RATING FOR CONCRETE ARCH - CROWN POINT

Existing Bridge Data

Bridge ID: 019901

Location: Hopkinton, RI

Route Carried: RI 3 (Main Street)

Feature Intersected: Pawcatuck River

Year Built: 1924

Bridge Type: Reinforced Concrete Arch

Skew: 26º - 34'

Span Length: $L_{span} := 80 \cdot ft$

Overall Width: $W_{slab} := 46 \cdot ft + 2 \cdot in$

Roadway Width: $W_{roadway} := 30 \cdot ft$

Data, Assumptions, and Material Properties

$h := 18 \quad in$ (Depth of Arch Section Cut)

$b_{arc} := 83$ (Width of Arch)

$b := 16.248 \quad in$ (Width of CSi Section Cut)

$c_r := 1 \quad in$ (Rebar Cover for Intrados and Extrados)

$d_r := 0.75 \quad in$ (3/4" Diameter)

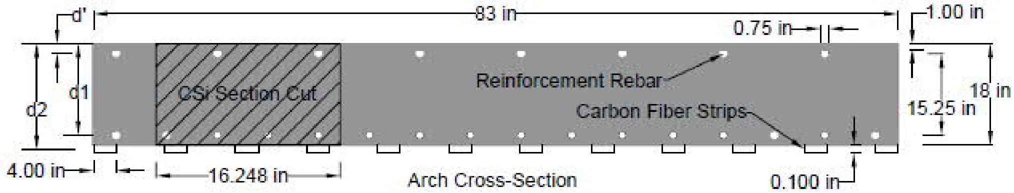
$w_{cs} := 4 \quad in$ (Width of Carbon Strips)

$d_{cs} := 0.1 \quad in$ (Thickness of Carbon Fiber Strips, Two Layers of 0.05'' Carbon Fiber)

$d' := c_r + \frac{d_r}{2} = 1.375 \quad in$ (Depth of Top Layer, Extrados Reinforcement)

$d_1 := h - c_r - \frac{d_r}{2} = 16.625 \quad in$ (Depth of First Bottom Layer, Intrados Reinforcement)

$d_2 := h + \frac{d_{cs}}{2} = 18.050 \quad in$ (Depth of Second Bottom Layer, Intrados Carbon Fiber Strips)



$A'_s := 8\cdot\pi\cdot\left(\frac{d_r}{2}\right)^2\cdot\left(\frac{1}{b_{arc}}\right)\cdot b = 0.692$ in^2 (Area of Extrados Steel, 8 - 3/4" Diameter Bars)

$A_{s1} := 16\cdot\pi\cdot\left(\frac{d_r}{2}\right)^2\cdot\left(\frac{1}{b_{arc}}\right)\cdot b = 1.384$ in^2 (Area of Introdos Steel, 16 - 3/4" Diameter Bars)

$A_{s2} := 12\cdot d_{cs}\cdot w_{cs}\cdot\left(\frac{1}{b_{arc}}\right)\cdot b = 0.940$ in^2 (Area of Carbon Fiber Strips, 12 Strips)

$f'_c := 2.5$ ksi	(Compression Strength of Concrete)	[Ref. MBE - 6A.5.2.1-1]
$f_y := 33$ ksi	(Yield Strength of Steel)	[Ref. MBE - 6A.5.2.2-1]
$f_{cs} := 255$ ksi	(Tensile Strength of Carbon Fiber)	[Ref. Zocon Consulting Engineers]
$\varepsilon_u := 0.003$	(Ultimate Strain of Concrete)	[Ref. ACI - Fig R21.2.2b]
$E_s := 29000$ ksi	(Modulus of Elasticity of Steel)	
$E_{cs} := 20000$ ksi	(Modulus of Elasticity Carbon Fiber)	[Ref. Zocon Consulting Engineers]
$E_c := 2850$ ksi	(Modulus of Elasticity Concrete, f'_c=2.5 ksi)	

LRFR Rating Factors for Limit States

$\phi_c := 0.95$	(Condition Factor for Concrete Arch Bridge)	[Ref. MBE - 6A.4.2.3]
$\phi_s := 1.0$	(System Factor for Concrete Arch Bridge)	[Ref. MBE - 6A.4.2.4]

Load Factors for Strength Limit States I & II:

$\gamma_{DC.st} := 1.25$	(Dead Load Factor for Strength I & II)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{DW.st} := 1.50$	(Wearing Surface Factor for Strength I & II)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{L.st.inv} := 1.75$	(Inventory Design Load Factor for Strength I)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{L.st.op} := 1.35$	(Operating Design Load Factor for Strength I)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{L.st.l} := 1.30$	(Legal Load Factor for Strength I)	[Ref. MBE - Table 6A.4.4.2.3a-1]
$\gamma_{L.st.l.SU} := 1.30$	(SU Load Factor for Strength I)	[Ref. MBE - Table 6A.4.5.4.3b-1]
$\gamma_{L.st.bp} := 1.15$	(Blanket Permit Load Factor for Strength II)	[Ref. MBE - Table 6A.4.5.4.2a-1]
$\gamma_{L.st.bp.6} := 1.20$	(Blanket Permit Load Factor for Strength II)	[Ref. MBE - Table 6A.4.5.4.2a-1]
$\gamma_{L.st.stp} := 1.20$	(Single Trip Permit Load Factor for Strength II)	[Ref. MBE - Table 6A.4.5.4.2a-1]
$\gamma_{L.st.l.EV} := 1.30$	(EV Load Factor for Strength I)	[Ref. MBE - 6A.5.12.10.3]

Load Factors for Service Limit State I:

$\gamma_{DC.sv} := 1.0$	(Dead Load Factor for Service I)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{DW.sv} := 1.0$	(Wearing Surface Factor for Service I)	[Ref. MBE - Table 6A.4.2.2-1]
$\gamma_{L.sv.p} := 1.0$	(Permit Load Factor for Service I)	[Ref. MBE - Table 6A.4.2.2-1]

DEVELOPMENT OF INTERACTION DIAGRAM FOR CONCRETE ARCH RATING

The interaction diagram will be created by varying the value for the neutral axis distance. From that, the steel strains and stresses and the concrete forces were calculated as shown from the equations listed below. From the output we are able to plot the values and create the interaction diagram for the given section.

Strength Reduction Factors $\phi_a := 0.75$ Compression Control [Ref. AASHTO LRFD 5.5.4.2-1]

$\phi_b := 0.9$ Tension Control [Ref. AASHTO LRFD 5.5.4.2]

Service Reduction Factor $\varphi := 1.0$ [Ref. RIDOT LRFR Guide. 4.3]

Equivalent Compression Zone Factor [Ref. AASHTO 5.7.2.2]

$$\beta_1 := \max\left[0.65, \text{if}\left[f'_c \le 4, 0.85, 0.85 - 0.05\cdot\left(f'_c - 4\right)\right]\right] = 0.850$$

Gross cross section: $A_g := b\cdot h = 292.464$ $\left(in^2\right)$

Steel Ratio: $\rho_s := \left(\frac{A'_s}{A_g}\right) = 0.00237$ $\rho'_{s.1} := \left(\frac{A_{s1}}{A_g}\right) = 0.00473$ $\rho'_{s.2} := \frac{A_{s2}}{A_g} = 0.00321$

Maximum Axial Compression: [Ref. AASHTO Eq. 5.7.4.4-3]

$$Pc_{max} := 0.80\cdot\left[0.85\cdot f'_c\cdot\left(A_g - A'_s - A_{s1}\right) + f_y\cdot\left(A'_s + A_{s1}\right)\right] = 548.456$$ (kips, unfactored)

Maximum Axial Tension:

$$Pt_{max} := f_y\cdot\left(A'_s + A_{s1}\right) + f_{cs}\cdot\left(A_{s2}\right) = 308.104$$ (kips, unfactored)

Modulus of Rupture: $f_r := \frac{7.5}{1000}\cdot\sqrt{f'_c\cdot 1000} = 0.375$ (ksi)

Crack Bending Moment: $M_{cr} := f_r\cdot\frac{1}{6}\cdot b\cdot h^2 = 329.022$ (kip-in)

Limits of Neutral Axis $x := 0, 0.1 .. \frac{h}{\beta_1}$

<u>Concrete Components</u>

Concrete Stress Block $a(x) := \beta_1\cdot x$

Nominal Concrete Axial Capacity $P_c(x) := 0.6\cdot f'_c\cdot a(x)\cdot b$ [Ref. ACI 440 10.2.8b]

Since the carbon fiber was installed post dead load application, subtract the dead load strain present in the concrete from the calculated carbon fiber strain

INTERACTION OF POSITIVE MOMENTS

Steel Components

$$\sigma_{cf} := \frac{P_{DC} + P_{DW}}{A_g} - \frac{\left(M_{DC} + M_{DW}\right)\frac{h}{2}}{\left(\frac{1}{12}\right)\cdot b\cdot h^3} \qquad \varepsilon_{cf} := \frac{\sigma_{cf}}{E_c}$$

Steel Strains

$$\varepsilon'_{s.p}(x) := \varepsilon_u\cdot\frac{x - d'}{x}$$

$$\varepsilon_{s1.p}(x) := \varepsilon_u\cdot\frac{d_1 - x}{x}$$

$$\varepsilon_{s2.p}(x) := \left(\varepsilon_u\cdot\frac{d_2 - x}{x}\right) - \varepsilon_{cf}$$

Steel Stresses For Strength

$$f'_{s.st}(x) := \min\left(E_s\cdot\varepsilon'_{s.p}(x), f_y\right)$$

$$f_{s1.st}(x) := \min\left(E_s\cdot\varepsilon_{s1.p}(x), f_y\right)$$

$$f_{s2.st}(x) := \min\left(E_{cs}\cdot\varepsilon_{s2.p}(x), f_{cs}\right)$$

Steel Stresses For Service

$$f'_{s.sv}(x) := \min\left(E_s\cdot\varepsilon'_{s.p}(x), 0.8f_y\right)$$

$$f_{s1.sv}(x) := \min\left(E_s\cdot\varepsilon_{s1.p}(x), 0.8f_y\right)$$

$$f_{s2.sv}(x) := \min\left(E_{cs}\cdot\varepsilon_{s2.p}(x), 0.9f_{cs}\right)$$

Nominal Ten./Comp. Steel (Axial Capacity - Strength)

$$P'_{s.st}(x) := A'_s\cdot f'_{s.st}(x)$$

$$P_{s1.st}(x) := A_{s1}\cdot f_{s1.st}(x)$$

$$P_{s2.st}(x) := A_{s2}\cdot f_{s2.st}(x)$$

Nominal Ten./Comp. Steel (Axial Capacity - Service)

$$P'_{s.sv}(x) := A'_s\cdot f'_{s.sv}(x)$$

$$P_{s1.sv}(x) := A_{s1}\cdot f_{s1.sv}(x)$$

$$P_{s2.sv}(x) := A_{s2}\cdot f_{s2.sv}(x)$$

Nominal Strength:

$$Pn_p(x) := P_c(x) - P_{s1.st}(x) - P_{s2.st}(x) + P'_{s.st}(x)$$

$$Mn_p(x) := P_c(x)\cdot\left(\frac{h}{2} - \frac{a(x)}{2}\right) + P'_{s.st}(x)\cdot\left(\frac{h}{2} - d'\right) + P_{s1.st}(x)\cdot\left(d_1 - \frac{h}{2}\right) + P_{s2.st}(x)\cdot\left(d_2 - \frac{h}{2}\right)$$

Compression Control Limit: $\varepsilon s_c := 0.002$ $\quad x1 := \frac{d_2\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_c} = 10.830$ [Ref. ACI - Fig. R21.2.2a-b]

Tension Control Limit: $\varepsilon s_t := 0.005$ $\quad x2 := \frac{d_2\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_t} = 6.769$ [Ref. ACI - Fig. R21.2.2a-b]

Plastic Limit: $\varepsilon s_p := 0.1$ $\quad x3 := \frac{d_2\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_p}$

Strength Reduction Factor:

$$\phi(x) := \left|\begin{array}{l} \text{return } \phi_b \text{ if } x \le x2 \\ \text{return } \phi_a \text{ if } x \ge x1 \\ \text{return } \phi_a + \frac{\phi_b - \phi_a}{\varepsilon s_t - \varepsilon s_c}\cdot\left(\varepsilon_{s2.p}(x) - \varepsilon s_c\right) \text{ otherwise} \end{array}\right.$$

$\phi(x2) = 0.900$ [Ref. AASHTO LRFD 5.5.4.2]

$\phi(x1) = 0.750$ [Ref. AASHTO LRFD 5.5.4.2-1]

$\phi\left(\frac{x1 + x2}{2}\right) = 0.810$

Design Capacities

Design Axial Capacity (Strength) $\phi P_{n.p.st}(x) := \phi(x)\cdot Pn_p(x)$

Design Moment Capacity (Strength) $\phi M_{n.p.st}(x) := \phi(x)\cdot Mn_p(x)$

Design Axial Capacity (Service) $\phi P_{n.p.sv}(x) := \varphi\cdot Pn_p(x)$

Design Moment Capacity (Service) $\phi M_{n.p.sv}(x) := \varphi\cdot Mn_p(x)$

INTERACTION OF NEGATIVE MOMENTS

Steel Components

$$\sigma_{cf} := \frac{P_{DC} + P_{DW}}{A_g} - \frac{(M_{DC} + M_{DW})\frac{h}{2}}{\left(\frac{1}{12}\right)\cdot b\cdot h^3} \qquad \varepsilon_{cf} := \frac{\sigma_{cf}}{E_c}$$

Steel Strains

$$\varepsilon'_{s.n}(x) := \varepsilon_u\cdot\frac{(h - d') - x}{x}$$

$$\varepsilon_{s1.n}(x) := \varepsilon_u\cdot\frac{x - (h - d_1)}{x}$$

$$\varepsilon_{s2.n}(x) := \left[\varepsilon_u\cdot\frac{x - (h - d_2)}{x}\right] - \varepsilon_{cf}$$

Steel Stresses For Strength

$$f'_{s.n.st}(x) := \min(E_s\cdot\varepsilon'_{s.n}(x), f_y)$$

$$f_{s1.n.st}(x) := \min(E_s\cdot\varepsilon_{s1.n}(x), f_y)$$

$$f_{s2.n.st}(x) := \min(E_{cs}\cdot\varepsilon_{s2.n}(x), f_{cs})$$

Steel Stresses For Service

$$f'_{s.n.sv}(x) := \min(E_s\cdot\varepsilon'_{s.n}(x), 0.8f_y)$$

$$f_{s1.n.sv}(x) := \min(E_s\cdot\varepsilon_{s1.n}(x), 0.8f_y)$$

$$f_{s2.n.sv}(x) := \min(E_{cs}\cdot\varepsilon_{s2.n}(x), 0.9f_{cs})$$

Nominal Ten./Comp. Steel (Axial Capacity - Strength)

$$P'_{s.n.st}(x) := A'_s\cdot f'_{s.n.st}(x)$$

$$P_{s1.n.st}(x) := A_{s1}\cdot f_{s1.n.st}(x)$$

$$P_{s2.n.st}(x) := A_{s2}\cdot f_{s2.n.st}(x)$$

Nominal Ten./Comp. Steel (Axial Capacity - Service)

$$P'_{s.n.sv}(x) := A'_s\cdot f'_{s.n.sv}(x)$$

$$P_{s1.n.sv}(x) := A_{s1}\cdot f_{s1.n.sv}(x)$$

$$P_{s2.n.sv}(x) := A_{s2}\cdot f_{s2.n.sv}(x)$$

Nominal Strength:

$$Pn_n(x) := P_c(x) - P'_{s.n.st}(x) + P_{s1.n.st}(x) + P_{s2.n.st}(x)$$

$$Mn_n(x) := P_c(x)\cdot\left(\frac{h}{2} - \frac{a(x)}{2}\right) + P'_{s.n.st}(x)\cdot\left(\frac{h}{2} - d'\right) + P_{s1.n.st}(x)\cdot\left(d_1 - \frac{h}{2}\right) + P_{s2.n.st}(x)\cdot\left(d_2 - \frac{h}{2}\right)$$

Compression Control Limit:

$$x1n := \frac{(h - d')\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_c} = 9.975$$ [Ref. ACI - Fig. R21.2.2a-b]

Tension Control Limit:

$$x2n := \frac{(h - d')\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_t} = 6.234$$ [Ref. ACI - Fig. R21.2.2a-b]

Plastic Limit:

$$x3n := \frac{(h - d')\cdot\varepsilon_u}{\varepsilon_u + \varepsilon s_p} = 0.484$$

Strength Reduction Factor:

$$\phi n(x) := \begin{array}{|l} \text{return } \phi_b \text{ if } x \le x2n \\ \text{return } \phi_a \text{ if } x \ge x1n \\ \text{return } \phi_a + \frac{\phi_b - \phi_a}{\varepsilon s_t - \varepsilon s_c}\cdot(\varepsilon'_{s.n}(x) - \varepsilon s_c) \text{ otherwise} \end{array}$$

$\phi n(x2n) = 0.900$ [Ref. AASHTO LRFD 5.5.4.2]

$\phi n(x1n) = 0.750$ [Ref. AASHTO LRFD 5.5.4.2-1]

$\phi n\left(\frac{x1n + x2n}{2}\right) = 0.808$

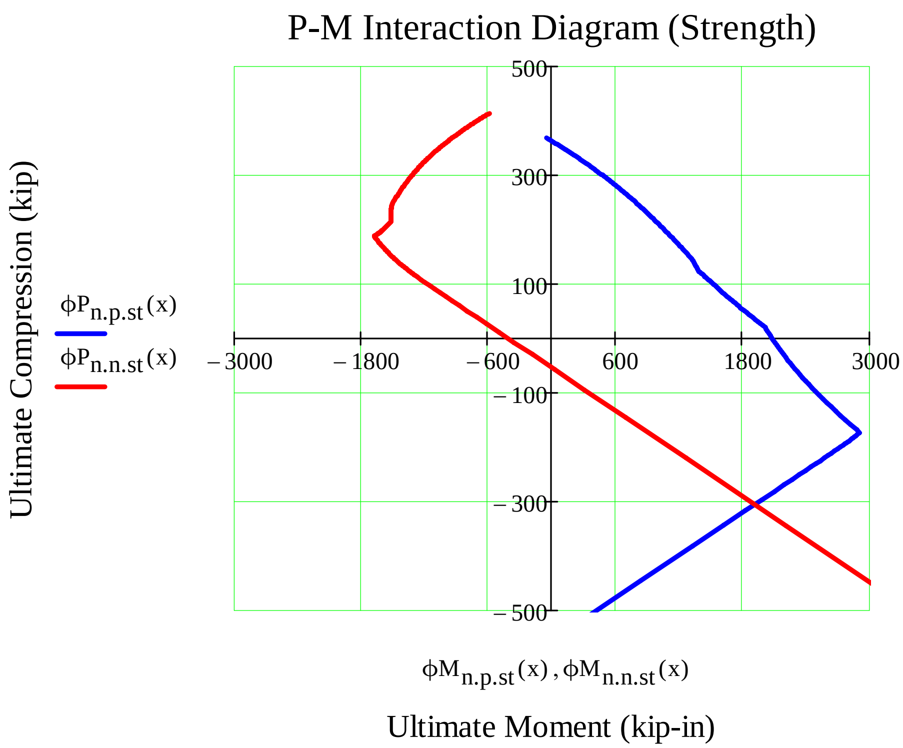
Design Capacities

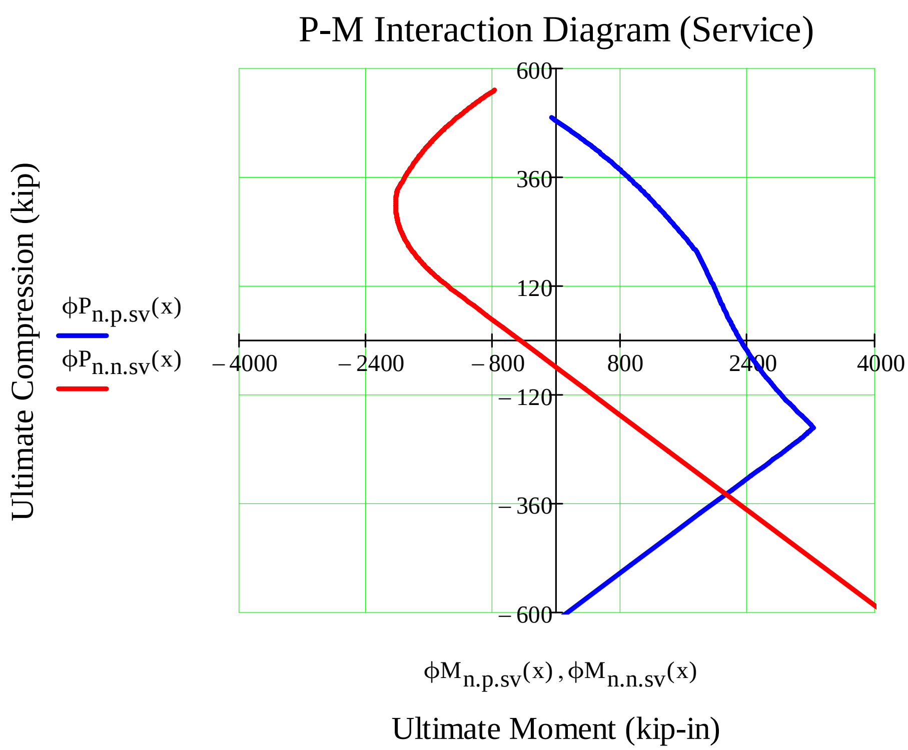
Design Axial Capacity (Strength) $\phi P_{n.n.st}(x) := \phi n(x) \cdot Pn_{n}(x)$

Design Moment Capacity (Strength) $\phi M_{n.n.st}(x) := -\phi n(x) \cdot Mn_{n}(x)$

Design Axial Capacity (Service) $\phi P_{n.n.sv}(x) := \varphi \cdot Pn_{n}(x)$

Design Moment Capacity (Service) $\phi M_{n.n.sv}(x) := -\varphi \cdot Mn_{n}(x)$


P-M Interaction Diagram (Strength)
Ultimate Compression (kip)
ϕPn.p.st(x)
ϕPn.n.st(x)
500
300
100
− 100
− 300
− 500
− 3000
− 1800
− 600
600
1800
3000
ϕMn.p.st(x), ϕMn.n.st(x)
Ultimate Moment (kip-in)


P-M Interaction Diagram (Service)
Ultimate Compression (kip)
ϕPn.p.sv(x)
ϕPn.n.sv(x)
600
360
120
− 120
− 360
− 600
− 4000
− 2400
− 800
800
2400
4000
ϕMn.p.sv(x), ϕMn.n.sv(x)
Ultimate Moment (kip-in)