3D SCATTER PLOT OF HEART-SHAPED SURFACE

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Reference: http://www.mathematische-basteleien.de/heart.htm

$\varepsilon := 0.01$ Set tolerance for f-function.

Define functions to calculate points on surface of heart. ("Cor" is Latin for "heart.")

```
Cor1(tol) := | m ← 0
             | for i ∈ 1 .. 101
             |   | z ← 1.5·(−1 + (i − 1)/50)
             |   | for j ∈ 1 .. 101
             |   |   | x ← 1.0·(−1 + (j − 1)/50)
             |   |   | n ← 0
             |   |   | for k ∈ 1 .. 601
             |   |   |   | y ← 1.25·(−1 + (k − 1)/1000)
             |   |   |   | f ← (2·x^2 + y^2 + z^2 − 1)^3 − (x^2·z^3)/10 − y^2·z^3
             |   |   |   | if |f| < tol
             |   |   |   |   | n ← n + 1
             |   |   |   |   | F_n ← |f|
             |   |   |   |   | y1_n ← y
             |   |   | if n ≥ 1
             |   |   |   | m ← m + 1
             |   |   |   | f ← min(F)
             |   |   |   | for p ∈ 1 .. n
             |   |   |   |   Row ← (x  y1_p  z)  if F_p = f
             |   |   |   | H ← Row  if m = 1
             |   |   |   | H ← stack(H, Row)  otherwise
             | H
```

```
Cor2(tol) := | m ← 0
             | for i ∈ 1 .. 101
             |   | z ← 1.5·(−1 + (i − 1)/50)
             |   | for j ∈ 1 .. 101
             |   |   | x ← 1.0·(−1 + (j − 1)/50)
             |   |   | n ← 0
             |   |   | for k ∈ 602 .. 1001
             |   |   |   | y ← 1.25·(−1 + (k − 1)/1000)
             |   |   |   | f ← (2·x^2 + y^2 + z^2 − 1)^3 − (x^2·z^3)/10 − y^2·z^3
             |   |   |   | if |f| < tol
             |   |   |   |   | n ← n + 1
             |   |   |   |   | F_n ← |f|
             |   |   |   |   | y1_n ← y
             |   |   | if n ≥ 1
             |   |   |   | m ← m + 1
             |   |   |   | f ← min(F)
             |   |   |   | for p ∈ 1 .. n
             |   |   |   |   Row ← (x  y1_p  z)  if F_p = f
             |   |   |   | H ← Row  if m = 1
             |   |   |   | H ← stack(H, Row)  otherwise
             | H
```

$Heart1 := Cor1(\varepsilon)$ $\quad$ $Heart2 := Cor2(\varepsilon)$

$Heart3 := augment\left(augment\left(Heart1^{\langle 1\rangle}, -Heart1^{\langle 2\rangle}\right), Heart1^{\langle 3\rangle}\right)$

$Heart4 := augment\left(augment\left(Heart2^{\langle 1\rangle}, -Heart2^{\langle 2\rangle}\right), Heart2^{\langle 3\rangle}\right)$

$X := stack\left(Heart1^{\langle 1\rangle}, Heart2^{\langle 1\rangle}\right)$

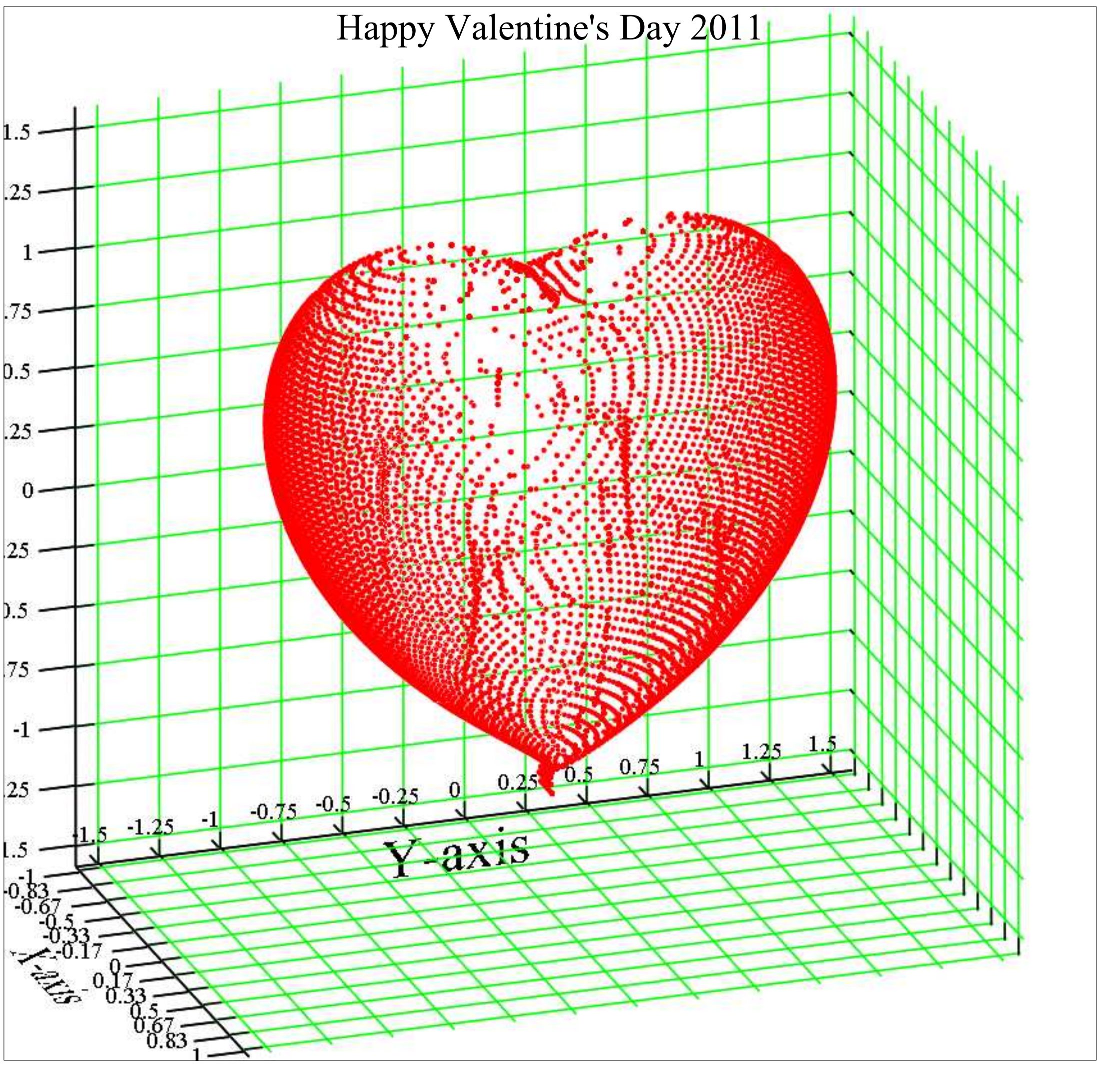
$X := stack\left(X, Heart3^{\langle 1\rangle}\right)$ $\quad$ $X := stack\left(X, Heart4^{\langle 1\rangle}\right)$

$Y := \mathrm{stack}\left(Heart1^{\langle 2\rangle}, Heart2^{\langle 2\rangle}\right)$

$Y := \mathrm{stack}\left(Y, Heart3^{\langle 2\rangle}\right)$ $Y := \mathrm{stack}\left(Y, Heart4^{\langle 2\rangle}\right)$

$Z := \mathrm{stack}\left(Heart1^{\langle 3\rangle}, Heart2^{\langle 3\rangle}\right)$

$Z := \mathrm{stack}\left(Z, Heart3^{\langle 3\rangle}\right)$ $Z := \mathrm{stack}\left(Z, Heart4^{\langle 3\rangle}\right)$



(X, Y, Z)

Notes on Construction

1. Basic Approach

The basic approach use herein is to calculate values of f = f(x,y,z) with z ranging from -1.5 to 1.5, x ranging from -1 to 1, and y ranging from -1.25 to 1.25. Whenever the absolute value of f is less than a pre-specified tolerance, the y-value is saved and a counter (n) is incremented.

When all of the y-values have been computed for a given z and x, then a row of th H matrix, Row = (x,y,z) is saved. The y-value is that y for which | f | is the smallest. This insures that of all the possible y values in an ε neighborhood of the true point on the heart's surface, the saved (x,y,z) point is the one closest to the true surface point. The H matrix contains all of the saved surface points; it is an mx3 matrix.

2. Implicit Function Theorem

The heart surface defines y implicitly as a function of z and x, piecewise in two parts. The first part is the surface piece swept out by an xz plane starting at y = -1.25 and moving to y = -0.5. The second part is the surface piece swept out by the xz plane starting at y = -0.5 and moving to y = 0.0. The implicit function theorem ensures that y is a single-valued function of z and x on each of the two surface pieces. This is why there are two Cor functions, Cor1 and Cor2. Each Cor function is assured of finding at most exactly one y on or near the heart surface as y sweeps to the right along each of the two surface pieces.

3. Bilateral Symmetry

Cor1 and Cor2 only construct the left half of the heart. The right half is, and must be obtained by invoking the bilateral symmetry of the heart, because y would not be a function of z and x if we swept from y = -1.25 all the way to y = +1.25. Given the mx3 matrix H-left that describes the left side of the heart, H-right is just H-left with the second column, the column of y values, multiplied by -1.

Cor2 is needed so as to be able to construct y as a piecewise function of z and x on the left half of the heart. On the first surface piece, y ranges from -1.25 to -0.5. On the second surface piece, y ranges from -0.5 to 0.0.

Heart1 and Heart 2 comprise the left side of the heart. Heart3 and Heart4 comprise the right side of the heart. By bilateral symmetry, Heart3 is Heart1 with its second column (the y values) negated. Heart4 is Heart2 with its y values negated.

So the heart surface of the Reference can indeed be constructed as a Mathcad 3D scatter plot.

This construction verifies numerically that the equation f(x,y,z) = 0, as taken from the Reference, does indeed describe a heart-shaped surface.