INSTITUTO TECNOLÓGICO DE AERONÁUTICA

MP-288 - Exercises on Conjugate Gradient Method

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1) Consider the function $f(\mathbf{x}) = f(x_1, x_2) = 10x_1^4 - 20x_1^2x_2 + 10x_2^2 - 2x_1 + 5$.

Starting from $\mathbf{x}^0 = (-1.5, 3)$, use both the steepest descent and conjugate gradient methods to find a minimum point of $f(\mathbf{x})$. Perform exact line search (use $df/d\alpha = 0$ at $\mathbf{x}_k$ to find all the α^{*k}). Plot a graph showing $f(\mathbf{x})$ and the $\alpha^{*k}\mathbf{d}^k$ directions used over iterations of both methods.

2) Consider the function

$f(\mathbf{x}) = f(x_1, x_2) = 4\left[\sqrt{x_1^2 + (10 - x_2)^2} - 10\right]^2 + \frac{1}{2}\left[\sqrt{x_1^2 + (10 + x_2)^2} - 10\right]^2 - 5(x_1 + x_2)$.

Starting from $\mathbf{x}^0 = (-4, 4)$, use the conjugate gradient method to find a minimum point of $f(\mathbf{x})$. Perform numerical line search by the golden section method. Use the routine to be developed in Problem 3. Plot a graph showing $f(\mathbf{x})$ and the $\alpha^{*k}\mathbf{d}^k$ directions used over iterations.

3) Implement a Matlab routine called `conjugate_gradient.m` based on the conjugate gradient method. Solve Problem 2 with that and optionally Problem 1. Define it as `[xo]=conjugate_gradient(f,x0,tol)`.

In other words, define a routine in which the inputs are the function `f`, the initial point `x0` and the tolerance for zero gradient `tol`; the output is `xo`, a local minimum of $f(\mathbf{x})$. Use your previously implemented routine `golden_section.m` as line search.