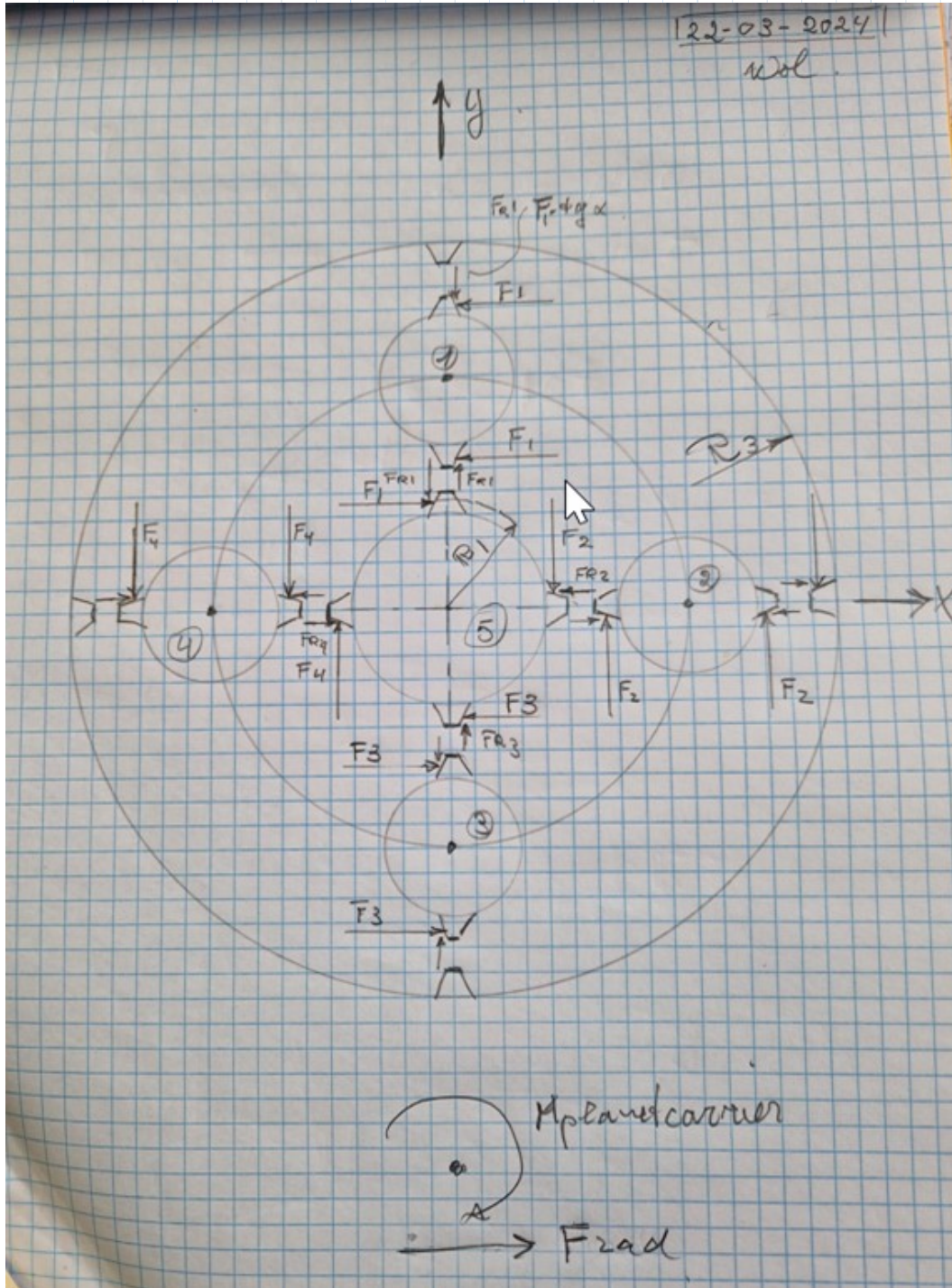


Gear ring forces

The purpose of this spreadsheet is to calculate the forces between the sunwheel and the planet gears. The geometry of the tooth profile is not considered. The problem here is that we have 3 equations and 4 unknowns. Often, these equations are solved by addressing the geometry issue.



Pitch diameter of the gear ring	$D_{gearing} := 1.2 \text{ m}$	$R_{gearing} := \frac{D_{gearing}}{2}$
---------------------------------	--------------------------------	--

Pitch diameter of the pinion	$D_{pinion} := 0.3 \text{ m}$	$R_{pinion} := \frac{D_{pinion}}{2}$
------------------------------	-------------------------------	--------------------------------------

Moment planet carrier	$M_{pl} := 2500 \text{ kN} \cdot \text{m}$	
-----------------------	--	--

Pressure angle	$\alpha_1 := 20 \text{ deg}$	
----------------	------------------------------	--

Average tooth load

Force of on the planet carrier divided over 4 planet wheels	$F_{pl} := \frac{M_{pl}}{R_{gearing} \cdot 4}$	$F_{pl} = 1041.667 \text{ kN}$
---	--	--------------------------------

Radial load on the planet carrier	$F_{rad} := 290 \text{ kN}$	
-----------------------------------	-----------------------------	--

The radial load on the tooth of the sun wheel

Add. load on planet 1	$F_1 := 122 \text{ kN}$	
-----------------------	-------------------------	--

Add. load on planet 2	$F_2 := -40 \text{ kN}$	
-----------------------	-------------------------	--

Add. load on planet 3	$F_3 := -134 \text{ kN}$	
-----------------------	--------------------------	--

Add. load on planet 4	$F_4 := 53 \text{ kN}$	
-----------------------	------------------------	--

Force with added load 1	$F_{1n} := F_1 + F_{pl}$	$F_{1n} = (1.164 \cdot 10^3) \text{ kN}$
-------------------------	--------------------------	--

Force with added load 2	$F_{2n} := F_2 + F_{pl}$	$F_{2n} = (1.002 \cdot 10^3) \text{ kN}$
-------------------------	--------------------------	--

Force with added load 3	$F_{3n} := F_3 + F_{pl}$	$F_{3n} = 907.667 \text{ kN}$
-------------------------	--------------------------	-------------------------------

Force with added load 4	$F_{4n} := F_4 + F_{pl}$	$F_{4n} = (1.095 \cdot 10^3) \text{ kN}$
-------------------------	--------------------------	--

	$F_{avx} := \frac{(F_{1n} + F_{2n} + F_{3n} + F_{4n})}{4}$	$F_{avx} = 1041.917 \text{ kN}$
--	--	---------------------------------

Setup first equations of motion

We now have three equations: the moment balance and force balances in the x and y directions. To solve this system, we need four equations. However, the moment balance provides only one additional equation.

To obtain the fourth equation, we can leverage the geometric symmetry of the gear system. We can assume that the forces are evenly distributed across the planetary gears, and that the radial load is equal on each tooth. With this geometric assumption, we can derive a fourth equation.

By solving these four equations, we can determine the forces on the teeth of the gear system and thereby solve the problem.

The three equations

Moment balance around the axis of the planetary carrier:

The moment balance around the axis of the planetary carrier tells us that the sum of all moments around this axis must be equal to the total moment on the planetary carrier. So:

$$M_{pl} = (F_1 + F_2 + F_3 + F_4) \times (2/3)$$

Here:

$M_{pl} = 2500$ kNm, the moment on the planetary carrier.

$D = 1.2$ m, the diameter of the gear ring.

F_1, F_2, F_3, F_4 are the unknown forces on the teeth of the gear system.

Force balance in the x-direction:

$$F_1 - F_2 \times \tan(\alpha) - F_3 + F_4 \times \tan(\alpha) = F_{rad}$$

Here, $F_{rad} = 290$ kN is the radial load on the planetary carrier. See the freebody diagram

Force balance in the y-direction:

$$- F_1 \times \tan(\alpha) - F_2 + F_3 \times \tan(\alpha) + F_4 = 0$$

Step 1: Adding extra geometric symmetry for uniform force distribution

We utilize the geometric symmetry of the gear system by assuming that the forces are evenly distributed across the planetary gears, and that the radial load is equal on each tooth. Thus, each tooth experiences

$$F_r = (F_1 + F_2 + F_3 + F_4) / 4$$

F_r is used to indicate the even distribution of force across the four teeth of the planet carrier. This force is the sum of all forces on the teeth divided by the number of teeth, and is used to check if the forces are evenly distributed across the teeth. F_1 , F_2 , F_3 , and F_4 are the forces on the planet gears.

4 Equations assuming that the forces are evenly distributed across the planetary gears

Moment balance around the axis of the planetary carrier:

$$M_{pl} = (F_1 + F_2 + F_3 + F_4) \times (2D/3)$$

Here, $M_{pl} = 2500$ kNm is the moment on the planetary carrier, and $D = 1.2$ m is the diameter of the gear ring.

Equation for uniform force distribution:

$$F_r = (F_1 + F_2 + F_3 + F_4) / 4$$

Here, F_r is the radial force evenly distributed over each tooth.

Force balance in the x-direction:

$$F_1 - F_2 \times \tan(\alpha) - F_3 + F_4 \times \tan(\alpha) = F_{rad}$$

Here, $F_{rad} = 290$ kN is the radial load on the planetary carrier.

Force balance in the y-direction:

$$- F_1 \times \tan(\alpha) - F_2 + F_3 \times \tan(\alpha) + F_4 = 0$$

With these four equations, we have established a system of equations describing the forces on the teeth of the gear system. We can now use numerical methods to solve this system and find the values of F_1 , F_2 , F_3 , F_4 .

Solving the forces F1, F2, F3 and F4

The coefficient matrix A and the right-hand vector b are formed as follows:

coefficient matrix A

$$A := \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 1 & -\tan(\alpha_1) & -1 & \tan(\alpha_1) \\ -\tan(\alpha_1) & -1 & \tan(\alpha_1) & 1 \end{bmatrix}$$

vector matrix

$$b := \begin{bmatrix} \frac{2 \cdot M_{pl}}{D_{gearing}} \\ F_{avx} \\ F_{rad} \\ 0 \end{bmatrix}$$

Found a singularity while evaluating this expression. You may divide by 0 or inverting a singularity matrix

$$bb := \|A\| \quad bb = 0$$

$$Inv := A^{-1}$$

$$xx := A_{coeff}^{-1} \cdot b$$