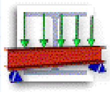
Converted from "**Finding the Shear Force and Bending Moment Along a Beam**" created by **Mathcad Staff**



WORKSHEET 9: EXAMPLE 7.5

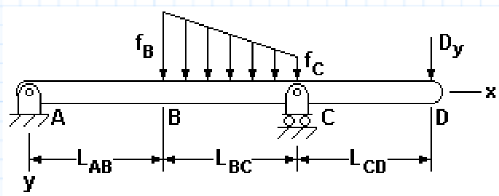
Relations Between Distributed Load, Shear Force, and Bending Moment

This example shows how the shear force and the bending moment along a simply supported beam can be determined as a function of the distance from one end. The method used is based on the differential equations that relate the shear force, the bending moment, and the distributed load. This example and its set of equations can be used to solve many problems in Section 7.3. These include Problems 7-3.3. 7-3.6, 7-3.7, 7-3.10,7-3.13, and 7-3.15.

Statement

Use Eqs. (7.4) and (7.5) to determine the shear force and bending moment diagrams for the beam in the figure below.

$$\frac{d}{dx}V = -w \qquad \text{Eq. (7.4)}$$



$$\frac{d}{dx}M = V \qquad \text{Eq. (7.5)}$$

Parameters

$f_B := 300 \cdot \frac{kN}{m}$ end limit of linearly varying distributed load at B

$f_C := 100 \cdot \frac{kN}{m}$ end limit of linearly varying distributed load at C

$D_y := 300 \cdot kN$ vertical load at D

$L_{AB} := 6 \cdot m$ distance between A and B

$L_{BC} := 6 \cdot m$ distance between B and C

$L_{CD} := 6 \cdot m$ distance between C and D

Solution

Equivalent Concentrated Load Representation of Distributed Load

We begin by determining the expression of the distributed load w(x) as a function of position x. Since w(x) is a linear function, we can express it in the form w(x) = cx + d, where c and d are constants. We know w at x = LAB and x = LAB + LBC:

$f_B = c \cdot L_{AB} + d$ and $f_C = c \cdot (L_{AB} + L_{BC}) + d$

These can be solved simultaneously to give

$$c := \frac{f_C - f_B}{L_{BC}} \qquad c = -33.333 \, \frac{kN}{m^2}$$

$$d := f_B - c \cdot L_{AB} \qquad d = 500 \, \frac{kN}{m}$$

Thus, the linearly varying distributed load can be written as

$$w(x) := c \cdot x + d$$

We check here that indeed the function gives the right values at the known limits:

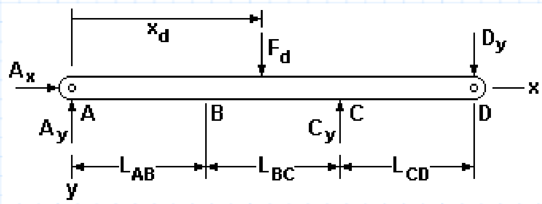
$$w(L_{AB}) = 300 \, \frac{kN}{m} \qquad w(L_{AB} + L_{BC}) = 100 \, \frac{kN}{m}$$

To go further, this distributed load can be represented by one resultant force Fd acting at a specific location x = xd, where

$$F_d := \int_{L_{AB}}^{L_{AB}+L_{BC}} w(x)\,\mathrm{d}x \qquad F_d = \left(1.2 \cdot 10^3\right)\ kN$$

$$x_d := \frac{\displaystyle\int_{L_{AB}}^{L_{AB}+L_{BC}} x \cdot w(x)\,\mathrm{d}x}{F_d} \qquad x_d = 8.5\ m$$

The free-body diagram of the entire beam with the distributed load replaced by the resultant force Fd is shown on the right.



We now obtain the reactions Ax, Ay, and Cy from the equilibrium equations.

Since Σ Fx = 0, $A_x := 0$

Since Σ M(point A) = 0 $\left(L_{AB}+L_{BC}\right) \cdot C_y - x_d \cdot F_d - \left(L_{AB}+L_{BC}+L_{CD}\right) \cdot D_y = 0$

$$C_y := \frac{\left(x_d \cdot F_d + D_y \cdot L_{AB} + D_y \cdot L_{BC} + D_y \cdot L_{CD}\right)}{\left(L_{AB}+L_{BC}\right)}$$

$$C_y = \left(1.3 \cdot 10^3\right)\ kN$$

Since Σ Fy = 0 $A_y + C_y - F_d - D_y = 0$

$$A_y := F_d + D_y - C_y \qquad A_y = 200\ kN$$

We now proceed to determine the shear force and bending moment as functions of x for the entire beam, using Eqs. (7.4) and (7.5).

Shear Force Diagram

From A to B There is no load between A and B, so the shear force increases by Ay at A and then remains constant from A to B:

$$V_{AB}(x) := A_y$$

From B to C From our solution between A and B, $V_{AB}(L_{AB})=200\ \text{kN}$. Integrating Eq. (7.4) from x = LAB to an arbitrary value of x between B and C:

$$\int_{V_{AB}(L_{AB})}^{V_{BC}(x)} 1\, \mathrm{d}V = \int_{L_{AB}}^{x} -w\, \mathrm{d}x = \int_{L_{AB}}^{x} -(c \cdot x + d)\, \mathrm{d}x$$

we obtain an equation for V between B and C:

$$V_{BC}(x) := V_{AB}(L_{AB}) - \left(\frac{c \cdot \left(x^2 - L_{AB}^{\ 2}\right)}{2} + d \cdot (x - L_{AB}) \right)$$

From C to D At C, V undergoes an increase of $C_Y=1300\ \text{kN}$ due to the force exerted by the pin support. Adding this change to the value of V at C obtained from our solution from B to C, the value of V just to the right of C is

$$V_{BC}(L_{AB}+L_{BC}) + C_y = 300\ kN$$

There is no loading between C and D, so V remains constant from C to D:

$$V_{CD}(x) := V_{BC}(L_{AB}+L_{BC}) + C_y$$

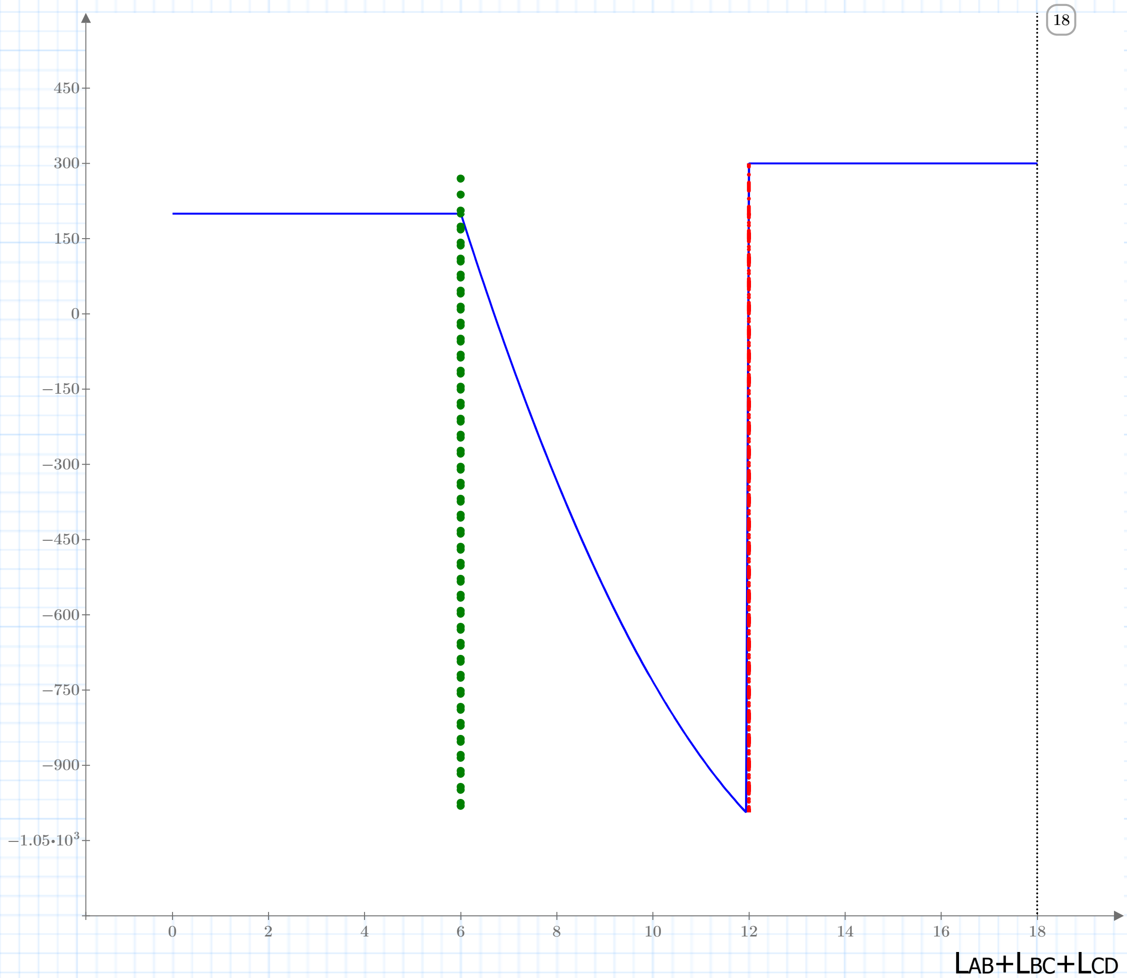
We combine the results for all three sections using Mathcad's if function:

$$V(x) := \mathbf{if}\left(x < L_{AB}, V_{AB}(x), \mathbf{if}\left(x < L_{AB}+L_{BC}, V_{BC}(x), V_{CD}(x)\right)\right)$$

The shear force diagram is shown below, after defining a range variable for the distance from the left end:

$i := 0 .. 300$

$$x_i := \frac{i}{300} \cdot \left(L_{CD} + L_{AB} + L_{BC}\right)$$



Bending Moment Diagram

From A to B Integrating Eq. (7.5) from x = 0 to an arbitrary value of x between A and B:

$$\int_{0}^{M_{AB}(x)} 1\,\mathrm{d}M = \int_{0}^{x} V_{AB}(x)\,\mathrm{d}x = \int_{0}^{x} A_y\,\mathrm{d}x$$

we obtain:

$$M_{AB}(x) := A_y \cdot x \qquad\qquad M_{AB}(L_{AB}) = \left(1.2 \cdot 10^{3}\right)\ kN \cdot m$$

From B to C Integrating Eq. (7.5) from x = LAB to an arbitrary value of x between B and C:

$$\int_{M_{AB}(L_{AB})}^{M_{BC}(x)} 1\,\mathrm{d}M = \int_{L_{AB}}^{x} V_{BC}(x)\,\mathrm{d}x$$

$$\int_{L_{AB}}^{x} \left(\frac{-c}{2} \cdot x^{2} - d \cdot x + \left(\frac{c}{2} \cdot L_{AB} + d \cdot L_{AB} + V_{AB}(L_{AB})\right)\right) \mathrm{d}x$$

we obtain an equation for V between B and C:

$$M_{BC}(x) := M_{AB}(L_{AB}) + \frac{-c}{6} \cdot \left(x^{3} - L_{AB}{}^{3}\right) - \frac{d}{2} \cdot \left(x^{2} - L_{AB}{}^{2}\right) + \left(\frac{c}{2} \cdot L_{AB}{}^{2} + d \cdot L_{AB} + V_{AB}(L_{AB})\right) \cdot \left(x - L_{AB}\right)$$

Note that at x, x:=Lab, $\qquad M_{BC}(L_{AB}) = \left(1.2 \cdot 10^{3}\right)\ kN \cdot m$

$$M_{BC}(L_{AB} + L_{BC}) = -1.8 \cdot 10^{3}\ kN \cdot m$$

From C to D Integrating Eq. (7.5) from x = LAB + LBC to an arbitrary value of x between C and D:

$$\int_{M_{BC}(L_{AB}+L_{BC})}^{M_{CD}(x)} 1 \, \mathrm{d}M = \int_{L_{AB}+L_{BC}}^{x} V_{CD}(x) \, \mathrm{d}x$$

$$\int_{L_{AB}+L_{BC}}^{x} \left(V_{BC}(L_{AB}+L_{BC})+C_y\right) \mathrm{d}x$$

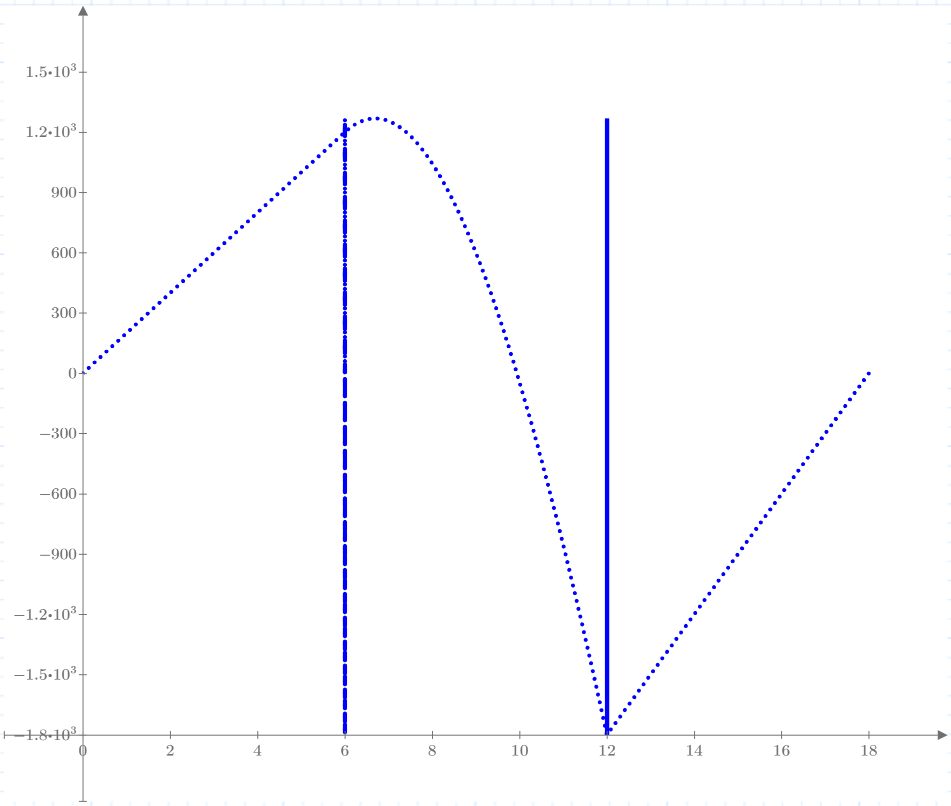
we obtain:

$$M_{CD}(x) := M_{BC}(L_{AB}+L_{BC}) + \left(V_{BC}(L_{AB}+L_{BC})+C_y\right) \cdot \left(x-L_{AB}-L_{BC}\right)$$

We combine the results for all three sections:

$$M(x) := \mathbf{if}\left(x<L_{AB}, M_{AB}(x), \mathbf{if}\left(x<L_{AB}+L_{BC}, M_{BC}(x), M_{CD}(x)\right)\right)$$

The bending moment diagram is shown below:



Discussion

Compare this example with Example 7.3, in which we use free-body diagrams to determine the shear force and bending moment as functions of x for this beam and loading.