▶ Units, functions, etc

Filling The Pond

The volume flow rate in a pipe, assuming laminar flow (we'll check this applies later) is given by

$$\Phi(D, P_{TOT}, \eta, L) := \frac{\pi \cdot D^4 \cdot P_{TOT}}{128 \cdot \eta \cdot L}$$

where D is the pipe diameter, P_{TOT} is the total pressure drop from inlet to outlet (i.e. including pressure due to elevation change), η is the dynamic viscosity, and L is the length of the pipe.

The Reynolds number is given by $R_e = \frac{V \cdot D \cdot \rho}{\mu}$

Where V is the linear flow velocity. If the Reynolds number is less than 2000 then the flow is laminar, so we need to check that the linear flow rate is less than $\frac{2000 \cdot \eta}{D \cdot \rho}$

We can now look at some various "pond scenarios". In all scenarios we will assume the following:

Temperature of water in hose: $T := 20\,°C$

Dynamic viscosity of water: $\eta(T) := \left[1.509 \cdot e^{-0.034 \cdot (T/°C)} + 0.2547\right] \cdot Pa \cdot s$ (Valid for 0-100°C)

$\eta := \eta(T)$ $\eta = 1.0192\,Pa \cdot s$

Density of water: $\rho(T) := \left[999.93 + 0.023 \cdot (T/°C) - 5.439 \cdot 10^{-3} \cdot (T/°C)^2\right] \cdot \frac{kg}{m^3}$ (Valid for 0-40°C)

$\rho := \rho(T)$ $\rho = 998.2144 \frac{kg}{m^3}$

1) Mike's pond filling method.

Mike's pond is unusual, because it is so deep that if a typical person stood on the bottom the water level would come up to their necks. It is also rather "boxy", because it apparently has no shelves for plants, etc. I guess Mike likes very deep water plants. This gives it a large volume.

$Depth := 1.5 \cdot m$ $Length := 2 \cdot m$ $Width := 1.5 \cdot m$

$Volume := Depth \cdot Length \cdot Width$ $Volume = 1188.7742\,gal$

The large volume is presumably why Mike chooses to use a really humongous hose to fill it:

Hose diameter: $D := 50 \cdot mm$

There is obviously little point in connecting such a hose to a standard household tap, so we will assume that Mike actually goes out to the street and taps into the water main via the nearest fire hydrant. This gives him some high water pressure, but he needs a long hose

Water pressure: $P_W := 120 \cdot psi$

Hose length: $L := 150 \cdot ft$

For the sake of argument we will assume no elevation change from the hydrant to the pond

Change in elevation from start to end of hose (positive means outlet is lower than inlet: $\Delta z := 0 \cdot m$

Total pressure drop over length of hose: $P_{TOT} := P_W + \rho \cdot g \cdot \Delta z$ $\quad P_{TOT} = 120\,psi$

So Mike fills his pond at a rate of

$VolumeRate := \Phi(D, P_{TOT}, \eta, L)$ $\quad VolumeRate = 0.1634\,\frac{m^3}{min}$

The linear flow rate is

$$\frac{4 \cdot VolumeRate}{\pi \cdot D^2} = 1.3872\,\frac{m}{s}$$

which is less than $\frac{2000 \cdot \eta}{D \cdot \rho} = 41\,\frac{m}{s}$ so our assumption of laminar flow is justified.

So Mike fills his pond in

$$\frac{Volume}{VolumeRate} = 27.536\,min$$

This is an excellent time, and might even be quick enough that the pond is full before the police arrive, turn off the water, and arrest Mike for tapping into the fire hydrant and stealing water.

2) Richard's pond filling method.

Richard fortunately has a pond that is more like a pond and less like a very small swimming pool, so he will use a garden hose.

$Depth := 2.5 \cdot ft$ $\quad Length := 8 \cdot ft$ $\quad Width := 5 \cdot ft$

Richard's pond is actually not a rectangle (it's roughly elliptical), and it has shelves for plants. We can introduce a "shape factor" as an approximate way to account for this

$$ShapeFactor := \frac{1}{3}$$

$Volume := Depth \cdot Length \cdot Width \cdot ShapeFactor$ $\quad Volume = 249.3506\,gal$

A typical garden hose in the US is $D := \frac{5}{8} \cdot in$ $\quad L := 25 \cdot ft$

The house water pressure is $P_W := 60 \cdot psi$

The pond is a bit lower than the outside faucet the hose is connected to

Change in elevation from start to end of hose (positive means outlet is lower than inlet: $\Delta z := 1 \cdot m$

Total pressure drop over length of hose: $P_{TOT} := P_W + \rho \cdot g \cdot \Delta z$ $\quad P_{TOT} = 61.4198\,psi$

So Richard fills his pond at a rate of

$$\text{VolumeRate} := \Phi\left(D, P_{TOT}, \eta, L\right) \qquad \text{VolumeRate} = 1.3473\,\frac{\text{gal}}{\text{min}}$$

The linear flow rate is

$$\frac{4\cdot\text{VolumeRate}}{\pi\cdot D^2} = 0.4294\,\frac{\text{m}}{\text{s}}$$

which is less than $\frac{2000\cdot\eta}{D\cdot\rho} = 129\,\frac{\text{m}}{\text{s}}$ so our assumption of laminar flow is justified.

So Richard fills his pond in

$$\frac{\text{Volume}}{\text{VolumeRate}} = 3.0847\,\text{hr}$$

This is not as fast as Mike's method, but Richard will be sitting drinking a beer in the sun as the pond fills, whereas Mike will be trying to fend off his cell mate, who keeps telling him he looks like a "nice boy"...