Beam calculation using singularity functions (#2)

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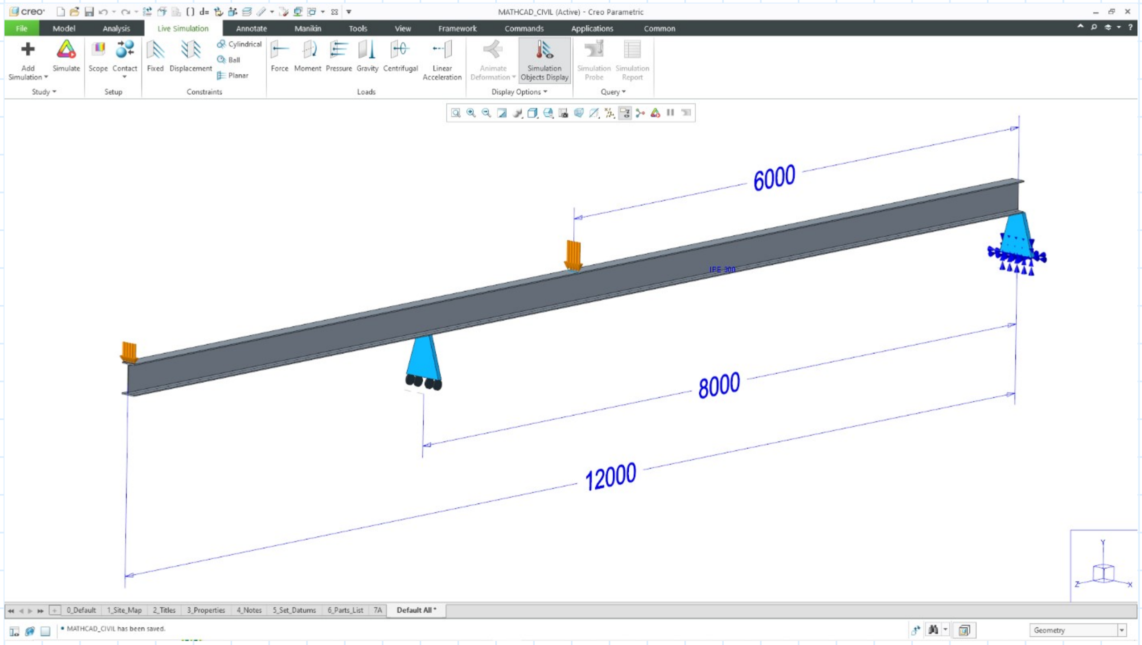
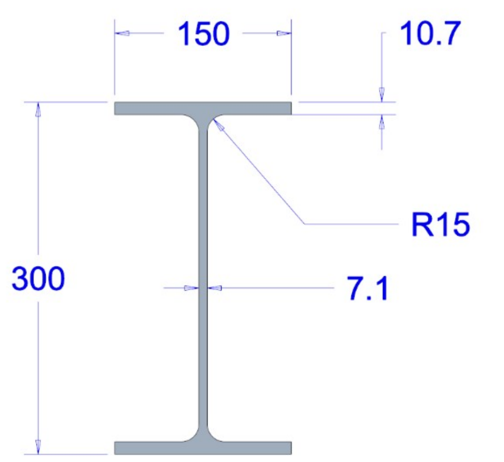




Scenario: You have a standard 300mm steel I-beam with a length of 12 meters. It is simply supported at one end and at 8 meters from that end. There is a load of 5000 Newtons in the middle and 10000 Newtons at the unsupported end.

- Calculate the cross-sectional area of the I-beam.
- Calculate the cross-sectional moment of inertia about the midpoint of the beam. (You may choose to ignore the contribution of the fillets. If you want the additional challenge, you may include them.)
- Use the Chart Component to display shear and bending moment diagrams.
- Calculate the deflection at the end of the I-beam.

You may ignore the effect of the fillets as necessary for shear, bending moment, and deflection.

on a beam must be zero. A simply supported beam is one that rests on two supports and is free to move horizontally. Typical practical applications of simply supported beams with point loadings include bridges, beams in buildings, and beds of machine tools.

2. Input data

matrix and vector index origin parameter

$ORIGIN := 1$

beam dimensions

$w_b := 150\ mm$

$h_b := 300\ mm$

$tv_b := 10.7\ mm$

$th_b := 7.1\ mm$

material properties (copied from steel.mtl in Creo Parametric 9 if applicable; otherwise https://www.tribology-abc.com/sub15.htm)

$\rho_b := 7850\ \frac{kg}{m^3}$

$E_b := 210\ GPa$

input table

- index 1 at supported end
- X_IN: horizontal beam coordinate, measured along beam; positive X oriented towards deflected side
- F_IN: external applied force; positive Y oriented upwards

index	X_IN (mm)	F_IN (N)
1	0	0
2	6000	5000
3	8000	0
4	12000	10000

$dim := \text{rows}(X\_IN) = 4$

$l_b := X\_IN_{dim} = 12\ m$

3. Calculation

calculation will be executed with unitless parameter values, scaled to SI

$$ul(x) := \frac{x}{\text{SIUnitsOf}(x)}$$

$w_b := ul(w_b)$ $h_b := ul(h_b)$ $tv_b := ul(tv_b)$ $th_b := ul(th_b)$ $l_b := ul(l_b)$

$\rho_b := ul(\rho_b)$ $E_b := ul(E_b)$

$X := ul(X\_IN)$ $F := ul(F\_IN)$

cross-sectional area of beam (value in m²)

$$A_b := w_b \cdot h_b - 2 \cdot (h_b - 2 \cdot tv_b) \cdot \left(\frac{w_b - th_b}{2}\right) = 0.005$$

second moment of area around bending axis of beam section (value in m^4)
(= area moment of inertia; source: https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html)

$$I_b := \frac{th_b \cdot (h_b - 2 \cdot tv_b)^3}{12} + \frac{w_b}{12} \cdot \left(h_b^{\ 3} - (h_b - 2 \cdot tv_b)^3\right) = 7.999 \cdot 10^{-5}$$

bending stiffness of the beam
(= flexural rigidity; source: https://en.wikipedia.org/wiki/Bending_stiffness)

$$EI_b := E_b \cdot I_b = 1.68 \cdot 10^7$$

distributed beam weight

$$W_b := \frac{\rho_b \cdot A_b \cdot l_b \cdot ul(g)}{l_b} = 399.388$$

assumption: the beam weight is negligible compared to the applied deformation load > will be neglected

$$W_b := 0$$

singularity function definition (https://en.wikipedia.org/wiki/Singularity_function)

$$f_s(x, a, n) := (x - a)^n \cdot (x > a) \cdot (n \ge 0)$$

beam equations for load, shear, moment, slope, deflection (elastic curve)

the integration constants c1 and c2 for shear and moment will always be zero if the reaction forces and moments acting on the beam are included in the loading function, because the shear and moment diagrams must close to zero at the end of the beam (source: "Machine Design - Fourth Edition" by Robert L. Norton, p. 118)

$$q(x)=R_1\cdot f_s(x,X_1,-1)-F_2\cdot f_s(x,X_2,-1)+R_3\cdot f_s(x,X_3,-1)-F_4\cdot f_s(x,X_4,-1)$$

$$V(x)=R_1\cdot f_s(x,X_1,0)-F_2\cdot f_s(x,X_2,0)+R_3\cdot f_s(x,X_3,0)-F_4\cdot f_s(x,X_4,0)$$

$$M(x)=R_1\cdot f_s(x,X_1,1)-F_2\cdot f_s(x,X_2,1)+R_3\cdot f_s(x,X_3,1)-F_4\cdot f_s(x,X_4,1)$$

$$\theta(x)=\frac{1}{EI_b}\cdot\left(\frac{R_1}{2}\cdot f_s(x,X_1,2)-\frac{F_2}{2}\cdot f_s(x,X_2,2)+\frac{R_3}{2}\cdot f_s(x,X_3,2)-\frac{F_4}{2}\cdot f_s(x,X_4,2)+c_1\right)$$

$$y(x)=\frac{1}{EI_b}\cdot\left(\frac{R_1}{6}\cdot f_s(x,X_1,3)-\frac{F_2}{6}\cdot f_s(x,X_2,3)+\frac{R_3}{6}\cdot f_s(x,X_3,3)-\frac{F_4}{6}\cdot f_s(x,X_4,3)+c_1\cdot x+c_2\right)$$

$$x_{off}:=l_b+ul(1\ mm)$$

$$V(x_{off})=0$$

$$M(x_{off})=0$$

$$y(X_1)=0$$

$$y(X_3)=0$$

4 equations, 4 unknown (R1, R3, c1, c2)

Guess Values

$R_1:=400$ $R_3:=400$ $c_1:=1$ $c_2:=1$

Constraints

$$R_1\cdot f_s(x_{off},X_1,0)+R_3\cdot f_s(x_{off},X_3,0)=F_2\cdot f_s(x_{off},X_2,0)+F_4\cdot f_s(x_{off},X_4,0)$$

$$R_1\cdot f_s(x_{off},X_1,1)+R_3\cdot f_s(x_{off},X_3,1)=F_2\cdot f_s(x_{off},X_2,1)+F_4\cdot f_s(x_{off},X_4,1)$$

$$\frac{R_1}{6}\cdot f_s(X_1,X_1,3)+\frac{R_3}{6}\cdot f_s(X_1,X_3,3)+c_1\cdot X_1+c_2=\frac{F_2}{6}\cdot f_s(X_1,X_2,3)+\frac{F_4}{6}\cdot f_s(X_1,X_4,3)$$

$$\frac{R_1}{6}\cdot f_s(X_3,X_1,3)+\frac{R_3}{6}\cdot f_s(X_3,X_3,3)+c_1\cdot X_3+c_2=\frac{F_2}{6}\cdot f_s(X_3,X_2,3)+\frac{F_4}{6}\cdot f_s(X_3,X_4,3)$$

Solver

$$RES:=\text{find}(R_1,R_3,c_1,c_2)$$

$R_1:=RES_1=-3.75\cdot10^3$ $c_1:=RES_3=4.083\cdot10^4$

$R_3:=RES_2=1.875\cdot10^4$ $c_2:=RES_4=2.821\cdot10^{-28}$

4. Results

$$V(x) := \left(R_1 \cdot f_s\left(x, X_1, 0\right) - F_2 \cdot f_s\left(x, X_2, 0\right) + R_3 \cdot f_s\left(x, X_3, 0\right) - F_4 \cdot f_s\left(x, X_4, 0\right)\right) \cdot N$$

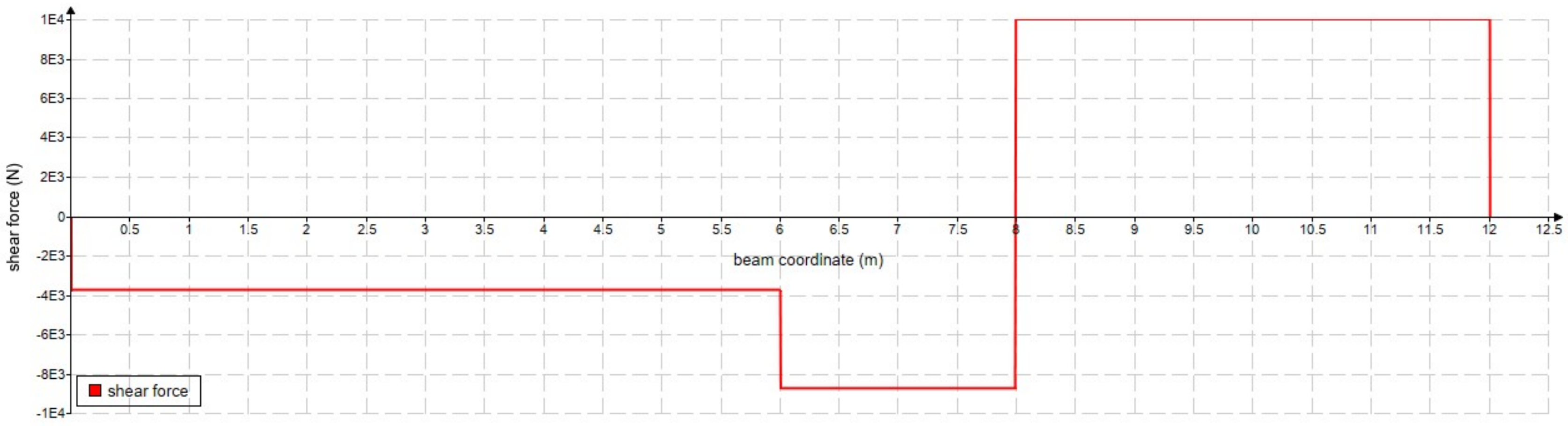
$$M(x) := \left(R_1 \cdot f_s\left(x, X_1, 1\right) - F_2 \cdot f_s\left(x, X_2, 1\right) + R_3 \cdot f_s\left(x, X_3, 1\right) - F_4 \cdot f_s\left(x, X_4, 1\right)\right) \cdot N \cdot m$$

$$y(x) := \left(\frac{1}{EI_b} \cdot \left(\frac{R_1}{6} \cdot f_s\left(x, X_1, 3\right) - \frac{F_2}{6} \cdot f_s\left(x, X_2, 3\right) + \frac{R_3}{6} \cdot f_s\left(x, X_3, 3\right) - \frac{F_4}{6} \cdot f_s\left(x, X_4, 3\right) + c_1 \cdot x + c_2\right)\right) \cdot m$$

$range := 0, 10^{-3} .. l_b \cdot 1.0001$ $\qquad i := 1, 2 .. dim$ $\qquad Y_i := y\left(X_i\right)$

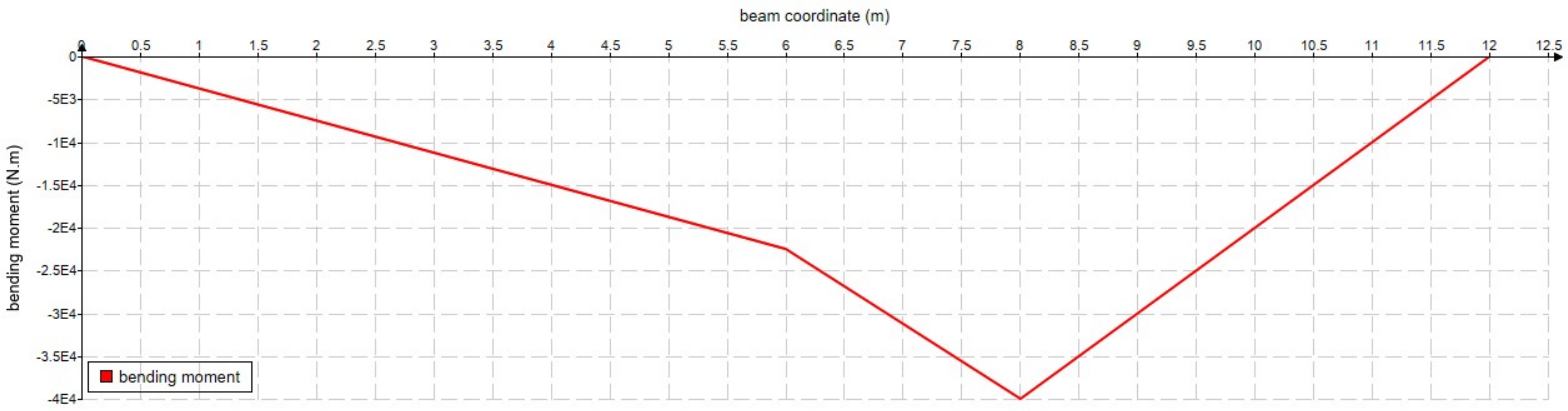
Inputs

$\mathbf{X}_1 := range$ $\qquad \mathbf{Y}_1 := V(range)$



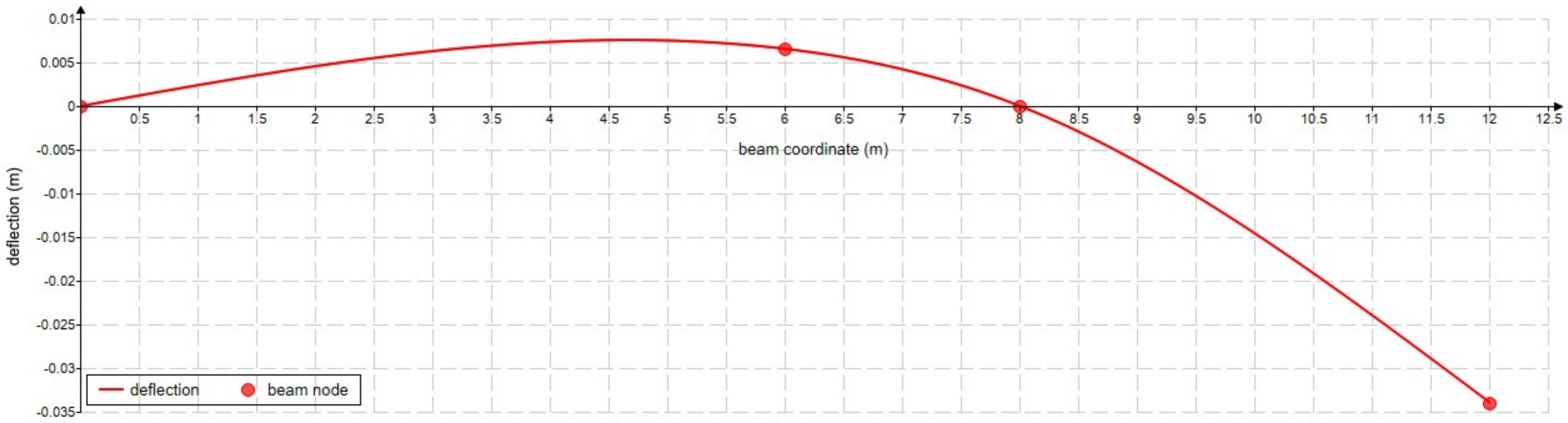
Inputs

$\mathbf{X}_1 := range$ $\qquad \mathbf{Y}_1 := M(range)$



Inputs

$\mathbf{X}_1 := range$ $\qquad \mathbf{Y}_1 := y(range)$ $\qquad \mathbf{X}_2 := X$ $\qquad \mathbf{Y}_2 := Y$



$y_{extr} := y\left(l_b\right) = -33.933\ mm$