

|implicitplot2d()

|Draghilev's method

Finding extremes: Himmelblau's function  $f(x, y) := (x^2 + y - 11)^2 + (x + y^2 - 7)^2$

$$\frac{d}{dx} f(x, y) \text{ simplify } \rightarrow 4 \cdot x^3 + 4 \cdot x \cdot y - 42 \cdot x + 2 \cdot y^2 - 14 \quad \frac{d}{dy} f(x, y) \text{ simplify } \rightarrow 2 \cdot x^2 + 4 \cdot x \cdot y + 4 \cdot y^3 - 26 \cdot y - 22$$

$$F(X) := \begin{pmatrix} (x \ y) \leftarrow (X_0 \ X_1) \\ 4 \cdot x^3 + 4 \cdot x \cdot y - 42 \cdot x + 2 \cdot y^2 - 14 \\ 2 \cdot x^2 + 4 \cdot x \cdot y + 4 \cdot y^3 - 26 \cdot y - 22 \end{pmatrix}$$

$$D(t, x) := \text{Draghilev}(F(x)) \rightarrow \begin{pmatrix} 16 \cdot x_0 \cdot (x_3)^3 - 504 \cdot (x_1)^2 \cdot x_3 - 8 \cdot x_0 \cdot (x_3)^2 + 8 \cdot x_0 \cdot (x_4)^2 - 8 \cdot x_1 \cdot (x_3)^2 - 16 \cdot x_0 \cdot (x_4)^3 - 16 \cdot x_1 \cdot (x_4)^3 + 48 \cdot (x_1)^2 \cdot (x_3) \\ 8 \cdot x_1 \cdot (x_3)^2 - 16 \cdot x_0 \cdot (x_3)^3 - 8 \cdot x_0 \cdot (x_4)^2 - 312 \cdot (x_0)^2 \cdot x_4 - 16 \cdot x_1 \cdot (x_3)^3 - 8 \cdot x_1 \cdot (x_4)^2 + 16 \cdot x_1 \cdot (x_4)^3 + 24 \cdot (x_0)^2 \cdot (x_3) \\ 48 \cdot (x_0)^3 + 144 \cdot (x_0)^2 \end{pmatrix}$$

given  $F(\text{stack}(x, y))_1 = 0$        $\text{GetX0}(x, y) := \text{Find}(x, y)$

$X0 := \text{GetX0}(-5, -4)$      $\text{tmin} := 0$      $\text{tmax} := -0.007$      $N := 100$      $ii := 0..N - 1$

$\text{result} := \text{rkfixed}(\text{stack}(X0, 1, X0), \text{tmin}, \text{tmax}, N, D)$

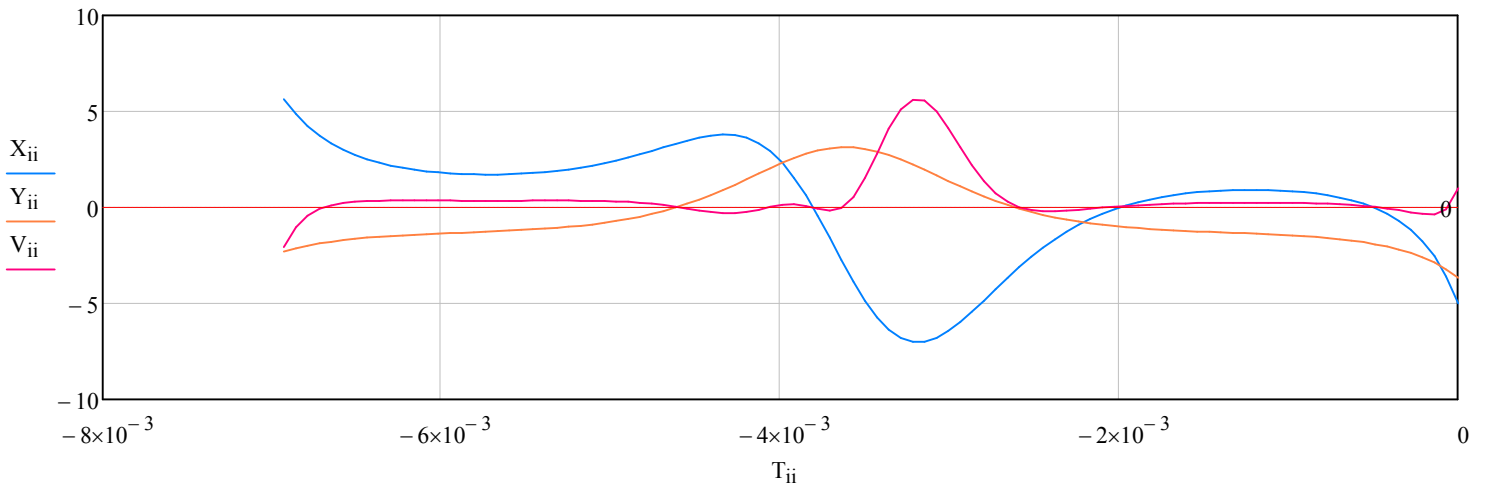
$T := \text{result}^{(0)}$      $X := \text{result}^{(1)}$      $Y := \text{result}^{(2)}$      $V := \text{result}^{(3)}$

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Roots(res) :=
  n ← cols(res) / 2
  v ← res<n>
  nv ← Search(v)
  for ii ∈ 0..length(nv) - 1
    for jj ∈ 1..n - 1
      out<ii, jj-1> ← Interpol[ (res<jj>)<nv<ii>, (res<jj>)<nv<ii>+1>, v<nv<ii>, v<nv<ii>+1> ]
  out
  
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$\text{roots} := \text{Roots}(\text{result})$      $\text{rows}(\text{roots}) = 9$

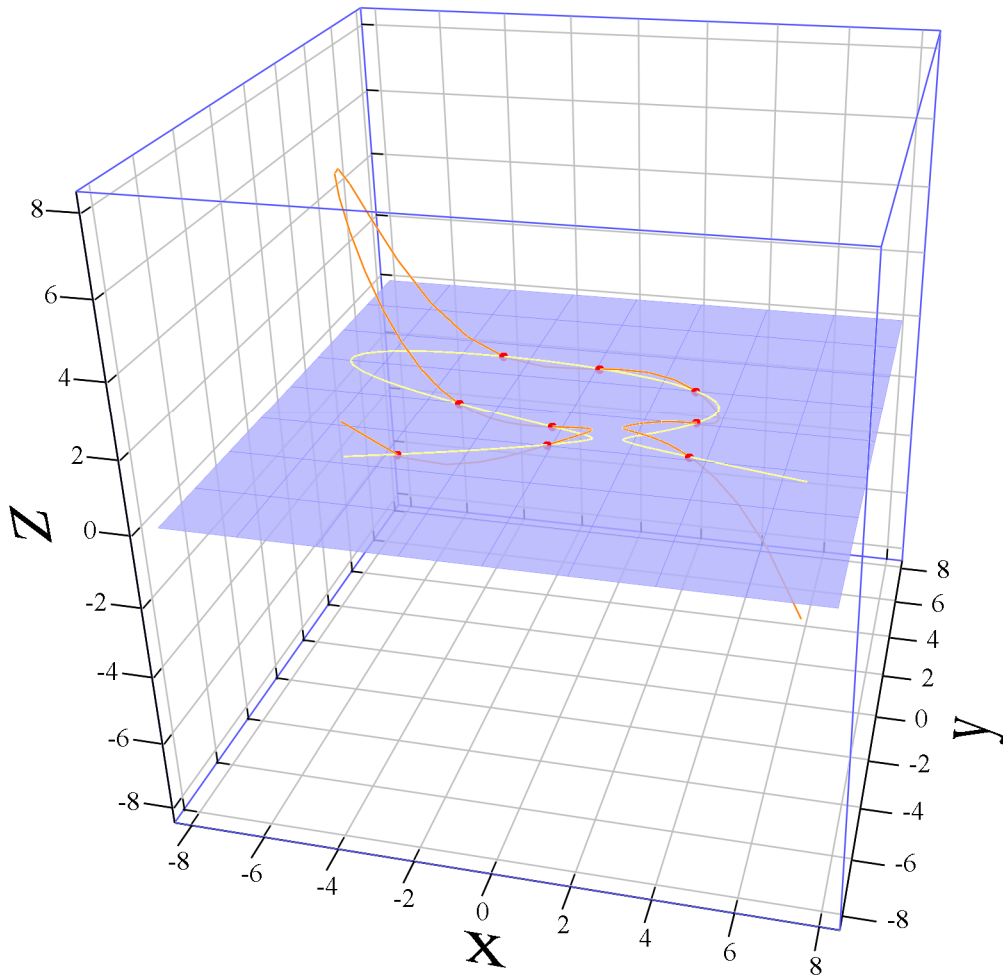
$$\text{roots}^T = \begin{pmatrix} -3.711 & -0.121 & -0.263 & -3.068 & -2.78 & 0.1 & 2.983 & 3.378 & 3.564 \\ -3.26 & -1.951 & -0.925 & -0.083 & 3.129 & 2.874 & 2.003 & 0.071 & -1.843 \end{pmatrix}$$



$$\text{Curve} := (X \ Y \ V)^T \quad \text{Plane}(x,y) := 0$$

$$\text{CurveProj} := (X \ Y \ V \cdot 0)^T \quad \text{RootsPoints} := (\text{roots}^{(0)} \ \text{roots}^{(1)} \ \text{roots}^{(0)}.0)^T$$

$$F(\text{stack}(x,y)) \rightarrow \begin{pmatrix} 4 \cdot x^3 + 4 \cdot x \cdot y - 42 \cdot x + 2 \cdot y^2 - 14 \\ 2 \cdot x^2 + 4 \cdot x \cdot y + 4 \cdot y^3 - 26 \cdot y - 22 \end{pmatrix}$$



xmin := -8    ymin := -8    xmax := 8    ymax := 8

coords :=  $\begin{pmatrix} \text{xmax} & \text{ymax} \\ \text{xmin} & \text{ymin} \end{pmatrix}$

(nx ny) := (150 150)    grids := (nx ny)<sup>T</sup>

f(x,y) := F(stack(x,y))<sub>0</sub>    S1 := implicitplot2d(f, coords, grids)

(nx ny) := (150 150)    grids := (nx ny)<sup>T</sup>

f(x,y) := F(stack(x,y))<sub>1</sub>    S2 := implicitplot2d(f, coords, grids)

