▶ Text Syle definitions
▶ Maths Style definitions
▶ Code of Practice

Angles as a Dimension

Author / Contact

Philip Oakley
philipoakley@iee.org

▶ Process

▼ Description

Description

This worksheet shows how the use of Angles as a dimension would work. The recent inclusion of the money dimension allows us to simulate the effect.

▲ Description

▼ Units of Angle

Update the built-in unit definitions that (should) contain the angle dimension.

$\text{rad} := \text{rad}\,¤$ $\qquad$ $\text{rad} = 1\,¤$ $\qquad$ << see the extra ¤ symbol for the dimension

$\text{deg} := \text{deg}\cdot\text{rad}$ $\qquad$ $° := °\cdot\text{rad}$

$\text{rev} := \text{rev}\cdot\text{rad}$

$\text{cy} := 2\pi\,\text{rad}$ $\qquad$ $\text{turn} := \text{rev}$

$\text{sr} := \text{sr}\cdot\left(¤^2\right)$ $\qquad$ Solid angle is $\frac{L_y\cdot L_z}{{L_x}^2}$ which refactors to $\left(\frac{L_y}{L_x}\right)\cdot\left(\frac{L_z}{L_x}\right)$ which is Angle squared.

Though they are on independent axes, however we have the 3 axis cycle whereby there is no choice as to the second axis

$\text{Hza} := \text{cy}\cdot\text{Hz}$ $\qquad$ $\text{Hza} = 6.283\,\frac{¤}{\text{s}}$

▲ Units of Angle

▼ Trigonometric Angle functions

a is an Angle and must have the dimensions of Angle to the unit power

$\text{Sine}(a) := \sin\left(\frac{a}{\text{rad}}\right)$ $\qquad$ $\text{Cosecant}(a) := \csc\left(\frac{a}{\text{rad}}\right)$

$\text{Cosine}(a) := \cos\left(\frac{a}{\text{rad}}\right)$

$\text{Secant}(a) := \sec\left(\frac{a}{\text{rad}}\right)$

$\text{Tangent}(a) := \tan\left(\frac{a}{\text{rad}}\right)$

$\text{Cotangent}(a) := \cot\left(\frac{a}{\text{rad}}\right)$

$\text{Sinc}(a) := \text{sinc}\left(\frac{a}{\text{rad}}\right)$

v is a numeric value and must have no dimensional powers

$\text{ArcSine}(v) := \text{asin}(v)\cdot\text{rad}$

$\text{ArcSecant}(v) := \text{asec}(v)\cdot\text{rad}$

$\text{ArcCosine}(v) := \text{acos}(v)\cdot\text{rad}$

$\text{ArcCosecant}(v) := \text{acsc}(v)\cdot\text{rad}$

$\text{ArcTangent}(v) := \text{atan}(v)\cdot\text{rad}$

$\text{ArcCotanget}(v) := \text{acot}(v)\cdot\text{rad}$

$\text{ArctTangent2}(x,y) := \text{atan2}(x,y)\cdot\text{rad}$

$\text{Angle}(x,y) := \text{angle}(x,y)\cdot\text{rad}$

Trigonometric Angle functions

Extended Trig Functions

$\text{Versine}(a) := \frac{1}{2}\cdot\text{Sine}\left(\frac{a}{2}\right)^2$

http://en.wikipedia.org/wiki/Haversine

$\text{Haversine}(a) := \text{Sine}\left(\frac{a}{2}\right)^2$

Extended Trig Functions

Exponential functions

$$\text{Exp}(i\theta) := \left|\begin{array}{l} \text{if } \operatorname{Re}\left(\frac{i\theta}{\text{rad}}\right) \neq 0 \\ \quad\left|\begin{array}{l} \text{"The neper scale is kept separate"} \\ \text{error("Must be a pure imaginary Angle, separate the real and imaginary parts")} \end{array}\right. \\ \exp\left(\frac{i\theta}{\text{rad}}\right) \end{array}\right.$$

$$\text{Ln}(z) := \left|\begin{array}{l} \text{if } |z| \neq 1 \\ \quad\left|\begin{array}{l} \text{"The neper scale is kept separate"} \\ \text{error("Must be unit magnitude, returns the imaginary phase with Angle dimensions")} \end{array}\right. \\ \ln(z)\cdot\text{rad} \end{array}\right.$$

The same functions but without constraints

$$\mathrm{Exp}(i\theta) := \exp\left(\frac{i\theta}{rad}\right)$$

$$\mathrm{Ln}(z) := \ln(z)\cdot rad$$

The one Argand complex plane to Angle function

$$\mathrm{Arg}(z) := \arg(z)\cdot rad$$

Exponential functions

Hyperbolic functions

$$\mathrm{HyperSine}(\theta) := \sinh\left(\frac{\theta}{rad}\right)$$

$$\mathrm{HyperSecant}(\theta) := \mathrm{asech}\left(\frac{\theta}{rad}\right)$$

$$\mathrm{HyperCosine}(\theta) := \cosh\left(\frac{\theta}{rad}\right)$$

$$\mathrm{HyperCosecant}(\theta) := \mathrm{acsch}\left(\frac{\theta}{rad}\right)$$

$$\mathrm{HyperTangent}(\theta) := \tanh\left(\frac{\theta}{rad}\right)$$

$$\mathrm{HyperCotangent}(\theta) := \coth\left(\frac{\theta}{rad}\right)$$

$$\mathrm{ArcHyperSine}(z) := \mathrm{asinh}(z)\cdot rad$$

$$\mathrm{ArcHyperSecant}(z) := \mathrm{asech}(z)\cdot rad$$

$$\mathrm{ArcHyperCosine}(z) := \mathrm{acosh}(z)\cdot rad$$

$$\mathrm{ArcHyperCosecant}(z) := \mathrm{acsch}(z)\cdot rad$$

$$\mathrm{ArcHyperTangent}(z) := \mathrm{atanh}(z)\cdot rad$$

$$\mathrm{ArcHyperCotangent}(z) := \mathrm{acoth}(z)\cdot rad$$

Hyperbolic functions

(Key) New Angle functions

$$\mathrm{Arc}(r,\theta) := r\cdot\frac{\theta}{rad}$$

$$\mathrm{ArcLength}(r,\theta) := \mathrm{Arc}(r,\theta)$$

$$\text{Moment}(\text{radius}, \text{tangential}) := \text{radius}\cdot\frac{\text{tangential}}{\text{rad}}$$

$$\text{Torque}(\text{radius}, \text{tangential}) := \text{Moment}(\text{radius}, \text{tangential})$$

$$\text{Phase}(t, T) := \frac{t}{T}\cdot\text{cy}$$

$$\text{cPhase}(t, T) := \text{Exp}(i\cdot\text{Phase}(t, T))$$

(Key) New Angle functions

Solid Angle & Chord

$$\text{Chord}(a, f) := 2f\cdot\text{Cosine}\left(\frac{a}{2}\right)$$

$$\text{ArcChord}(d, f) := 2\cdot\text{ArcCosine}\left[\frac{\left(\frac{d}{2}\right)}{f}\right]$$

The d and f parameters are informed by the f/# calculations for lenses.
The true f/# calculation is the diameter divided by the radial distance to the edge of the lens.

$$\text{SolidAngle}(\text{Area}, \text{radius}) := \frac{\text{Area}}{\text{radius}^2}\cdot\text{sr}$$

Solid Angle & Chord

VIM Vocabulary Comment/Errata

The International Vocabulary for Metrology is called the VIM. A copy can be found at http://www.bipm.org/en/publications/guides/vim.html

From the perspective of MathCAD's use of dimensions as an error checking feature and this worksheet's use of the Money dimension to detect user mistakes, slips and lapses in their formula construction and evaluation, an errata for the VIM is required.

Errata 1
Section $1.2 "**kind of quantity**", Note 1, Example 1.
The example suggests that a circumference is of the same kind as wavelength and diameter, that is, of the kind of quantity called length. This suggests that curved lines and straight lines are of the same kind of quantity. The note already states that the distinctions (or lack of) are somewhat arbitrary.

The Length examples should be of the straight line kind. These are defined simply by their end points.
- The circumference example should be deleted from the Example.

Errata 2
Section $1.2 "**kind of quantity**", Note 2.
The note states "Quantities of the same kind within a given **system of quantities** have the same **quantity dimension.**" The note then goes on to allow "quantities of the same dimension are not necessarily of the same kind". Examples are then given, however it does not indicate that "**off-system quantity dimensions**" may be used to aid the detection of errors in external calculations.

- Add "the detection of errors in external calculations may be aided by **off-system quantity dimensions**"

Errata 3
Section $1.3 "**system of quantities**", Add Note 2.
- "**off-system quantities** are external quantities and their dimension, not within the system of quantities used outside the scope of the VIM. For example use in error detection in external calculations."

Errata 4
Section $1.7 "**quantity dimension**", Add Note 4b.
- "Additional **off-system quantity dimensions** can be used in external calculations to discriminate between quantities, not of the same kind, would have the same in-system quantity dimensions."
- "Coherent off-system quantity dimensions are quantities of dimension one within the system of quantities."
- "Examples of off-system quantity dimensions could include: currency, angle, material type (1 apple; 1 orange), L_x,L_y, L_z Length distinctions."

Errata 5
Section $1.8 "**quantity of dimesion one**", Add Note 2b
- Coherent **off-system quantity dimensions** are of quantity dimension one and can be used in extrenal calculations to express such extra information and assist in error detection and consistency checking.

Errata 6
Section $1.9 "**measurement unit**", Note 2b
- off-system quantity dimensions may be used in external calculations to indentify quantity mesurements not of the same kind.

Errata 7
Section $1.10 "**base unit**", Note 1b
- there are off-system base units for each off-system base quantity.

Errata 8
Section $1.15 "**off-system measurement unit**", Add Example.
- Example 3 The measurement of torque in J/turn includes the off-system dimension of Angle and the off-system measurement unit of turn (one turn equals 2π radians).

Errata 9
Section $1.19 "**quantity value**", Ammend Example 3.

- "Curvature of a given arc: 112 m^{-1}, or 112 rad m^{-1} including the angle **off-system quantity dimension**."

VIM Vocabulary Comment/Errata

SI Comment/Errata

The International System of Units (SI) provides the units and corresponding system of quantities to be used in the International System of Quantities (ISQ) and the definition of the dimensions of those quantities. A copy can be found at http://www.bipm.org/en/si/si_brochure/.
The SI uses the vocabulary defined in the VIM

Errata 1
$1.3 **Dimensions of Quantities**, add paragraph 2, after table

- Additional dimensions, beyond the SI convention, are used in some external calculations for the detection of errors within quantity calculations. These additional dimenstions, such as money, or angle, material kind (1 apple, 1 orange) are off-system quantities within the extended VIM terminology. The detection and prevention of such errors is generally a good thing. When such off-system base quantities are coherent they are of dimension one within the SI.

Off-SI base quantity	Symbol for quantity	Symbol for dimension	Example DisplayName
Money	C,cost, price	¤	$ (dollar)
Angle	θ,ϕ,φ	A	Angle
User1	kgws	U_1	Wet_Steam
User2	kgg	U_2	Gas
User3	amount	U_3	Apples
User4	amount	U_4	Oranges

SI Comment/Errata

Note

Note: The use of 'Arc' in the inverse transforms is contradictory.
It is not the length of the arc, even for a radius=1, that is returned, rather it is the angle that is returned.
A better term may be 'Inv', such as InvSine (shortened from InverseSine).

$\mathrm{Exp}(\mathrm{rad}\cdot i) = 0.54 + 0.841i$

$\mathrm{Exp}\left(\frac{\mathrm{cy}}{2}\cdot i\right) = -1$

$\mathrm{Exp}(\pi\cdot i) = \blacksquare$

$\mathrm{Re}[(1+2i)\mathrm{lb}] = 0.454\,\mathrm{kg}$

$\mathrm{Im}[(1+2i)\mathrm{lb}] = 0.907\,\mathrm{kg}$

$\mathrm{Ln}\left(\frac{1+i}{\sqrt{2}}\right) = 0.785i$ ¤

$\mathrm{Ln}\left(\frac{1+i}{\sqrt{2}}\right) = 0.785i\cdot\mathrm{rad}$

Note : You can't have a complex value in the unit scaling placeholder (MathCAD V14 M030)

$\mathrm{Arc}(1\mathrm{m}, 45\mathrm{deg}) = 0.785\,\mathrm{m}$

$\mathrm{ArcLength}(1\mathrm{m}, 45\mathrm{deg}) = 0.785\,\mathrm{m}$

$\text{Moment}(1\text{m}, 1\text{N}) = 1\,\frac{\text{m}^2\cdot\text{kg}}{¤\cdot\text{s}^2}$

$\text{Torque}(1\text{m}, 1\text{N}) = 1\,\frac{\text{m}^2\cdot\text{kg}}{¤\cdot\text{s}^2}$

$\text{Phase}(1, 2\pi) = 1\,¤$

$\text{cPhase}(1, 2\pi) = 0.54 + 0.841i$