

Optics

OPTICS

Learning by Computing,
with Examples
Using Mathcad[®], Matlab[®],
Mathematica[®], and Maple[®]

Second Edition

K.D. Møller

With 308 Illustrations



Includes CD-ROM

With Mathcad

Matlab

Mathematica

 Springer

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*To
colleagues, staff, and students
of the
New Jersey Institute of Technology,
Newark, New Jersey*

Preface

The book is for readers who want to use model computational files for fast learning of the basics of optics. In the Second Edition, Matlab, Mathematica and Maples files have been added to the Mathcad files on the CD of the First Edition. The applications, given at the end of files to suggest different points of view on the subject, are extended to home work problems and are also on the CD of the Second Edition.

While the book is suited well for self learning, it was written over several years for a one semester course in optics for juniors and seniors in science and engineering. The applications provide a simulated laboratory where students can learn by exploration and discovery instead of passive absorption.

The text covers all the standard topics of a traditional optics course, including: geometrical optics and aberration, interference and diffraction, coherence, Maxwell's equations, wave guides and propagating modes, blackbody radiation, atomic emission and lasers, optical properties of materials, Fourier transforms and FT spectroscopy, image formation, and holography. It contains step by step derivations of all basic formulas in geometrical and wave optics.

The basic text is supplemented by over 170 Mathcad, Matlab, Mathematica and Maple files, each suggesting programs to solve a particular problem, and each linked to a topic in or application of optics. The computer files are dynamic, allowing the reader to see instantly the effects of changing parameters in the equations. Students are thus encouraged to ask "what . . . if" questions to assess the physical implications of the formulas. To integrate the files into the text, applications are listed connecting the formulas and the corresponding computer file, and problems for all 11 chapters are on the CD.

The availability of the numerical Fourier transform makes possible an introduction to the wave theory of imaging, spatial filtering, holography and Fourier transform spectroscopy.

The book is written for the study of particular projects but can easily be adapted to a variation of related studies. The three fold arrangement of text, applications and files makes the book suitable for “self-learning” by scientists and engineers who would like to refresh their knowledge of optics. All files are printed out and are available on a CD, (Mathcad 7) (Mathcad 2000) (Matlab 6.5) (Mathematica 4.1) (Maple 9.5) and may well serve as starting points to find solutions to more complex problems as experienced by engineers in their applications.

The book can be used in optical laboratories with faculty-student interaction. The files may be changed and extended to study the assigned projects, and the student may be required to hand in printouts of all assigned applications and summarize what he has been learned.

I would like to thank Oren Sternberg and Assaf Sternberg for the translation of the files into Matlab, Mathematica and Maples, Prof. Ken Chin and Prof. Haim Grebel of New Jersey Institute of Technology for continuous support, and my wife for always keeping me in good spirit.

Newark, New Jersey

K.D. Möller

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