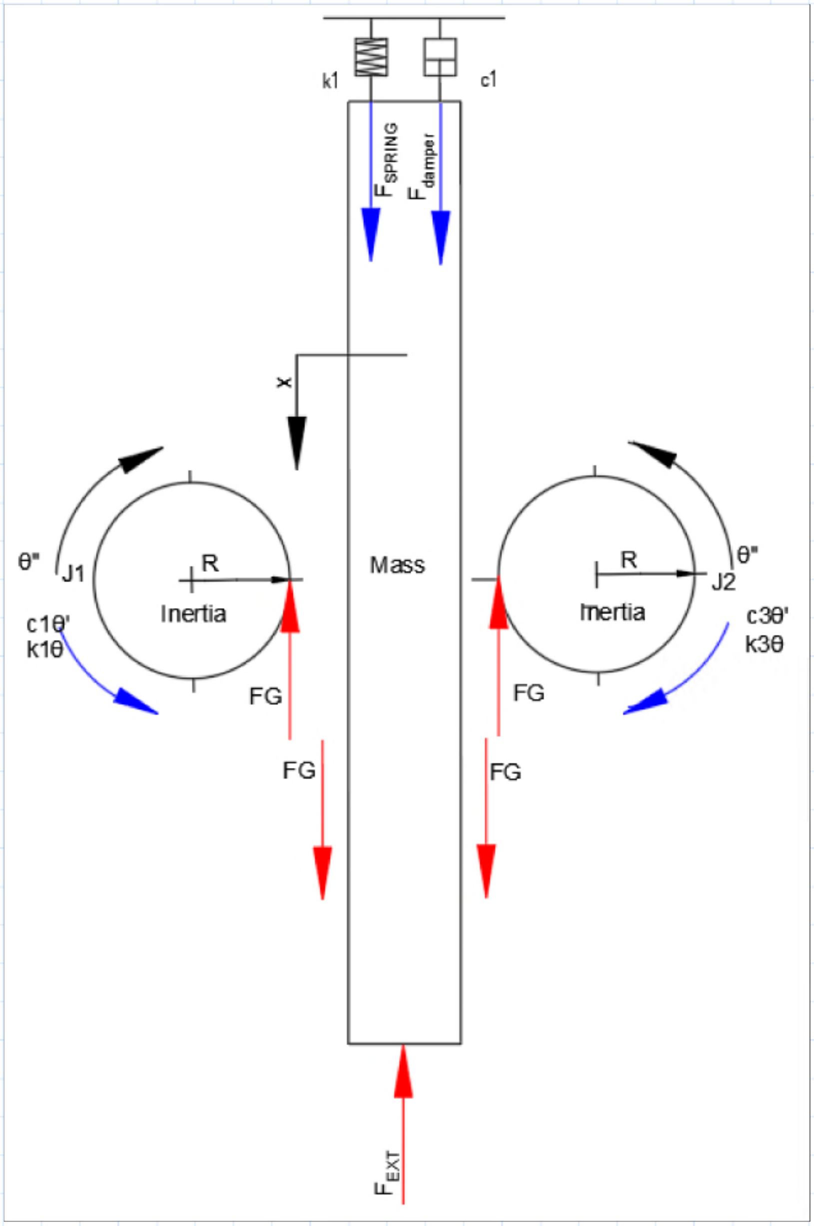
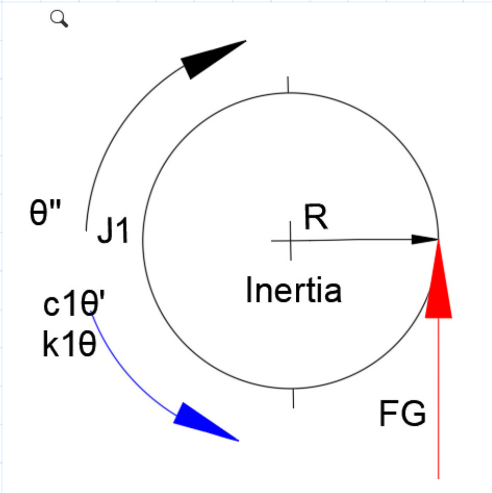
Second-order differential equation to model the system behavior



Second-order differential equation Newton rotational law drive 1

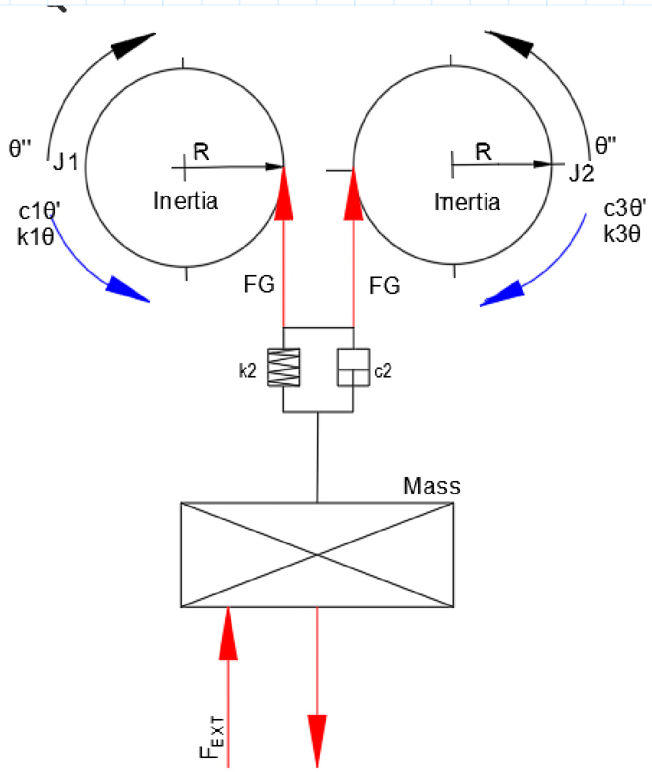


For the rotation of the first motor (drive 1) according to Newton's laws with damping and stiffness:

$$\theta_1''(t) = \frac{-c_1 \cdot \theta_1'(t) - k_1 \cdot \theta_1(t) + k_2 \cdot \left(\frac{x_2(t)}{R_1} - \theta_1(t)\right) + c_2 \cdot \left(\frac{x_2'}{R_2}(t) - \theta_1'(t)\right) + T_{brake1} + \frac{m_{rack} \cdot g}{2} \cdot R_1}{J_1 + J_{1gear}\left(\frac{R_1}{R_2}\right)^2}$$

Mass of inertia 1 pinion	$J_1 := 0.01$
Mass of inertia 1 gearbox	$J_{1gear} := 0.02$
Pitch diameter pinion 1	$R_1 := 100$
Radial Damping Coefficient drive 1	$c_1 := 22590$
Radial Stiffness Coefficient drive 1	$k_1 := 1 \cdot 5.7 \cdot 10^7$
Brake	$T_{brake1} := 10$

Second-order differential equation Newton rotational law rack



For the rack (mass-spring-damper system) according to Newton's laws with damping and stiffness:

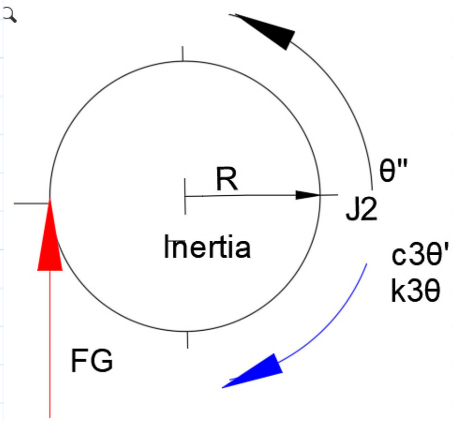
$$x_2''(t) = \frac{-k_2 \cdot (x_2\ (t) - \theta_1(t) \cdot R_1) - c_2 \cdot (x_2'\ (t) - \theta_1'(t) \cdot R_1) + k_2 \cdot (\theta_2(t) \cdot R_2 - x_2\ (t)) + c_2 \cdot}{m_{rack}}$$

Mass rack $m_{rack} := 5$

Radial Damping Coefficient gear-pinion 1 $c_2 := 22590$

Radial Stiffness Coefficient gear-pinion 1 $k_2 := 1 \cdot 5.7 \cdot 10^7$

Second-order differential equation Newton rotational law drive 2



For the rotation of the second motor (drive 2) according to Newton's laws with damping and stiffness:

$$\theta_2''(t) = \frac{-k_2 \cdot \left(\theta_3(t) - \frac{x_2}{R_2}\ (t)\right) - c_2 \cdot \left(\theta_3'(t) - \frac{x_2}{R_2}'(t)\right) + \frac{m_{rack} \cdot g}{2} \cdot R_2 - k_3 \cdot \theta_2(t) - c_3 \cdot \theta_2'(t}{J_2 + J_{2gear} \left(\frac{R_1}{R_2}\right)^2}$$

Mass of inertia 2 pinion	$J_2 := 250$
Mass of inertia 2 gearbox	$J_{2gear} := 250$
Pitch diameter pinion 2	$R_2 := 100$
Radial Damping Coefficient damper 2	$c_3 := 22590$
Radial Stiffness Coefficient 2	$k_3 := 1 \cdot 5.7 \cdot 10^7$
	$T_{brake2} := 20$
	$g_1 := 9.81$

Guess Values

$$\theta_1(0) = 0 \qquad z_2(0) = 0 \qquad \theta_3(0) = 0$$

$$\theta_1'(0) = 0 \qquad z_2'(0) = 0 \qquad \theta_3'(0) = 0$$

Constraints

$$\theta_1''(t) = \frac{-c_1 \cdot \theta_1'(t) - k_1 \cdot \theta_1(t) + k_2 \cdot \left(\frac{z_2(t)}{R_1} - \theta_1(t)\right) + c_2 \cdot \left(\frac{z_2'(t)}{R_2} - \theta_1'(t)\right)}{J_1}$$

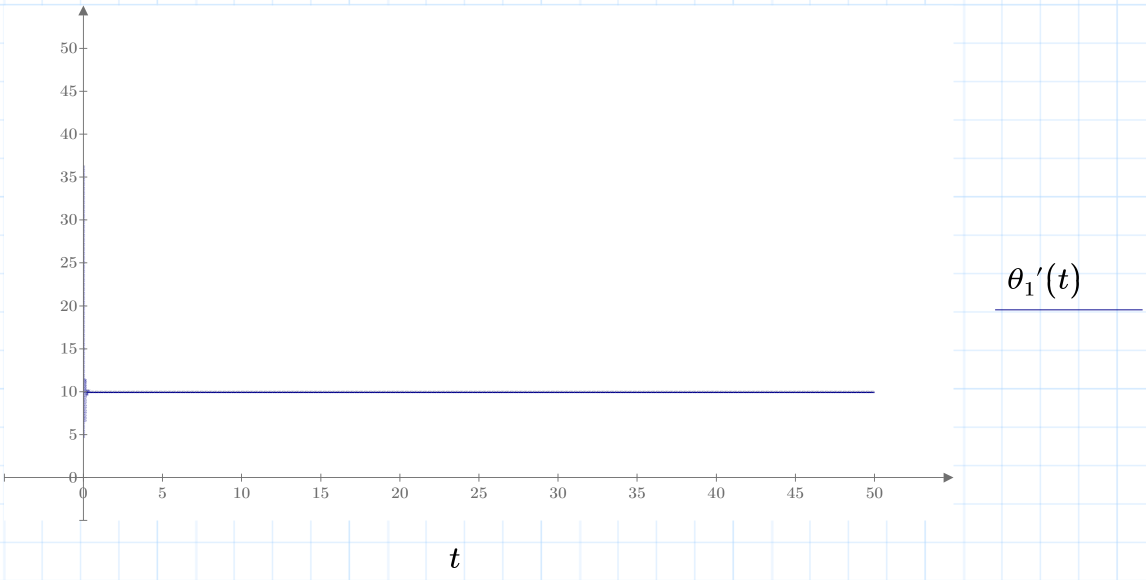
$$z_2''(t) = \frac{-k_2 \cdot \left(z_2(t) - \theta_1(t) \cdot R_1\right) - c_2 \cdot \left(z_2'(t) - \theta_1'(t) \cdot R_1\right) + k_2 \cdot \left(\theta_3(t) \cdot R_2 - z_2(t)\right) + c_2 \cdot}{m_{rack}}$$

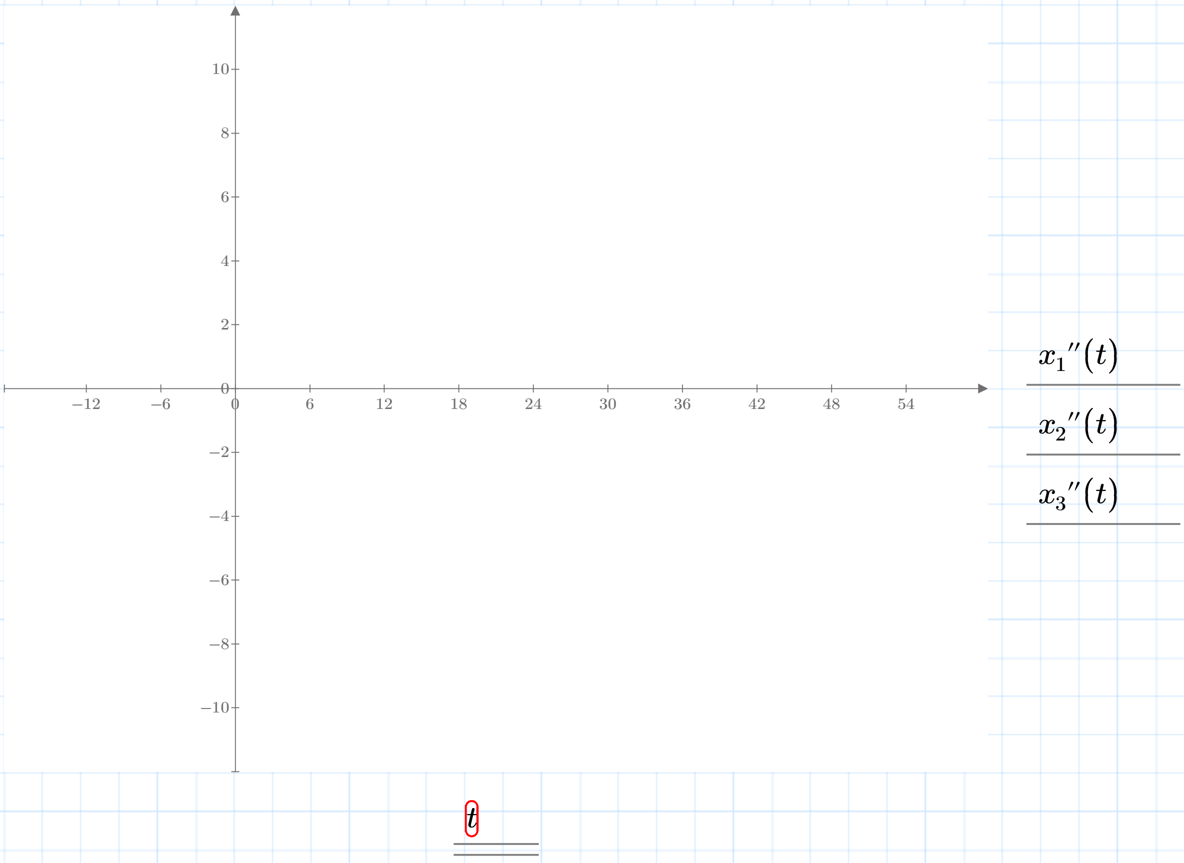
$$\theta_3''(t) = \frac{-k_2 \cdot \left(\theta_3(t) - \frac{z_2(t)}{R_2}\right) - c_2 \cdot \left(\theta_3'(t) - \frac{z_2'(t)}{R_2}\right) + \frac{m_{rack} \cdot g_1 \cdot R_2}{2} - k_3 \cdot \theta_3(t) - c_3 \cdot \theta_3'(}{J_2}$$

Solver

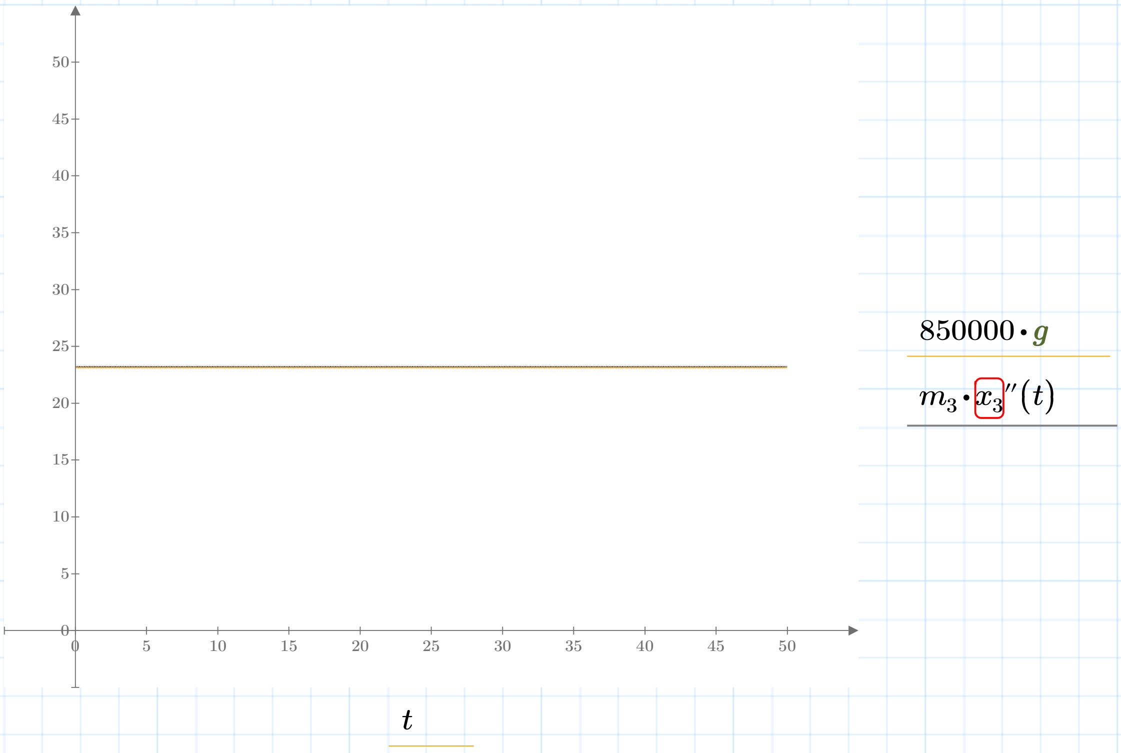
$$\begin{bmatrix}\theta_1 \\ z_2 \\ \theta_3\end{bmatrix} := \mathbf{odesolve}\left(\begin{bmatrix}\theta_1(t) \\ z_2(t) \\ \theta_3(t)\end{bmatrix}, 75\right)$$

$t := 0, 0.0001 .. 50$





10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−6
0
6
12
18
24
30
36
42
48
54
$x_1''(t)$
$x_2''(t)$
$x_3''(t)$
t



50
45
40
35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40
45
50
$850000 \cdot g$
$m_3 \cdot x_3''(t)$
t