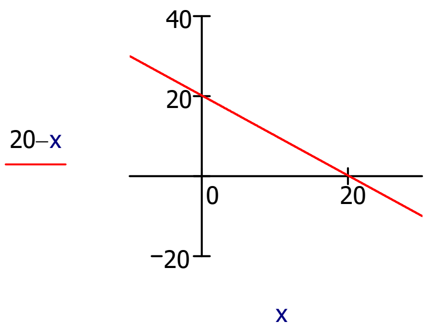
Problem from https://www.ptcusercommunity.com/thread/136984

$$f(x,y) := 20x + 16y - 2x^2 - y^2 - \frac{x \cdot y}{2}$$

$$B(x,y) := (x + y \le 20) \wedge (x \ge 0) \wedge (y \ge 0)$$



Domain of f restricted to the triangle between axis and red line.

Outside it f have not interest.

Critical points: Points where Grad f = 0 union borders union discontinuities.

Disconts None

Gradient

$$f_x(x,y) := \frac{\partial}{\partial x} f(x,y) \rightarrow 20 - 4 \cdot x - \frac{1}{2} \cdot y$$

Equate both components to zero.

$$f_y(x,y) := \frac{\partial}{\partial y} f(x,y) \rightarrow 16 - 2 \cdot y - \frac{1}{2} \cdot x$$

Solve the system:

$$\begin{pmatrix} x_o & y_o \end{pmatrix} := \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} \text{solve}, x, y \rightarrow \begin{pmatrix} \frac{128}{31} & \frac{216}{31} \end{pmatrix}$$

$$f(x_o, y_o) \rightarrow \frac{3008}{31} = 97.03$$ with $B(x_o, y_o) = 1$

Borders $y_1(x) := 20 - x$ $y_2(x) := 0$ $x_5(y) := 0$

Substitute into f to see maxs and mins of f

$$r_1(x) := f(x, y_1(x)) \text{ simplify } \rightarrow 34 \cdot x - 80 - \frac{5}{2} \cdot x^2$$ with 0<x<20

$$r_2(x) := f(x, y_2(x)) \text{ simplify } \rightarrow 20 \cdot x - 2 \cdot x^2$$ with 0<x<20

$r_5(y) := f(x_5(y), y) \rightarrow 16 \cdot y - y^2$ with 0<y<20

This is max and mins of f can appear along those curves and points, and not at any other place. For find the extremas, as all are polynoms, search for zero of derivatives and borders, and get the min by inspection

$x_1 := \frac{\partial}{\partial x} r_1(x) \text{ solve}, x \rightarrow \frac{34}{5}$ $y_1 := y_1(x_1) \rightarrow \frac{66}{5}$ $f_1 := f(x_1, y_1) \rightarrow \frac{178}{5}$

$x_2 := \frac{\partial}{\partial x} r_2(x) \text{ solve}, x \rightarrow 5$ $y_2 := y_2(x_2) \rightarrow 0$ $f_2 := f(x_2, y_2) \rightarrow 50$

$x_3 := 0$ $y_3 := 0$ $f_3 := f(x_3, y_3) \rightarrow 0$

$y_5 := \frac{\partial}{\partial y} r_5(y) \text{ solve}, y \rightarrow 8$ $x_5 := x_5(y_5) \rightarrow 0$ $f_5 := f(x_5, y_5) \rightarrow 64$

$x_6 := 20$ $y_6 := 0$ $f_6 := f(x_6, y_6) \rightarrow -400$

$x_7 := 0$ $y_7 := 20$ $f_7 := f(x_7, y_7) \rightarrow -80$

Numerically

$Nx := 100$ $Ny := 100$ $\Delta x := \frac{20}{Nx - 1}$ $\Delta y := \frac{20}{Ny - 1}$

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F := | x ← 0
     | for  m ∈ 1 .. Nx
     |   | y ← 0
     |   | for  n ∈ 1 .. Ny
     |   |   | F_{m,n} ← | f(x,y)  if  B(x,y)
     |   |   |           | f2  otherwise
     |   |   | y ← y + Δy
     |   | x ← x + Δx
     | F
```

$\max(F) = 97.01$

$\min(F) = -400$

Alvaro