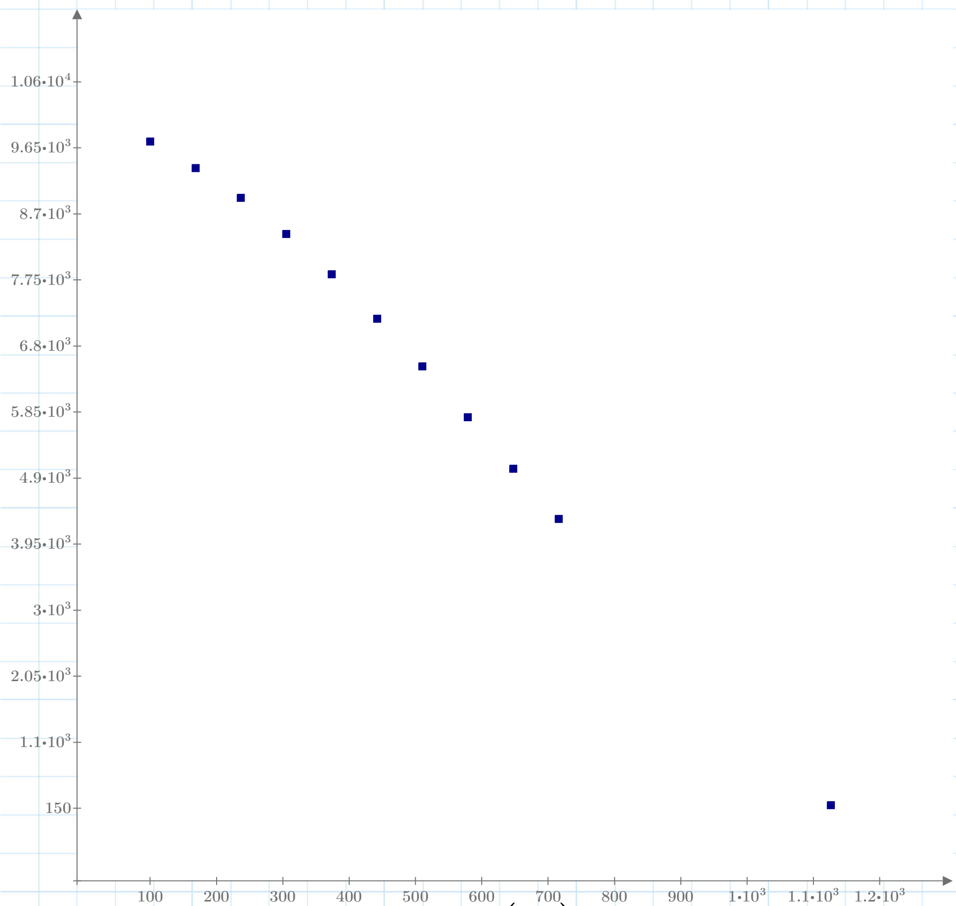
Example Curve Fits - Linear, Polynomial, Etc.

$$ps := \begin{bmatrix} 100 \\ 168.4 \\ 236.8 \\ 305.3 \\ 373.7 \\ 442.1 \\ 510.5 \\ 578.9 \\ 647.4 \\ 715.8 \\ 1126.3 \end{bmatrix} psi \qquad qres := \begin{bmatrix} 9742 \\ 9360 \\ 8929 \\ 8413 \\ 7833 \\ 7193 \\ 6507 \\ 5777 \\ 5035 \\ 4312 \\ 195 \end{bmatrix} m \cdot m \cdot \frac{m}{s}$$



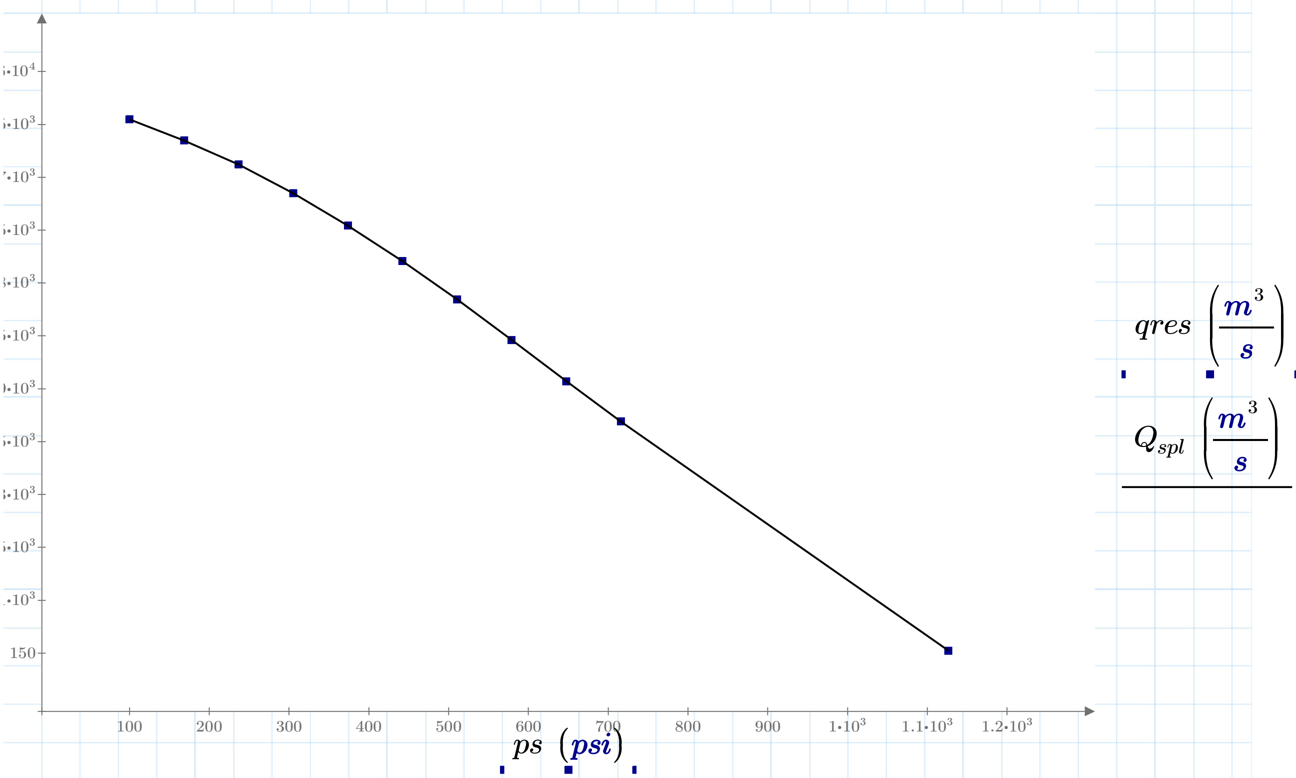
$qres \left(\frac{m^3}{s}\right)$

$ps \ (psi)$

CASE 1: Cubic Spline of Data

$$Q_{spline}(P) := \text{interp}(\text{cspline}(ps, qres), ps, qres, P)$$

$$ps = \begin{bmatrix} 100 \\ 168.4 \\ 236.8 \\ 305.3 \\ 373.7 \\ 442.1 \\ 510.5 \\ 578.9 \\ 647.4 \\ 715.8 \\ 1.126 \cdot 10^3 \end{bmatrix} psi \quad qres = \begin{bmatrix} 9.742 \cdot 10^3 \\ 9.36 \cdot 10^3 \\ 8.929 \cdot 10^3 \\ 8.413 \cdot 10^3 \\ 7.833 \cdot 10^3 \\ 7.193 \cdot 10^3 \\ 6.507 \cdot 10^3 \\ 5.777 \cdot 10^3 \\ 5.035 \cdot 10^3 \\ 4.312 \cdot 10^3 \\ 195 \end{bmatrix} \frac{m^3}{s} \qquad Q_{spl} := Q_{spline}(ps) = \begin{bmatrix} 9.742 \cdot 10^3 \\ 9.36 \cdot 10^3 \\ 8.929 \cdot 10^3 \\ 8.413 \cdot 10^3 \\ 7.833 \cdot 10^3 \\ 7.193 \cdot 10^3 \\ 6.507 \cdot 10^3 \\ 5.777 \cdot 10^3 \\ 5.035 \cdot 10^3 \\ 4.312 \cdot 10^3 \\ 195 \end{bmatrix} \frac{m^3}{s}$$



CASE 2: Straight Line Fits

$b \cdot x + a$

$X := \overrightarrow{ps}$ $\quad$ $Y := \overrightarrow{qres}$

$Y_{pred}(x) := \text{slope}(X, Y) \cdot x + \text{intercept}(X, Y)$

$Y_{lin1} := Y_{pred}(\overrightarrow{ps})$

Alternatively: use polyfit

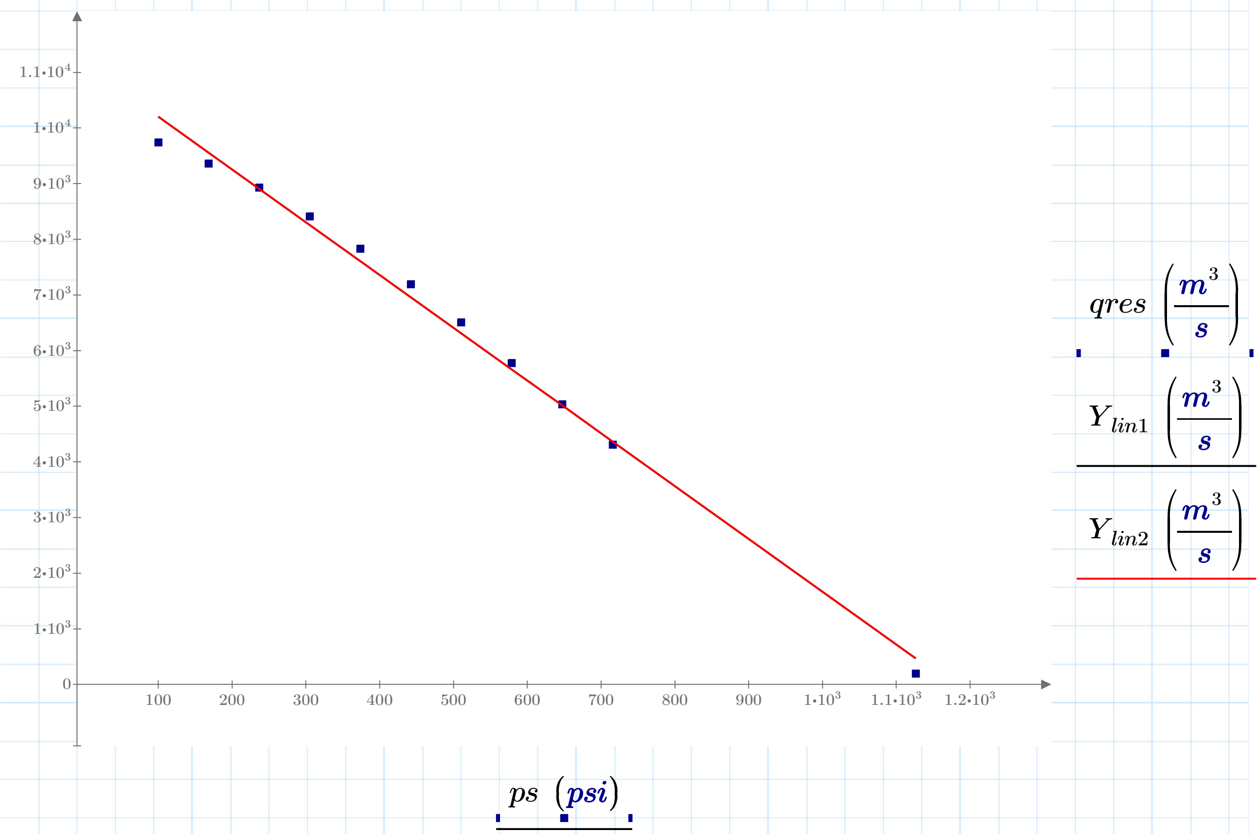
$pc := \text{polyfitc}(X, Y, 1)$ (implied confidence of 95%)

$coeff := \text{submatrix}(pc, 1, 2, 1, 1)$ rows 1,2 of column 1

$Y2(x) := \sum_{i=0}^{1} coeff_i \cdot x^i$

$Y_{lin2} := Y2(\overrightarrow{ps})$

$cc := \text{corr}(Y_{lin2}, Y) = 0.997$



CASE 3: Quadratic fit of Qres(ps) $a \cdot x^2 + b \cdot x + c$

Using the "linfit" function, to fit linear function to the data (in this case, linear in x^2, x and a constant)

$X := \overrightarrow{ps}$ $Y := \overrightarrow{qres}$

$vf(x) := \begin{bmatrix} x \cdot x \\ x \\ 1 \end{bmatrix}$ Basis Function: $a \cdot x^2 + b \cdot x + c$

$$c := \text{linfit}(X, Y, vf) = \begin{bmatrix} -3.841 \cdot 10^{-11} \dfrac{m^5 \cdot s^3}{kg^2} \\ -0.001 \dfrac{m^4 \cdot s}{kg} \\ \left(1.068 \cdot 10^4\right) \dfrac{m^3}{s} \end{bmatrix}$$ Solve for the constants: a, b, c

$f(x) := c \cdot vf(x)$ Multiply constants by the basis function to yield prediction

$Y_{quad1} := \overrightarrow{f(ps)}$ $cc := \text{corr}\left(Y_{quad1}, Y\right) = 0.999$

Repeat using polyc:

$pc := \text{polyfitc}(X, Y, 2)$ (implied confidence of 95%)

$coeff := \text{submatrix}(pc, 1, 3, 1, 1)$ rows 1,3 of column 1

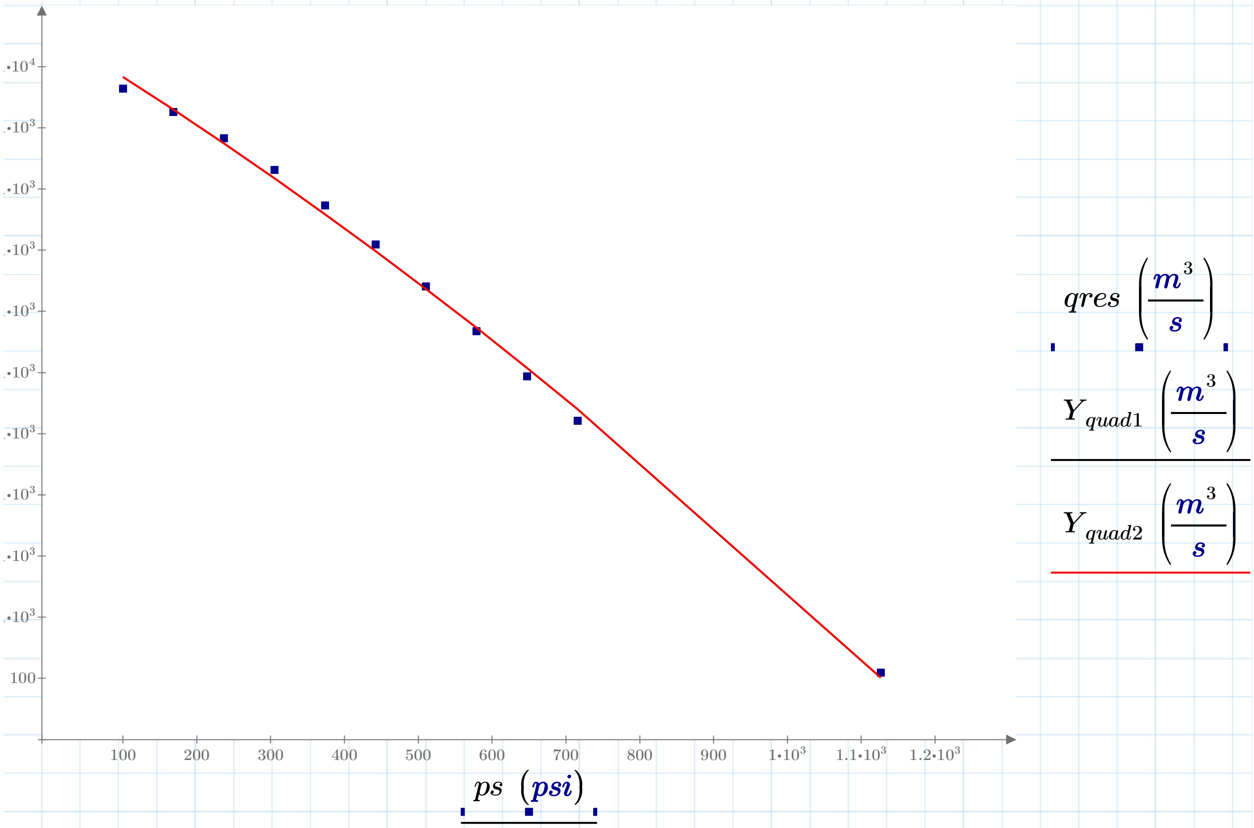
$Y2(x) := \sum_{i=0}^{2} coeff_i \cdot x^i$ $Y_{quad2} := Y2\left(\overrightarrow{ps}\right)$

$cc := \text{corr}\left(Y_{quad2}, Y\right) = 0.999$

$pstat := \text{polyfitstat}(X, Y, 2) =$

"Regression Analysis"	"Value"
"Standard Deviation"	$142.147 \dfrac{m^3}{s}$
"R2"	0.998
"Adjusted R2"	0.997
"Predicted R2"	0.953
"PRESS"	$\left(3.619 \cdot 10^6\right) \dfrac{m^6}{}$

“PRESS”	[illegible] s^2
“Durbin–Watson”	0.86
“Coefficients”	$[4 \times 8]$
“ANOVA”	$[8 \times 6]$
“Diagnostics”	$[12 \times 9]$



CASE 3: Cubic fit of Qres(ps)

$a \cdot x^3 + b \cdot x \cdot x + c \cdot x + d$

Using the "linfit" function, to fit linear function to the data (in this case, linear in x^3, X^2, etc)

$X := \overrightarrow{ps}$ $\quad Y := \overrightarrow{qres}$

$vf(x) := \begin{bmatrix} x^3 \\ x^2 \\ x \\ 1 \end{bmatrix}$ Basis function $\quad a \cdot x^3 + b \cdot x^2 + c \cdot x + d$

$c := \text{linfit}(X, Y, vf) = \begin{bmatrix} (2.188 \cdot 10^{-16}) \dfrac{m^6 \cdot s^5}{kg^3} \\ -2.946 \cdot 10^{-9} \dfrac{m^5 \cdot s^3}{kg^2} \\ 0.01 \dfrac{m^4 \cdot s}{kg} \\ (4.253 \cdot 10^{-9}) \dfrac{m^3}{s} \end{bmatrix}$ Solve for the constants: a, b, c, d

$f(x) := c \cdot vf(x)$ Multiply constants by the basis function to yield prediction

$Y_{cub1} := \overrightarrow{f(ps)}$ $\quad cc := \text{corr}(Y_{cub1}, Y) = 0.817$

This is clearly wrong!!!! Correlation coefficient should get better (at worst a=0)

Now use polyc:

$pc := \text{polyfitc}(X, Y, 3)$ (implied confidence of 95%)

$coeff := \text{submatrix}(pc, 1, 4, 1, 1)$ rows 1,4 of column 1

$coeff = \begin{bmatrix} (1.007 \cdot 10^{4}) \dfrac{m^3}{s} \\ -3.278 \cdot 10^{-4} \dfrac{m^4 \cdot s}{kg} \\ -2.561 \cdot 10^{-10} \dfrac{m^5 \cdot s^3}{kg^2} \\ (1.732 \cdot 10^{-17}) \dfrac{m^6 \cdot s^5}{kg^3} \end{bmatrix}$

$$Y2(x) := \sum_{i=0}^{3} coeff_i \cdot x^i$$

$$Y_{cub2} := \overrightarrow{Y2(ps)}$$

$$cc := \text{corr}\left(Y_{cub2}, Y\right) = 1$$