EXPLICIT DIFFERENCE TIME DOMAIN TRANSMISSION LINE CODE NO INSULATOR

REQUIRED INPUT VALUES:

Ground Conductivity: $\sigma_g := 1\cdot10^{-3}$

Cable Conductivity: $\sigma_c := 5.8\cdot10^{7}$

Ground relative diel const: $\varepsilon_{rg} := 16$

Cable Radii $a_o := 10^{-2}$

DRIVER INPUT:

$E_o := 2\cdot10^{5}$ $\quad E_1 := 3\cdot10^{2}$

$t_o := 3\cdot10^{-8}$ $\quad r := 120$ $\quad t_1 := 10^{-4}$ $\quad r2 := 10$

$\alpha := \frac{1}{t_o}\cdot\frac{\ln(r)}{r-1}$ $\quad \beta := \alpha\cdot r$ $\quad A := \frac{1}{t_1}\cdot\frac{\ln(r2)}{r2-1}$ $\quad B := A\cdot r2$

$\alpha = 1.341\times10^{6}$ $\quad A = 2.558\times10^{3}$

$\beta = 1.609\times10^{8}$ $\quad B = 2.558\times10^{4}$

Cable Cell Size & No. of Cells:

$Ncell := 10$ $\quad \Delta z := 100$

$Lz := Ncell\cdot\Delta z$ $\quad Lz = 1\times10^{3}$

IMPORTANT CONSTANTS:

$\mu_o := 4\cdot\pi\cdot10^{-7}$

$c := 3\cdot10^{8}$

$\varepsilon_o := \frac{1}{\mu_o\cdot c^2}$

DEFINE TIME STEPS: $\Delta t := 0.4\cdot\frac{\Delta z}{c}$ $\quad \Delta t = 1.333\times10^{-7}$

$Tmax := 1\cdot10^{-2}$ $\quad Nt := \mathrm{ceil}\left(\frac{Tmax}{\Delta t}\right)$ $\quad Nt = 7.5\times10^{4}$

$TSTART := \mathrm{time}(Tmax)$

$TSTART = 1.493\times10^{9}$

END OF INPUT

$t_o := \frac{\ln\left(\frac{\beta}{\alpha}\right)}{\beta-\alpha}$ $\quad t_o = 3\times10^{-8}$

$\varepsilon_g := \varepsilon_{rg}\cdot\varepsilon_o$

DO TIME ITERATION: $k := 0..Nt$ $\quad t_0 := \Delta t$ $\quad t_{k+1} := t_k + \Delta t$

DEFINE TRANSMISSION LINE PARAMETERS:

$Rc_k := \frac{1}{\pi\cdot a_o^{2}\cdot\sigma_c}$ $\quad Rs_k := \frac{\mu_o}{4\cdot\pi\cdot t_k}$ $\quad R_k := Rc_k + Rs_k$ $\quad \delta_k := \sqrt{\frac{2\cdot t_k}{\mu_o\cdot\sigma_g}}$

$$\delta_k := \begin{cases} \delta_k & \text{if } 0 < \delta_k < Lz \\ 1.00001\cdot a_o & \text{if } \delta_k \le 0 \\ Lz & \text{otherwise} \end{cases}$$

$$Lg_k := \frac{\mu_o}{2\cdot\pi}\cdot\ln\left(\frac{\delta_k}{a_o}\right) \qquad L_k := Lg_k$$

$$Gg_k := \frac{2\cdot\pi\cdot\sigma_g}{\ln\left(\frac{\delta_k}{a_o}\right)} \qquad CCg_k := \frac{2\cdot\pi\cdot\varepsilon_g}{\ln\left(\frac{\delta_k}{a_o}\right)}$$

Define Driving Field:

$$E1_k := E_o\cdot\left(e^{-\alpha\cdot t_k} - e^{-\beta\cdot t_k}\right)$$

$$E2_k := E_1\cdot\left(e^{-A\cdot t_k} - e^{-B\cdot t_k}\right)$$

$$E_k := E1_k + E2_k$$

Calculate Analytic Test Case For Shorts On Both Ends:

$$Itest_k := \left| \begin{array}{l} num \leftarrow e^{-\alpha\cdot t_k} - e^{\frac{-R_k\cdot t_k}{L_k}} \\ denom \leftarrow L_k\cdot\left(\frac{R_k}{L_k} - \alpha\right) \\ E_o\cdot\frac{num}{denom} \end{array} \right.$$

Define Short on both ends of cable:

$$j := 1..Ncell \qquad Cg_{j,k} := \left| \begin{array}{ll} 10^{20} & \text{if } j = 1 \\ 10^{20} & \text{if } j = Ncell \\ CCg_k & \text{otherwise} \end{array} \right.$$

Compute Finite Difference Equations:

Simple Uncoupled Examples:

$$II_0 := 0$$

$$II_k := \left| \begin{array}{ll} 0 & \text{if } k \le 0 \\ \dfrac{\frac{L_{k-1}}{L_k}\cdot II_{k-1} + \frac{\Delta t}{L_k}\cdot E_k}{1 + \frac{\Delta t}{L_k}\cdot R_k} & \text{otherwise} \end{array} \right.$$

$$III := \left| \begin{array}{l} \text{for } k \in 0..Nt \\ \quad \left| \begin{array}{ll} Ans_k \leftarrow 0 & \text{if } k \le 0 \\ Ans_k \leftarrow \dfrac{\frac{L_{k-1}}{L_k}\cdot Ans_{k-1} + \frac{\Delta t}{L_k}\cdot E_k}{1 + \frac{\Delta t}{L_k}\cdot R_k} & \text{otherwise} \end{array} \right. \\ Ans \end{array} \right.$$

Fully Coupled Equations:

$I :=$ for $a \in 1..Ncell$ <-------------------------Must initialize inside program!

for $b \in 1..Nt$

$AI_{a,b} \leftarrow 0$

$VG_{a,b} \leftarrow 0$

for $k \in 1..Nt$ <-------------------------Time Loop

for $j \in 1..Ncell$ <-------------------------Spatial Loop for Voltage

$\Delta I \leftarrow AI_{j,k-1} \quad \text{if } j \le 1$ <-------------------------Current from Previous Time

$\Delta I \leftarrow AI_{j,k-1} - AI_{j-1,k-1} \quad \text{otherwise}$

$$VG_{j,k} \leftarrow \frac{VG_{j,k-1} - \frac{\Delta I}{\Delta z} \cdot \frac{\Delta t}{Cg_{j,k}}}{1 + \frac{\Delta t}{Cg_{j,k}} \cdot Gg_k}$$

$VT \leftarrow VG$

for $n \in 1..Ncell - 1$ <-------------------------Spatial Loop for Current

$\Delta V \leftarrow VT_{n+1,k} - VT_{n,k}$ <-------------------------Voltage from Present Time

$$AI_{n,k} \leftarrow \frac{AI_{n,k-1} + \frac{\Delta t}{L_k} \cdot \left(E_k - \frac{\Delta V}{\Delta z}\right)}{1 + \frac{\Delta t}{L_k} \cdot R_k}$$

AI

AI

Summary of Answers from Various Models:

$$I_{exp_k} := I_{\frac{Ncell}{2},k}$$

$\max(I_{exp}) = 8.792 \times 10^4$

$\max(Itest) = 7.932 \times 10^4$

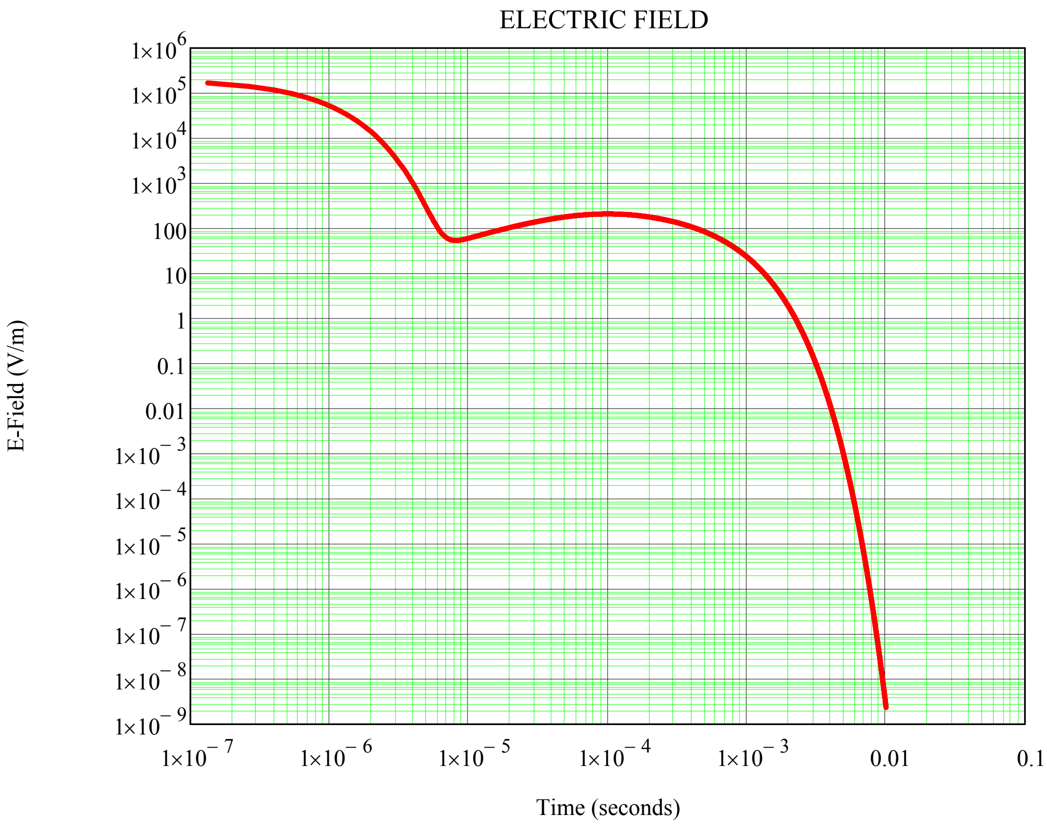
$\max(II) = 7.275 \times 10^4$

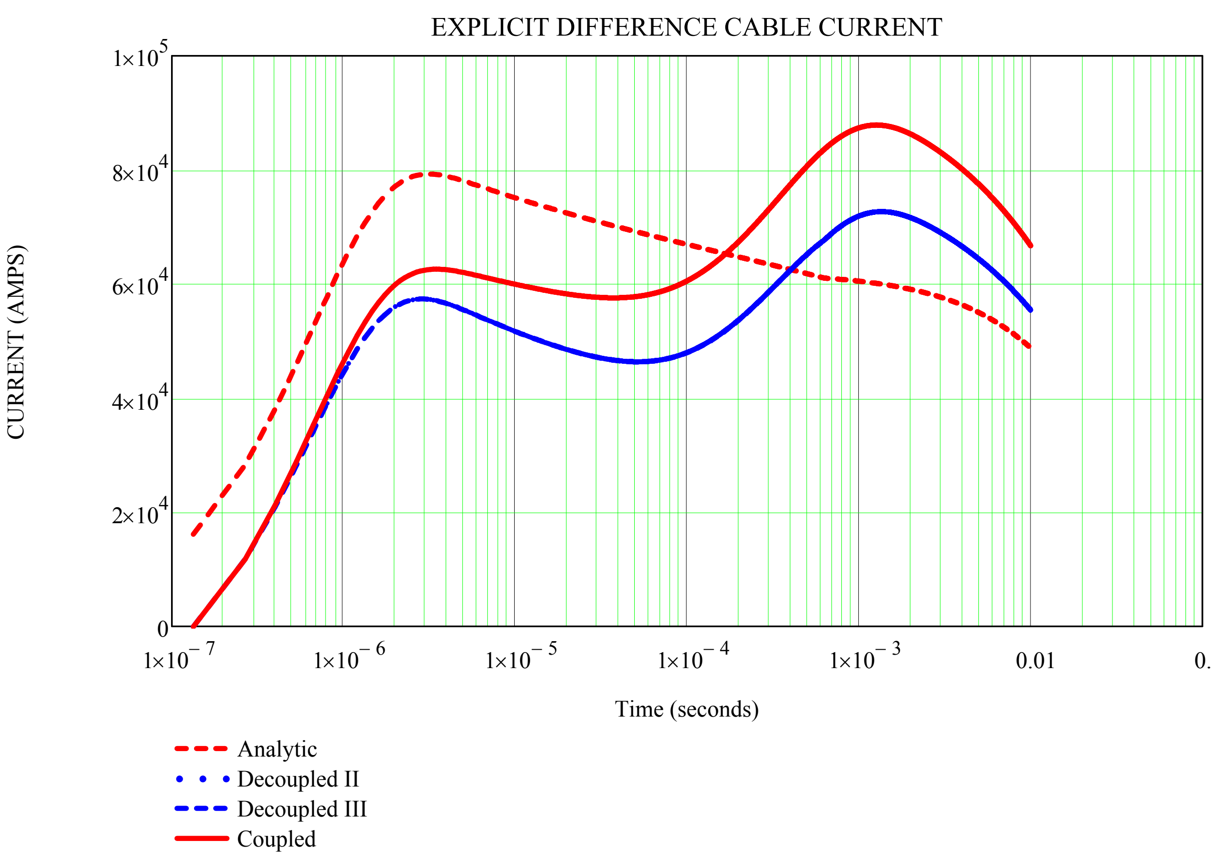
$\max(III) = 7.275 \times 10^4$

$TFINISH := time(Itest)$

$TFINISH = 1.493 \times 10^9$

$TFINISH - TSTART = 2.324 \times 10^3$


ELECTRIC FIELD
E-Field (V/m)
1×10^6
1×10^5
1×10^4
1×10^3
100
10
1
0.1
0.01
1×10^-3
1×10^-4
1×10^-5
1×10^-6
1×10^-7
1×10^-8
1×10^-9
1×10^-7
1×10^-6
1×10^-5
1×10^-4
1×10^-3
0.01
0.1
Time (seconds)


EXPLICIT DIFFERENCE CABLE CURRENT
CURRENT (AMPS)
1×10^5
8×10^4
6×10^4
4×10^4
2×10^4
0
1×10^-7
1×10^-6
1×10^-5
1×10^-4
1×10^-3
0.01
0.1
Time (seconds)
Analytic
Decoupled II
Decoupled III
Coupled

$AbsI_k := \left| I_{exp_k} \right|$ $\max(I_{exp}) = 8.792 \times 10^4$

CALCULATION OF WAVEFORM NORMS:

Rectified Charge (A-sec) === $\sum_k (AbsI_k \cdot \Delta t) = 769.415$ **COULOMBS**

Root Action (A-sec1/2) =====** $ROOT\_ACTION := \sqrt{\sum_k \left[(AbsI_k)^2 \cdot \Delta t \right]} = 7.724 \times 10^3$

$k := 0 .. Nt - 1$

$Idot_k := \frac{II_{k+1} - II_k}{\Delta t}$ $AbsIdot_k := \left| Idot_k \right|$

Maximum dI/dt (A/sec) === $\max(AbsIdot) = 8.874 \times 10^{10}$

ASSUME 10 OHM LOAD, THE ENERGY DELIVERED TO LOAD = $ROOT\_ACTION^2 \cdot 10 = 5.966 \times 10^8$

WHICH IS A FEW HUNDRED MEGAJOULES!!!!!