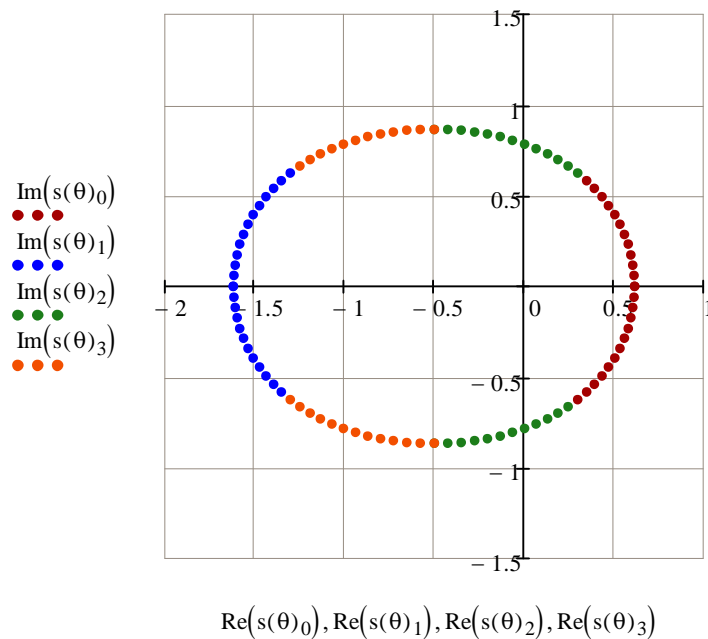


$$G(s) := \frac{1}{s^2 \cdot (1 + s)^2}$$

$$s(\theta) := G(s) = 1 \cdot \exp(j \cdot \theta) \begin{cases} \text{assume, } \theta = \text{real} \\ \text{solve, s, fully} \\ \text{factor} \end{cases} \rightarrow \begin{pmatrix} \frac{\sqrt{4 \cdot \sqrt{e^{-\theta \cdot j} + 1} - 1}}{2} \\ \frac{\sqrt{4 \cdot \sqrt{e^{-\theta \cdot j} + 1} + 1}}{2} \\ \frac{\sqrt{1 - 4 \cdot \sqrt{e^{-\theta \cdot j} - 1}}}{2} \\ \frac{\sqrt{1 - 4 \cdot \sqrt{e^{-\theta \cdot j} + 1}}}{2} \end{pmatrix} \xrightarrow{G(s(\theta)) \text{ simplify}} \begin{pmatrix} e^{\theta \cdot j} \\ e^{\theta \cdot j} \\ e^{\theta \cdot j} \\ e^{\theta \cdot j} \end{pmatrix}$$

$$\theta := 0, \frac{\pi}{12} .. 2\pi$$



$$s\left(\frac{\pi}{6}\right) = \begin{pmatrix} 0.609 - 0.117j \\ -1.609 + 0.117j \\ -0.349 + 0.859j \\ -0.651 - 0.859j \end{pmatrix}$$

$$G\left(s\left(\frac{\pi}{6}\right)\right) = \begin{pmatrix} 0.866 + 0.5j \\ 0.866 + 0.5j \\ 0.866 + 0.5j \\ 0.866 + 0.5j \end{pmatrix}$$



N := 12 i := 0..N $\theta_i := i \cdot \frac{2\pi}{N}$ **Some selected vsolutions**

$$s(\theta)^T = \begin{pmatrix} \begin{pmatrix} 0.618 \\ 0.609 - 0.117j \\ 0.581 - 0.231j \\ 0.536 - 0.341j \\ 0.474 - 0.445j \\ 0.395 - 0.54j \\ 0.3 - 0.625j \\ 0.395 + 0.54j \\ 0.474 + 0.445j \\ 0.536 + 0.341j \\ 0.581 + 0.231j \\ 0.609 + 0.117j \\ 0.618 \end{pmatrix} & \begin{pmatrix} -1.618 \\ -1.609 + 0.117j \\ -1.581 + 0.231j \\ -1.536 + 0.341j \\ -1.474 + 0.445j \\ -1.395 + 0.54j \\ -1.3 + 0.625j \\ -1.395 - 0.54j \\ -1.474 - 0.445j \\ -1.536 - 0.341j \\ -1.581 - 0.231j \\ -1.609 - 0.117j \\ -1.618 \end{pmatrix} & \begin{pmatrix} -0.5 + 0.866j \\ -0.349 + 0.859j \\ -0.202 + 0.839j \\ -0.061 + 0.806j \\ 0.071 + 0.759j \\ 0.192 + 0.698j \\ 0.3 + 0.625j \\ 0.192 - 0.698j \\ 0.071 - 0.759j \\ -0.061 - 0.806j \\ -0.202 - 0.839j \\ -0.349 - 0.859j \\ -0.5 - 0.866j \end{pmatrix} & \begin{pmatrix} -0.5 - 0.866j \\ -0.651 - 0.859j \\ -0.798 - 0.839j \\ -0.939 - 0.806j \\ -1.071 - 0.759j \\ -1.192 - 0.698j \\ -1.3 - 0.625j \\ -1.192 + 0.698j \\ -1.071 + 0.759j \\ -0.939 + 0.806j \\ -0.798 + 0.839j \\ -0.651 + 0.859j \\ -0.5 + 0.866j \end{pmatrix} \end{pmatrix}$$

$$G(s) := \frac{1}{s^2(s+1)^2}$$

$$M2(a,b) := (|G(a+j\cdot b)|)^2 \left| \begin{array}{l} \text{assume, a = real, b = real} \\ \text{simplify} \end{array} \right. \rightarrow \frac{1}{(a^2 + b^2)^2 \cdot (a+1+b\cdot j)^4}$$

Thats wrong! The square of the magnitude of any complex number is a positive (real) number.

$$M2(0.2,0.2) = 56.327 - 43.77j \quad \text{wrong!} \qquad |M2(0.2,0.2)| = 71.334$$

Mathcad should really know it better!

Taking additionally the maginitude should not be necessary!

$$(|G(a+j\cdot b)|)^2 \left| \begin{array}{l} \text{substitute, a = 0.2, b = 0.2} \\ \text{simplify} \end{array} \right. \rightarrow 71.334002921840759679$$

But with a simple and stupid trick we can convince Mathcad/muPad to do it the right way:

$$G(a,b) := \text{Re}(G(a+j\cdot b)) + j\cdot\text{Im}(G(a+j\cdot b)) \qquad \text{Redefinition of } G()$$

$$(|G(a,b)|)^2 \left| \begin{array}{l} \text{assume, a = real, b = real} \\ \text{simplify, factor} \end{array} \right. \rightarrow \frac{1}{(2\cdot a + a^2 + b^2 + 1) \cdot (a^2 + b^2)^2}$$

$$|G(a,b)| \left| \begin{array}{l} \text{assume, a = real, b = real} \\ \text{simplify, factor} \end{array} \right. \rightarrow \frac{1}{\sqrt{(a^4 + 2\cdot a^3 + 2\cdot a^2\cdot b^2 + a^2 + 2\cdot a\cdot b^2 + b^4 + b^2)^2}}$$

Goal was to convince Mathcad that $|G(a+j\cdot b)| = \frac{1}{(2\cdot a + a^2 + b^2 + 1) \cdot (a^2 + b^2)}$ but this seems not to be

possible. We can easily prove that the expression in the first pair of parantheses never gets negative as the minimum of that expression is at a=-1 and is b^2.

So I have to assign it manually :-)

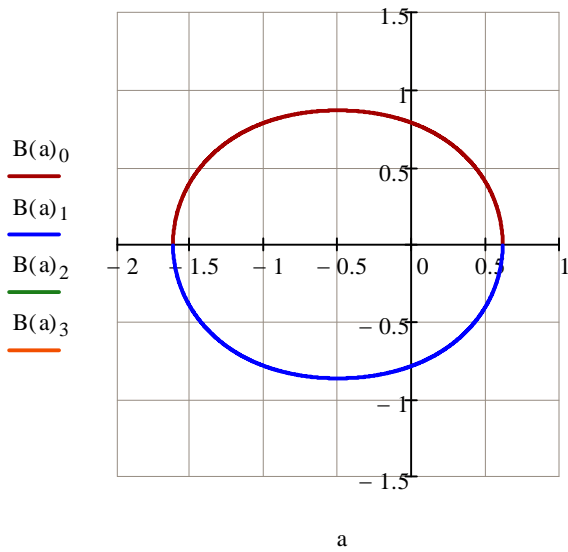
$$\text{MagnG}(a,b) := \frac{1}{(2\cdot a + a^2 + b^2 + 1) \cdot (a^2 + b^2)}$$

Now lets solve the given equation $|G(s)| = 1$

$$B(a) := \text{MagnG}(a,b) = 1 \text{ solve, b} \rightarrow \left(\begin{array}{l} \sqrt{\frac{\sqrt{4\cdot a^2 + 4\cdot a + 5}}{2} - a^2 - a - \frac{1}{2}} \\ -\sqrt{\frac{1}{2} \cdot \sqrt{4\cdot a^2 + 4\cdot a + 5} - a^2 - a - \frac{1}{2}} \\ \sqrt{-a - a^2 - \frac{\sqrt{4\cdot a^2 + 4\cdot a + 5}}{2} - \frac{1}{2}} \\ -\sqrt{-a - a^2 - \frac{1}{2} \cdot \sqrt{4\cdot a^2 + 4\cdot a + 5} - \frac{1}{2}} \end{array} \right)$$

It looks like the third and fourth solution won't yield a real result ever and the plot below seems to

$$a := -2, -1.999..2$$



Adding the additional assumption does not make it easier and simpler:

$$B(a) := \text{MagnG}(a, b) = 1 \left| \begin{array}{l} \text{assume, ALL = real} \\ \text{solve, b} \end{array} \right. \rightarrow \left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{\sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2} - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}{2} \\ \frac{\sqrt{2} \cdot \sqrt{\sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2} - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}{2} \end{array} \right) \text{ if } 0 \leq -a - a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5}$$

$$\left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2} - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2} - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}{2} \end{array} \right) \text{ if } 0 \leq -a - a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5}$$

$$\left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{\sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{\sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \end{array} \right) \text{ if } 0 > -a - a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5}$$

Adding the additional assumption does not make it easier and simpler:

$$\begin{array}{l}
 B(a) := \text{MagnG}(a, b) = 1 \quad \left| \begin{array}{l} \text{assume, ALL = real} \\ \text{solve, b} \end{array} \right. \rightarrow \\
 \left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{4 \cdot a^2 + 4 \cdot a + 5 - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}}{2} \\ \frac{-\sqrt{2} \cdot \sqrt{4 \cdot a^2 + 4 \cdot a + 5 - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}}{2} \end{array} \right) \quad \text{if } 0 \leq -a - a^2 - \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - \frac{1}{2} \wedge 0 \leq \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - a^2 - a - \frac{1}{2} \\
 \left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{-2 \cdot a - 2 \cdot a^2 - \sqrt{4 \cdot a^2 + 4 \cdot a + 5} - 1}}{2} \end{array} \right) \quad \text{if } 0 \leq -a - a^2 - \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - \frac{1}{2} \wedge 0 > \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - a^2 - a - \frac{1}{2} \\
 \left(\begin{array}{l} \frac{\sqrt{2} \cdot \sqrt{4 \cdot a^2 + 4 \cdot a + 5 - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \\ \frac{\sqrt{2} \cdot \sqrt{4 \cdot a^2 + 4 \cdot a + 5 - 2 \cdot a^2 - 2 \cdot a - 1}}{2} \end{array} \right) \quad \text{if } 0 > -a - a^2 - \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - \frac{1}{2} \wedge 0 \leq \frac{\sqrt{4 \cdot a^2 + 4 \cdot a + 5}}{2} - a^2 - a - \frac{1}{2}
 \end{array}$$