▶ Units, functions, etc

Filling The Pond

The pressure drop over a length of pipe is given by

$$P_{TOT} = f\cdot\frac{L}{D}\cdot\frac{\rho\cdot V^2}{2}$$

where P_{TOT} is the total pressure drop from inlet to outlet (i.e. including pressure due to elevation change), ρ is the density of the fluid, L is the length of the pipe, D is the pipe diameter, V is the linear flow velocity, and f is the friction factor.

If the Reynolds number is R_e, then the friction factor is given by

$$f = \frac{64}{R_e} \qquad \text{if} \qquad R_e < 3000 \qquad \text{(laminar flow)}$$

or

$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\varepsilon\cdot D^{-1}}{3.7} + \frac{2.51}{R_e\cdot\sqrt{f}}\right) \qquad \text{if} \qquad R_e > 3000 \qquad \text{(turbulent flow)}$$

where ε is the roughness of the inside of the pipe.

The Reynolds number is given by $R_e = \frac{V\cdot D\cdot\rho}{\eta}$

where η is the dynamic viscosity.

Substituting the formula for the Reynolds number into the equations for the friction factor gives us

$$\frac{1}{\sqrt{f}} = \text{if}\left(\frac{V\cdot D\cdot\rho}{\eta} < 3000, \frac{1}{8}\cdot\sqrt{\frac{V\cdot D\cdot\rho}{\eta}}, -2.0\cdot\log\left(\frac{\varepsilon\cdot D^{-1}}{3.7} + \frac{2.51\cdot\eta}{V\cdot D\cdot\rho\cdot\sqrt{f}}\right)\right)$$

If we rearrange the equation for P_{TOT} and substitute for f we have

$$\sqrt{\frac{L\cdot\rho}{2\cdot D\cdot P_{TOT}}}\cdot V = \text{if}\left(\frac{V\cdot D\cdot\rho}{\eta} < 3000, \frac{1}{8}\cdot\sqrt{\frac{V\cdot D\cdot\rho}{\eta}}, -2.0\cdot\log\left(\frac{\varepsilon\cdot D^{-1}}{3.7} + \frac{2.51\cdot\eta}{D\cdot\rho}\cdot\sqrt{\frac{L\cdot\rho}{2\cdot D\cdot P_{TOT}}}\right)\right)$$

We can solve this for V

Given

$$\sqrt{\frac{L\cdot\rho}{2\cdot D\cdot P_{TOT}}}\cdot V = \text{if}\left(\frac{V\cdot D\cdot\rho}{\eta} < 3000, \frac{1}{8}\cdot\sqrt{\frac{V\cdot D\cdot\rho}{\eta}}, -2.0\cdot\log\left(\frac{\varepsilon\cdot D^{-1}}{3.7} + \frac{2.51\cdot\eta}{D\cdot\rho}\cdot\sqrt{\frac{L\cdot\rho}{2\cdot D\cdot P_{TOT}}}\right)\right)$$

$$V_{lin}(P_{TOT}, L, D, \rho, \eta, \varepsilon, V) := \text{Find}(V)$$

where the V in the argument list is the guess value to be passed to the solve block

We can now look at some various "pond scenarios". In all scenarios we will assume the following:

Temperature of water in hose: $T := 25\,°C$

Dynamic viscosity of water: $\eta(T) := \left[1.509 \cdot e^{-0.034 \cdot (T/°C)} + 0.2547\right] \cdot 10^{-3} \cdot Pa \cdot s$ (Valid for 0-100°C)

$\eta := \eta(T)$ $\eta = 8.9967 \times 10^{-4}\, Pa \cdot s$

Density of water: $\rho(T) := \left[999.93 + 0.023 \cdot (T/°C) - 5.439 \cdot 10^{-3} \cdot (T/°C)^2\right] \cdot \frac{kg}{m^3}$ (Valid for 0-40°C)

$\rho := \rho(T)$ $\rho = 997.1056 \frac{kg}{m^3}$

Pipe roughness: $\varepsilon := 30 \cdot \mu m$

1) Mike's pond filling method.

Mike's pond is unusual, because it is so deep that if a typical person stood on the bottom the water level would come up to their necks. It is also rather "boxy", because it apparently has no shelves for plants, etc. I guess Mike likes very deep water plants. This gives it a large volume.

$Depth := 1.5 \cdot m$ $Length := 2 \cdot m$ $Width := 1.5 \cdot m$

$Volume := Depth \cdot Length \cdot Width$ $Volume = 1188.7742\, gal$

The large volume is presumably why Mike chooses to use a really humongous hose to fill it:

Hose diameter: $D := 50 \cdot mm$

There is obviously little point in connecting such a hose to a standard household tap, so we will assume that Mike actually goes out to the street and taps into the water main via the nearest fire hydrant. This gives him some high water pressure, but he needs a long hose

Water pressure: $P_W := 120 \cdot psi$

Hose length: $L := 150 \cdot ft$

For the sake of argument we will assume no elevation change from the hydrant to the pond

Change in elevation from start to end of hose (positive means outlet is lower than inlet: $\Delta z := 0 \cdot m$

Total pressure drop over length of hose: $P_{TOT} := P_W + \rho \cdot g \cdot \Delta z$ $P_{TOT} = 120\, psi$

The linear flow rate from Mike's hose is thus

$V := V_{lin}\left(P_{TOT}, L, D, \rho, \eta, \varepsilon, 1 \cdot \frac{m}{s}\right)$ $V = 9.9913 \frac{m}{s}$

The volume flow rate is therefore

$\Phi := \frac{V \cdot \pi \cdot D^2}{4}$ $\Phi = 310.9498 \frac{gal}{min}$

So the time taken for Mike to fill his pond is

$$\frac{Volume}{\Phi} = 3.823\,min$$

This is an amazing time, and is certainly quick enough that the pond is full before the police arrive, turn off the water, and arrest Mike for tapping into the fire hydrant and stealing water.

The Reynolds' number is

$$R_e := \frac{V \cdot D \cdot \rho}{\eta} \qquad R_e = 5.5367 \times 10^5$$

so the flow is turbulent

The friction factor is

$f := 0.2$

Given

$$\frac{1}{\sqrt{f}} = if\left(R_e < 3000, \frac{64}{R_e}, -2.0 \cdot \log\left(\frac{\varepsilon \cdot D^{-1}}{3.7} + \frac{2.51}{R_e \cdot \sqrt{f}}\right)\right)$$

$Find(f) = 0.0182$

2) Richard's pond filling method.

Richard fortunately has a pond that is more like a pond and less like a very small swimming pool, so he will use a garden hose.

$Depth := 2.5 \cdot ft \qquad Length := 8 \cdot ft \qquad Width := 5 \cdot ft$

Richard's pond is actually not a rectangle (it's roughly elliptical), and it has shelves for plants. We can introduce a "shape factor" as an approximate way to account for this

$$ShapeFactor := \frac{1}{3}$$

$Volume := Depth \cdot Length \cdot Width \cdot ShapeFactor \qquad Volume = 249.3506\,gal$

A typical garden hose in the US is $D := \frac{5}{8} \cdot in \qquad L := 25 \cdot ft$

The house water pressure is $P_W := 50 \cdot psi$

The pond is a bit lower than the outside faucet the hose is connected to

Change in elevation from start to end of hose (positive means outlet is lower than inlet: $\Delta z := 1 \cdot m$

Total pressure drop over length of hose: $P_{TOT} := P_W + \rho \cdot g \cdot \Delta z \qquad P_{TOT} = 51.4182\,psi$

The linear flow rate from Richard's hose is thus

$$V := V_{lin}\left(P_{TOT}, L, D, \rho, \eta, \varepsilon, 1 \cdot \frac{m}{s}\right) \qquad V = 7.7948\,\frac{m}{s}$$

The volume flow rate is therefore

$$\Phi := \frac{V \cdot \pi \cdot D^2}{4} \qquad \Phi = 24.4545 \frac{gal}{min}$$

So the time taken for Richard to fill his pond is

$$\frac{Volume}{\Phi} = 10.1965\,min$$

This is not as fast as Mike's method, but Richard will be sitting drinking a beer in the sun as the pond fills, whereas Mike will be trying to fend off his cell mate, who keeps telling him he looks like a "nice boy"...

The Reynolds' number is

$$R_e := \frac{V \cdot D \cdot \rho}{\eta} \qquad R_e = 1.3714 \times 10^5$$

so the flow is turbulent

The friction factor is

$$f := 0.2$$

Given

$$\frac{1}{\sqrt{f}} = if\left(R_e < 3000, \frac{64}{R_e}, -2.0 \cdot \log\left(\frac{\varepsilon \cdot D^{-1}}{3.7} + \frac{2.51}{R_e \cdot \sqrt{f}}\right)\right)$$

$$Find(f) = 0.0244$$

3) Addendum: Richard's pond emptying method.

Occasionally Richard has to drain the pond to clean out the sludge, along with any old beer bottles that he accidentally dropped in there when he fell over laughing thinking about Mike in his cell. He does this by siphoning the pond over a retaining wall, using the same hose that was used to fill it

The house water pressure is now of course $P_W := 0 \cdot psi$

Change in elevation from start to end of hose (positive means outlet is lower than inlet: $\Delta z := 1.5 \cdot m$

Total pressure drop over length of hose: $P_{TOT} := P_W + \rho \cdot g \cdot \Delta z$ $\qquad P_{TOT} = 2.1273\,psi$

The linear flow rate from Richard's hose is thus

$$V := V_{lin}\left(P_{TOT}, L, D, \rho, \eta, \varepsilon, 1 \cdot \frac{m}{s}\right) \qquad V = 1.4675 \frac{m}{s}$$

The volume flow rate is therefore

$$\Phi := \frac{V \cdot \pi \cdot D^2}{4} \qquad \Phi = 4.604 \frac{gal}{min}$$

So the time taken to empty the pond is

$$\frac{Volume}{\Phi} = 54.1601\,min$$

At least, it would be if the inlet to the hose didn't keep clogging up with bits of debris from the pond! It actually takes him an entire day to get the water out, and at this point Richard really wishes he had Mike's monster hose!

The Reynolds' number is

$R_e := \frac{V \cdot D \cdot \rho}{\eta}$ $R_e = 25819.5051$ so the flow is turbulent

The friction factor is

$f := 0.2$

Given

$$\frac{1}{\sqrt{f}} = \text{if}\left(R_e < 3000, \frac{64}{R_e}, -2.0 \cdot \log\left(\frac{\varepsilon \cdot D^{-1}}{3.7} + \frac{2.51}{R_e \cdot \sqrt{f}}\right)\right)$$

$\text{Find}(f) = 0.0285$