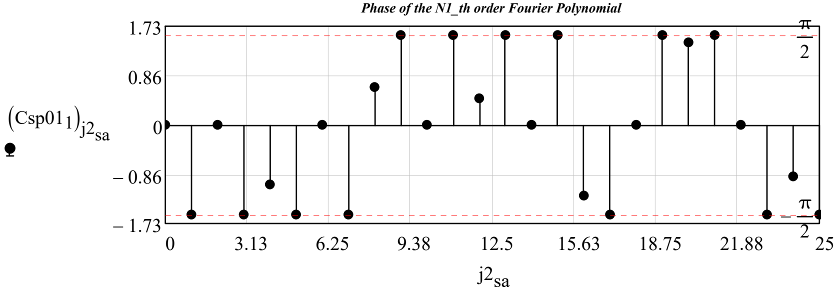


$$j2_{sa} := 0 .. rows(Csp01_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Csp01_3 \cdot Hz$

$Bw_{sa} = 0.046 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.092 \cdot MHz$

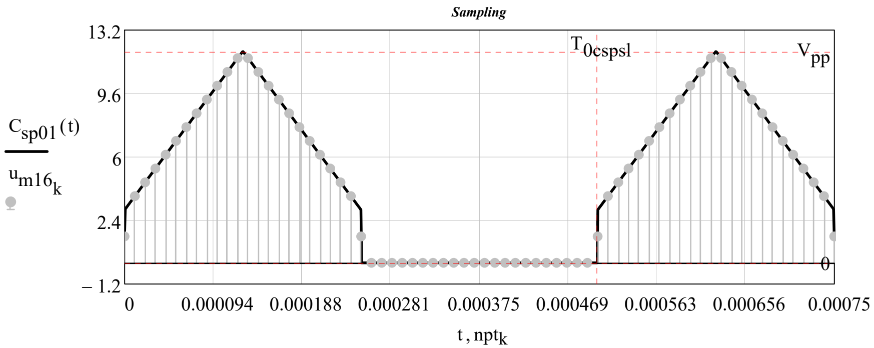
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T4_{stcp\_}} = 110.486$

$u_{m16_k} := C_{sp01}(npt_k)$

$u_{m16}^T =$

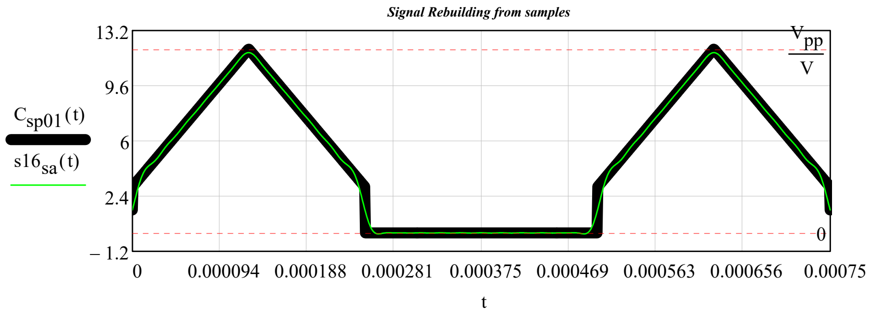
	0	1	2	3	4	5	6	7
0	1.5	3.783	4.565	5.348	6.13	6.913	7.696	...



$relerr = 10\cdot\%$ $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.289\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s16_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m16_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ $relerr = 10\cdot\%$

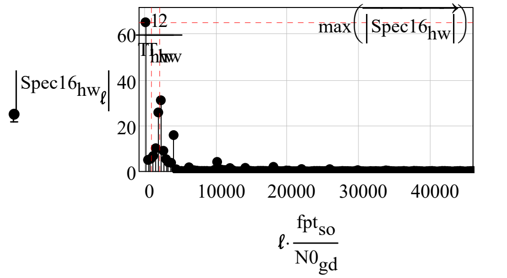


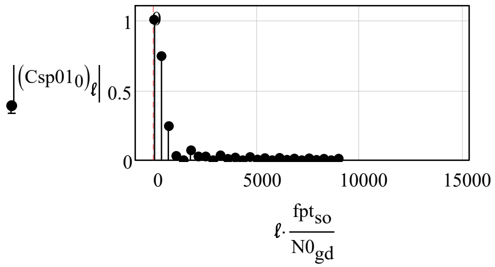
$length(u_{m16}) = 256$

$fpt_{so} = 92\cdot kHz$

$Spec16_{hw} := fft(u_{m16})$ $length(Spec16_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

17 Bipolar Sawtooth with positive slope Pulse Train

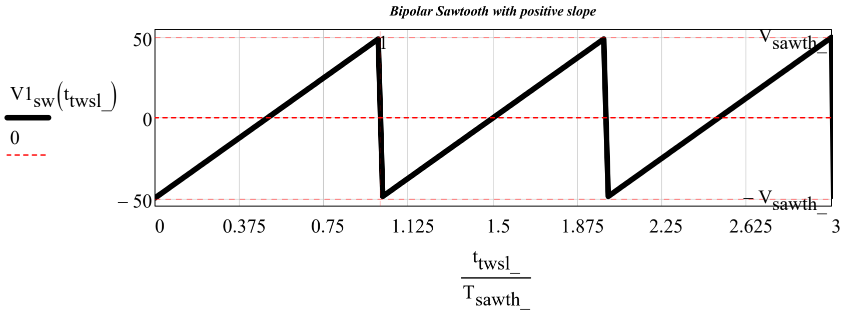
Period: $T_{sawth\_} := 1\cdot\delta_{sawth\_}$

Frequency: $f_{sawth\_} := \frac{1}{T_{sawth\_}}$ $f_{sawth\_} = 1\cdot MHz$

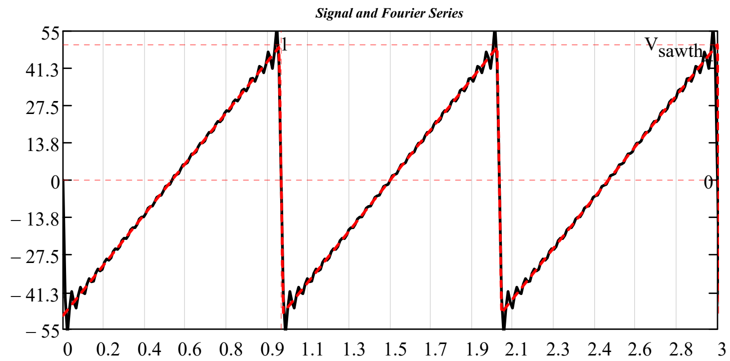
$T_{sawth\_} = 1\cdot\mu s$ $\omega_{sawth\_} := 2\cdot\pi\cdot f_{sawth\_}$ $\omega_{sawth\_} = 6.283\cdot\frac{Mrads}{sec}$

$t_{twsl\_} := 0, \frac{5\cdot T_{sawth\_}}{500} .. 5\cdot T_{sawth\_}$

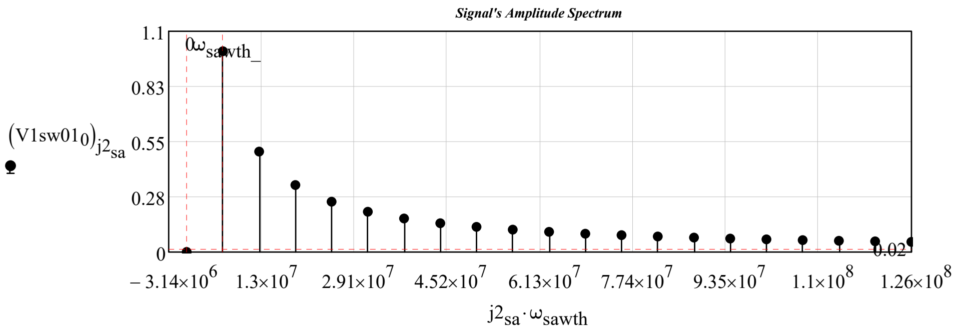
$V1_{sw}(t) := \frac{v1_{sw}(t, T_{sawth\_}, V_{sawth\_}, N0_{gd})}{V}$

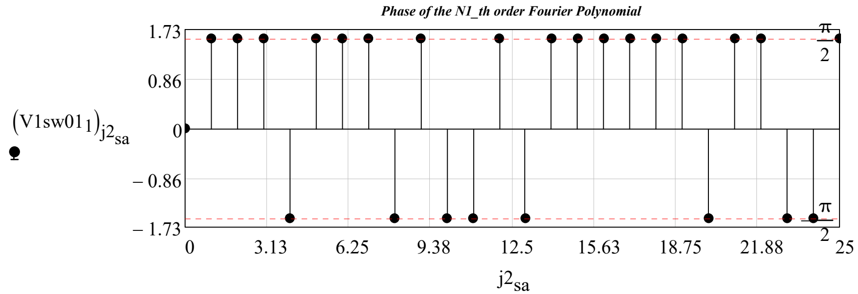


$$V1sw01 := SPCT\left(V1_{sw}, rt_{gd}, N1\_, 0\cdot s, T_{sawth\_}\right) \qquad N1\_ = 25$$



$$j2_{sa} := 0 .. rows\left(V1sw01_0\right) - 1 \qquad \omega_{ptd\_} = 6.283\times10^{-3}\cdot\frac{Mrads}{s}$$





$$Bw_{sa} := V1sw01_3\cdot Hz$$

$$Bw_{sa} = 23\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa} \qquad fpt_{so} = 46\cdot MHz$

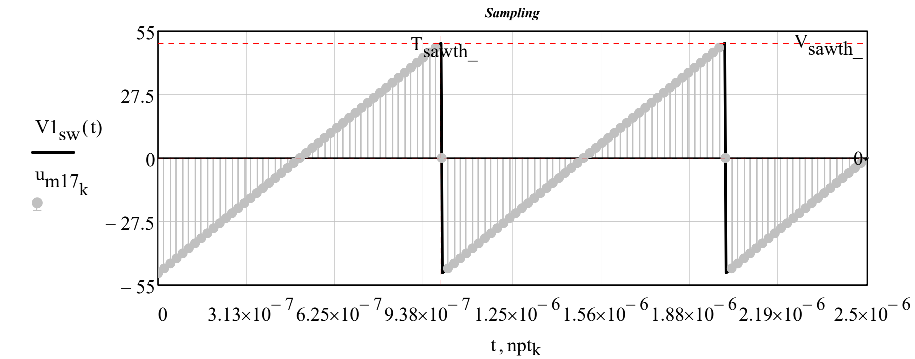
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{sawth\_}} = 5.565$

$$u_{m17_k} := V1_{sw}\left(npt_k\right)$$

$u_{m17}{}^T =$

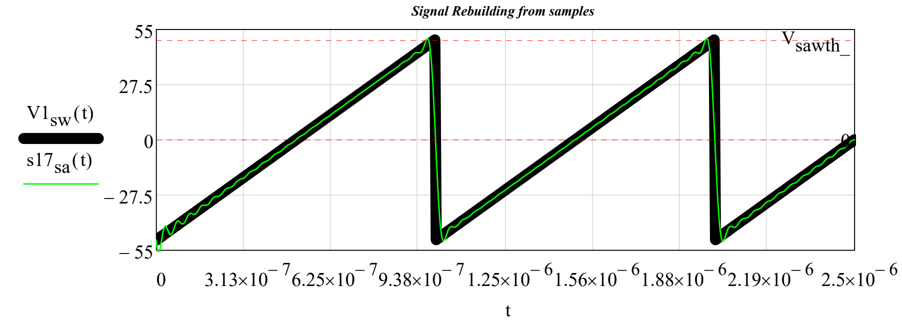
	0	1	2	3	4
0	-50	-47.826	-45.652	-43.478	...



relerr = 10·% $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa} \qquad \omega_{bwr} = 144.513\cdot\frac{Mrads}{sec} \qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s17_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m17_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10\cdot\%$

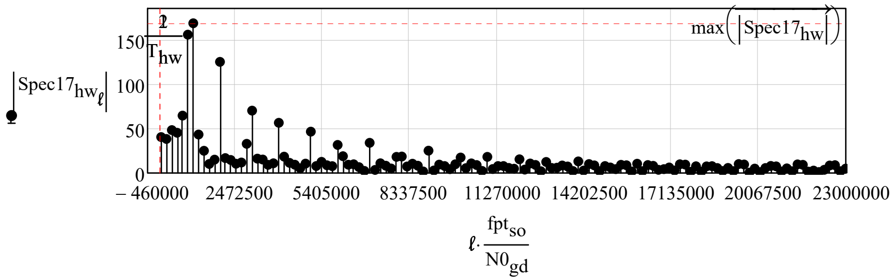


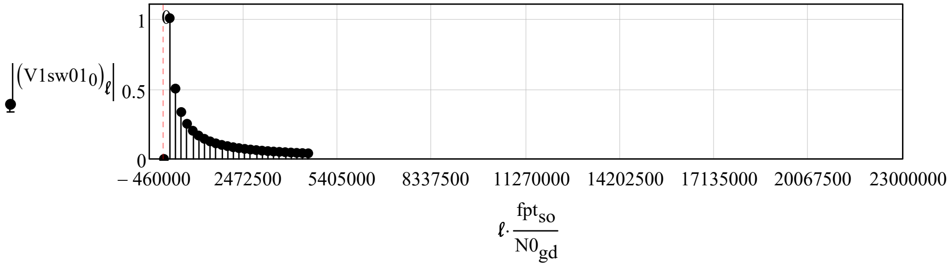
$$length(u_{m17}) = 256$$

$$fpt_{so} = 46 \cdot MHz$$

$$Spec17_{hw} := fft(u_{m17}) \quad length(Spec17_{hw}) = 129$$

$$\ell := 0..\frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$$





Periodic Waveforms

18 Bipolar Sawtooth with negative slope Pulse Train

Amplitude: $V_{sawth\_} = 50 \cdot V$

Sawtooth length: $\delta_{sawth\_} = 1 \cdot \mu s$

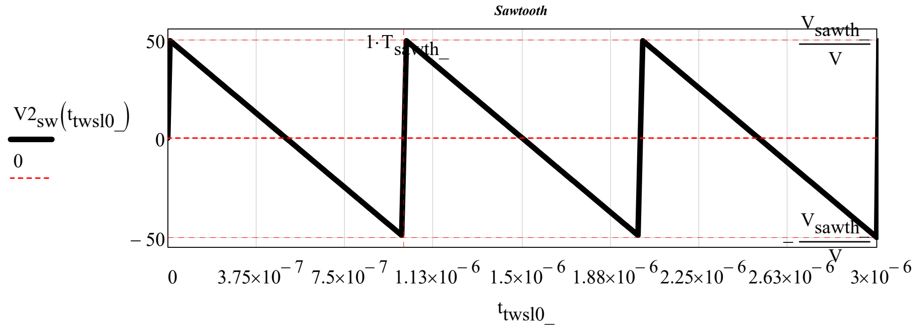
Slope: $sp_{sawth\_} = 50 \cdot \frac{V}{\mu s}$

Period: $T_{sawth\_} = 1 \cdot \mu s$

Frequency: $\frac{1}{T_{sawth\_}} = 1 \cdot MHz$

$$t_{twsl0\_} := -T_{sawth\_} \cdot 0, T_{sawth\_} \cdot 0 + \frac{5 \cdot T_{sawth\_} + T_{sawth\_} \cdot 0}{500} .. 5 \cdot T_{sawth\_}$$

$$V2_{sw}(t) := \frac{v2_{sw}(t, T_{sawth\_}, V_{sawth\_}, N0_{gd})}{V}$$



Dirichlet conditions

A periodic function s(t)=s(t+T), can be expressed by the Fourier series provided that (Dirichlet conditions):
(1) it is discontinuous and presents a finite number of discontinuities in the period T;
(2) has a limited average value in the period T;
(3) it has a finite number of maximum positive or negative.
If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

1) signals of energy for which holds: $\int_{-\infty}^{\infty} \left(|s_{fs}(t)|\right)^2 dt < \infty$,

2) power signals for which holds: $\lim_{T \to \infty} \left[\frac{1}{T} \cdot \int_{-T}^{T} \left(|s_{fs}(t)|\right)^2 dt\right] < \infty$

Fourier series definition

$$s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(\omega \cdot k \cdot t) + b_k \cdot \sin(\omega \cdot k \cdot t)\right)$$

The coefficients are defined as follows:

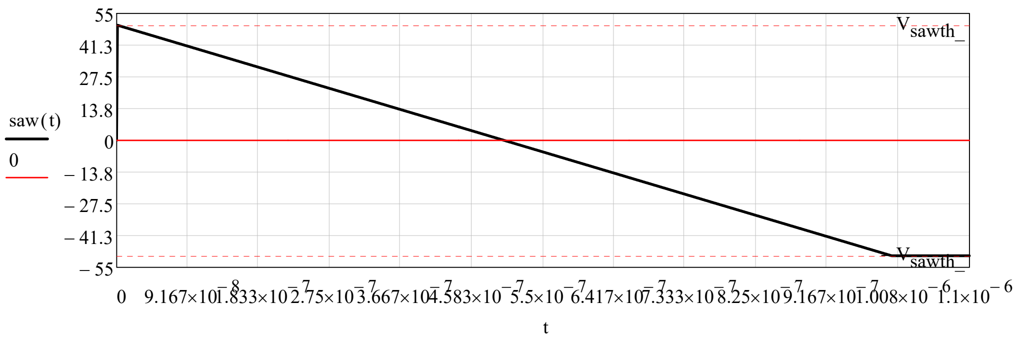
$$\frac{a_0}{2} = A_{fs} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t)\, dt$$

$$a_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t)\, dt$$

$$b_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t)\, dt$$

$V_{sawth\_} = 50\,V$

$$saw(t) := 2 \cdot V_{sawth\_} \cdot \left[\left(\frac{-t}{T_{sawth\_}} + 1\right) \cdot \left(\Phi(t) - \Phi\left(t - T_{sawth\_}\right)\right) - \frac{1}{2}\right]$$



$$\frac{a_0}{2} = A_{fs} = \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{t,0}^{t_0+T_{so}} \left(\frac{-t}{T_{sawth\_}} + 1\right) \cdot \left(\Phi(t) - \Phi\left(t - T_{sawth\_}\right)\right) - \frac{1}{2}\, dt = \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} \ldots\right.$$

$$\frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) dt = 0$$

$$a_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t)\, dt = 2 \cdot \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) \cdot \cos(\omega \cdot k \cdot t)\, dt$$

$$2 \cdot \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) \cdot \cos(\omega \cdot k \cdot t)\, dt = \frac{2 \cdot V_{sawth\_} \cdot \left(4 \cdot \sin\left(\frac{T_{sawth\_} \cdot \omega \cdot k}{2}\right)^2 - T_{sawth\_} \cdot \omega \cdot k \cdot \sin\left(T_{sav} \ldots\right.\right.}{T_{sawth\_}{}^2 \cdot \omega^2 \cdot k^2}$$

$$a_k = \frac{2 \cdot V_{sawth\_} \cdot \left(4 \cdot \sin\left(\frac{T_{sawth\_} \cdot \omega \cdot k}{2}\right)^2 - T_{sawth\_} \cdot \omega \cdot k \cdot \sin\left(T_{sawth\_} \cdot \omega \cdot k\right)\right)}{T_{sawth\_}{}^2 \cdot \omega^2 \cdot k^2}$$

$$b_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t)\, dt = 2 \cdot \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{t_0}^{t_0+T} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) \cdot \sin(\omega \cdot k \cdot t)\, dt$$

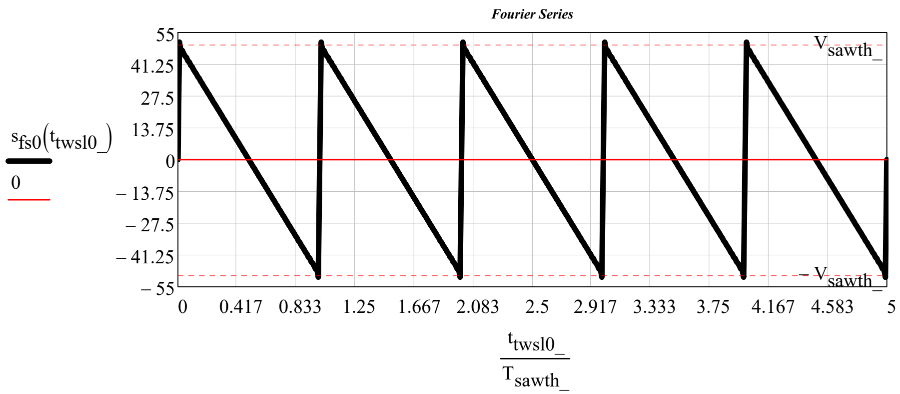
$$2 \cdot \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{t_0}^{t_0+T} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) \cdot \sin(\omega \cdot k \cdot t)\, dt = \left(\cos\left(\frac{T_{sawth\_} \cdot \omega \cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_} \cdot \omega \cdot k\right)}{T_{sawth\_} \cdot \omega \cdot k}\right) \cdot \frac{4 \cdot V_{saw} \ldots}{\left(T_{sawth\_} \ldots\right.}$$

$$b_k = \left(\cos\left(\frac{T_{sawth\_} \cdot \omega \cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_} \cdot \omega \cdot k\right)}{T_{sawth\_} \cdot \omega \cdot k}\right) \cdot \frac{4 \cdot V_{sawth\_}}{\left(T_{sawth\_} \cdot \omega \cdot k\right)}$$

$\omega_{sf} := \omega_{sawth\_}$

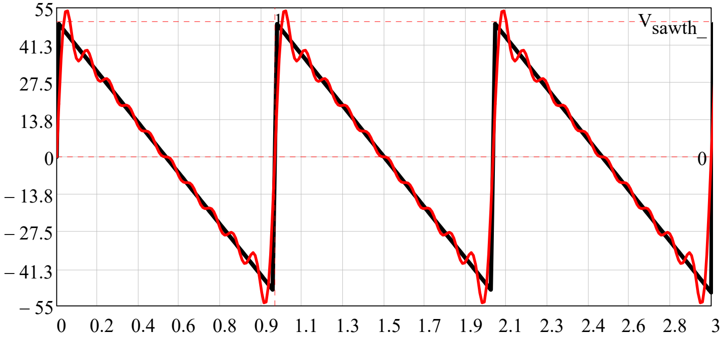
$$s_{fs0}(t) := \frac{2 \cdot V_{sawth\_}}{\left(T_{sawth\_} \cdot \omega_{sf}\right)} \cdot \sum_{k=1}^{N0_{gd}} \left[\frac{\left(4 \cdot \sin\left(\frac{T_{sawth\_} \cdot \omega_{sf} \cdot k}{2}\right)^2 - T_{sawth\_} \cdot \omega_{sf} \cdot k \cdot \sin\left(T_{sawth\_} \cdot \omega_{sf} \cdot k\right)\right)}{T_{sawth\_} \cdot \omega_{sf} \cdot k} \cos\left(\omega_{sf} \cdot k \ldots\right. + \left(\cos\left(\frac{T_{sawth\_} \cdot \omega_{sf} \cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_} \cdot \omega_{sf} \cdot k\right)}{T_{sawth\_} \cdot \omega_{sf} \cdot k}\right) \cdot \frac{2}{k} \cdot \sin\left(\omega_{sf} \cdot k \cdot t\right)\right.$$

$N0_{gd} = 256 \quad V_{sawth\_} = 50\,V$



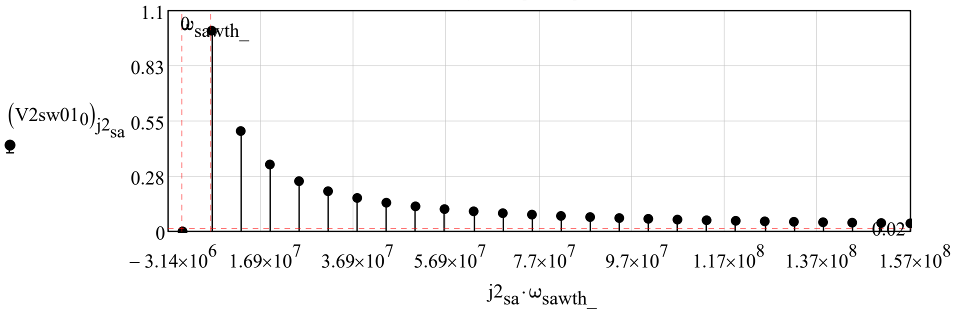
$V2sw01 := SPCT\left(V2_{sw}, rt_{gd}, N1\_, 0 \cdot s, T_{sawth\_}\right)$ $\qquad N1\_ = 25$

Signal and Fourier Series

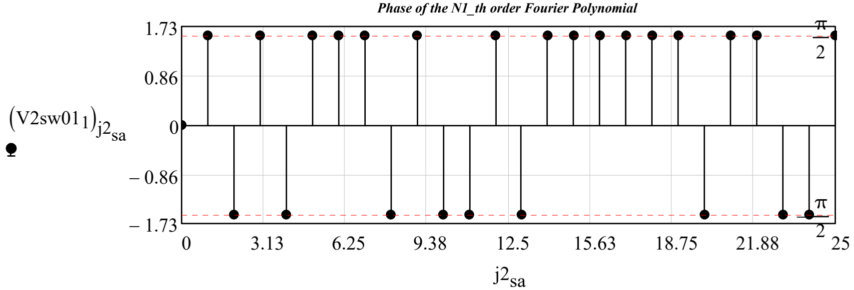


$j2_{sa} := 0 .. rows\left(V2sw01_0\right) - 1$ $\qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := V2sw01_3 \cdot Hz$

$Bw_{sa} = 23 \cdot MHz$

sampling frequency: $\qquad fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 46 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

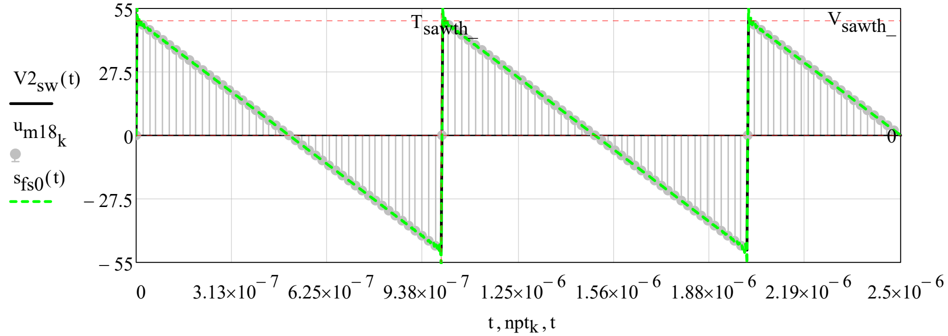
Frequency resolution: $\qquad \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{sawth\_}} = 5.565$

$u_{m18_k} := V2_{sw}\left(npt_k\right)$

$u_{m18}^T =$

	0	1	2	3	4
0	0	47.826	45.652	43.478	...

Comparison among the given signal, the sampled and the calculated series

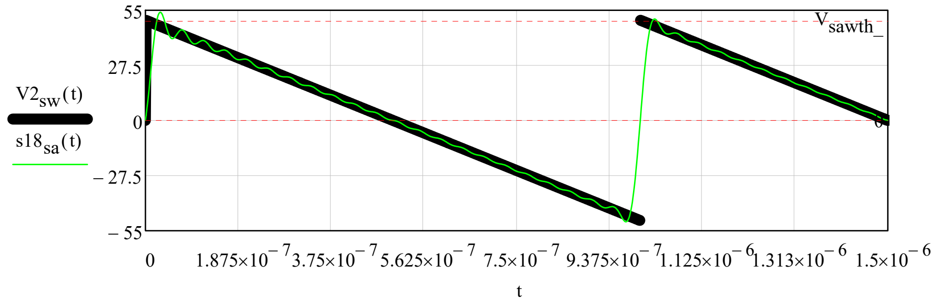


$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 144.513 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\qquad s18_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m18_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad relerr = 10 \cdot \%$

Signal Rebuilding from samples

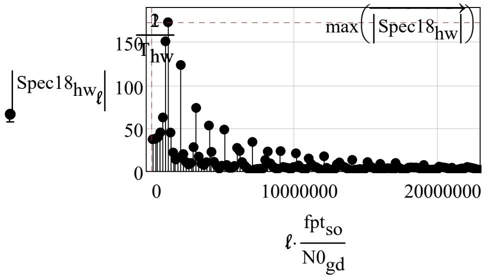


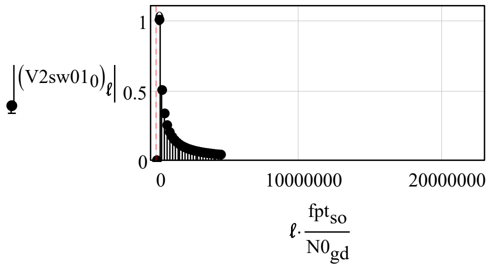
$length\left(u_{m18}\right) = 256$

$fpt_{so} = 46 \cdot MHz$

$Spec18_{hw} := fft\left(u_{m18}\right)$ $\qquad length\left(Spec18_{hw}\right) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

19 Bipolar Sawtooth with adjustable rising and falling edges Pulse Train

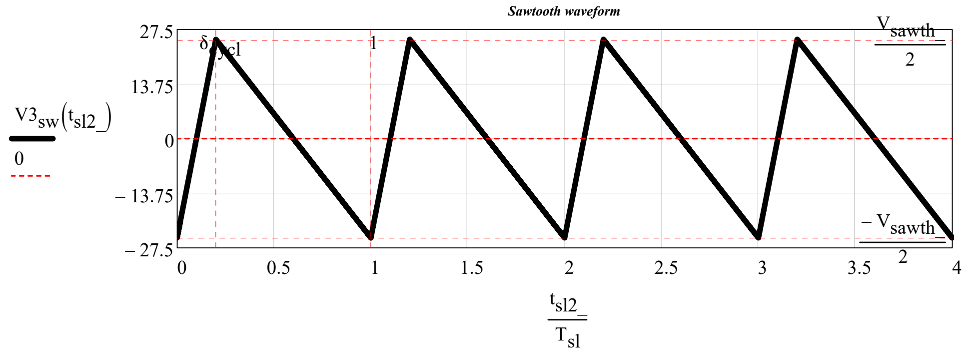
$$\delta_{cycl} \cdot T_{sl} = 220 \cdot ns$$

$$T_{sl} = 1.1 \cdot \mu s \qquad f_{3sw} := \frac{1}{T_{sl}} \qquad \omega_{3sw} := 2 \cdot \pi \cdot f_{3sw} \qquad \omega_{3sw} = 5.712 \cdot \frac{Mrads}{sec}$$

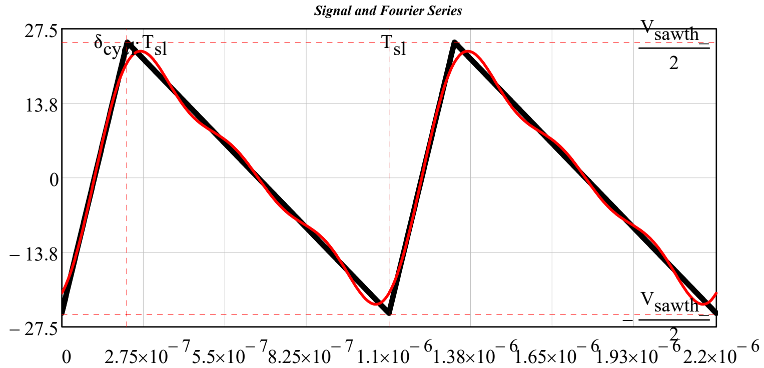
$$\delta_{cycl} = 20 \cdot \%$$

$$t_{sl2\_} := 0 \cdot T_{sl}, 0 \cdot T_{sl} + \frac{4 \cdot T_{sl}}{10000} .. 4 \cdot T_{sl}$$

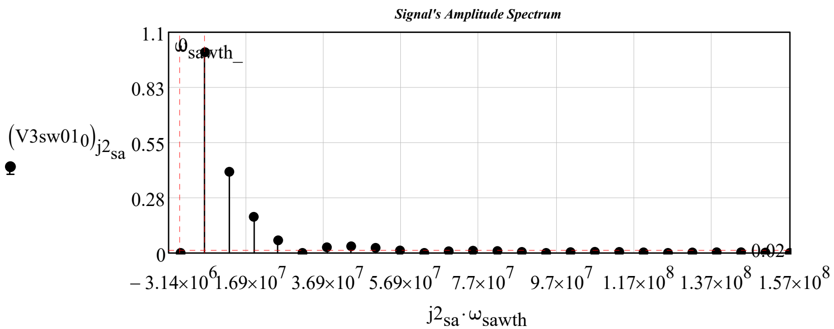
$$V3_{sw}(t) := \frac{V_s\left(t \cdot sec^{-1}, T_{sl} \cdot sec^{-1}, \delta_{cycl}, V_{sawth\_}, N_{gd}\right)}{V}$$

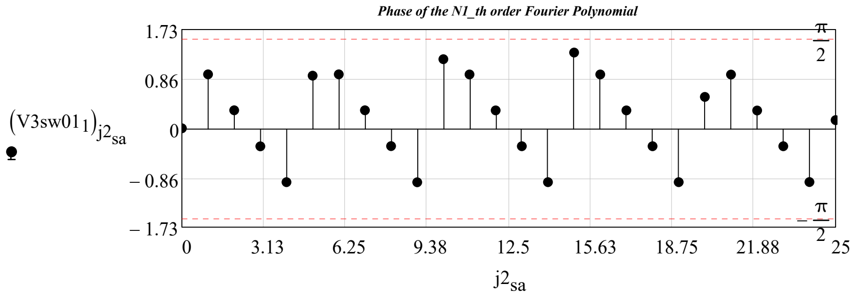


$$V3sw01 := SPCT\left(V3_{sw}, rt_{gd}, N1\_, 0 \cdot s, T_{sl}\right) \qquad N1\_ = 25$$



$$j2_{sa} := 0 .. rows\left(V3sw01_0\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$





$Bw_{sa} := V3sw01_3 \cdot Hz$

$Bw_{sa} = 11.818 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 23.636 \cdot MHz$

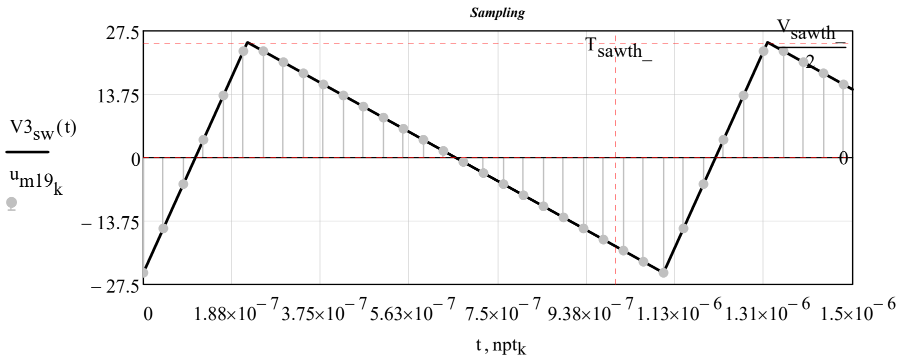
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{sawth\_}} = 10.831$

$$u_{m19_k} := V3_{sw}\left(npt_k\right)$$

$u_{m19}^T =$

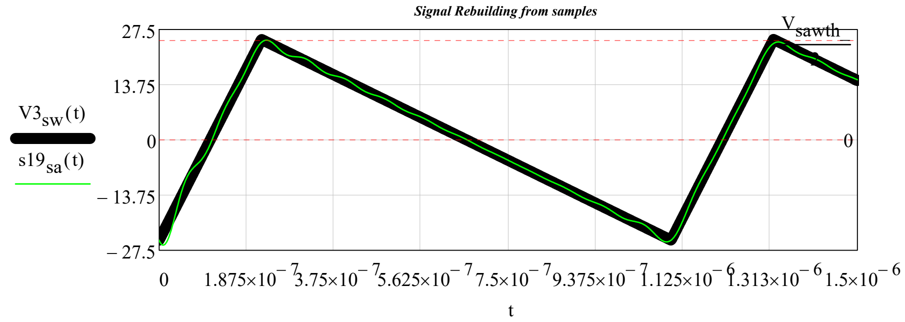
	0	1	2	3	4	5	6
0	-25	-15.385	-5.769	3.846	13.462	23.077	...



relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 74.256 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s19_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m19_n} \cdot \mathrm{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%



Periodic Waveforms

20 AM test signal (single tone)

Carrier Amplitude: $A1_{sl} := 20 \cdot volt$

Modulating signal's amplitude: $B1_{sl} := 12 \cdot volt$

Carrier pulsation: $\omega1_{csl} := 15 \cdot \omega_{0gd}$ Carrier period: $T1_{csl} := \frac{2 \cdot \pi}{\omega1_{csl}}$

Carrier frequency: $f_{1csl} := \frac{\omega1_{csl}}{2 \cdot \pi}$

Modulating signal's pulsation: $\omega1_{msl} := \frac{\omega1_{csl}}{20}$ Modulating signal's period: $T1_{msl} := \frac{2 \cdot \pi}{\omega1_{msl}}$

Modulating signal's frequency: $f1_{msl} := \frac{\omega1_{msl}}{2 \cdot \pi}$

$\omega1_{csl} = 94.248 \cdot \frac{krads}{sec}$ $\frac{\omega1_{csl}}{\omega1_{msl}} = 20$ $\omega_{0gd} = 6.283 \cdot \frac{krads}{s}$

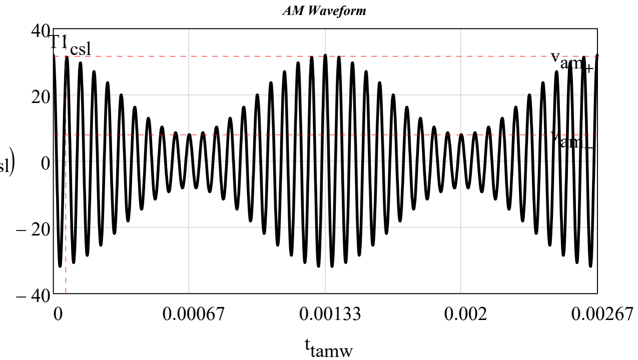
$v_{am_+} := A1_{sl} + B1_{sl}$ $v_{am\_} := A1_{sl} - B1_{sl}$ $A1_{sl} = v_{am_+} + v_{am\_}$ $B1_{sl} = v_{am_+} - v_{am\_}$

$v_{am_+} = 32 \cdot volt$

$v_{am\_} = 8 \cdot volt$ AM modulation index: $m_{amsl} := \frac{v_{am_+} - v_{am\_}}{v_{am_+} + v_{am\_}}$ $m_{amsl} = 60 \cdot \%$ $\frac{B1_{sl}}{A1_{sl}} = 60 \cdot \%$

$$t_{tamw} := -T_{0gd} \cdot 3, -T_{0gd} \cdot 3 + \frac{40 \cdot T1_{csl} + T_{0gd} \cdot 3}{5000} .. 40 \cdot T1_{csl}$$

$V2am(t_{tamw}, \omega1_{msl}, \omega1_{csl}, A1_{sl}, B1_{sl})$

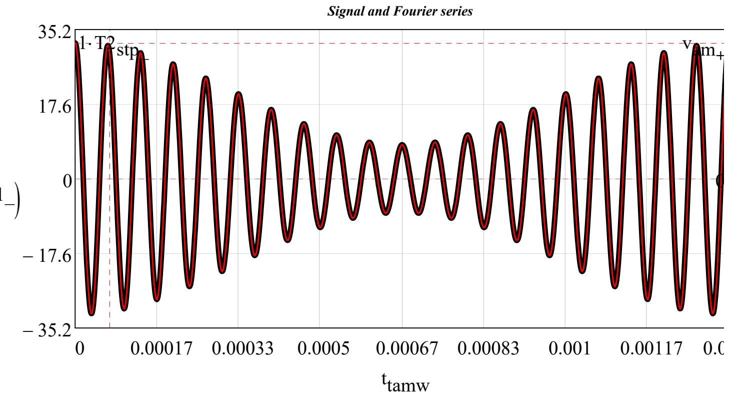


$V2_{am}(t) := V2am(t, \omega1_{msl}, \omega1_{csl}, A1_{sl}, B1_{sl})$

$V2_{ams} := SPCT(V2_{am}, rt_{gd}, N1\_, 0 \cdot s, T1_{msl})$ $N1\_ = 25$

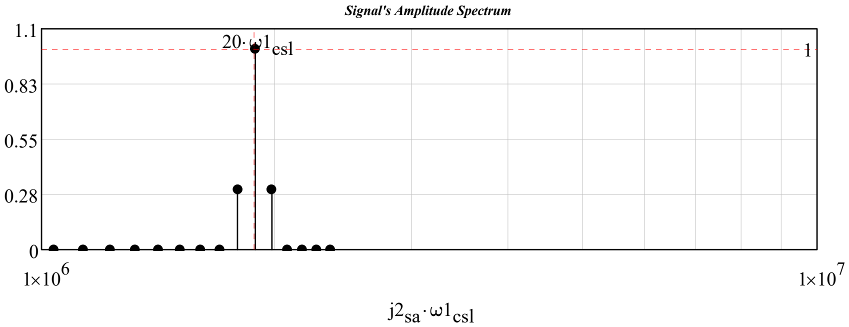
$V2_{am}(t_{tamw})$

$fs(t_{tamw}, V2_{ams_9}, V2_{ams_{10}}, T1_{msl}, N1\_)$

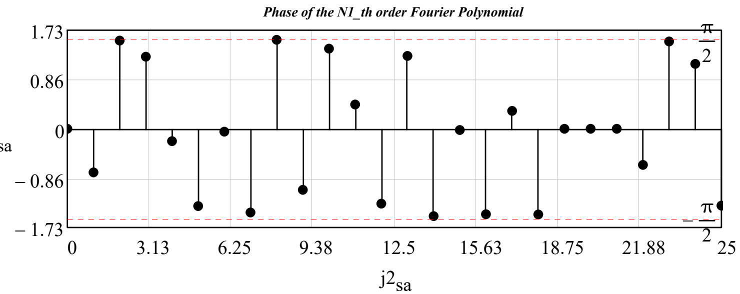


$j2_{sa} := 0 .. rows(V2_{ams_0}) - 1$ $\omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

$(V2_{ams_0})_{j2_{sa}}$



$(V2_{ams_1})_{j2_{sa}}$



$Bw_{sa} := V2_{ams_3} \cdot Hz$

$Bw_{sa} = 16.5 \cdot kHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 33 \cdot kHz$

$npt_k := \frac{k}{fpt_{so}}$

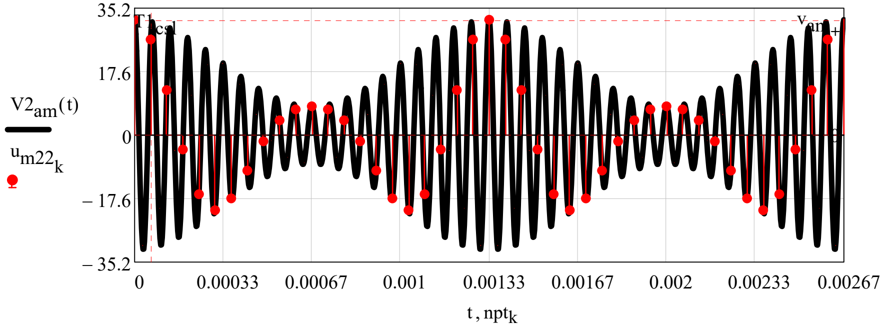
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T1_{csl}} = 116.364$

$$u_{m22_k} := V2_{am}\left(npt_k\right)$$

$u_{m22}^{T} =$

	0	1	2	3	4	5	6
0	32	-30.587	26.511	-20.245	12.502	-4.137	...

Sampling

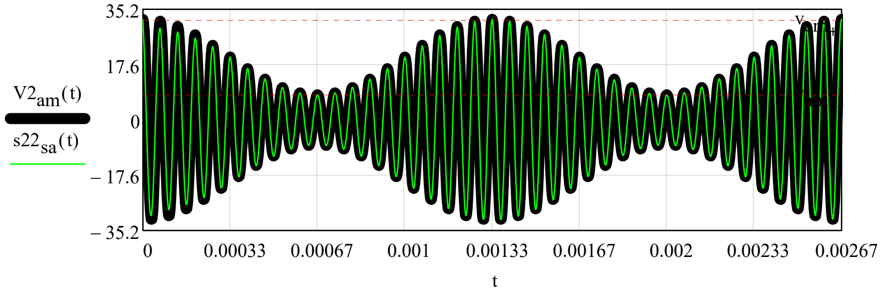


$relerr = 10\cdot\%$ $\quad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.104\cdot\frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot B}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s22_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m22_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ rel

Signal Rebuilding from samples



TEST Waveforms

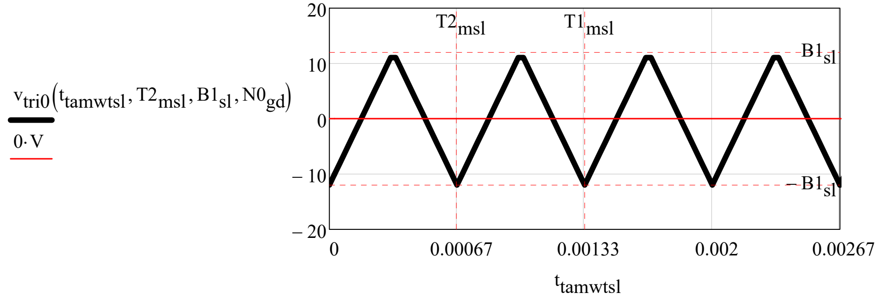
Periodic Waveforms

21 AM test signal (triangular wave)

$$\omega_{2msl} := \frac{\omega 1_{csl}}{10} \qquad T2_{msl} := \frac{2\cdot\pi}{\omega_{2msl}}$$

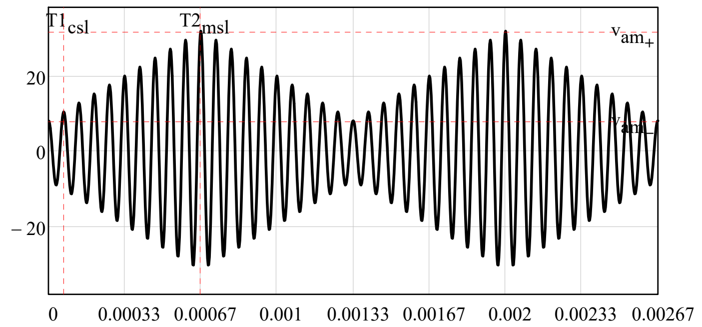
$$t_{tamwtsl} := 0\cdot sec, 40\cdot\frac{T2_{msl}}{1000} .. 40\cdot T2_{msl}$$

Madulating Signal



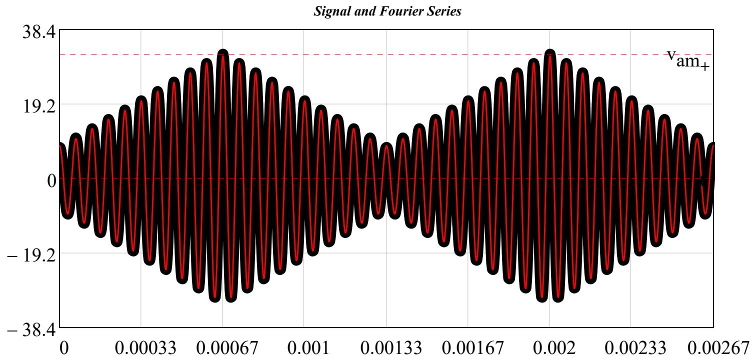
$$t_{twsl\_} := -T_{0gd}\cdot 3, -T_{0gd}\cdot 3 + \frac{8\cdot T2_{msl} + T_{0gd}\cdot 3}{500} .. 8\cdot T2_{msl}$$

AM Waveform

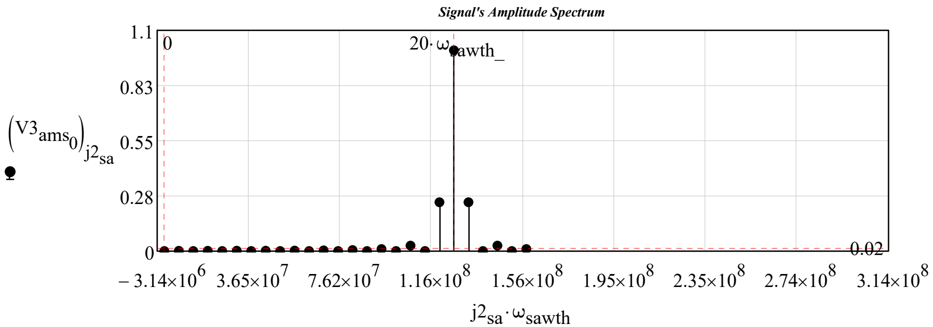


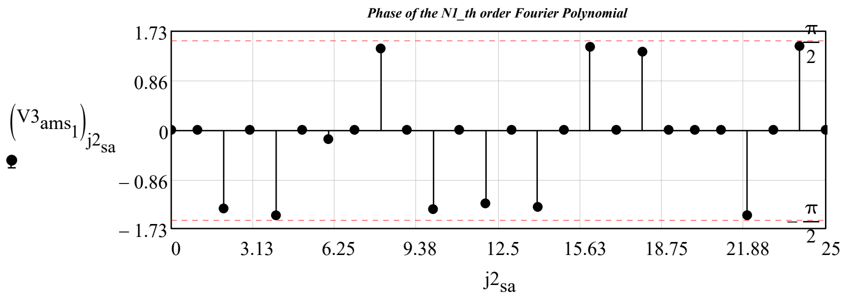
$$V3_{am}(t) := V3am\left(t, \omega 1_{msl}, \omega 1_{csl}, m_{amsl}, A1_{sl}, B1_{sl}, N0_{gd}\right)$$

$$V3_{ams} := SPCT\left(V3_{am}, rt_{gd}, N1\_, 0\cdot s, 2\cdot T2_{msl}\right) \qquad N1\_ = 25$$



$j2_{sa} := 0 .. rows\left(V3_{ams_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := V3_{ams_3} \cdot Hz$

$Bw_{sa} = 0.017 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.035 \cdot MHz$

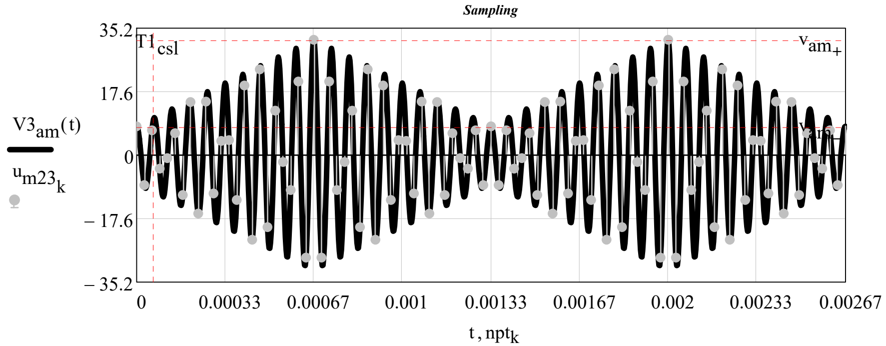
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 11.13$

$u_{m23_k} := V3_{am}\left(npt_k\right)$

$u_{m23}^T =$

	0	1	2	3	4	5	6
0	8	-8.295	6.885	-3.727	-0.831	6.081	...

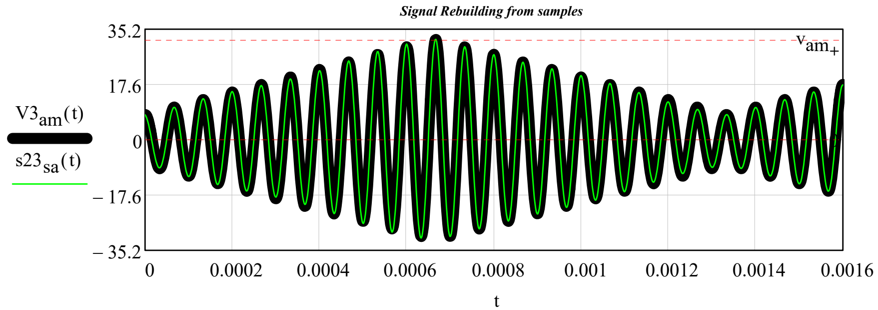


$relerr = 10 \cdot \% \qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.108 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s23_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m23_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right] \qquad N0_{gd} - 1 = 255$

$relerr = 10 \cdot \%$



Periodic Waveforms

22 AM DSBSC test signal (single tone)

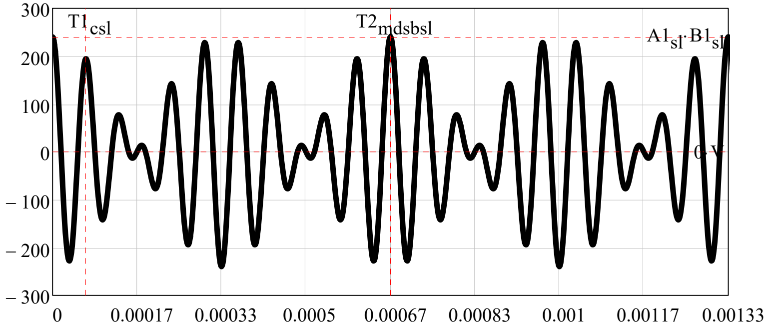
$\omega 2_{msl} := \frac{\omega 1_{csl}}{10}$ $T2_{mdsbsl} := \frac{2\cdot\pi}{\omega 2_{msl}}$ $\omega 2_{msl} = \frac{2\cdot\pi}{T2_{mdsbsl}}$ $\frac{A1_{sl}\cdot B1_{sl}}{2} = 120\cdot volt^2$

$\omega 1_{csl} = 94.248\cdot\frac{krads}{sec}$ $\omega 2_{msl} = 9.425\cdot\frac{krads}{sec}$ $f2_{msl} := \frac{1}{T2_{msl}}$ $f1_{csl} := \frac{\omega 1_{csl}}{2\cdot\pi}$

$T_{1csl} := \frac{1}{f1_{csl}}$ $\nu_{sl} := 40$

$$t_{tdsbw} := 0\cdot sec, \nu_{sl}\cdot\frac{T2_{mdsbsl}}{20000} .. \nu_{sl}\cdot T2_{mdsbsl}$$

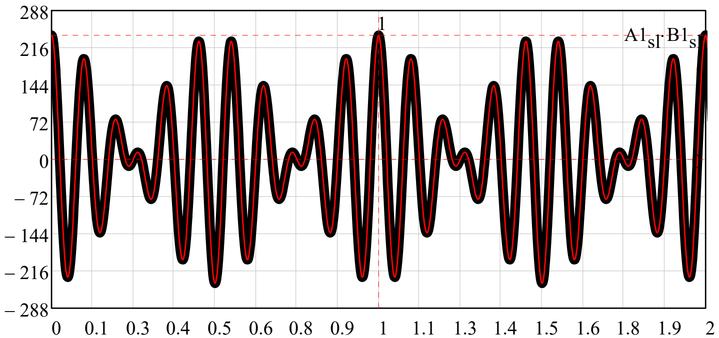
DSBSC single tone



$$V4_{am}(t) := V4dsbsc\left(t, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl}\right)$$

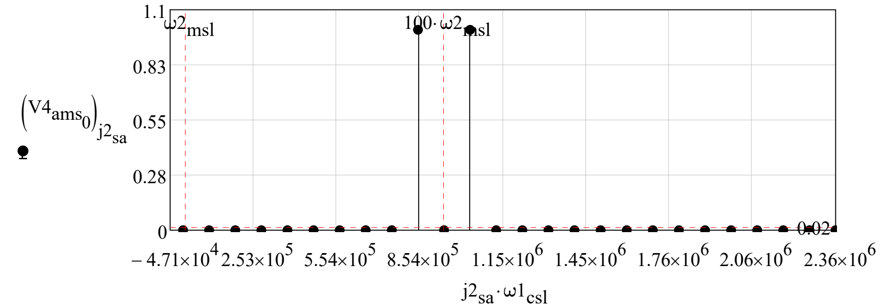
$$V4_{ams} := SPCT\left(V4_{am}, rt_{gd}, N1\_, 0\cdot s, T2_{mdsbsl}\right) \qquad N1\_ = 25$$

Signal and Fourier Series

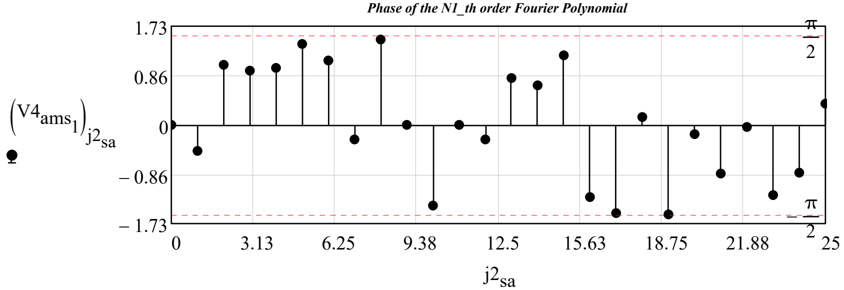


$j2_{sa} := 0 .. rows\left(V4_{ams_0}\right) - 1$ $\omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := V4_{ams_3}\cdot Hz$

$Bw_{sa} = 0.024\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $fpt_{so} = 0.048\cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

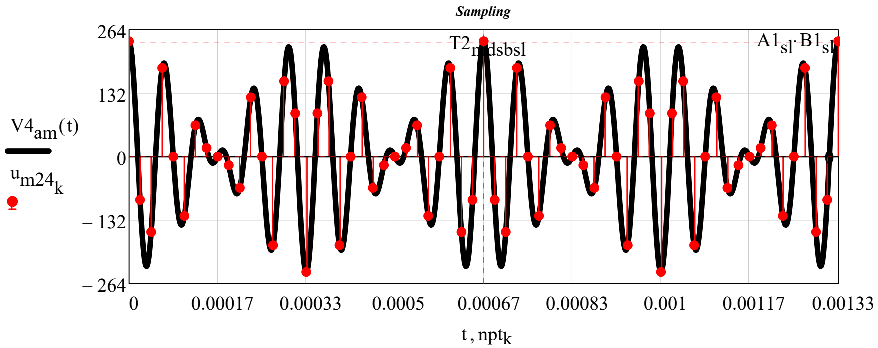
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T2_{mdsbsl}} = 8$

$u_{m24_k} := V4_{am}\left(npt_k\right)$

$u_{m24}^T =$

	0	1	2	3	4
0	240	-90.079	-156.788	184.363	...

Sampling



relerr = 10·%

$\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.151\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

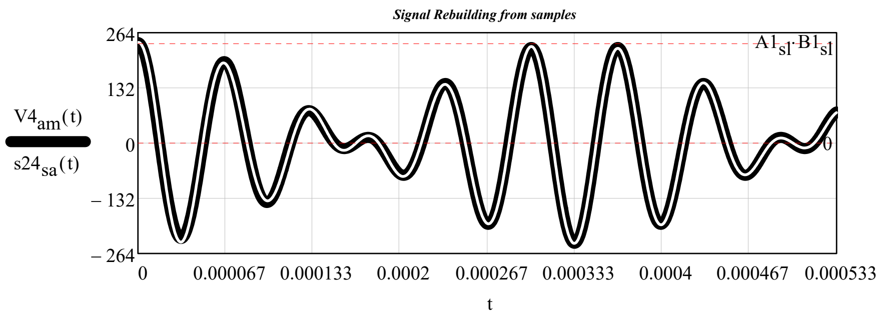
interpolation formula:

$$s24_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m24_n}\cdot\mathrm{sinc}\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$$

$N0_{gd} - 1 = 255$

relerr = 10·%

Signal Rebuilding from samples



TEST Waveforms

Periodic Waveforms

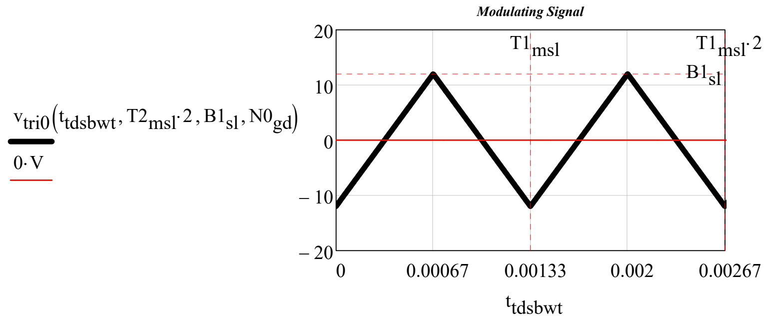
23 AM DSBSC test signal (triangular wave)

$T_{18} := T2_{mdsbsl}$

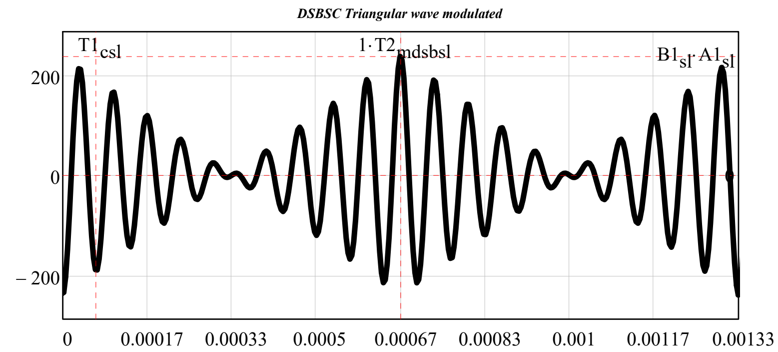
$f_{18} := \frac{1}{T_{18}}$

$t_{tdsbwt} := -T_{18}\cdot 3, -T_{18}\cdot 3 + \frac{8\cdot T_{18} + T_{18}\cdot 3}{2000} .. 8\cdot T_{18}$

Modulating Signal



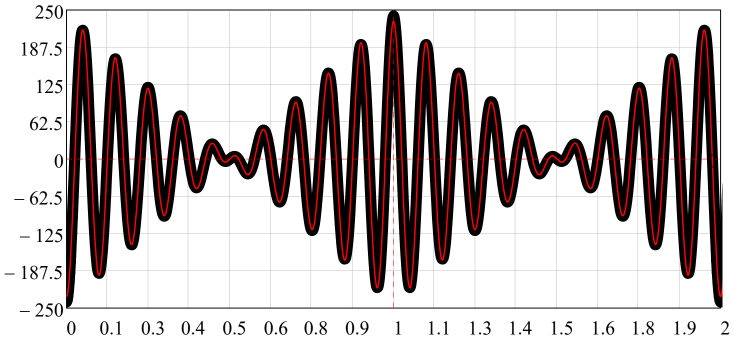
DSBSC Triangular wave modulated



$N1\_ := 25$ $v5_{dsbsc}(t) := V5_{dsbsc}\left(t, T2_{mdsbsl}, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd}\right)$

$f_{cfm} = 3\cdot MHz$ $V5dsbsc := SPCT\left(v5_{dsbsc}, rt_{gd}, N1\_, 0\cdot s, 2\cdot T2_{mdsbsl}\right)$ $N1\_ = 25$

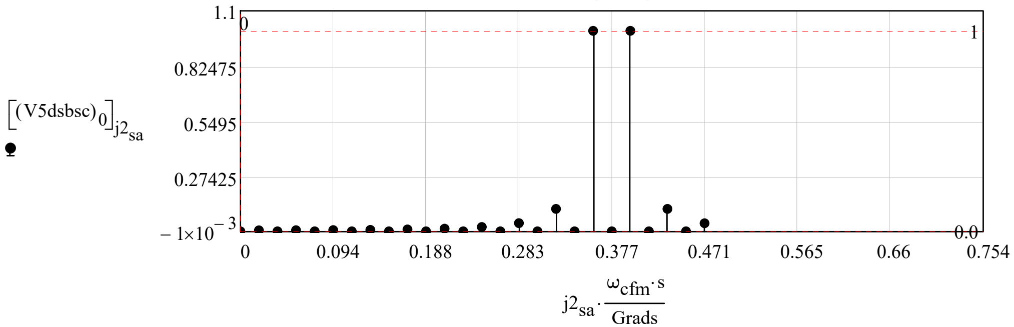
Signal and Fourier Series



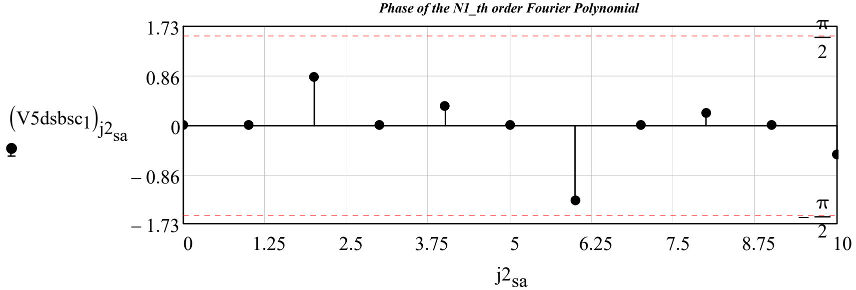
$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$

$j2_{sa} := 0 .. rows(V5dsbsc_0) - 1$ $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



Bandwidth: $Bw_{sa} := V5dsbsc_3 \cdot Hz$

$Bw_{sa} = 0.017 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.035 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

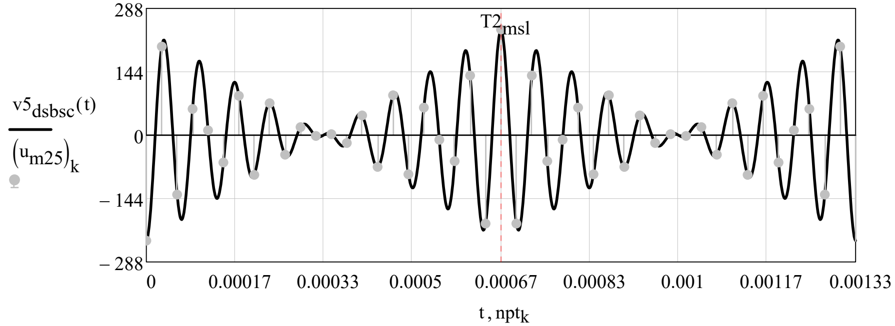
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{mdsbsl}} = 11.13$

$(u_{m25})_k := v5_{dsbsc}(npt_k)$

$u_{m25}^T =$

	0	1	2	3	4	5
0	-240	200.989	-135.324	59.405	10.681	...

Sampling



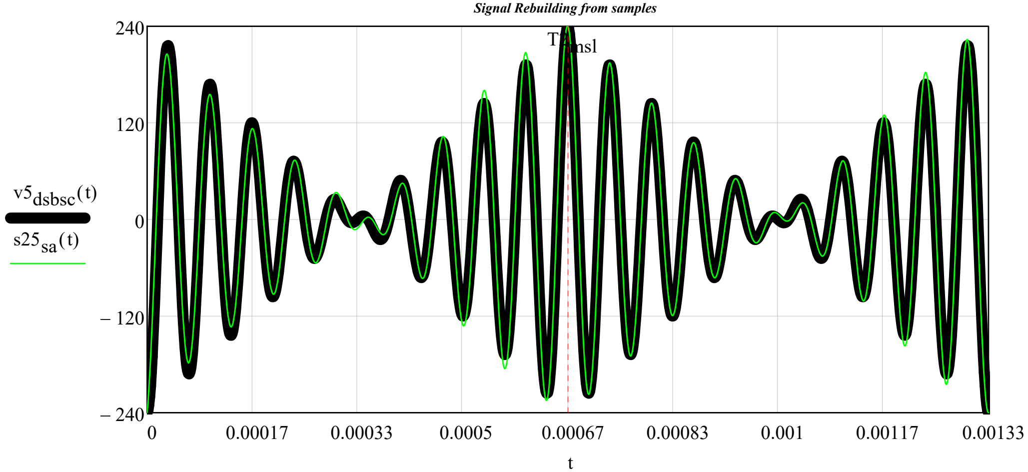
relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.108 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s25_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m25_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $N0_{gd} - 1 = 255$

relerr = 10·%

Signal Rebuilding from samples



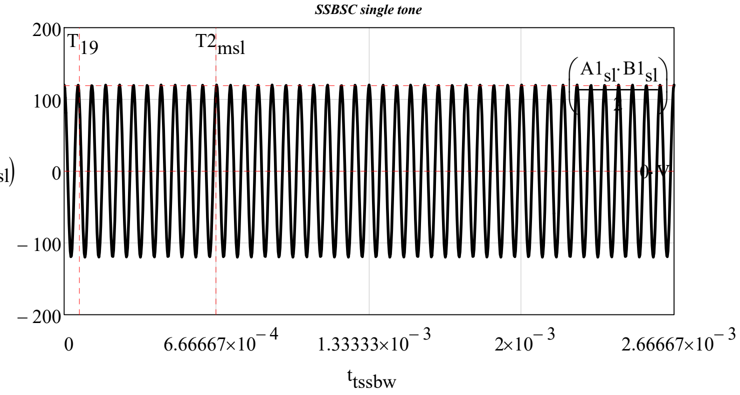
TEST Waveforms

Periodic Waveforms

24 AM SSBSC test signal (single tone)

$f_{19} := \frac{\omega 1_{csl}}{2 \cdot \pi}$ $\qquad T_{19} := \frac{1}{f_{19}}$

$t_{ssbw} := 0 \cdot sec, \frac{4 \cdot T2_{msl}}{1000} .. 4 \cdot T2_{msl}$

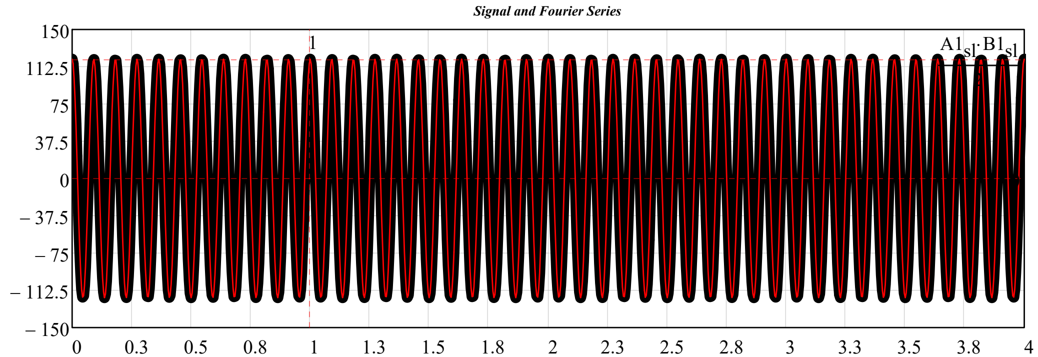


$\frac{A1_{sl} \cdot B1_{sl}}{2} = 120\,V^2$

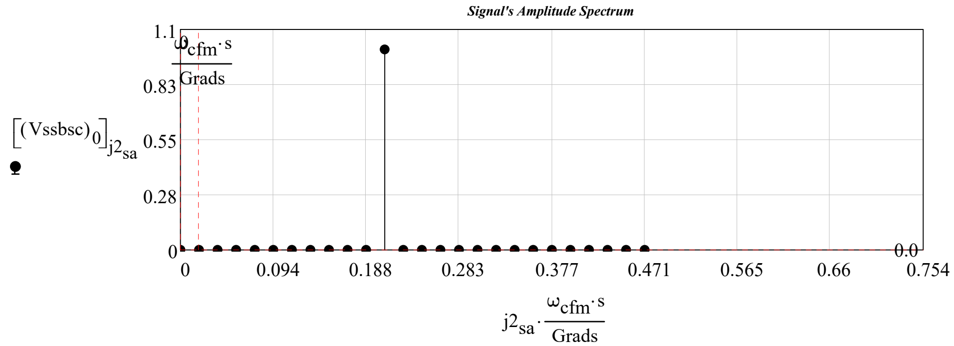
$v_{ssbsc}(t) := \frac{V_{ssbsc}(t, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl})}{V^2}$ $\qquad v_{ssbsc}(T_{19}) = 97.082$

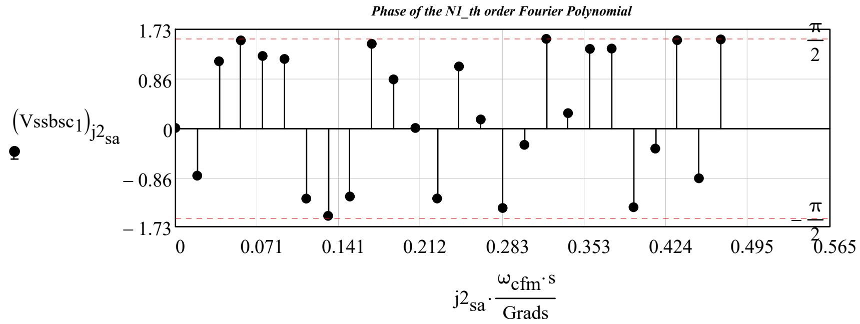
$N1\_ := 25$

$f_{cfm} = 3 \cdot MHz$ $\qquad Vssbsc := SPCT(v_{ssbsc}, rt_{gd}, N1\_, 0 \cdot s, T2_{mdsbsl})$ $\qquad N1\_ = 25$



$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$ $\qquad j2_{sa} := 0 .. rows(Vssbsc_0) - 1$ $\qquad \omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$





$Bw_{sa} := Vssbsc_3 \cdot Hz$

$Bw_{sa} = 0.023 \cdot MHz$

sampling frequency: $\qquad fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.045 \cdot MHz$

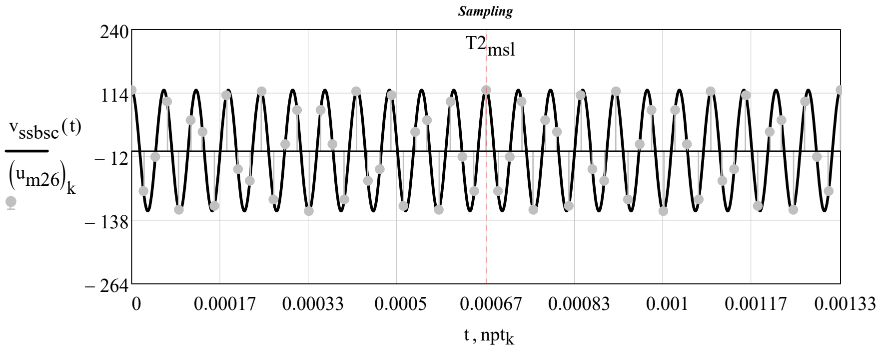
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\qquad \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 8.533$

$(u_{m26})_k := v_{ssbsc}(npt_k)$

$u_{m26}^T =$

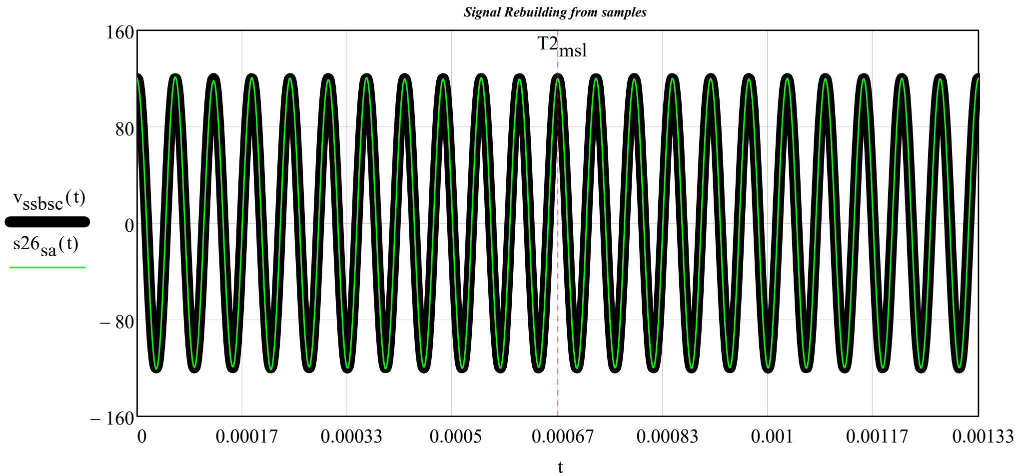
	0	1	2	3	4	5
0	120	-80.296	-12.543	97.082	-117.378	...



$relerr = 10\cdot\%$ $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.141\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s26_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m26_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ $relerr = 10\cdot\%$

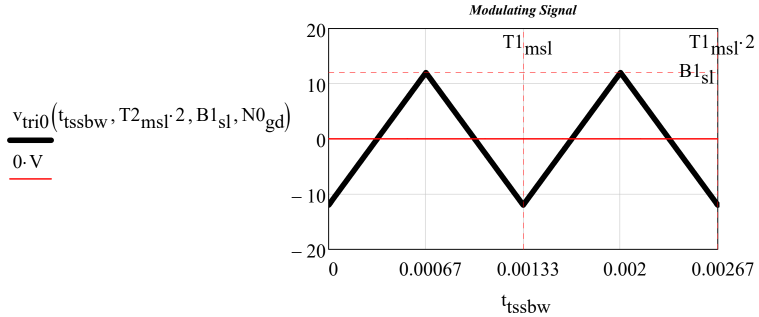


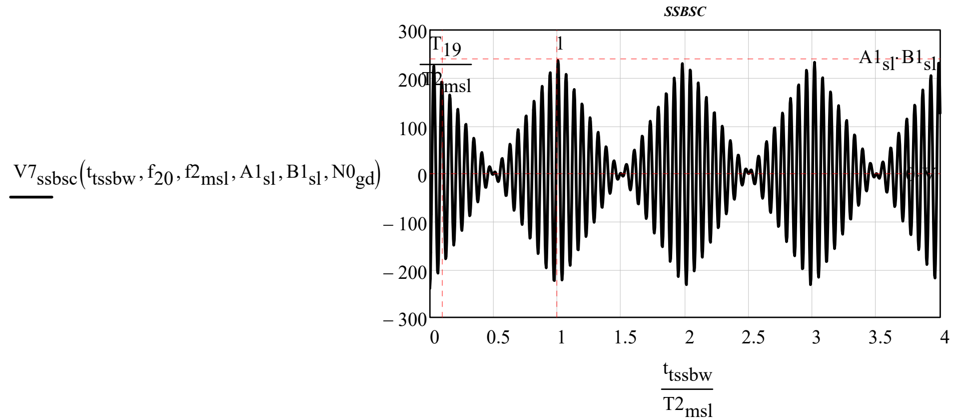
TEST Waveforms

Periodic Waveforms

25 AM SSBSC test signal (triangular wave)

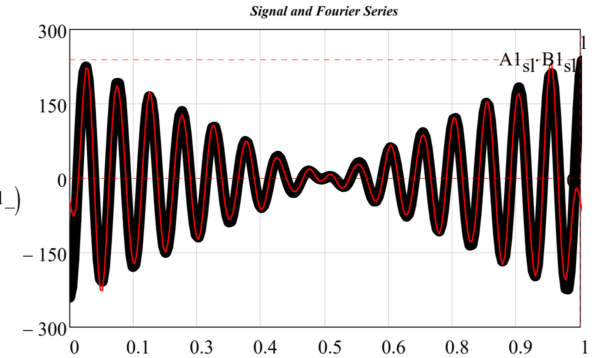
$f_{20} := \frac{10}{T1_{csl}}$ $\frac{A1_{sl}\cdot A1_{sl}}{2} = 200\,V^2$





$N1\_ := 25$ $v7_{ssbsc}(t) := V7_{ssbsc}\left(t, f_{20}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd}\right)$

$f_{cfm} = 3\cdot MHz$ $V7ssbsc := SPCT\left(v7_{ssbsc}, rt_{gd}, N1\_, 0\cdot s, T2_{msl}\right)$ $N1\_ = 25$

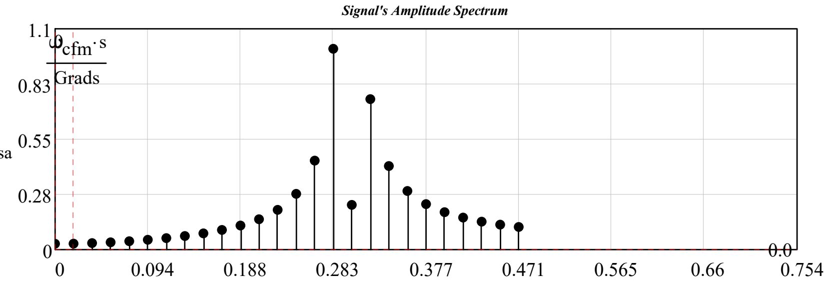


$\frac{v7_{ssbsc}(t_{tssbw})}{fs(t_{tssbw}, V7ssbsc_9, V7ssbsc_{10}, T2_{msl}, N1\_)}$

$\frac{t_{tssbw}}{T2_{msl}}$

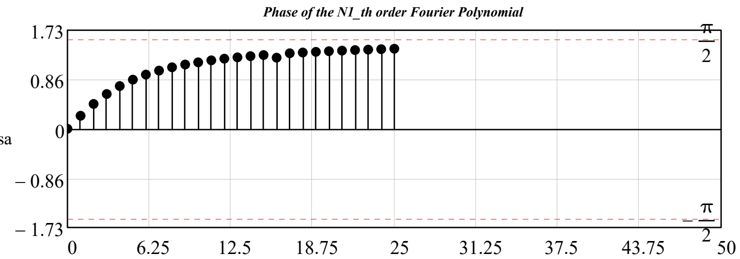
$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$

$j2_{sa} := 0 .. rows(V7ssbsc_0) - 1 \qquad \omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$



$\left[(V7ssbsc)_0\right]_{j2_{sa}}$

$j2_{sa} \cdot \frac{\omega_{cfm} \cdot s}{Grads}$



$(V7ssbsc_1)_{j2_{sa}}$

$j2_{sa}$

$Bw_{sa} := V7ssbsc_3 \cdot Hz$

$Bw_{sa} = 0.035 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.069 \cdot MHz$

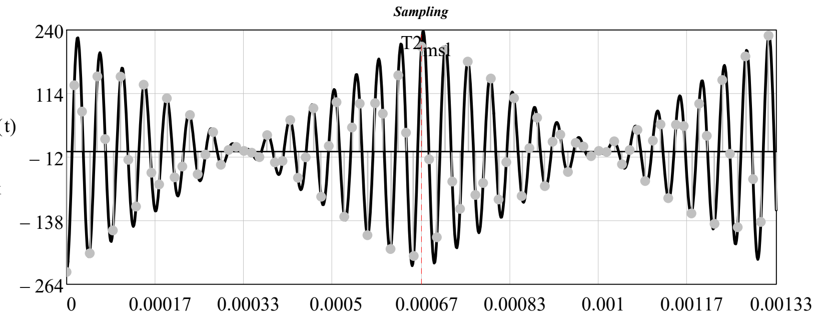
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 5.565$

$(u_{m27})_k := v7_{ssbsc}(npt_k)$

$u_{m27}^T =$

	0	1	2	3	4
0	-240	130.212	78.129	-202.786	...



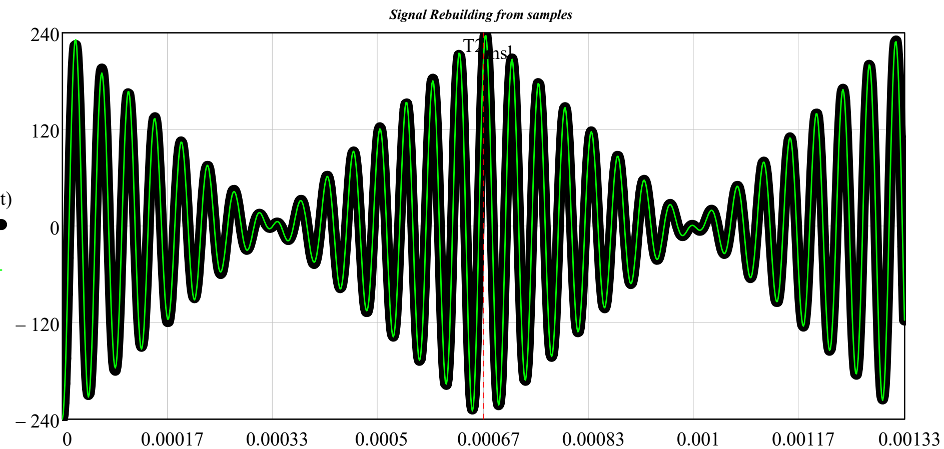
$\frac{v7_{ssbsc}(t)}{(u_{m27})_k}$

t, npt_k

relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.217 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s27_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m27_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10 \cdot \%$



$\frac{v7_{ssbsc}(t)}{s27_{sa}(t)}$

t

Periodic Waveforms

26 FM test signal (single tone) (change data in FM data.xmcd)

Carrier Amplitude:...: $A_{fm} = 20 \cdot V$

Carrier Frequency...: $f_{cfm} = 3 \cdot MHz$

Carrier period...: $T_{cfm} = 333.333 \cdot ns$

Angular frequency of the carrier....................................: $\omega_{cfm} = 18.85 \cdot \frac{Mrads}{sec}$

Amplitude of the single tone modulating signal............: $B_{fmm} = 15 \cdot V$

Period of the modulating signal...: $T_{fmm} = 8.333 \cdot \mu s$

Frequency of the single tone modulating signal........ .....: $f_{fmm} := \frac{1}{T_{fmm}}$ $f_{fmm} = 0.12 \cdot MHz$

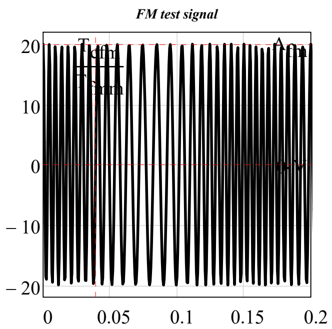
Angular frequency of the single tone modulating signal: $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{sec}$

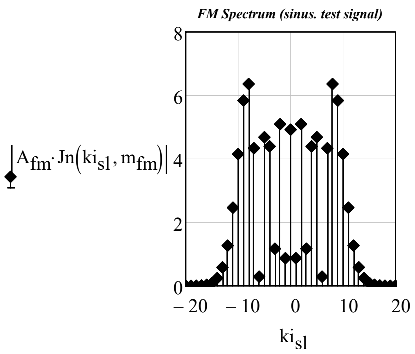
Frequency modulation index: $m_{fm} = 10$

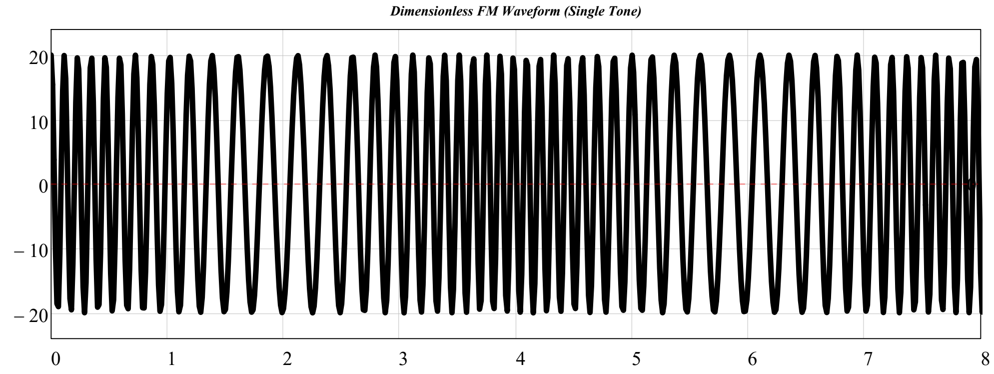
$\frac{T_{fmm}}{T_{cfm}} = 25$ $\frac{\omega_{cfm}}{\omega_{fmm}} = 25$ $k_{fm} = 8 \times 10^4 \frac{1}{Wb}$

$m_{fm} = 10$ $ki_{sl} := -30 .. 30$

$$t_{fmsl} := T_{fmm} \cdot 0, T_{fmm} \cdot 0 + \frac{10 \cdot T_{fmm} - T_{fmm} \cdot 0}{20000} .. 10 \cdot T_{fmm}$$



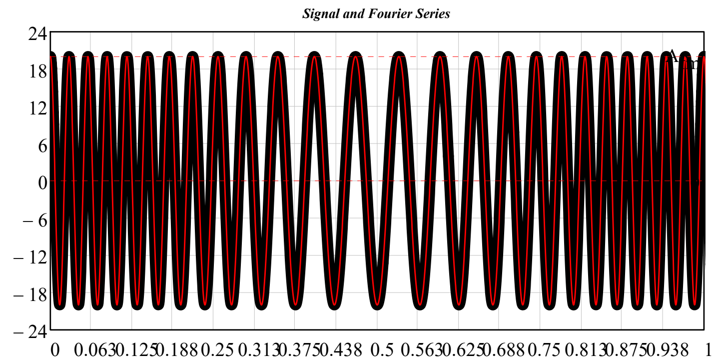




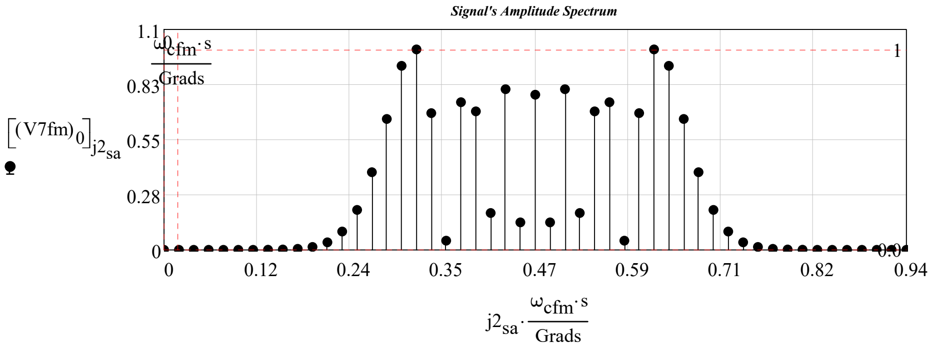
$m_{fm} = 10$ $A_{fm} = 20 V$ $B_{fmm} = 15 V$ $f_{cfm} = 3 \times 10^6 \frac{1}{s}$

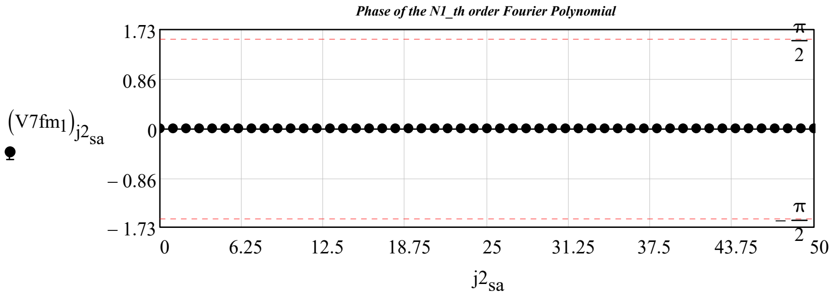
$N1\_ := 50$ Dimensionless: $v7_{fm}(t) := V7_{fm}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd})$ $T_{fmm} = 8.333 \cdot \mu s$

$f_{cfm} = 3 \cdot MHz$ $V7fm := SPCT(v7_{fm}, rt_{gd}, N1\_, 0 \cdot s, T_{fmm})$ $N1\_ = 50$



$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$ $j2_{sa} := 0 .. rows(V7fm_0) - 1$ $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$





$Bw_{sa} := V7fm_3 \cdot Hz$

$Bw_{sa} = 4.68 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 9.36 \cdot MHz$

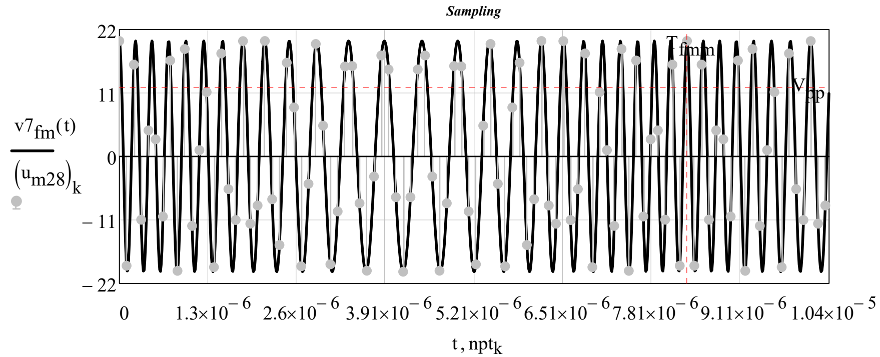
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{fmm}} = 3.282$

$(u_{m28})_k := v7_{fm}(npt_k)$

$u_{m28}^T =$

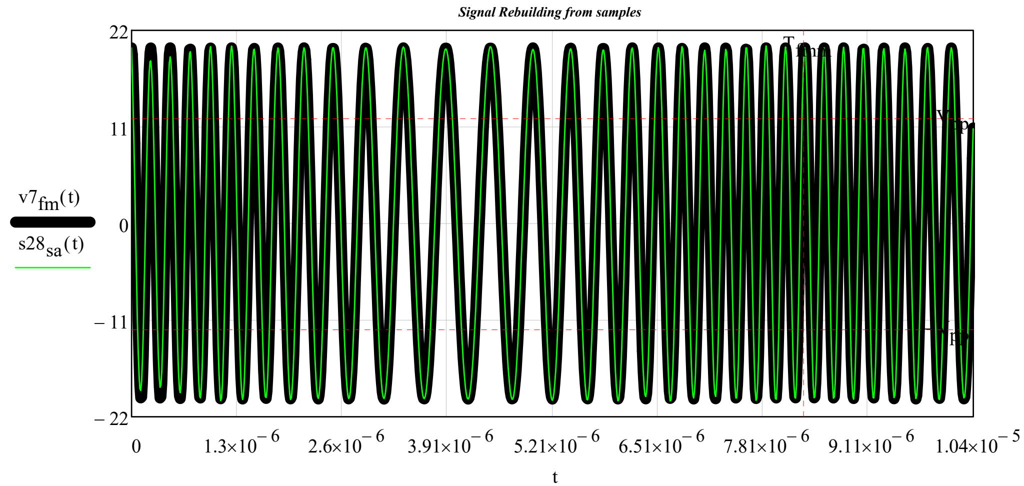
	0	1	2	3	4	5	6
0	20	-18.965	15.905	-10.972	4.491	2.955	...



relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 29.405 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s28_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m28_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%

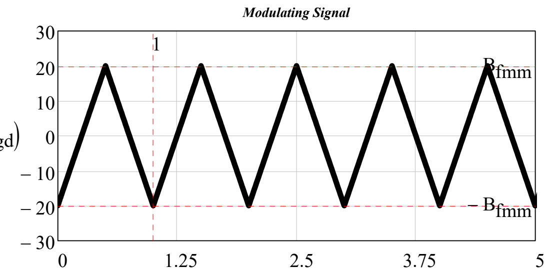


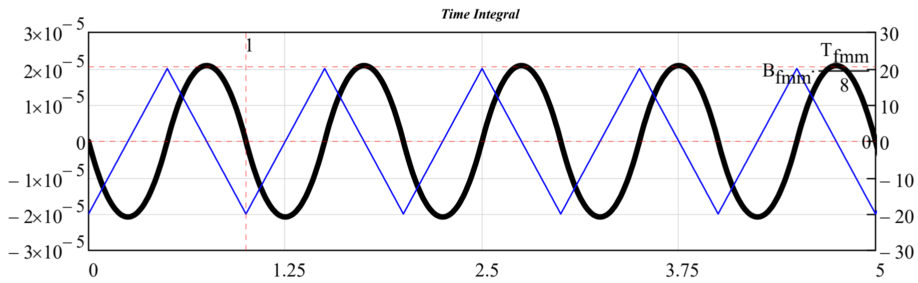
Periodic Waveforms

27 FM test signal (triangular wave)

$B_{fmm} := 20 \cdot V \qquad m_{fm} := 80 \qquad t_{fmwsl} := 0 \cdot T_{fmm}, \frac{10 \cdot T_{fmm} - 0 \cdot T_{fmm}}{1000} .. 10 \cdot T_{fmm}$

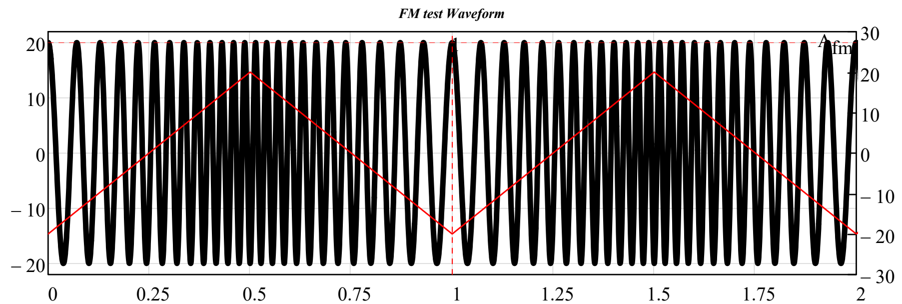
$v_{tri0}(t_{fmwsl}, T_{fmm}, B_{fmm}, N_{gd})$





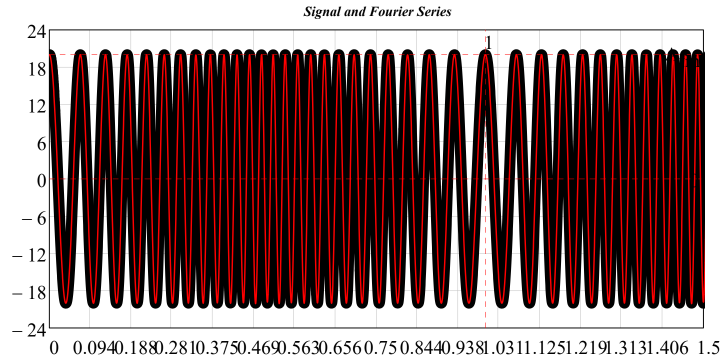
$k_{fm} := \frac{m_{fm} \cdot \omega_{fmm}}{2 \cdot \pi \cdot B_{fmm}} \qquad f_{fmm} := \frac{1}{T_{fmm}} \qquad k_{fm} = 0.48 \cdot (\mu V \cdot s)^{-1} \qquad f_{fmm} = 120 \cdot kHz \qquad f_{cfm} = 3 \cdot MHz$

$m_{fm} = 80$

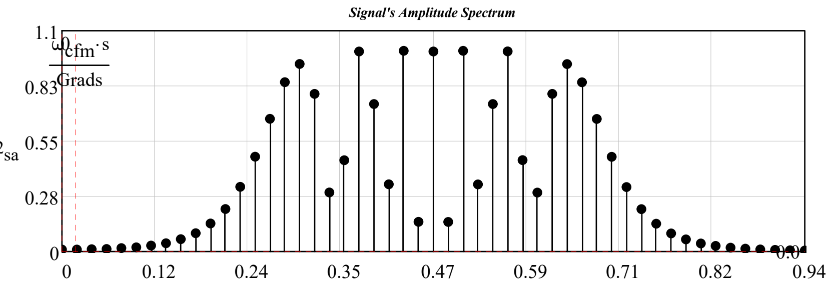


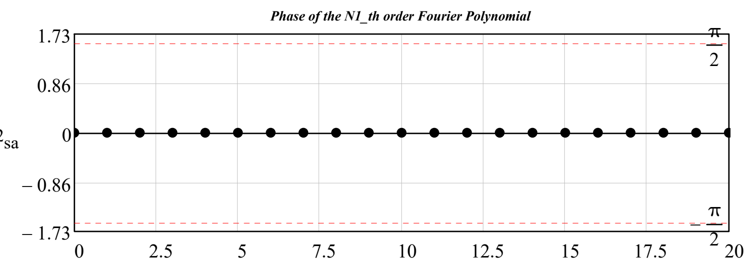
$N1\_ := 50 \qquad v8_{fm}(t) := V8_{fm}(t, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, k_{fm}, N1\_)$

$f_{cfm} = 3 \cdot MHz \qquad V8fm := SPCT(v8_{fm}, rt_{gd}, N1\_, 0 \cdot s, T_{fmm})$



$\omega_{cpm} = 3.77 \cdot \frac{Grads}{s} \qquad j2_{sa} := 0 .. rows(V8fm_0) - 1 \qquad \omega_{pmm} = 94.248 \cdot \frac{Mrads}{s}$





$Bw_{sa} := V8fm_3 \cdot Hz$

$Bw_{sa} = 5.16 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 10.32 \cdot MHz$

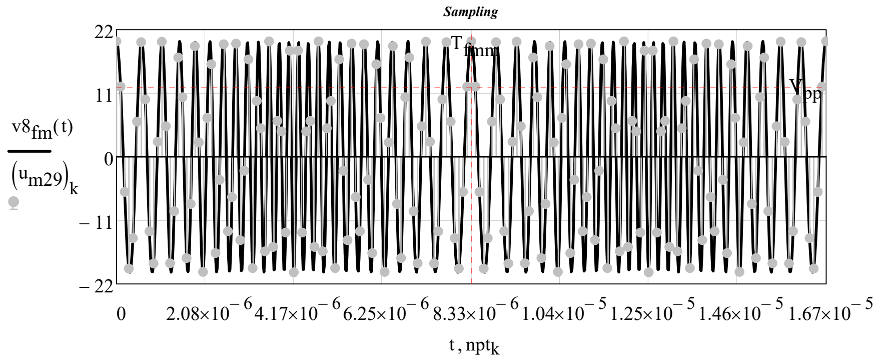
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{fmm}} = 2.977$

$$\left(u_{m29}\right)_k := v8_{fm}\left(npt_k\right)$$

$u_{m29}^T =$

	0	1	2	3	4	5	6
0	20	12.15	-6.069	-19.338	-14.082	6.098	...

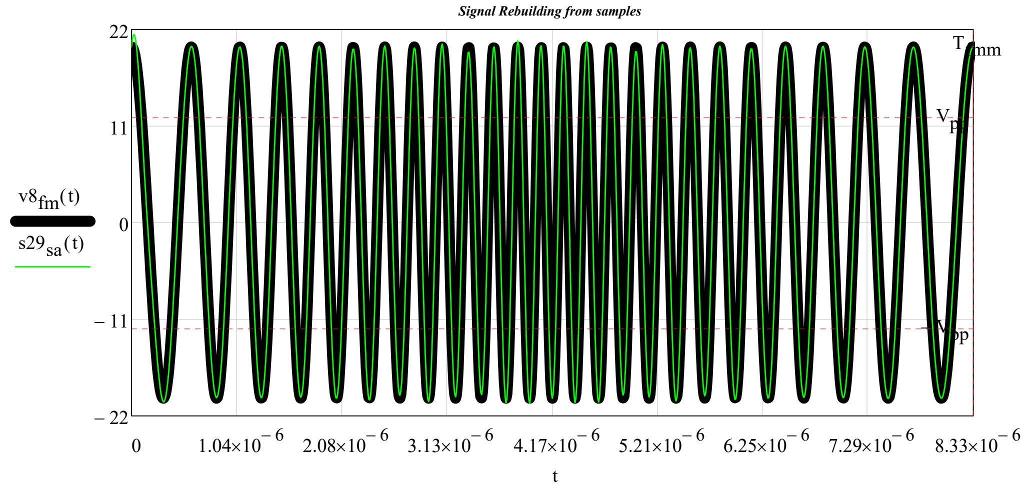


relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 32.421 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s29_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m29_n} \cdot \text{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $N0_{gd} - 1 = 255$

relerr = 10·%



TEST Waveforms

Periodic Waveforms

28 PM test signal (single tone)

Carrier Amplitude..: $A_{pm} := 20\cdot V,$ $A_{pm} = 20\cdot V$

Carrier Frequency..: $f_{cpm} = 600\cdot MHz,$

Carrier period..: $T_{cpm} = 1.667\cdot ns,$

Angular frequency of the carrier................................: $\omega_{cpm} = 3.77\cdot\frac{Grads}{sec},$

Amplitude of the modulating signal.................... ....: $B_{pm} = 30\,V,$

Modulating signal period..: $T_{pmm} = 0.067\cdot\mu s$,

Frequency of the harmonic modulating signal.........: $f_{pmm} = 15\cdot MHz,$ $\frac{T_{pmm}}{T_{cpm}} = 40$,

Angular frequency of the modulating signal:...........: $\omega_{pmm} = 94.248\cdot\frac{Mrads}{sec}.$

Phase modulation index..: $m_{pm} = 30\cdot rad$

Phase-sensitivity factor..: $k_{pm} = 1\cdot\frac{rad}{V}$

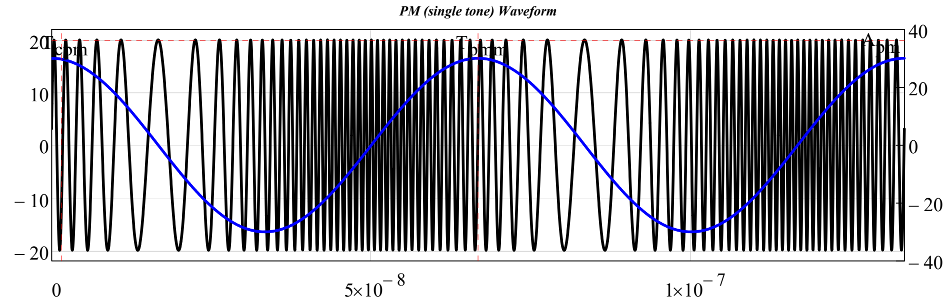
$$k_{pm} = \frac{m_{pm}}{B_{pm}}$$

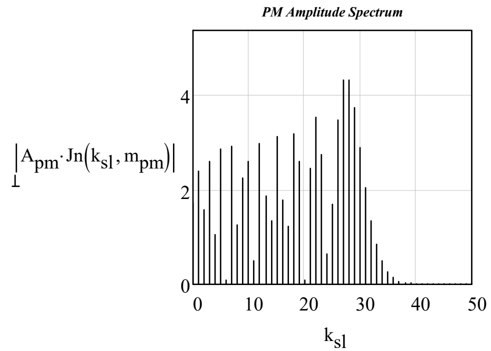
$$v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \mathrm{Re}\left[A_{pm}\cdot e^{j\cdot 2\cdot\pi\cdot f_{cpm}\cdot t}\cdot\sum_{k=-N_{gd}}^{N_{gd}}\left(e^{j\cdot\frac{k\cdot\pi}{2}}\cdot Jn(k, m_{pm})\cdot\cos(k\cdot 2\cdot\pi\cdot f_{pmm}\cdot t)\right)\right]$$

Dimensionless function: $V9_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \frac{v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})}{V}$

$f_{cpm} = 600\cdot MHz$ $t_{pm} := T_{cpm}\cdot 0, T_{cpm}\cdot 0 + \frac{80\cdot T_{cpm} - 0\cdot T_{cpm}}{4000} .. 80\cdot T_{cpm}$

$f_{pmm} = 15\cdot MHz$ $m_{pm} = 30$ $A_{pm} = 20\,V$





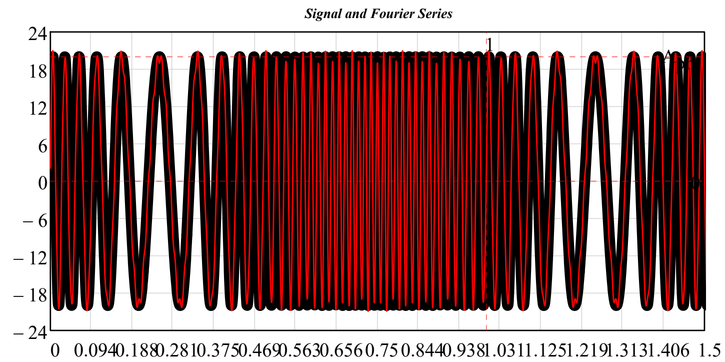
$m_{pm} = 30$

$N1\_ := 100$

$f_{cpm} = 600\cdot MHz$

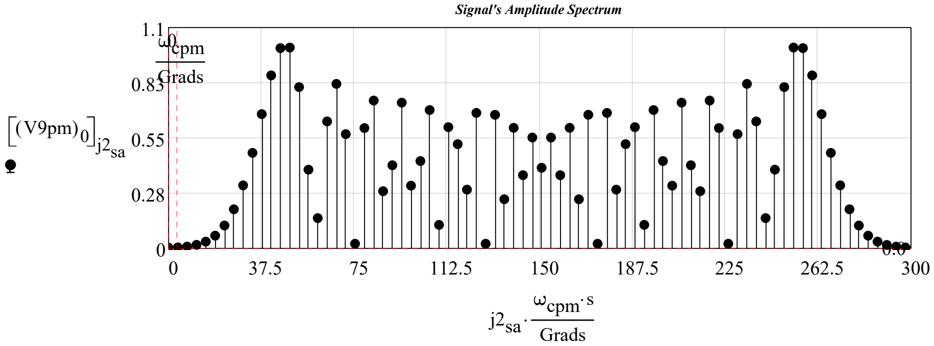
$v9_{pm}(t) := V9_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N1\_)$

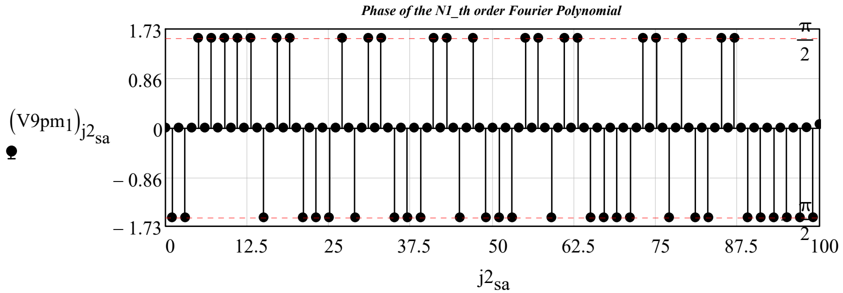
$V9pm := SPCT(v9_{pm}, rt_{gd}, N1\_, 0\cdot s, T_{pmm})$ $N1\_ = 100$



$\omega_{cpm} = 3.77 \cdot \frac{Grads}{s}$

$j2_{sa} := 0 .. rows\left(V9pm_0\right) - 1 \qquad \omega_{pmm} = 94.248 \cdot \frac{Mrads}{s}$





$Bw_{sa} := V9pm_3 \cdot Hz$

$Bw_{sa} = 1.125 \times 10^3 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 2.25 \times 10^3 \cdot MHz$

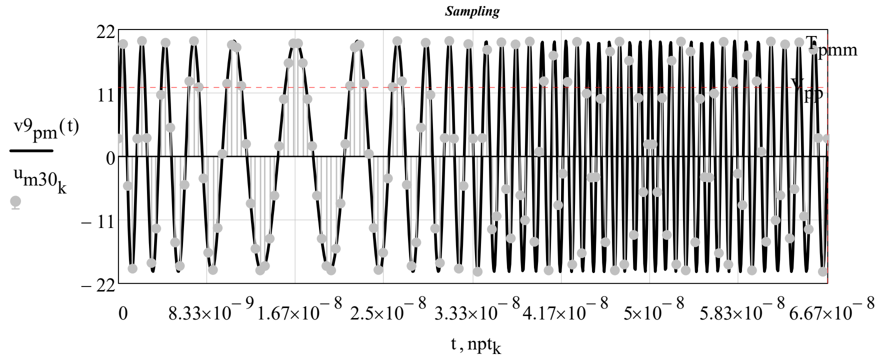
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pmm}} = 1.707$

$\left(u_{m30}\right)_k := v9_{pm}\left(npt_k\right)$

$u_{m30}^T =$

	0	1	2	3	4	5	6
0	3.085	19.458	-5.124	-19.462	3.061	19.995	...

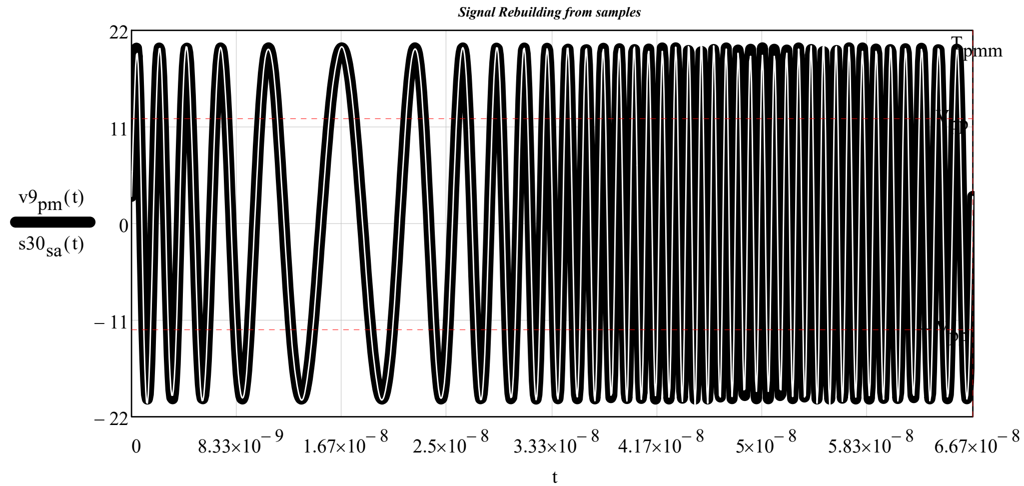


$relerr = 10 \cdot \%$

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 7.069 \times 10^3 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s30_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m30_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right) \right] \qquad N0_{gd} - 1 = 255 \qquad relerr$



TEST Waveforms

Periodic Waveforms

29 PM test signal (triangular wave)

$$T_{tri} := \frac{T_{pmm}}{2}$$

$$v_{mtri}(t_{sl}) := v_{tri0}(t_{sl}, T_{tri}, A_{pm}, N0_{gd})$$

$$v_{pmtri}(t_{sl}) := A_{pm} \cdot \cos(\omega_{cpm} \cdot t_{sl} + k_{pm} \cdot v_{mtri}(t_{sl}))$$

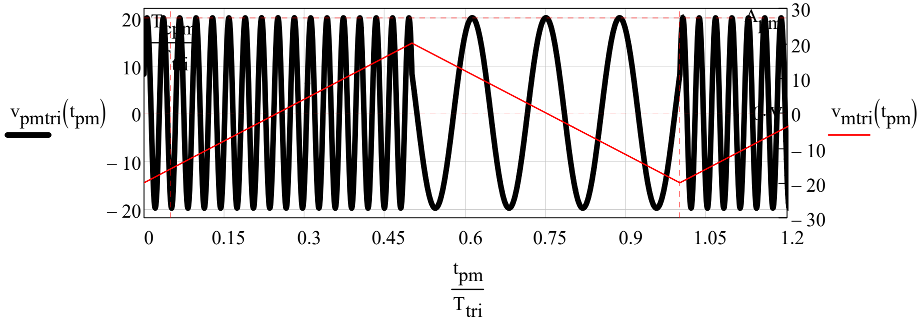
$$k_{pm} = \frac{m_{pm}}{B_{pm}} \qquad k_{pm} = 1 \cdot V^{-1}$$

$$v_{pmtri}(t, T_{pmm}, f_{cpm}, k_{pm}, A_{pm}, B_{pm}, N0_{gd}) = A_{pm} \cdot \cos(2 \cdot \pi \cdot f_{cpm} \cdot t + k_{pm} \cdot v_{tri0}(t, T_{tri}, B_{pm}, N0_{gd}))$$

$$V10_{pm}(t, T_{pmm}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd}) = \frac{v_{pmtri}(t, T_{tri}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd})}{V}$$

$$t_{pm} := T_{tri} \cdot 0, T_{tri} \cdot 0 + \frac{5 \cdot T_{tri} - 0 \cdot T_{tri}}{10000} .. 5 \cdot T_{tri}$$

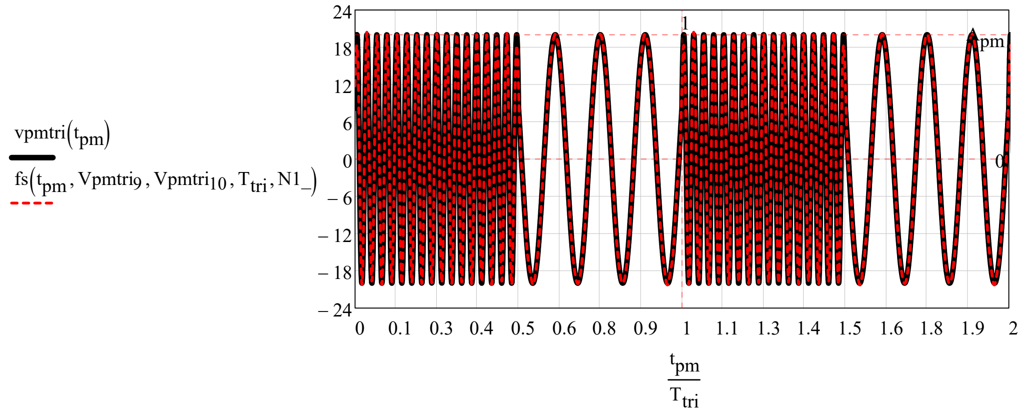
Carrier & Triangular Modulating Waveform



$$N1\_ := 50 \qquad vpmtri(t) := \frac{v_{pmtri}(t)}{V}$$

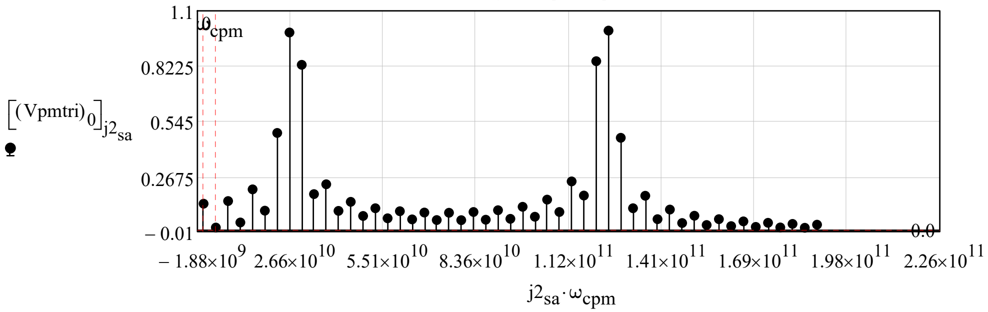
$$Vpmtri := SPCT(vpmtri, rt_{gd}, N1\_, 0 \cdot s, T_{tri}) \qquad N1\_ = 50$$

Signal and Fourier Series

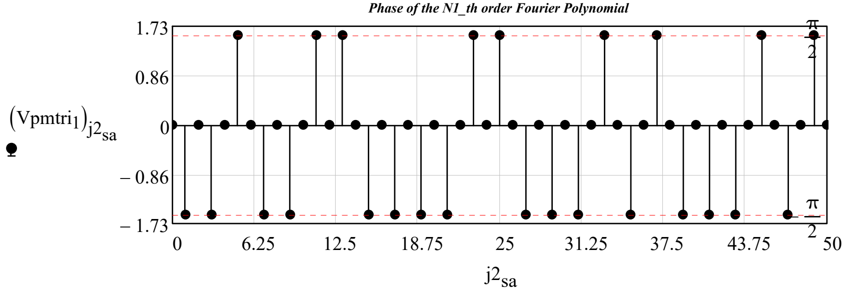


$$relerr := Vpmtri_7 \qquad j2_{sa} := 0 .. rows(Vpmtri_0) - 1 \qquad \omega_{pmm} = 94.248 \cdot \frac{Mrads}{s} \qquad relerr = 10 \cdot \%$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := Vpmtri_3 \cdot Hz$$

$$Bw_{sa} = 1.44 \times 10^3 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 2.88 \cdot GHz$

$$k := 0 .. 2^8 - 1 \qquad npt_k := \frac{k}{fpt_{so}}$$

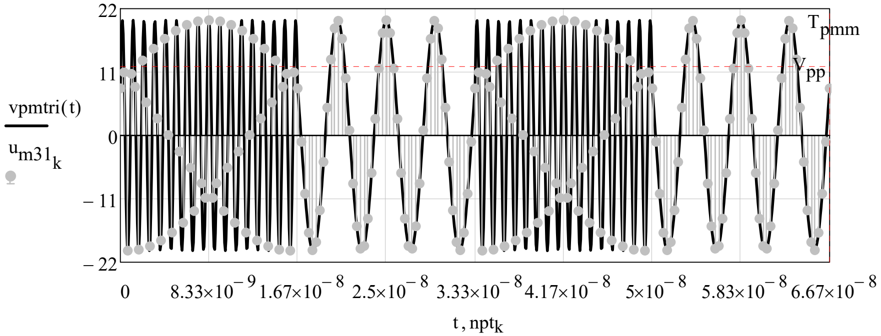
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pmm}} = 1.333$

$$\left(u_{m31}\right)_k := \mathrm{vpmtri}\left(\mathrm{npt}_k\right)$$

$u_{m31}{}^T =$

	0	1	2	3	4	5	6
0	8.162	10.942	-19.999	10.694	8.43	-19.814	...

Sampling

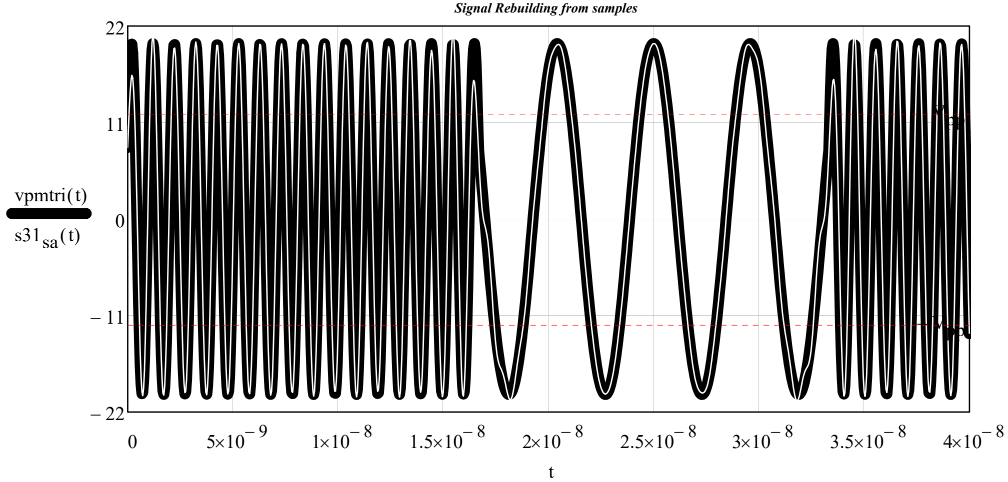


$\mathrm{relerr} = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 9.048 \times 10^3 \cdot \frac{\mathrm{Mrads}}{\mathrm{sec}}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s31_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m31_n} \cdot \mathrm{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $\qquad N0_{gd} - 1 = 255$

Signal Rebuilding from samples



TEST Waveforms

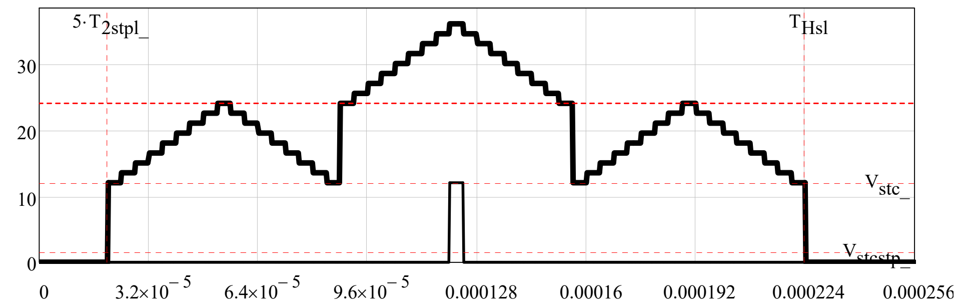
Periodic Waveforms

30 Staircase based test signal

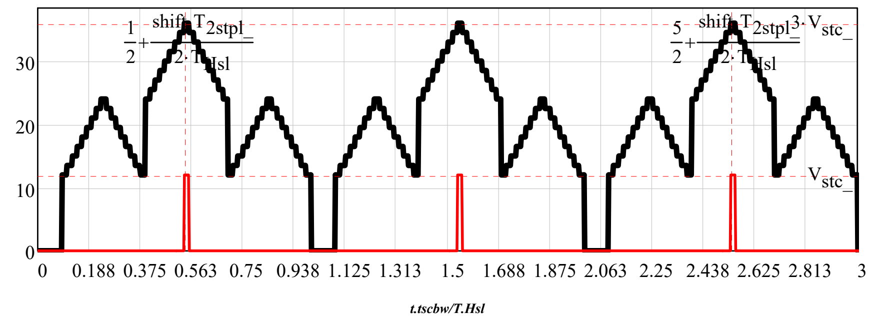
$\mathrm{shift} := 5$

$$T_{Hsl} := \left(6 \cdot m2_{steps\_} + \mathrm{shift} + 3\right) \cdot T_{2stpl\_} \qquad t_{tscbw} := 0 \cdot T_{Hsl}, 0 \cdot T_{Hsl} + \frac{5 \cdot T_{Hsl}}{2000} .. 5 \cdot T_{Hsl}$$

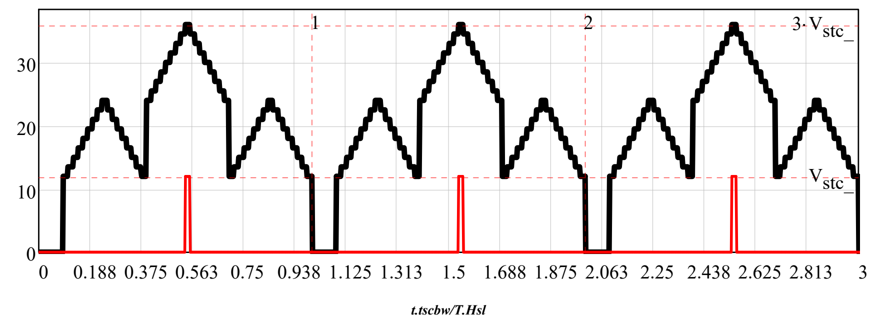
(-.-)



Staircase based test signal...



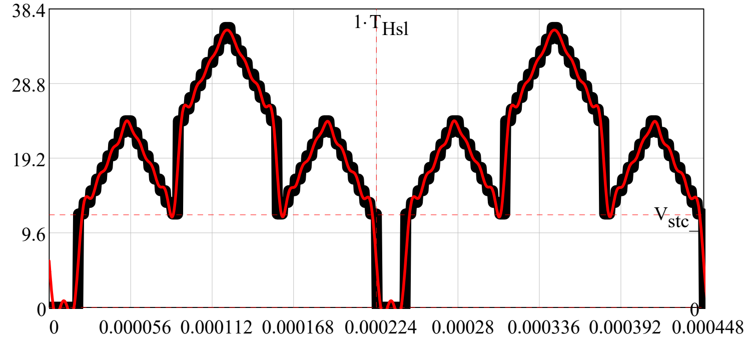
Staircase based test signal...



$N1\_ := 25$ $\qquad \omega_H := \frac{2\cdot\pi}{T_{Hsl}}$ $\qquad VH(t) := V_H(t, T_{Hsl}, T_{2stpl\_}, V_{stc\_}, mstc3_{steps\_}, shift, N_{gd})$

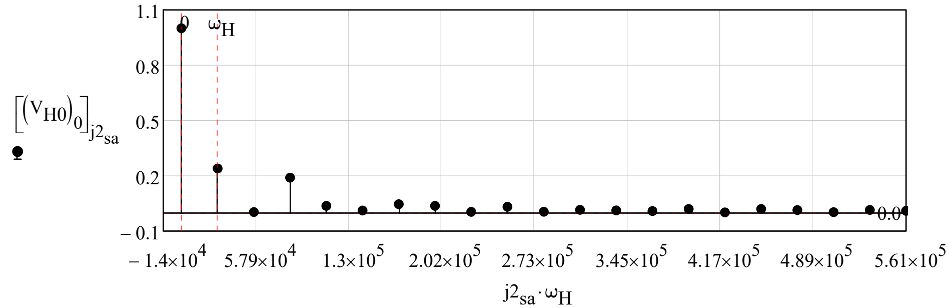
$5\cdot T_{2stpl\_} = 20\cdot\mu s$ $\qquad V_{H0} := SPCT(VH, rt_{gd}, N1\_, 5\cdot T_{2stpl\_}, T_{Hsl})$ $\qquad N1\_ = 25$

Signal and Fourier series

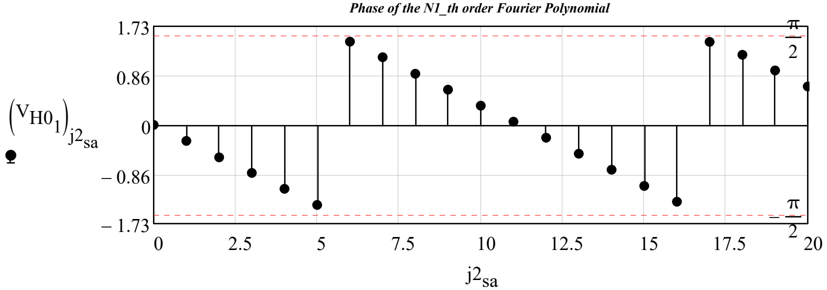


$j2_{sa} := 0..rows(V_{H0_0}) - 1$ $\qquad \omega_H = 28.05\cdot\frac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := V_{H0_3}\cdot Hz$

$Bw_{sa} = 0.103\cdot MHz$

sampling frequency: $\qquad fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 0.205\cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

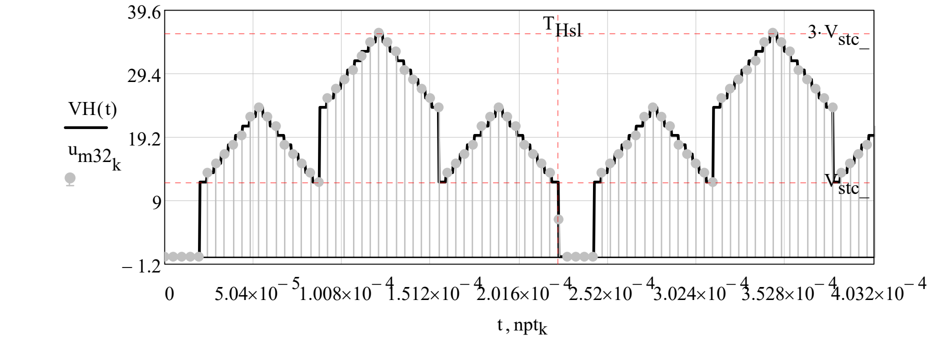
Frequency resolution: $\qquad \frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{Hsl}} = 5.565$

$(u_{m32})_k := VH(npt_k)$

$u_{m32}^T =$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	13.5	15	16.5	...

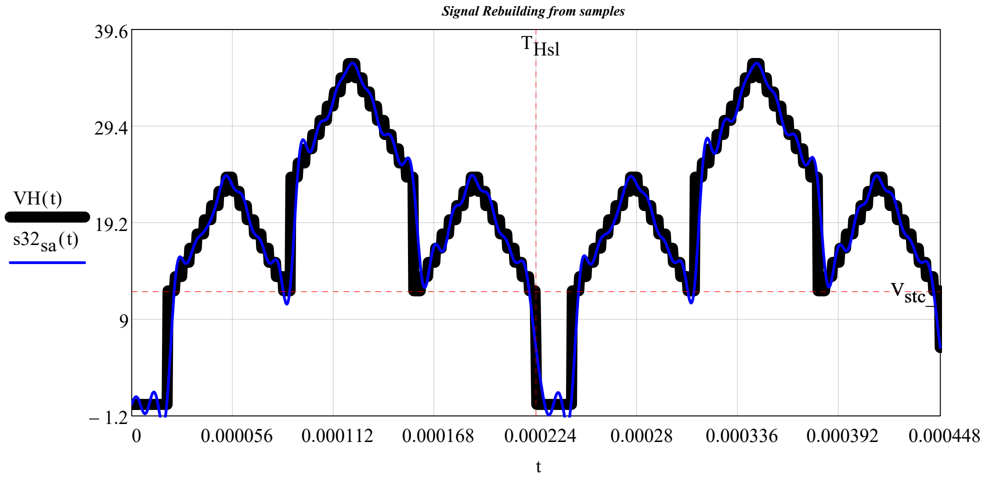
Sampling



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.645\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\qquad s32_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m32_n}\cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad relerr = 10\cdot\%$

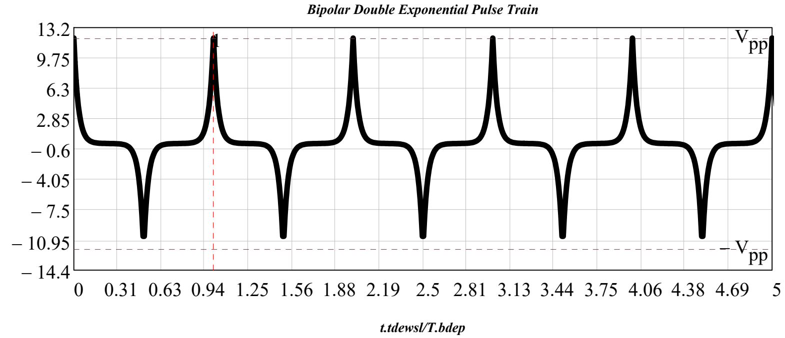


Periodic Waveforms

31 Bipolar Double Exponential Pulse Train

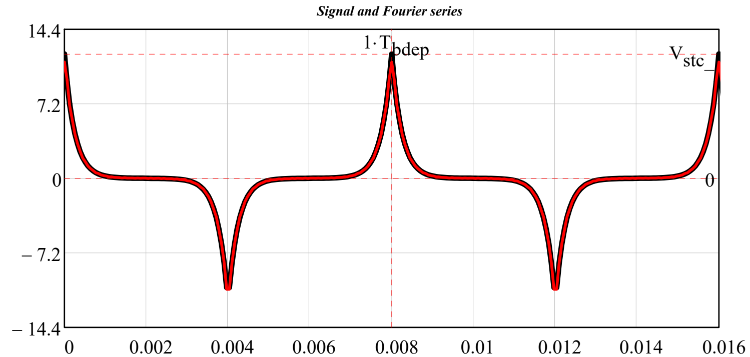
$$T_{bdep} := 32 \cdot \tau_{ptd\_}$$

$$t_{tdewsl} := -20 \cdot T_{bdep}, -20 \cdot T_{bdep} + \frac{20 \cdot T_{bdep} + 20 \cdot T_{bdep}}{5000} .. 20 \cdot T_{bdep}$$

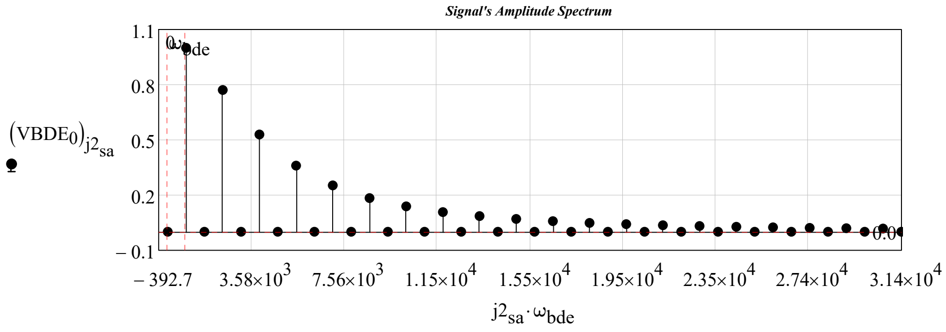


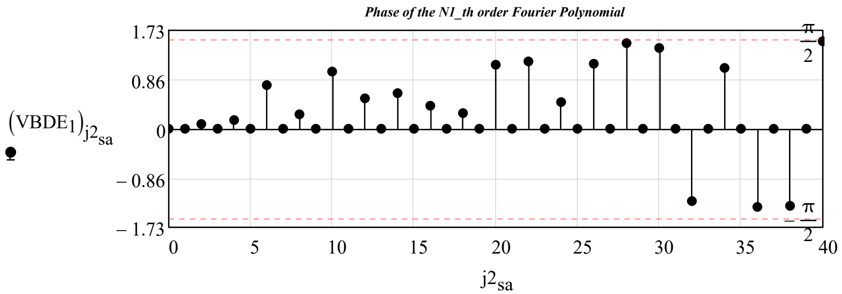
$N1\_ := 50$ $\qquad \omega_{bde} := \frac{2 \cdot \pi}{T_{bdep}}$ $\qquad Vbdept(t) := \frac{V_{bdept}\left(t, \tau_{ptd\_}, T_{bdep}, V_{pp}, N0_{gd}\right)}{V}$

$VBDE := SPCT\left(Vbdept, rt_{gd}, N1\_, 0 \cdot s, T_{bdep}\right) \qquad N1\_ = 50$



$j2_{sa} := 0 .. rows\left(VBDE_0\right) - 1 \qquad \omega_{bde} = 0.785 \cdot \frac{krads}{s}$





$Bw_{sa} := VBDE_3 \cdot Hz$

$Bw_{sa} = 6 \times 10^{-3} \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.012 \cdot MHz$

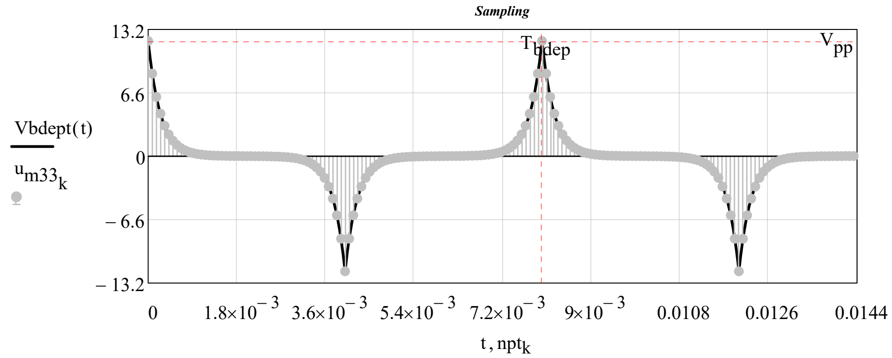
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{bdep}} = 2.667$

$(u_{m33})_k := Vbdept(npt_k)$

$u_{m33}^T =$

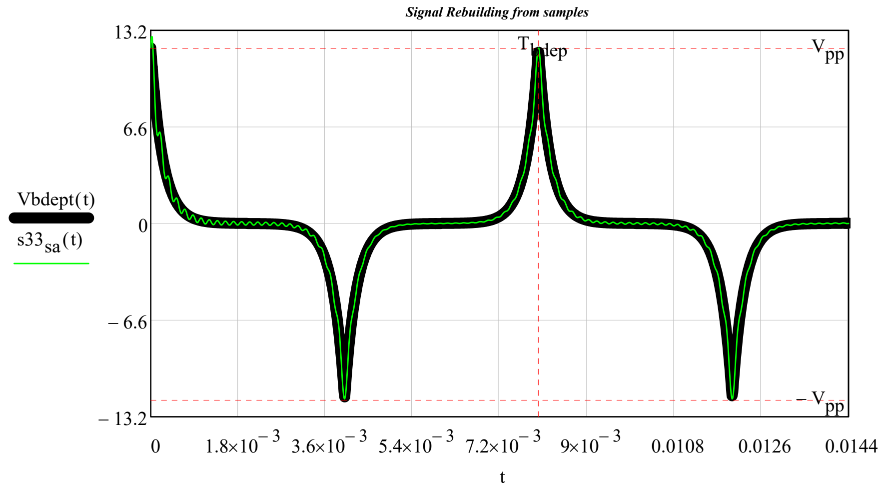
	0	1	2	3	4
0	12	8.598	6.161	4.415	...



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.038 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s33_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m33_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$



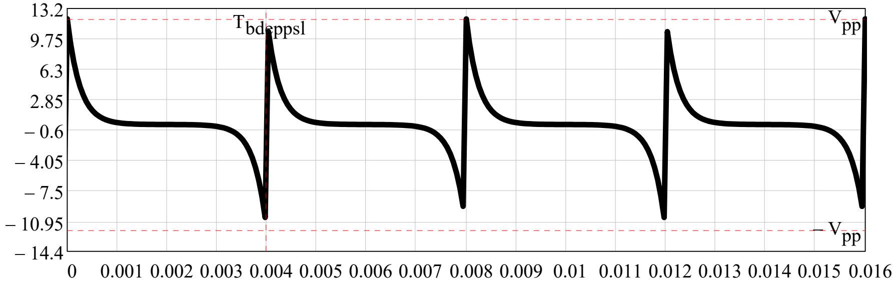
TEST Waveforms

Periodic Waveforms

32 Bipolar Double Exponential Odd symmetric Pulse Train

$$T_{bdeppsl} := 16 \cdot \tau_{ptd\_}$$

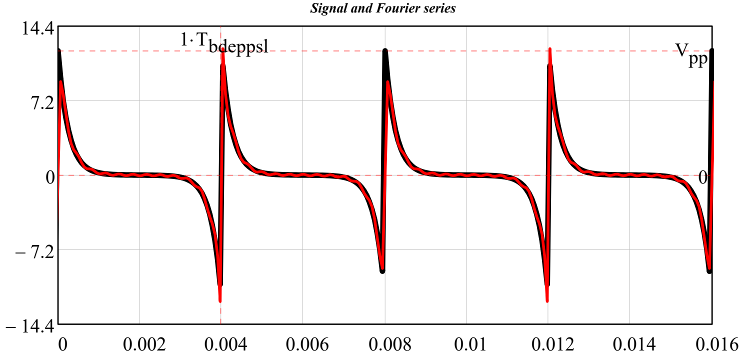
Bipolar Double Exponential Pulse Train



$N1\_ := 50$ $\qquad \omega_{bde} := \dfrac{2 \cdot \pi}{T_{bdeppsl}}$ $\qquad Vbdeospp(t) := \dfrac{V_{bdeospp}\left(t, \tau_{ptd\_}, T_{bdeppsl}, V_{pp}, N0_{gd}\right)}{V}$

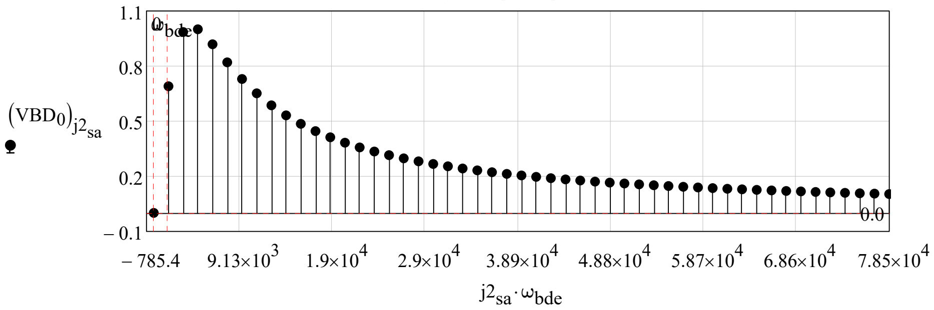
$VBD := SPCT\left(Vbdeospp, rt_{gd}, N1\_, 0 \cdot s, T_{bdeppsl}\right) \qquad N1\_ = 50$

Signal and Fourier series

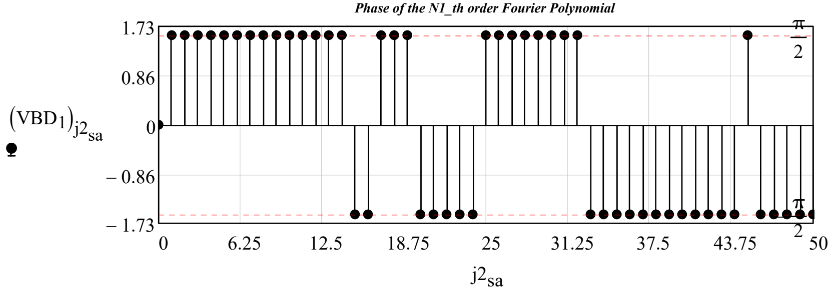


$j2_{sa} := 0 .. rows\left(VBD_0\right) - 1 \qquad \omega_{bde} = 1.571 \cdot \dfrac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := VBD_3 \cdot Hz$

$Bw_{sa} = 0.012 \cdot MHz$

sampling frequency: $\qquad fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.024 \cdot MHz$

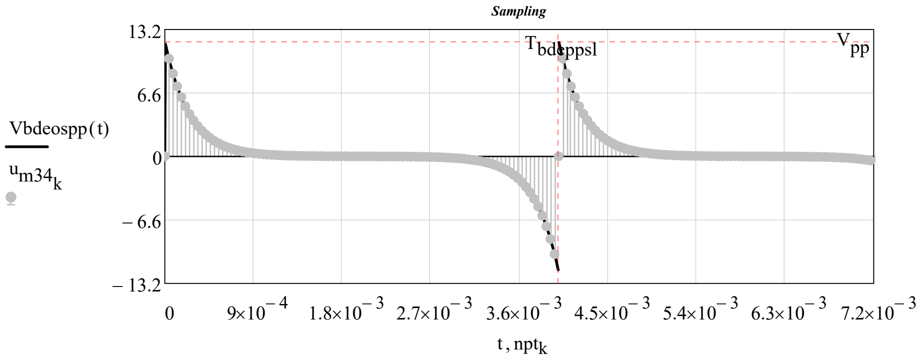
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\qquad \dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{bdeppsl}} = 2.667$

$$\left(u_{m34}\right)_k := Vbdeospp\left(npt_k\right)$$

$u_{m34}{}^T =$

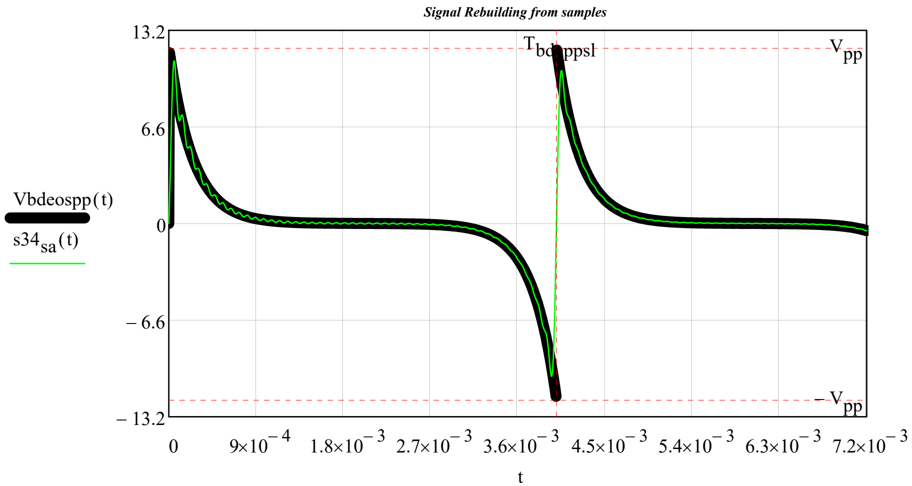
	0	1	2	3	4
0	-1.35·10^-6	10.158	8.598	7.278	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa} \qquad \omega_{bwr} = 0.075\cdot\frac{Mrads}{sec} \qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

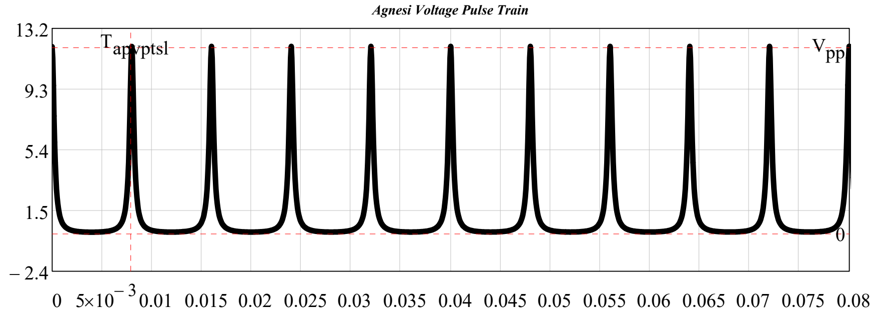
interpolation formula: $\quad s34_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m34_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right] \qquad N0_{gd} - 1 = 255$



Periodic Waveforms

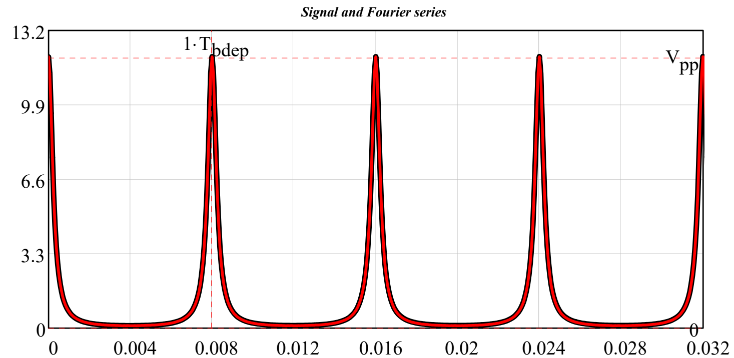
33 Agnesi Profile Voltage Pulse Train

$$T_{apvptsl} := 32\cdot\tau_{ptd\_}$$

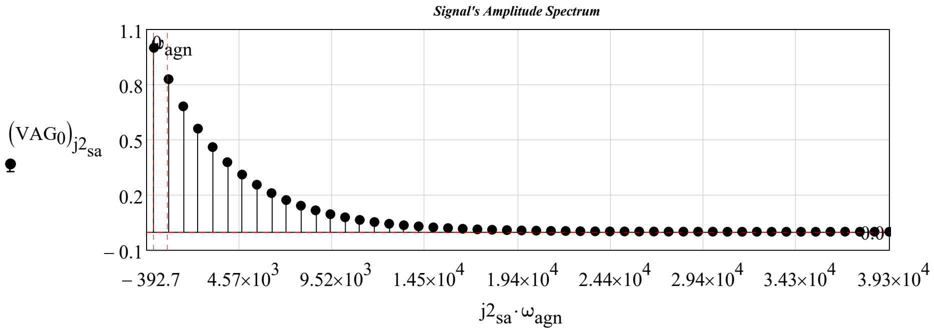


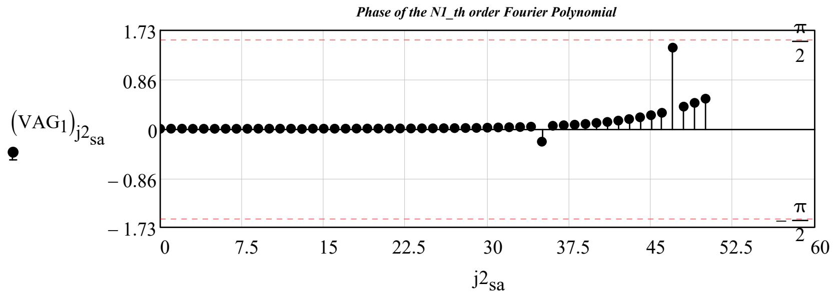
$N1\_ := 50 \qquad \omega_{agn} := \frac{2\cdot\pi}{T_{apvptsl}} \qquad Vagnp(t) := \frac{V_{agnp}\left(t,\tau_{ptd\_},T_{apvptsl},V_{pp},N0_{gd}\right)}{V}$

$VAG := SPCT\left(Vagnp, rt_{gd}, N1\_, 0\cdot s, T_{apvptsl}\right) \qquad N1\_ = 50$



$j2_{sa} := 0..rows\left(VAG_0\right) - 1 \qquad \omega_{agn} = 0.785\cdot\frac{krads}{s}$





$Bw_{sa} := VAG_3 \cdot Hz$

$Bw_{sa} = 2.875 \times 10^{-3} \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 5.75 \times 10^{-3} \cdot MHz$

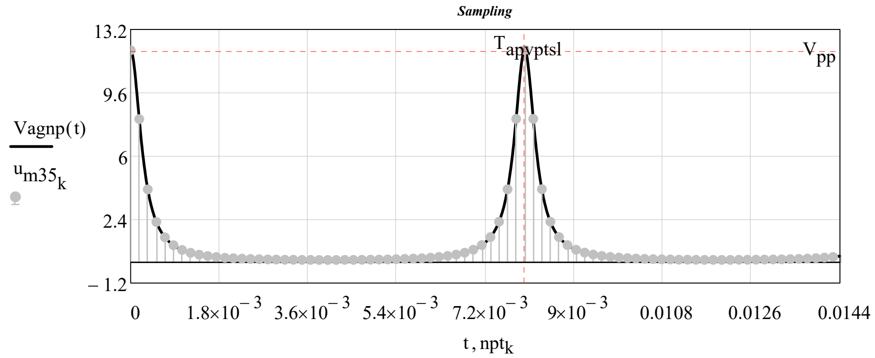
$npt_k := \dfrac{k}{fpt_{so}}$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{bdeppsl}} = 11.13$

$(u_{m35})_k := Vagnp(npt_k)$

$u_{m35}^{T} =$

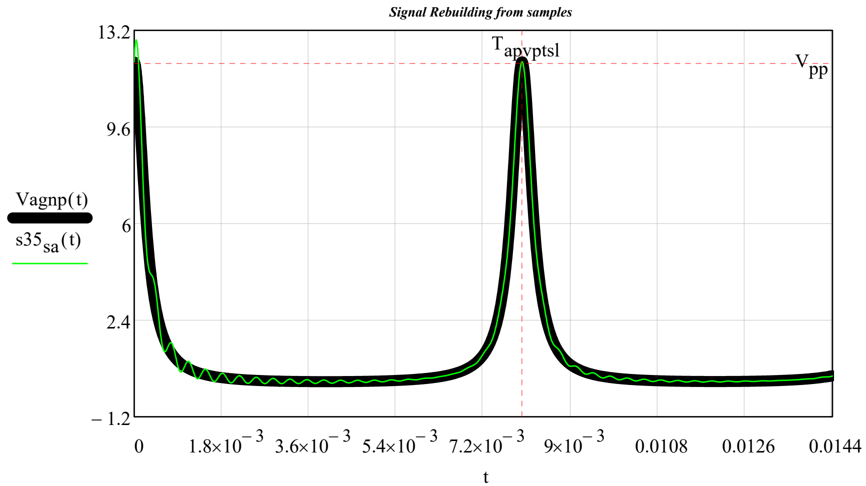
	0	1	2	3	4	5	6	7
0	12.019	8.106	4.108	2.262	1.395	0.939	0.675	...



$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.018 \cdot \dfrac{Mrads}{sec}$ $\quad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

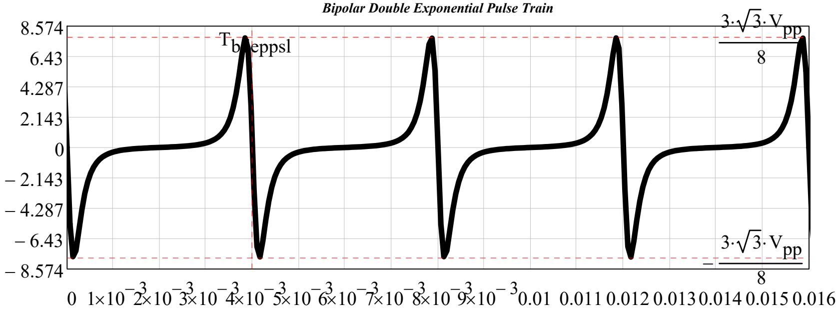
interpolation formula: $s35_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m35_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\quad N0_{gd} - 1 = 255$



Periodic Waveforms

34 Agnesi Derivative Profile Voltage Pulse Train

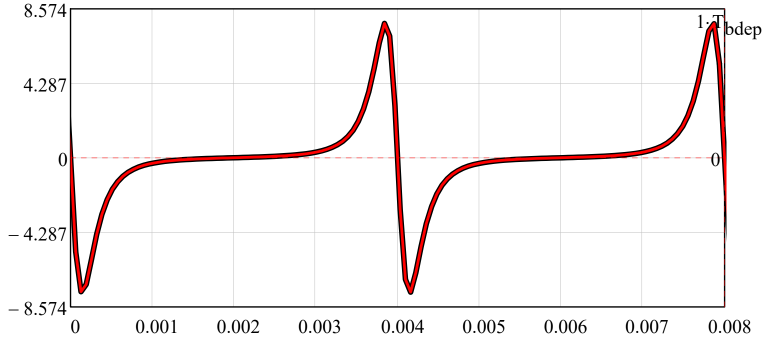
Bipolar Double Exponential Pulse Train



$N1\_ := 50$ $\omega_{bde} := \frac{2\cdot\pi}{T_{bdeppsl}}$ $VDagnp(t) := \frac{V_{Dagnp}(t, \tau_{ptd\_}, T_{bdeppsl}, V_{pp}, N0_{gd})}{V}$

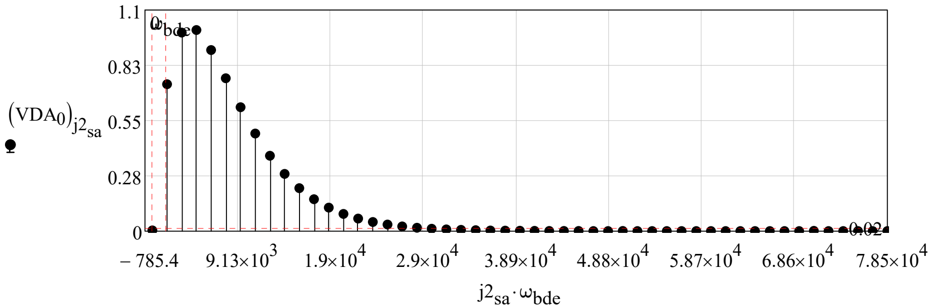
$VDA := SPCT(VDagnp, rt_{gd}, N1\_, 0\cdot s, T_{bdeppsl})$ $N1\_ = 50$

Signal and Fourier series

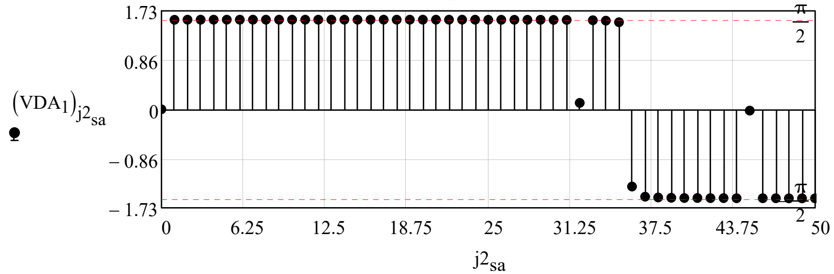


$j2_{sa} := 0 .. rows(VDA_0) - 1$ $\omega_{bde} = 1.571\cdot\frac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := VDA_3\cdot Hz$

$Bw_{sa} = 4.5\times 10^{-3}\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $fpt_{so} = 9\times 10^{-3}\cdot MHz$

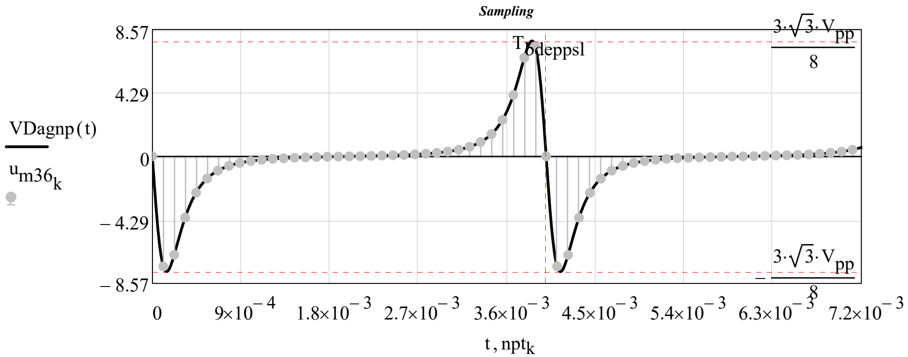
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{bdeppsl}} = 7.111$

$(u_{m36})_k := VDagnp(npt_k)$

$u_{m36}^T =$

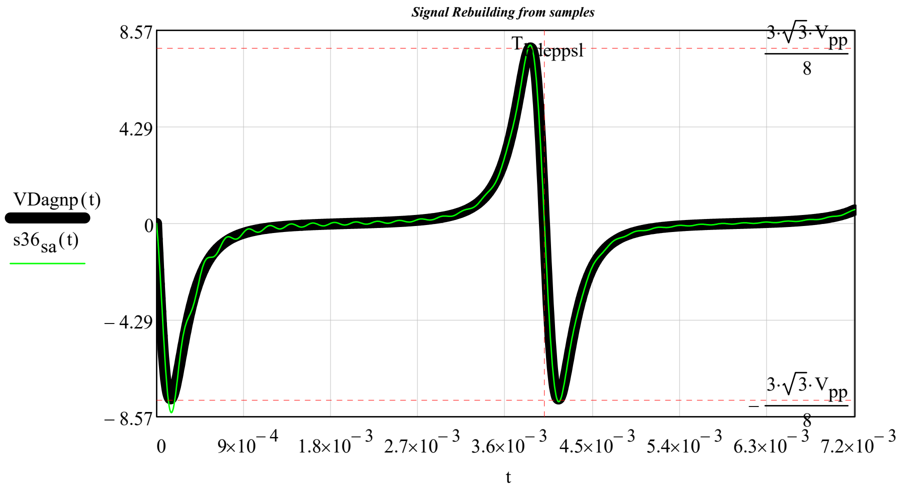
	0	1	2	3	4
0	$6.996\cdot 10^{-3}$	-7.43	-6.649	-4.138	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.028\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s36_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m36_n}\cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$



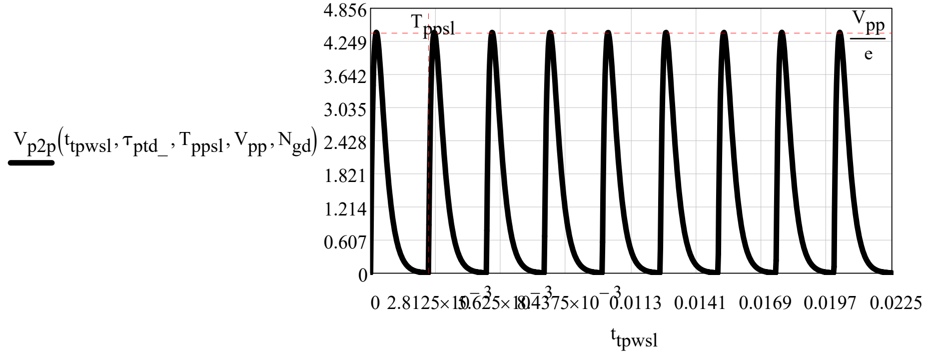
TEST Waveforms

Periodic Waveforms

35 Poisson Profile Voltage Pulse Train

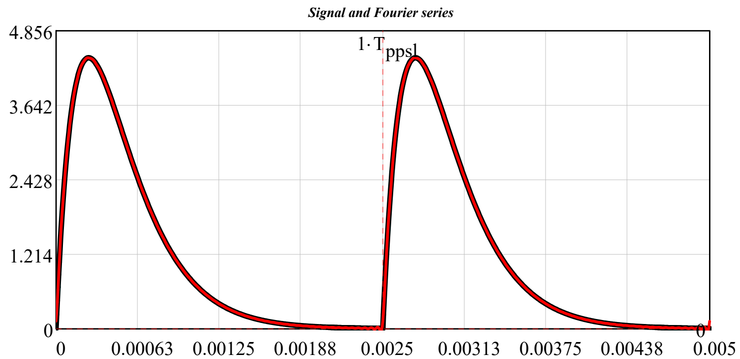
$$t_{tpw} := 0\cdot\tau_{ptd\_}, 0\cdot\tau_{ptd\_} + \frac{200\cdot\tau_{ptd\_}}{2000} .. 200\cdot\tau_{ptd\_}$$

$$T_{ppsl} := 10\cdot\tau_{ptd\_}$$



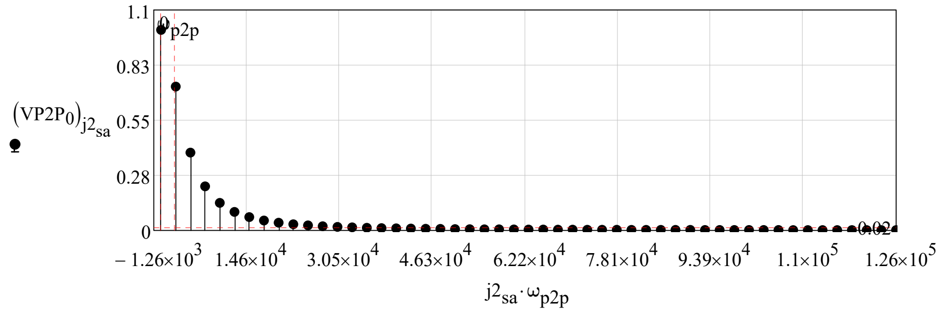
$\omega_{p2p} := \frac{2\cdot\pi}{T_{ppsl}}$ $\qquad V_{p2p}(t) := \frac{V_{p2p}(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd})}{V}$

$VP2P := SPCT(V_{p2p}, rt_{gd}, N1\_, 0\cdot s, T_{ppsl})$ $\qquad N1\_ = 50$

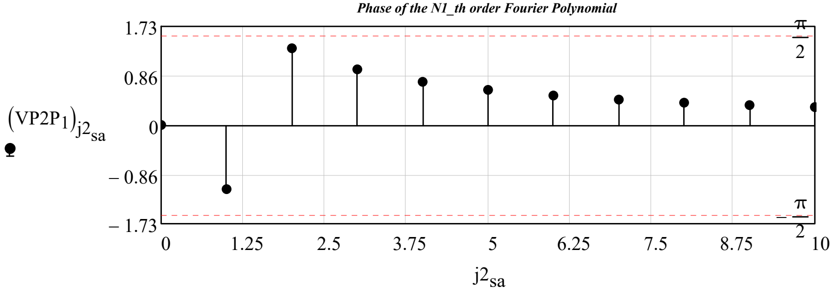


$j2_{sa} := 0 .. rows(VP2P_0) - 1$ $\qquad \omega_{p2p} = 2.513\cdot\frac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := VP2P_3\cdot Hz$

$Bw_{sa} = 0.01\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $fpt_{so} = 0.02\cdot MHz$

$npt_k := \dfrac{k}{fpt_{so}}$

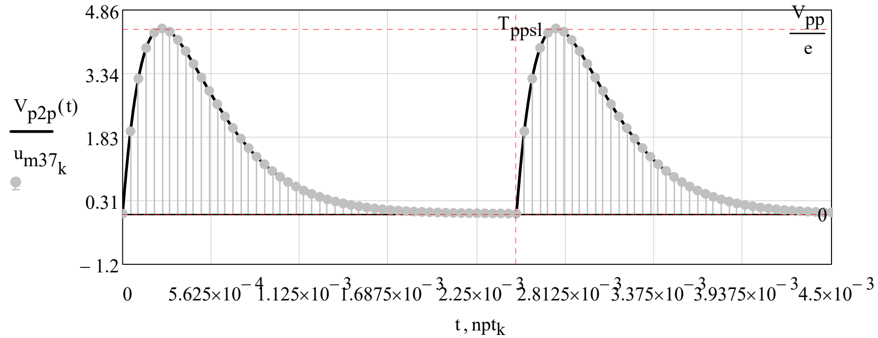
Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}}\cdot\dfrac{1}{T_{ppsl}} = 5.12$

$(u_{m37})_k := V_{p2p}(npt_k)$

$u_{m37}^{T} =$

	0	1	2	3	4
0	0	1.965	3.218	3.951	...

Sampling

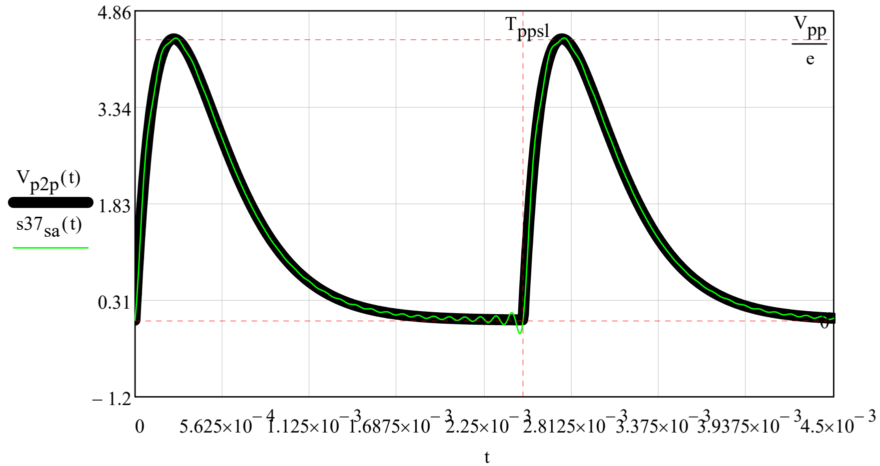


$relerr = 10\cdot\%$ $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.063\cdot\dfrac{Mrads}{sec}$ $n\cdot\dfrac{\pi}{\omega_{bwr}} = n\cdot\dfrac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s37_{sa}(t) := \left[\displaystyle\sum_{n=0}^{N0_{gd}-1}\left(u_{m37_n}\cdot \mathrm{sinc}(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $N0_{gd} - 1 = 255$ relerr =

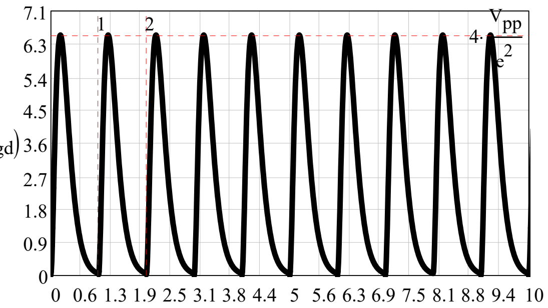
Signal Rebuilding from samples



Periodic Waveforms

36 Poisson Derivative Profile Voltage Pulse Train

$V_{2pDp}(t_{tpw}, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd})$



$\frac{t_{tpw}}{T_{ppsl}}$

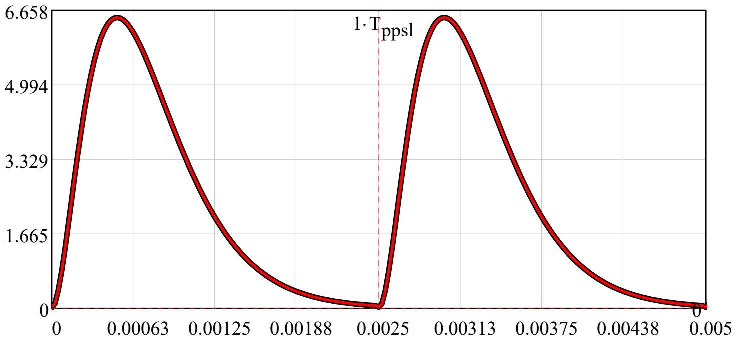
$$\omega_{2pDp} := \frac{2\cdot\pi}{T_{ppsl}} \qquad V2pDp(t) := \frac{V_{2pDp}(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd})}{V}$$

$$V2P := SPCT(V2pDp, rt_{gd}, N1\_, 0\cdot s, T_{ppsl}) \qquad N1\_ = 50$$

Signal and Fourier series

$V2pDp(t_{tpw})$

$fs(t_{tpw}, V2P_9, V2P_{10}, T_{ppsl}, N1\_)$

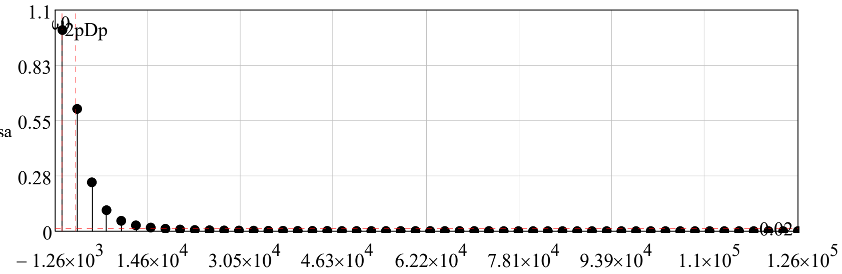


t_{tpw}

$$j2_{sa} := 0..rows(V2P_0) - 1 \qquad \omega_{2pDp} = 2.513\cdot\frac{krads}{s}$$

Signal's Amplitude Spectrum

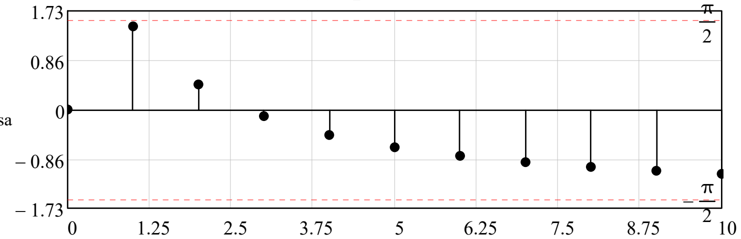
$(V2P_0)_{j2_{sa}}$



$j2_{sa}\cdot\omega_{2pDp}$

Phase of the N1_th order Fourier Polynomial

$(V2P_1)_{j2_{sa}}$



$j2_{sa}$

$$Bw_{sa} := V2P_3\cdot Hz$$

$$Bw_{sa} = 4.8\times10^{-3}\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 9.6\times10^{-3}\cdot MHz$

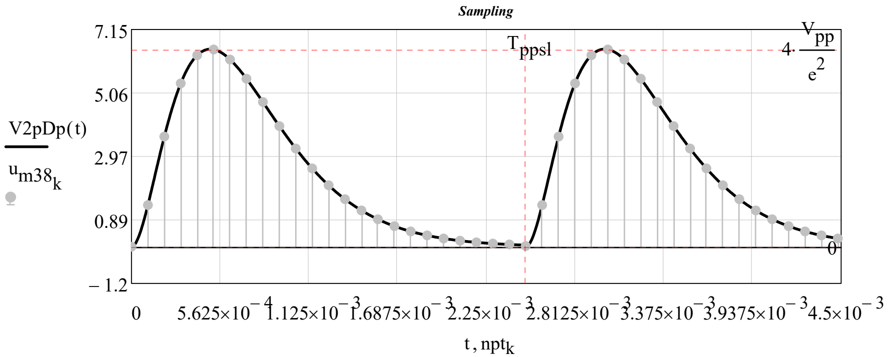
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{ppsl}} = 10.667$

$$(u_{m38})_k := V2pDp(npt_k)$$

$u_{m38}^T =$

	0	1	2	3	4	5	6	7	8
0	0	1.373	3.622	5.372	6.296	6.485	6.156	5.524	...

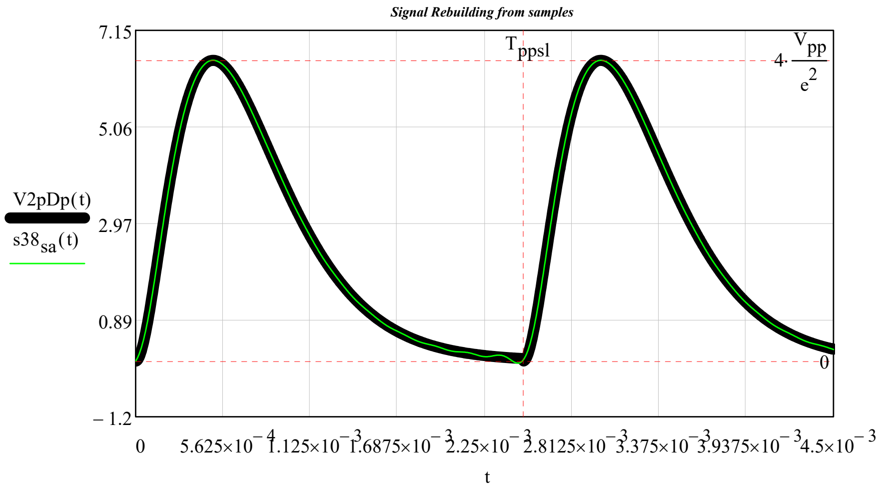


relerr = 10·%

$\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.03\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

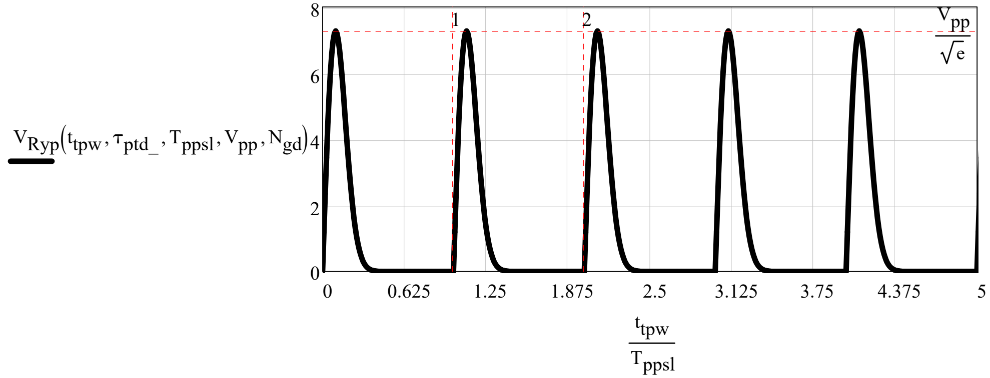
interpolation formula: $s38_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m38_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ relerr =



TEST Waveforms

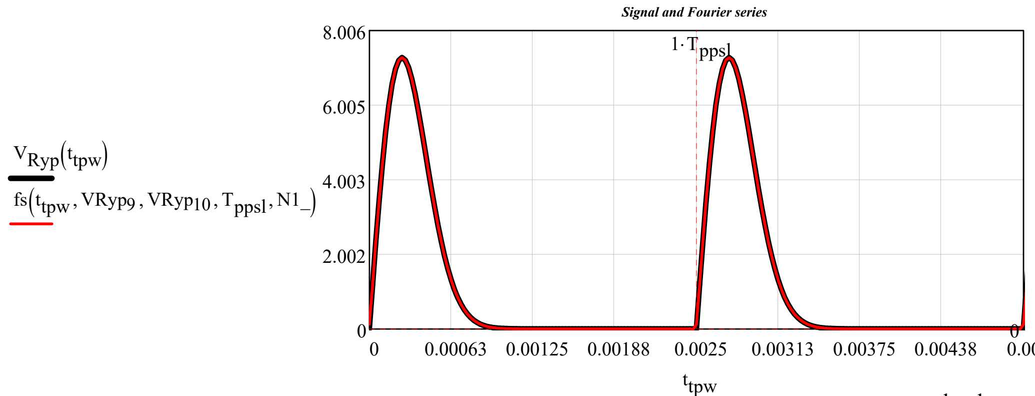
Periodic Waveforms

37 Rayleigh Profile Voltage Pulse Train

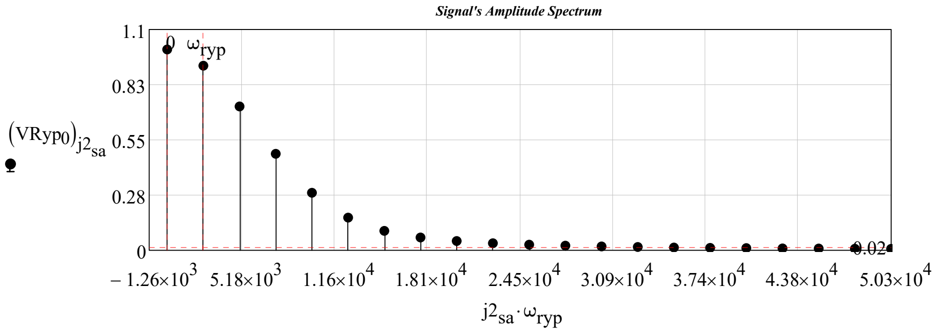


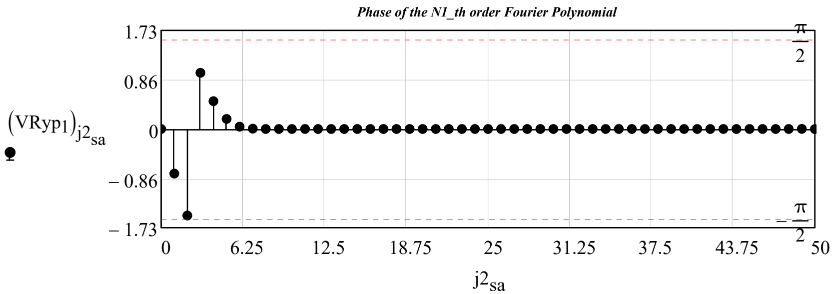
$\omega_{ryp} := \frac{2\cdot\pi}{T_{ppsl}}$ $V_{Ryp}(t) := \frac{V_{Ryp}\left(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd}\right)}{V}$

$VRyp := SPCT\left(V_{Ryp}, rt_{gd}, N1\_, 0\cdot s, T_{ppsl}\right)$ $N1\_ = 50$



$j2_{sa} := 0 .. rows\left(VRyp_0\right) - 1$ $\omega_{ryp} = 2.513\cdot\frac{krads}{s}$





$Bw_{sa} := VRyp_3 \cdot Hz$

$Bw_{sa} = 8 \times 10^{-3} \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.016 \cdot MHz$

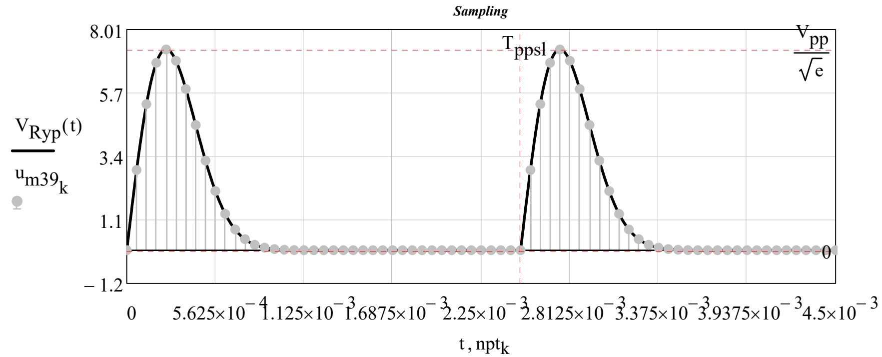
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{ppsl}} = 6.4$

$$(u_{m39})_k := V_{Ryp}(npt_k)$$

$u_{m39}^T =$

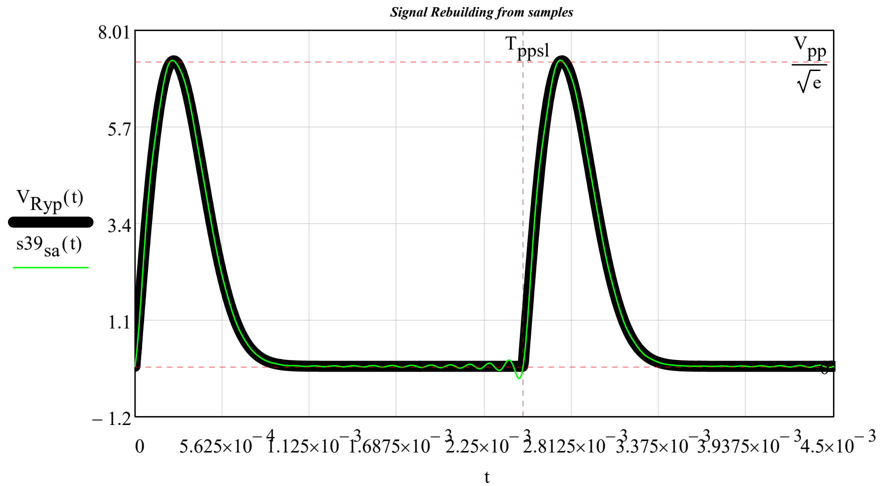
	0	1	2	3	4
0	0	2.908	5.295	6.794	...



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.05 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $$s39_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m39_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$$ $N0_{gd} - 1 = 255 \quad$ relerr = 10·%



Periodic Waveforms

38 Cap. Charge and Discharge Pulse Train

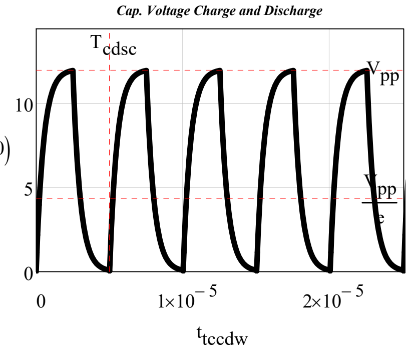
$\tau_{end} = 2.5 \cdot \mu s$ $\quad V_{pp} = 12\,V$

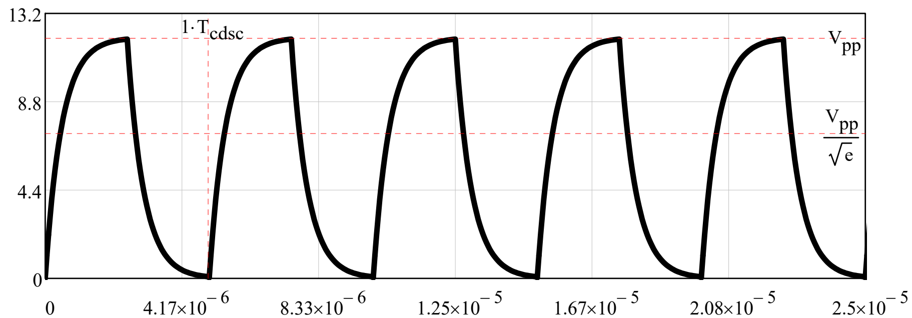
pulse width: $\quad 2 \cdot \tau_{end}$

time constant $\quad \tau_c = 0.5 \cdot \mu s$

Period: $\quad T_{cdsc} := 2 \cdot \tau_{end} \qquad \omega_{cdsc} := \dfrac{2 \cdot \pi}{T_{cdsc}}$

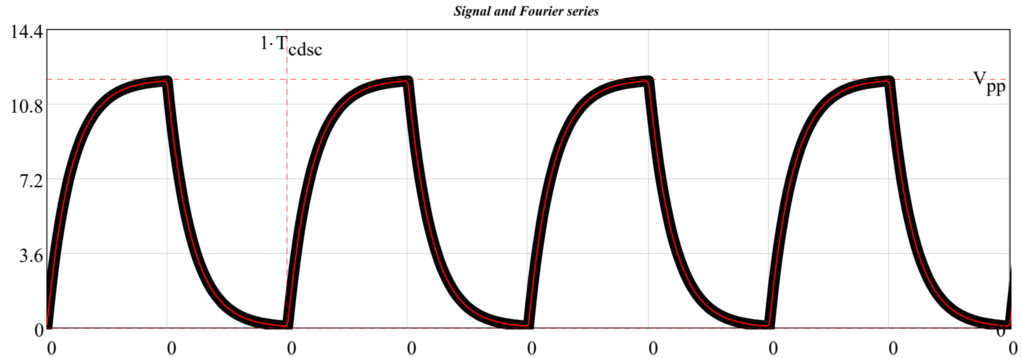
$$t_{tccdw} := 0 \cdot T_{cdsc}, 0 \cdot T_{cdsc} + \frac{100 \cdot T_{cdsc}}{10000} .. 100 \cdot T_{cdsc}$$



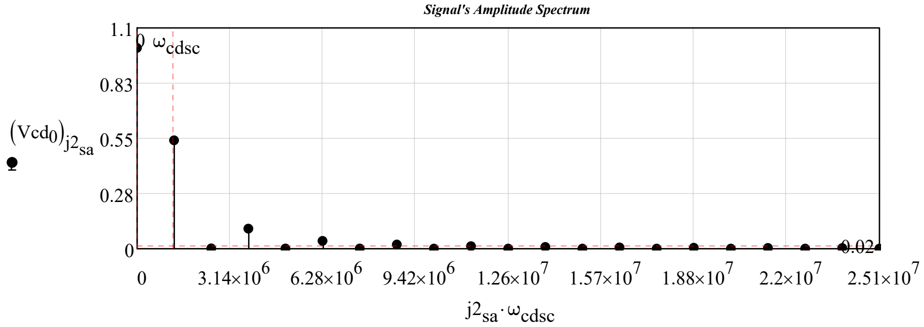


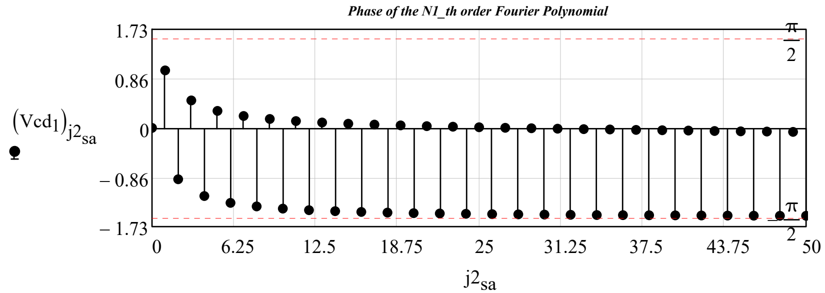
$$VCcd(t) := \frac{V_{Ccd}(t, T_{cdsc}, \tau_{end}, \tau_c, V_{pp}, N1\_)}{V}$$

$$Vcd := SPCT(VCcd, rt_{gd}, N1\_, 0 \cdot s, T_{cdsc}) \qquad N1\_ = 50$$



$$j2_{sa} := 0 .. rows(Vcd_0) - 1 \qquad \omega_{cdsc} = 1.257 \cdot \frac{Mrads}{s}$$





$Bw_{sa} := Vcd_3 \cdot Hz$

$Bw_{sa} = 5.2 \cdot MHz$

sampling frequency: $\quad fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 10.4 \cdot MHz$

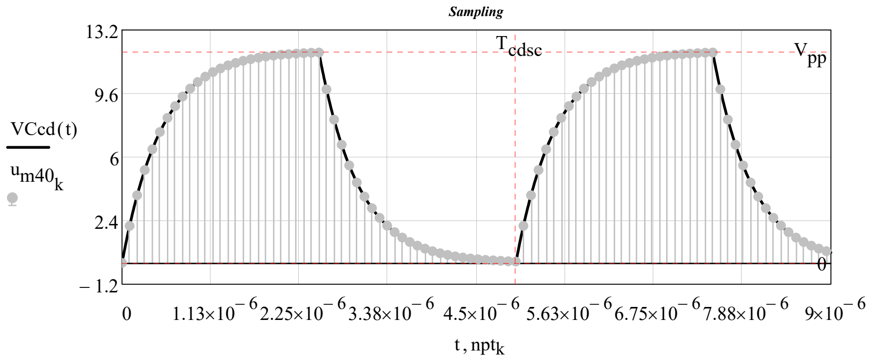
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{cdsc}} = 4.923$

$$u_{m40_k} := VCcd\left(npt_k\right)$$

$u_{m40}^{T} =$

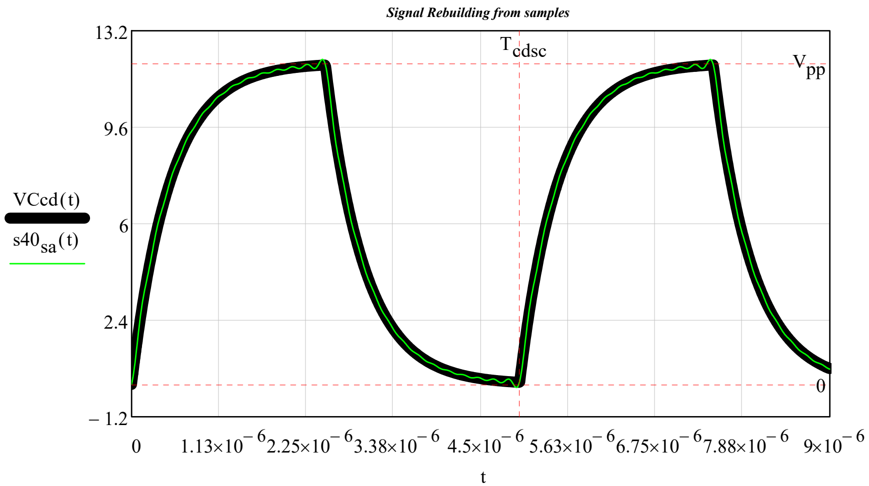
	0	1	2	3	4	5	6	7
0	0	2.099	3.831	5.261	6.44	7.412	8.215	...



relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 32.673 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

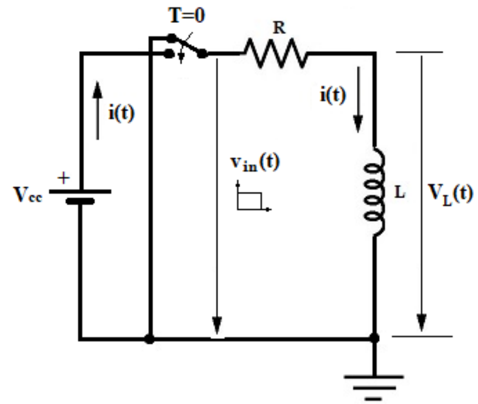
interpolation formula: $s40_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m40_n} \cdot \mathrm{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi \right) \right) \right]$ $N0_{gd} - 1 = 255$ relerr =

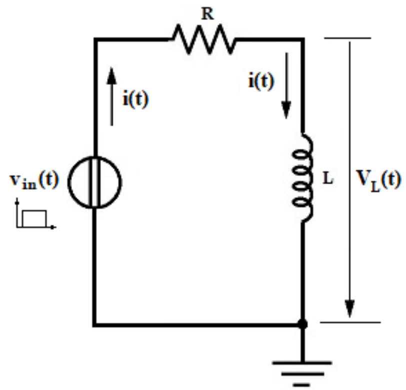


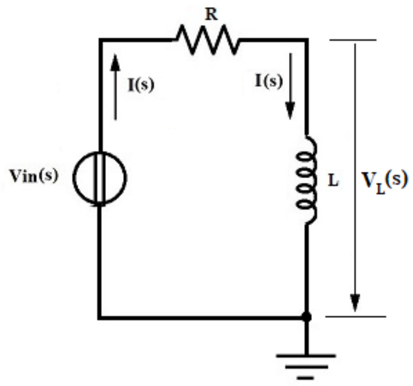
Periodic Waveforms

39 Induct Charge and Discharge Pulse Train

$R := 1 \cdot k\Omega$ $L := 1 \cdot \mu H$







$$V_L(s) = \frac{s \cdot L}{R + s \cdot L} \cdot V_{in}(s) \qquad I_L(s) = \frac{V_L(s)}{s \cdot L}$$

$$V_{in}(s) = V_{pp} \cdot \mathcal{L}\{\Phi(t) - \Phi(t-\tau)\} = V_{pp} \cdot \left(\mathcal{L}\{\Phi(t)\} - \mathcal{L}\{\Phi(t-\tau)\} \right) = V_{pp} \cdot \left(\frac{1}{s} - \frac{1}{s} \cdot e^{-\tau \cdot s} \right)$$

$$\Phi(t) - \Phi(t-\tau) \begin{vmatrix} \text{assume}, \tau > 0 \\ \text{laplace} \end{vmatrix} \rightarrow -\frac{e^{-\tau \cdot s} - 1}{s}$$

$\omega_0 := \frac{R}{L}$ $\tau_L := \frac{1}{\omega_0}$ $\tau_L = 1 \cdot ns$ $\tau := \frac{5}{\omega_0}$ $\omega_0 = 1 \cdot \frac{Grads}{s}$ $\tau = 5 \cdot ns$

$\omega_0 := \omega_0$ $\tau := \tau$

$$\boxed{V_{in}(s) = V_{pp} \cdot \frac{1 - e^{-\tau \cdot s}}{s}}$$

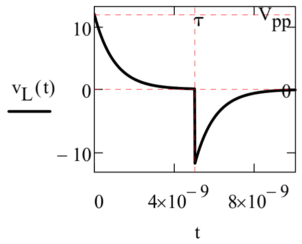
$$V_L(s) = \frac{s\cdot L}{R+s\cdot L}\cdot\left(V_{pp}\cdot\frac{1-e^{-\tau\cdot s}}{s}\right) = V_{pp}\cdot\frac{s}{\frac{R}{L}+s}\cdot\frac{1-e^{-\tau\cdot s}}{s} = V_{pp}\cdot\frac{s}{\omega_0+s}\cdot\frac{1-e^{-\tau\cdot s}}{s}$$

$$\frac{s}{\omega_0+s}\cdot\frac{1-e^{-\tau\cdot s}}{s}$$

assume, ALL = real
assume, $\omega_0 > 0$
assume, $\tau > 0$
invlaplace, s
simplify

$$\rightarrow e^{-t\cdot\omega_0}\cdot\left(e^{\tau\cdot\omega_0}\cdot\Phi(\tau-t) - e^{\tau\cdot\omega_0} + 1\right)$$

$$v_L(t) := V_{pp}\cdot e^{\frac{-t}{\tau_L}}\cdot\left[e^{\frac{\tau}{\tau_L}}\cdot(\Phi(\tau-t)-1)+1\right]$$



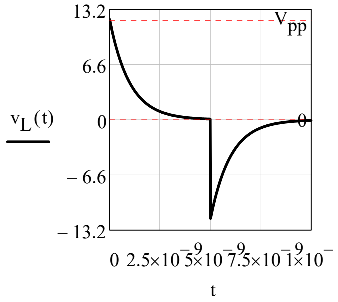
$$I_L(s) = \frac{V_L(s)}{s\cdot L} = \frac{V_{pp}}{L}\cdot\frac{1}{(\omega_0+s)}\cdot\frac{1-e^{-\tau\cdot s}}{s}$$

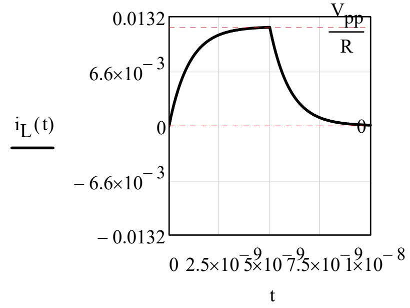
$$\frac{1}{\omega_0+s}\cdot\frac{1-e^{-\tau\cdot s}}{s}$$

assume, ALL = real
assume, $\omega_0 > 0$
assume, $\tau > 0$
invlaplace, s
simplify
collect, $e^{-t\cdot\omega_0}$

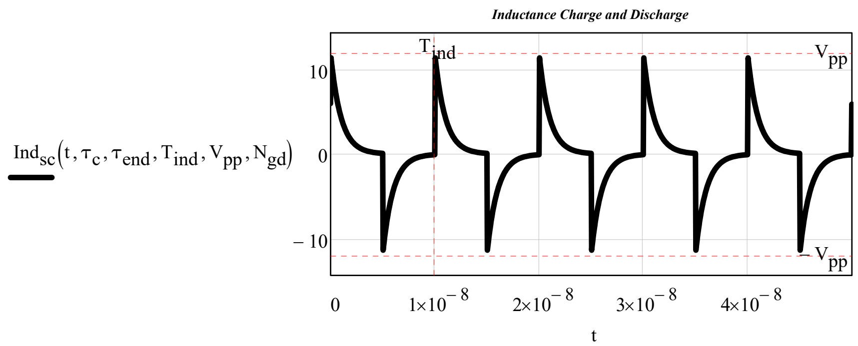
$$\rightarrow \left(-\frac{e^{\tau\cdot\omega_0}\cdot\Phi(\tau-t) - e^{\tau\cdot\omega_0}+1}{\omega_0}\right)\cdot e^{-t\cdot\omega_0} + \frac{\Phi(\tau-t)}{\omega_0}$$

$$i_L(t) := \frac{V_{pp}}{R}\cdot\left[\left[1-e^{\left(\frac{\tau-t}{\tau_L}\right)}\right]\cdot\Phi(\tau-t) + e^{\frac{-t}{\tau_L}}\cdot\left(e^{\frac{\tau}{\tau_L}}-1\right)\right]$$



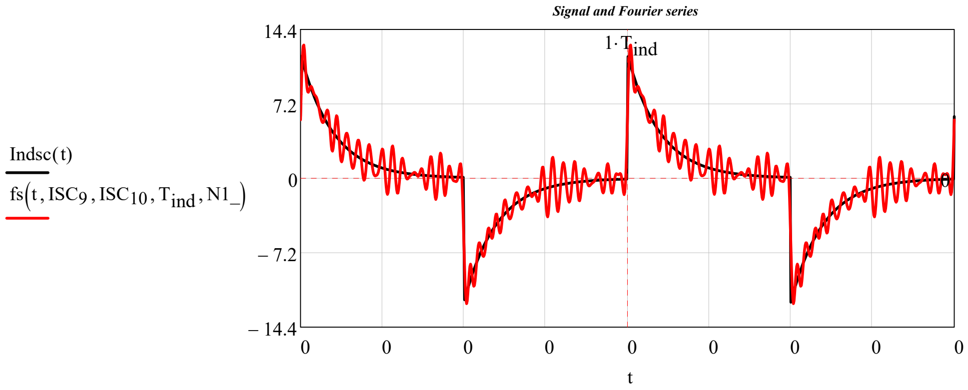


$T_{ind} := 2\cdot\tau$ $\tau_{end} := \tau$ $\tau_c = 1\cdot ns$

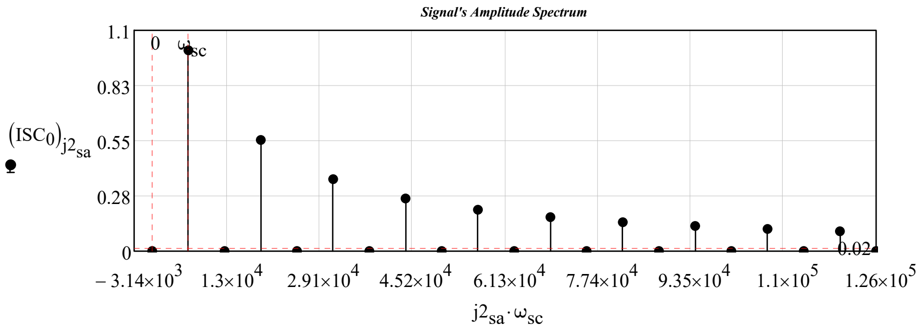


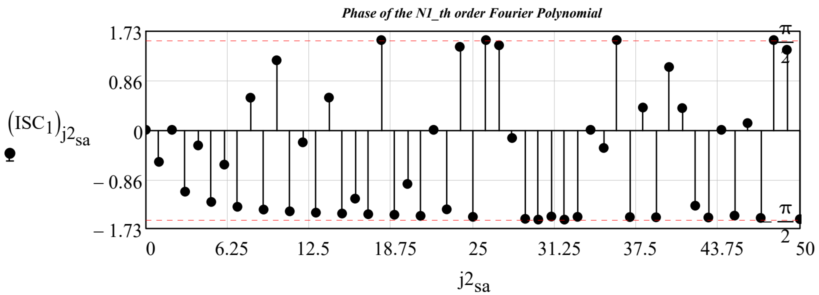
$$Indsc(t) := \frac{Ind_{sc}(t, \tau_c, \tau_{end}, T_{ind}, V_{pp}, N_{gd})}{V}$$

$\omega_{sc} := \frac{2\cdot\pi}{T_{0gd}}$ $ISC := SPCT(Indsc, rt_{gd}, N1\_, 0\cdot s, T_{ind})$ $N1\_ = 50$



$j2_{sa} := 0..\,rows(ISC_0) - 1$ $\qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := ISC_3 \cdot Hz$

$Bw_{sa} = 4.8 \times 10^3 \cdot MHz$

sampling frequency: $\quad fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 9.6 \times 10^3 \cdot MHz$

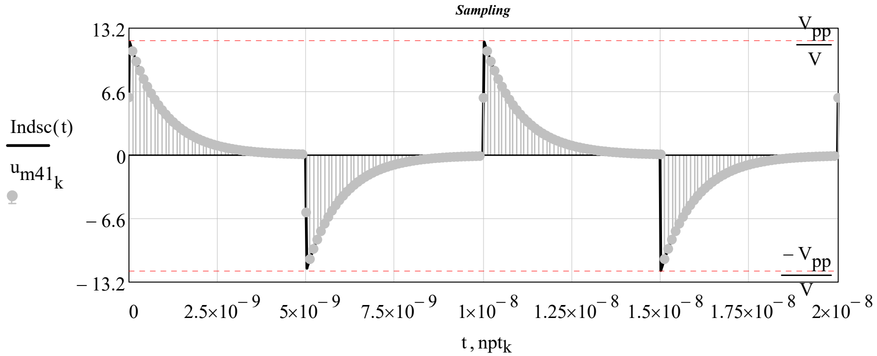
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\quad \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{ind}} = 2.667$

$u_{m41_k} := Indsc(npt_k)$

$u_{m41}^T =$

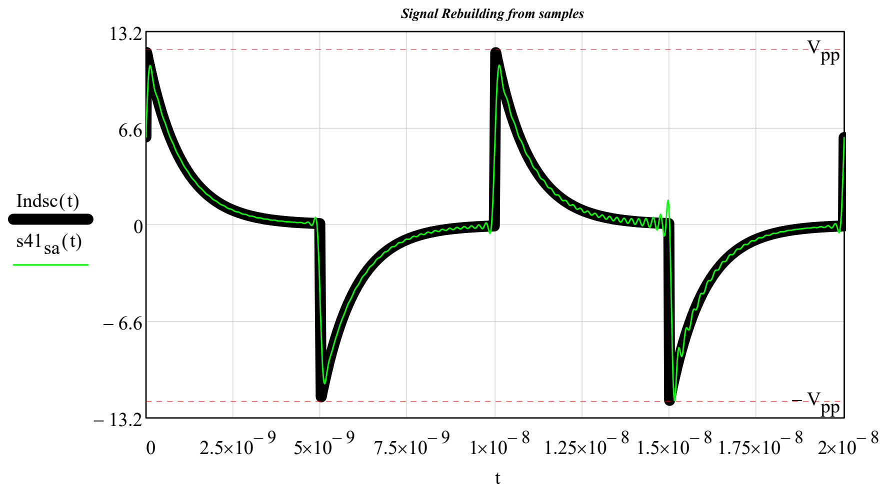
	0	1	2	3	4	5	6
0	6	10.813	9.743	8.779	7.911	7.128	...



$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 3.016 \times 10^4 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s41_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m41_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10 \cdot \%$



Periodic Waveforms

40 Parabolic Cusps Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$

Pulse width: $p_{wsl} = 250 \cdot \mu s$

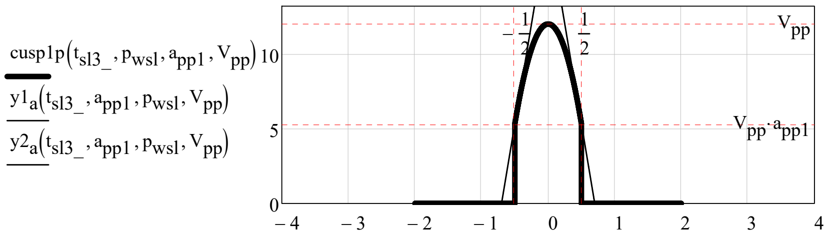
Duty cycle: $\delta_{cysl} := \gamma$

Period: $T_{pcsp} := \frac{p_{wsl}}{\delta_{cysl}}$ $\quad \omega_{pcsp} := \frac{2 \cdot \pi}{T_{pcsp}}$

Max pulse amplitude and cusp ratio: $a_{pp1} := \frac{4}{9}$

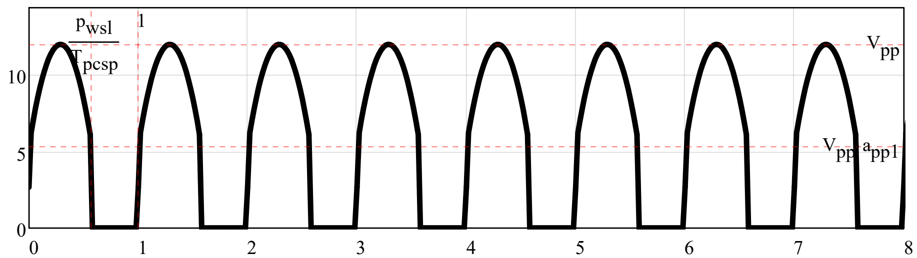
$$t_{sl3\_} := -2 \cdot p_{wsl}, -2 \cdot p_{wsl} + \frac{(2 \cdot p_{wsl} + 2 \cdot p_{wsl})}{10000} .. 2 \cdot p_{wsl}$$

Parabolic Cusp Pulse



$$t_{t11w} := 0 \cdot T_{pcsp}, 0 \cdot T_{pcsp} + \frac{10 \cdot T_{pcsp} - 0 \cdot T_{pcsp}}{500} .. 10 \cdot T_{pcsp}$$

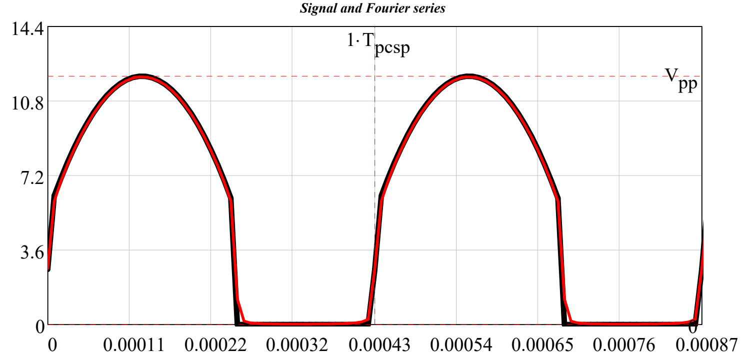
Parabolic Cusp Pulse Train



$$Csp11(t) := \frac{csp11(t, p_{wsl}, a_{pp1}, T_{pcsp}, V_{pp}, N0_{gd})}{V}$$

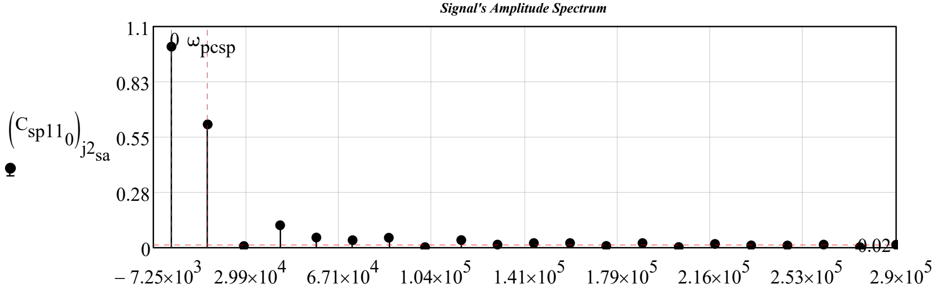
$C_{sp11} := SPCT(Csp11, rt_{gd}, N1\_, 0 \cdot s, T_{pcsp})$ $\quad N1\_ = 50$

Signal and Fourier series

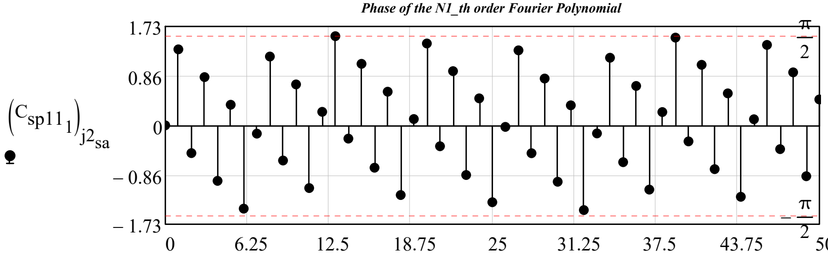


$j2_{sa} := 0 .. rows(C_{sp11_0}) - 1$ $\quad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := C_{sp11_3} \cdot Hz$

$Bw_{sa} = 0.111 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.222 \cdot MHz$

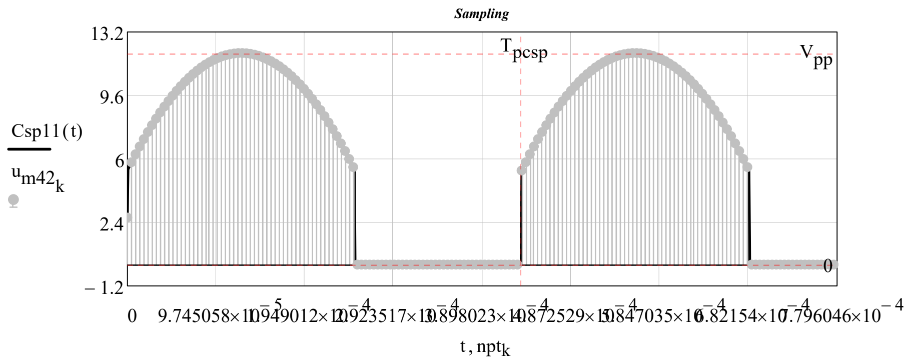
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pcsp}} = 2.667$

$$u_{m42_k} := Csp11(npt_k)$$

$u_{m42}{}^T =$

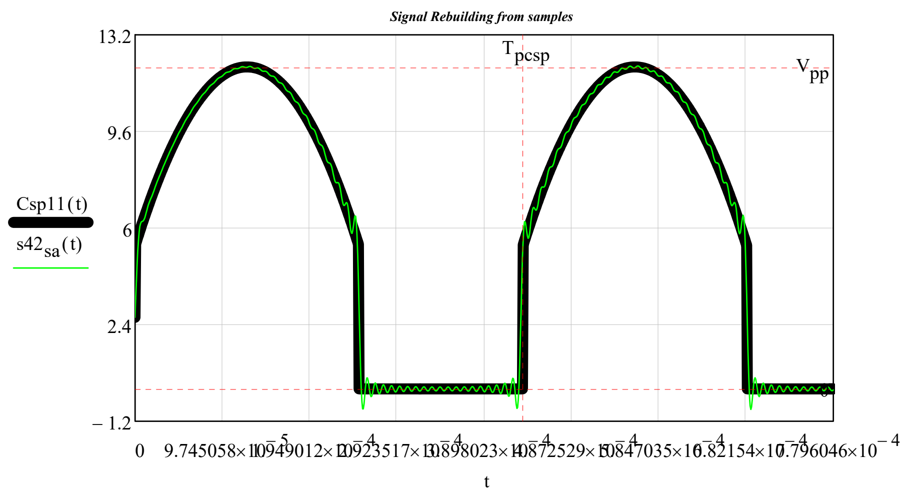
	0	1	2	3	4	5	6	7
0	2.667	5.806	6.261	6.699	7.119	7.522	7.908	...



relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.696 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s42_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m42_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $N0_{gd} - 1 = 255$



TEST Waveforms

Periodic Waveforms

41 Elliptic Cusps Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$

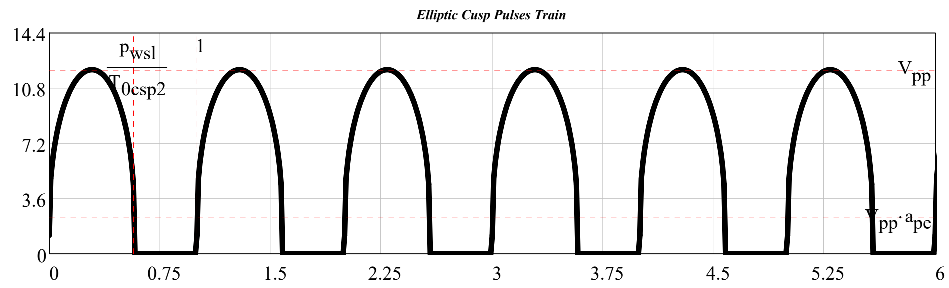
Pulse width: $p_{wsl} = 250 \cdot \mu s$

Duty cycle: $\delta_{cysl} := \gamma$

Period: $T_{0csp2} := \frac{p_{wsl}}{\delta_{cysl}}$

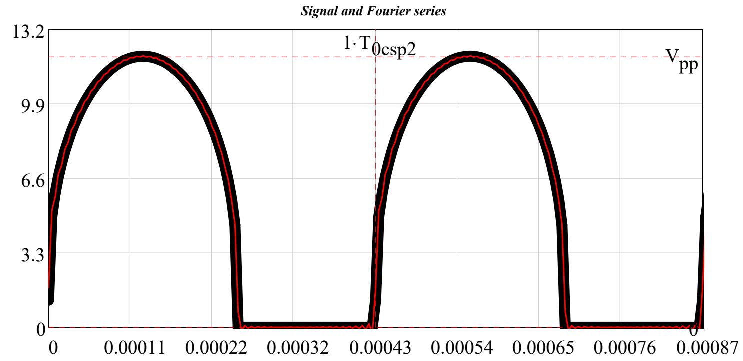
Max pulse amplitude and cusp ratio: $a_{pe} := \frac{2}{10}$

$$t_{22sl} := 0 \cdot T_{0csp2}, 0 \cdot T_{0csp2} + \frac{10 \cdot T_{0csp2} - 0 \cdot T_{0csp2}}{1000} .. 10 \cdot T_{0csp2}$$

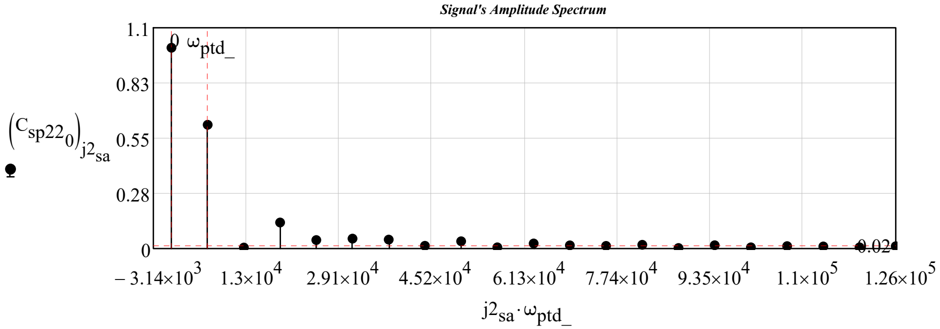


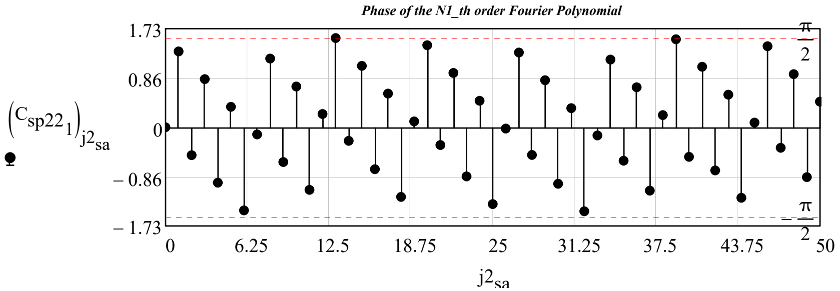
$$Csp22(t) := \frac{csp22(t, p_{wsl}, a_{pe}, T_{0csp2}, V_{pp}, N0_{gd})}{V}$$

$C_{sp22} := SPCT(Csp22, rt_{gd}, N1\_, 0 \cdot s, T_{0csp2})$ $N1\_ = 50$



$j2_{sa} := 0..rows\left(C_{sp22_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := C_{sp22_3} \cdot Hz$

$Bw_{sa} = 0.111 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.222 \cdot MHz$

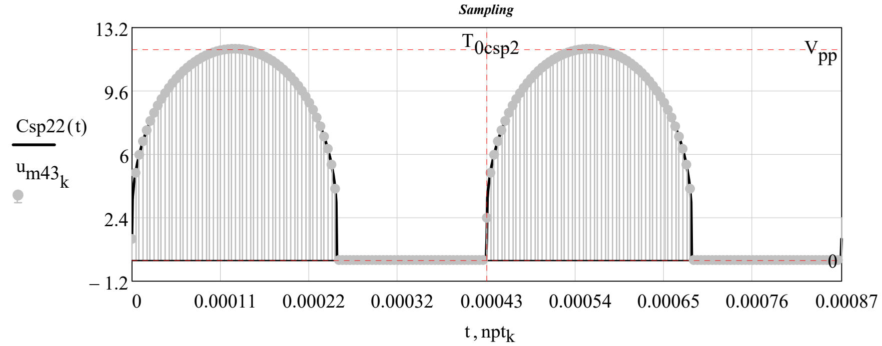
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0csp2}} = 2.667$

$u_{m43_k} := Csp22\left(npt_k\right)$

$u_{m43}^T =$

	0	1	2	3	4	5	6	7
0	1.2	4.956	5.981	6.745	7.369	7.901	8.366	...

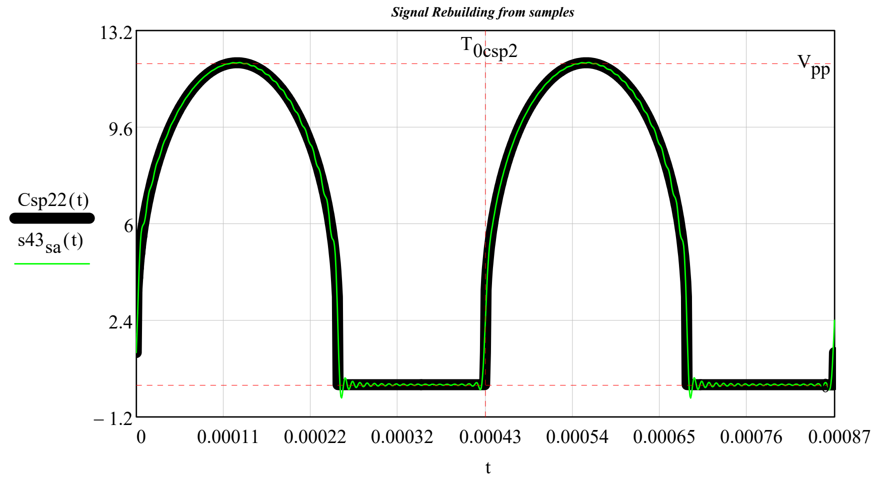


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.696 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s43_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m43_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10 \cdot \%$



$t1\_ := 0 \qquad \tau_{end\_} := time(t1\_)$

$$\tau_{end\_} = 1.618 \times 10^{9}$$

$$\frac{\tau_{end\_} - \tau_{init\_}}{3600} = 0.801$$

Fine