THE TEMPERATURE OF THE UNIVERSE NOW AND IN THE DISTANT FUTURE

Roger L. Mansfield, 2011 February 10

I did not mean in my PlanetPTC post of February 3 to shift the discussion away from Edwin Hubble and toward John Mather. It is just that the age and temperature of the universe are intimately related.

If Edwin Hubble can be said to have determined the age of the universe by means of his galactic recessional velocity vs. distance graph, then John Mather and his COBE colleagues can be said to have determined its temperature.

Because what the COBE team showed is that the universe, as it exists today, behaves *exactly* like a black body radiating at 2.725 degrees Kelvin.

Given that it took the universe 13.7 billion years to cool down to its present temperature of 2.725 degrees Kelvin as measured by the COBE team, how long will it take for the universe to cool down by one more degree, to 1.725 degrees Kelvin?

Using Hubble's and Mather's results, we can answer this question, at least to a first approximation, as follows.

A first-order, ordinary differential equation (ODE) is used to model radioactive decay. But well before radioactivity was discovered, Newton had used a similar equation to describe the cooling of a body not at thermal equilibrium with its surroundings. Let's assume that Newton's law of cooling applies here (even though we haven't properly established why the ODE should hold on the scale of the universe). We have

$$\frac{dT}{dt} = \alpha \cdot T \qquad \text{so that} \qquad \frac{dT}{T} = \alpha \cdot dt$$

Integrating,

$$\ln(T) - \ln(T_o) = \ln\left(\frac{T}{T_o}\right) = \alpha \cdot (t - t_o)$$

Raising e to equal powers,

$$\frac{T}{T_o} = e^{\alpha \cdot (t - t_o)}$$

so that

$$T = T_o \cdot e^{\alpha \cdot (t - t_o)}$$

We can select now α, then Symbolics>Variable> Solve, to arrive at the following expression for α ...

$$\alpha = \frac{\ln\left(\frac{T}{T_o}\right)}{t - t_o}$$

We have a good value for the age of the universe, $13.7\cdot10^9$ years. We need to determine the cooling constant, α.

Ralph Alpher, Robert Herman, and George Gamow calculated in the late 1940s that, if indeed the universe did originate in a Big Bang, then there should now still exist cosmic microwave background (CMB) radiation detectable in every direction of space, radiation with a spectrum characteristic of a black body at a temperature of approximately 5 degrees Kelvin.

In 1950 Gamow wrote (in *The Creation of the Universe,* still available as a Dover paperback), that the temperature of the universe shortly after the Big Bang was given by

$$T_o = \frac{1.5\cdot10^{10}}{\sqrt{t}}$$ degrees Kelvin, with t measured in seconds.

Therefore, one second after the Big Bang we have $T_o = 1.5\cdot10^{10}$ degrees Kelvin. I tried using this value in a previous version of this worksheet, but it led to a Kelvin temperature that was way too high 4.5 billion years ago, when Earth is believed to have formed. Modern cosmology calculates that the age of the universe at recombination* was 380 thousand years, or $13.7\cdot10^9 - 380\cdot10^3 = 1.36996\times10^{10}$ years ago. (*When the primordial fog lifted and light was able to travel freely.)

The temperature of the universe at recombination is currently believed to have been about 3000 degrees Celsius, or 3000 + 273 = 3273 degrees Kelvin. So let's start with that:

$$T_o := 3273$$ degrees Kelvin.

Using the expression for α obtained above, we have

$$\alpha := \frac{\ln\left(\frac{2.725}{T_o}\right)}{13.6996\cdot10^9} \qquad \alpha = -5.176\times10^{-10}$$ reciprocal years.

Now to find out how long that it will take, starting from the present epoch, for the universe to cool to 1.725 degrees Kelvin, we write again

$$T = T_o\cdot e^{\alpha\cdot(t-t_o)}$$

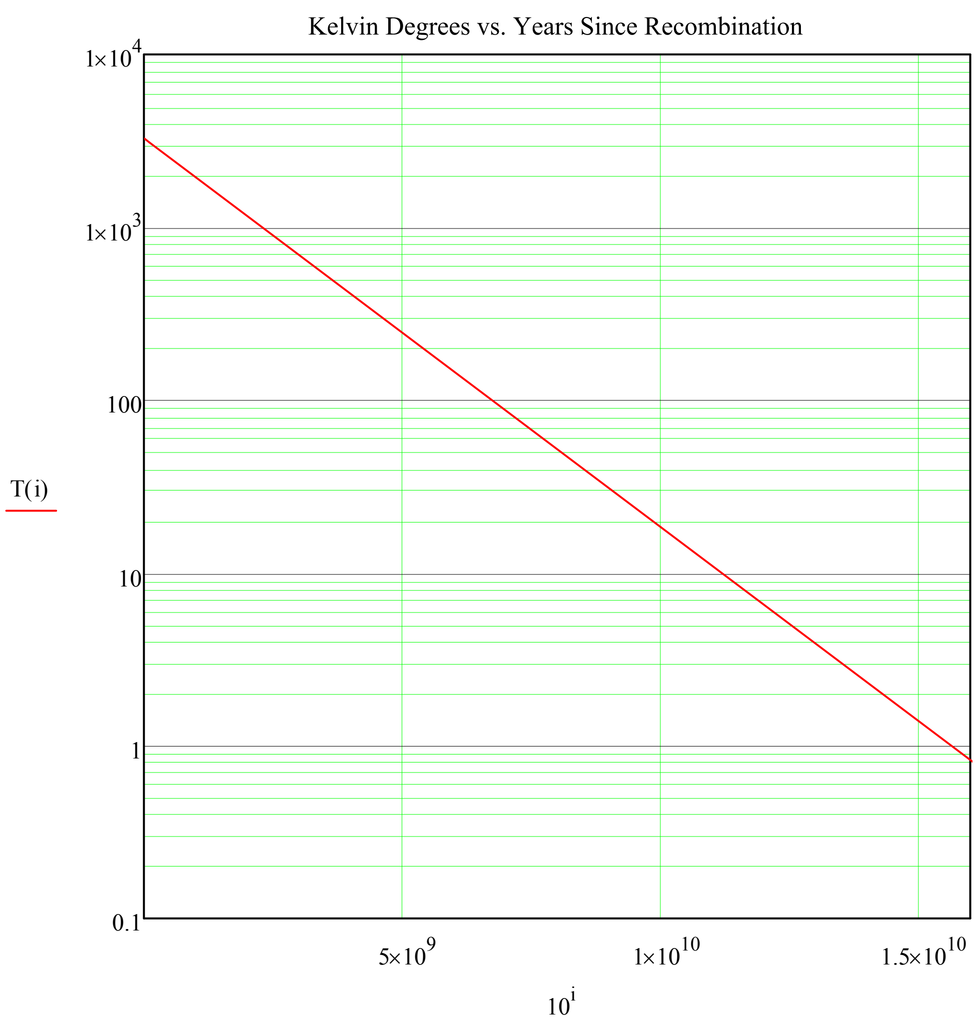
We can select t, then Symbolics>Variable>Solve, and set $t_o := 13.6996\cdot10^9$, $T_o := 2.725$, and $T := 1.725$, to arrive at the following expression for t ...

$$t := \frac{\alpha \cdot t_o + \ln\left(\frac{T}{T_o}\right)}{\alpha} \qquad t = 1.458 \times 10^{10} \text{ years.}$$

That's $14.58 \cdot 10^9 - 13.7 \cdot 10^9 = 8.8 \times 10^8$ or about 880 million years from now. So even if the ODE associated with radioactive decay and Newton's law of cooling doesn't hold on the scale of the universe, it seems likely that the universe will take a good while longer to cool down by even one more degree. Here's what the graph of temperature vs. time looks like according to the exponential decay model.

$$i := 1 .. 11 \qquad T_o := 3273$$

$$T(i) := T_o \cdot e^{\alpha \cdot \left(10^i\right)}$$



Given this cooling model, what do we now get for the temperature of the universe 4.5 billion years ago? We have

$$T := T_o \cdot e^{\alpha \cdot \left[(13.7-4.5)\cdot 10^9\right]}$$

$T = 27.98$ degrees Kelvin.

Gamow's equation yields

$t := (13.7 - 4.5)\cdot 10^9 \cdot 365.25 \cdot 86400$ seconds.

$$T := \frac{1.5\cdot 10^{10}}{\sqrt{t}}$$

$T = 27.838$ degrees Kelvin.

SUMMARY AND ACKNOWLEDGMENTS

1. Modern cosmology holds that the Big Bang occurred 13.7 billion years ago, and that the "let there be light" phase ("the primordial fog lifts") started some 380 thousand years afterward, at which time the temperature of the universe is believed to have been about 3000 degrees Celsius, or 3273 degrees Kelvin.

Using the COBE team's value of 2.725 degrees Kelvin for the present age of the universe, with a decaying exponential model of cooling, we are able to calibrate a cooling constant. With this cooling constant, we arrive at a value of about 28 degrees for the temperature of the universe 4.5 billion years ago, when Earth is believed to have formed.

2. Using Gamow's equation, which he derived more than 61 years ago, we again get about 28 degrees Kelvin for the temperature of the universe 4.5 billion years ago.

George Gamow (1904-1968) was a great and prolific physicist whose native tongue was Russian. See the Wikipedia entry on Gamow for more information about his life and scientific career.

Gamow's *The Creation of the Universe* and *Matter, Earth, and Sky,* are highly recommended, as much for Gamow's creativity and lucid exposition as for the historical value of these two popular, but important books.

3. I would like to thank Stuart Bruff and Alan Stevens, Mathcad experts whose comments at PlanetPTC were quite helpful in eliminating the errors in previous versions of this worksheet. See "Computing the Age of the Universe with Mathcad" at http://communities.ptc.com/community/mathcad.