

# BOLTED JOINT DESIGN AND ANALYSIS

Primarily from:

Bickford, John H., *An Introduction to the Design and Behavior of Bolted Joints*, Marcel Dekker, Inc., NY, 1990.

Adapted to Mathcad by Dan  
Frey.

NOTE: in many instances, a single variable can be calculated by one of several equations. Read the accompanying text, "toggle" on the appropriate equation, and "toggle" off the alternative equations using Format/Properties/Calculation. A box will appear to the right of equations that are disabled.

This Mathcad sheet is one integrated model. If you cut out some portions to paste them elsewhere, be aware of which variables have been defined in other parts of the sheet.

# Table of Contents

## Preliminary Definitions

- Derived Units
- Material Properties
- Behavior of Bolt Material
- Bolt Terminology and

Notation

- Define Bolt Geometry

## Strength

- Stress Area of Bolts
- Strength of Bolts
- Strength of the Threads

## Stiffness

- Stiffness of the Bolt
- Stiffness of the Joint

Material

- Total Joint Stiffness
- Slocum's Approach

## Tightening Bolted Joints

- Torque to set Preload
- Turn to set Preload

## Behavior of Joints in Service

- Externally Applied Forces
- The Joint Diagram
- Thermal Expansion
- Stress Corrosion Cracking

## Preliminary Definitions

### Define some useful derived units

$$\text{psi} := \frac{\text{lbf}}{\text{in}^2} \quad \text{ksi} := 1000 \cdot \text{psi} \quad \text{MPa} := 10^6 \cdot \text{Pa} \quad \text{mm} := 10^{-3} \cdot \text{m}$$

### Tabulate some common material properties:

$E_{\text{steel}} := 30 \cdot 10^6 \cdot \text{psi}$	Young's modulus of steel	$\nu_{\text{steel}} := 0.3$
$E_{\text{alum}} := 10 \cdot 10^6 \cdot \text{psi}$	Young's modulus of aluminum	$\nu_{\text{alum}} := 0.3$

### Define behavior of bolt material:

e.g. J429 grade 2 steel  
(pg.54)

$$E := E_{\text{steel}}$$

$$\sigma_y := 57 \cdot \text{ksi}$$

Yield strength

$$\epsilon_y := \frac{\sigma_y}{E}$$

$$\sigma_u := 74 \cdot \text{ksi}$$

Ultimate strength

$$\epsilon_p := 6 \cdot \%$$

Strain at peak stress

check on the last three

$$\sigma_f := 70 \cdot \text{ksi}$$

Stress at failure

$$\epsilon_f := 12 \cdot \%$$

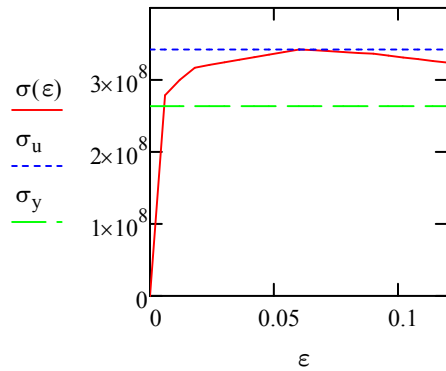
Strain at failure

The function below is a piece wise linear approximation of the stress strain curve as defined above.

$$vx := \left( 0 \quad \varepsilon_y \quad \varepsilon_y + \frac{\varepsilon_p - \varepsilon_y}{4} \quad \varepsilon_p \quad \varepsilon_p + \frac{\varepsilon_f - \varepsilon_p}{2} \quad \varepsilon_f \right)^T$$

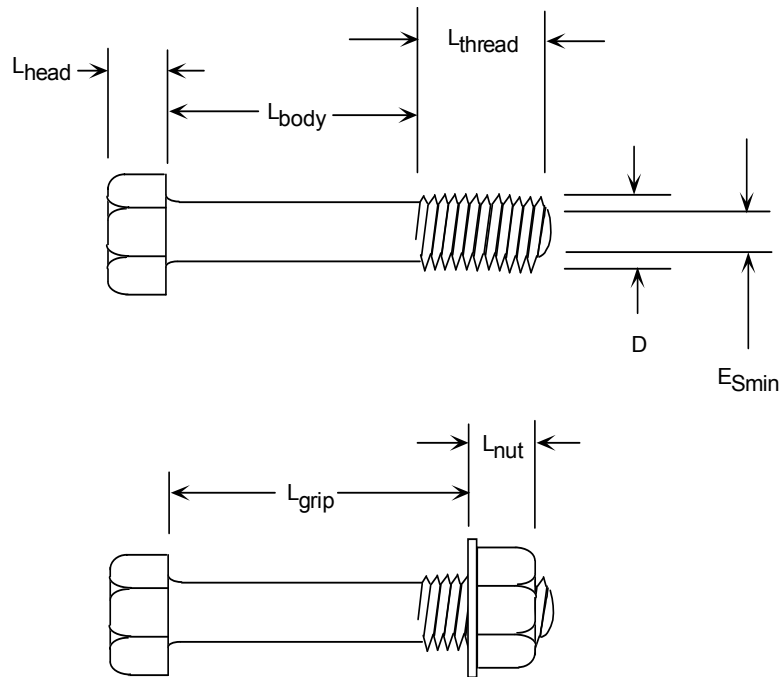
$$vy := \left( 0 \cdot \text{ksi} \quad \sigma_y \quad \sigma_y + \frac{\sigma_u - \sigma_y}{1.5} \quad \sigma_u \quad \sigma_f + \frac{\sigma_u - \sigma_f}{1.5} \quad \sigma_f \right)^T$$

$$\sigma(\varepsilon) := \text{linterp}(vx, vy, \varepsilon) \quad \varepsilon := 0, \frac{\varepsilon_f}{20} .. \varepsilon_f$$



This plot shows the stress/strain behavior of the bolt material.

## Bolt Terminology / Notation



$L_{grip}$  -- Grip length

$L_{nut}$  -- Length of the nut

$E_{Smin}$  -- Minimum pitch diameter of the threads

$D$  -- Nominal diameter of the threads

$L_{head}$  -- Length of the head of the bolt

$L_{body}$  -- Length of the unthreaded portion of the shank

$L_{thread}$  -- Length of the threaded portion of the bolt

## Define Bolt Geometry

Nominal  
Diameter

$$D := 16 \cdot \text{mm}$$

Thread  
pitch

$$p := 2 \cdot \text{mm}$$

Threads per  
inch

$$n := \frac{1}{p} \quad n = 152.4 \text{ ft}^{-1}$$

$$A_{\text{body}} := \frac{\pi}{4} \cdot D^2$$

$$A_{\text{head}} := 2 \cdot A_{\text{body}}$$

$$A_{\text{nut}} := A_{\text{head}}$$

$$L_{\text{thread}} := 50 \cdot \text{mm}$$

$$L_{\text{head}} := 24 \cdot \text{mm}$$

$$L_{\text{nut}} := 24 \cdot \text{mm}$$

$$L_{\text{body}} := 50 \cdot \text{mm}$$

$$L_{\text{grip}} := L_{\text{body}} + \frac{1}{2} \cdot L_{\text{thread}}$$

# Strength

## Stress Area of Bolts

The area of the threaded portion of the bolt that sees the stress (sometimes called the stress area) is critical. There are several ways to calculate it. Select the appropriate equation below and ensure that it is the only active equation (use "Toggle Equation" in the Math menu).

### English Units

$$A_s := 0.785 \cdot \left( D - \frac{0.985}{n} \right)^2$$

(Bickford, pg. 23) - Based on the mean of the pitch and root diameters for 60 degree threads.

$$E_{Smin} := .89 \cdot \text{in}^3$$

Minimum pitch diameter of the threads.

$$A_s := \pi \cdot \left( \frac{E_{Smin}}{2} - \frac{0.16238}{n} \right)^2$$

Recommended for bolt materials with yield strengths >100,000psi. Use the minimum pitch diameter of the threads.

$$A_s := 0.7854 \cdot \left( D - \frac{1.3}{n} \right)^2$$

Root area. More Conservative.

$$A_s := 0.7854 \cdot \left( D - \frac{1.3}{n} \right)^2$$

ASME Boiler and pressure vessel code.

## Metric

$$p = 2 \text{ mm}$$

Pitch of the  
threads

$$A_s := 0.7854 \cdot (D - 0.938 \cdot p)^2$$

(Bickford, pg. 25) - For metric  
threads.

$$E_{Smin} := 10 \cdot \text{mm}$$

Minimum pitch diameter of the  
threads.

$$A_s := 0.7854 \cdot (E_{Smin} - 0.268867 \cdot p)^2$$

Recommended for bolt materials with yeild strei  
>100,000psi. Use the minimum pitch diameter  
threads.

$$A_s := 0.7854 \cdot (D - 1.22687 \cdot p)^2$$

Root area. More  
Conservative.



## Strength of the Bolt

### Define the approximate stress distribution in a bolt

$$F_p := 50 \cdot \text{lbf}$$

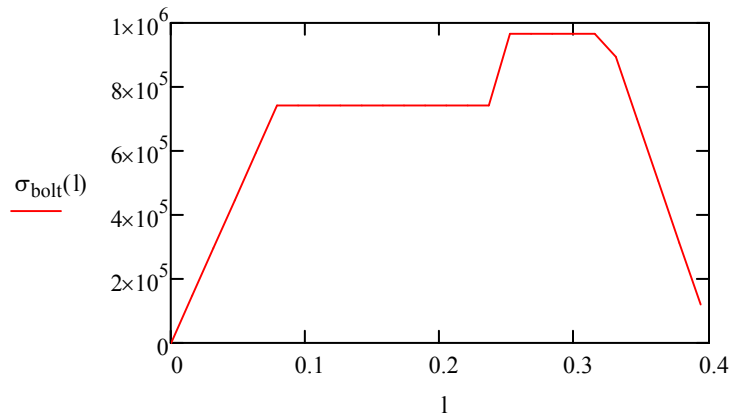
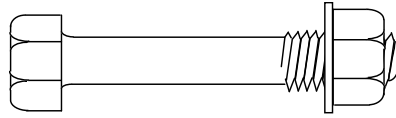
Tension in the bolt

$$\begin{array}{l}
 \text{vx} := \left[ \begin{array}{c} 0 \cdot \text{in} \\ L_{\text{head}} \\ L_{\text{head}} + L_{\text{body}} \\ (L_{\text{head}} + L_{\text{body}}) \cdot 1.01 \\ L_{\text{head}} + L_{\text{grip}} \\ L_{\text{head}} + L_{\text{grip}} + L_{\text{nut}} \\ L_{\text{head}} + L_{\text{body}} + L_{\text{thread}} \end{array} \right] \\
 \text{vy} := \left( \begin{array}{c} 0 \cdot \text{psi} \\ \frac{F_p}{A_{\text{body}}} \\ \frac{F_p}{A_{\text{body}}} \\ \frac{F_p}{A_s} \\ \frac{F_p}{A_s} \\ 0 \cdot \text{psi} \\ 0 \cdot \text{psi} \end{array} \right) \\
 \sigma_{\text{bolt}}(l) := \text{linterp}(\text{vx}, \text{vy}, l)
 \end{array}$$

Graph the stress distribution in a bolt.

$$l := 0 \cdot \text{in}, \frac{L_{\text{head}}}{5} .. L_{\text{head}} + L_{\text{body}} + L_{\text{thread}}$$

This is an approximate stress distribution across the length of a bolt. Left is the head of the bolt, right is the nut. The most highly stressed portion is the threaded area of the bolt under tension. This area therefore determines the strength of the bolt. The actual stress distribution is more complex.



The force (F) that a bolt can support before the shank (as opposed to the threads) fails is:

$$F_{ult} := \sigma_u \cdot A_s \quad F_{ult} = 1.772 \times 10^4 \text{ lbf}$$

$$F_y := \sigma_y \cdot A_s \quad F_y = 1.365 \times 10^4 \text{ lbf}$$

## Strength of Threads

### Nut material stronger than the bolt material

Bolt threads typically fail at the root. The total cross sectional area at that point is needed for bolt strength calculations.

$L_e := .2 \cdot \text{in}$  Length of thread engagement

$K_{nmax} := 0.257 \cdot \text{in}$  Maximum ID of nut

$E_{Smin} := 0.2 \cdot \text{in}$  Minimum PD of bolt

$n := 20 \cdot \text{in}^{-1}$  Threads per inch

This section continues to use a 1/4-20 bolt as an example.

#### Shear Area

$$A_{TS} := \pi \cdot n \cdot L_e \cdot K_{nmax} \cdot \left[ \frac{1}{n} + 0.57735 \cdot (E_{Smin} - K_{nmax}) \right]$$

$$A_{TS} = 0.055 \text{ in}^2 \quad A_s = 0.24 \text{ in}^2$$

According to military standard FED-STD-H28, when the nut material is much stronger than the bolt material, the shear area is approximated within 5% by the formula

$$A_{TS} := \frac{5}{8} \cdot \pi \cdot E_{Smin} \cdot L_e$$

$$A_{TS} = 0.079 \text{ in}^2$$

Rearranging, one can calculate the minimum thread engagement required to ensure that the bolt fails rather than the threads.

$$L_e := \frac{2 \cdot A_s}{\frac{5}{8} \cdot \pi \cdot E_{Smin}}$$

Where  $A_s$  is the stress area (computed in the bolt strength section)

$$L_e = 1.22 \text{ in}$$

$$A_s = 0.24 \text{ in} \cdot \text{in}$$

$$\frac{L_e}{D} = 1.936$$

Which is a fairly typical ratio for a nut one might purchase.

## Nut material weaker than the bolt material

If threads are tapped into a weak material (cast iron, Aluminum, plastic), the nut threads may fail first even though the shear area is greater.

$L_e := .2 \cdot \text{in}$	Length of thread engagement
$E_{nmax} := 0.257 \cdot \text{in}$	Maximum PD of nut
$D_{Smin} := 0.25 \cdot \text{in}$	Minimum OD of bolt threads
$n := 10 \cdot \text{in}^{-1}$	Threads per inch
$S_{st} := \sigma_u$	Tensile strength of bolt material
$S_{nt} := \frac{\sigma_u}{2}$	Tensile strength of nut material

This section continues to use a 1/4-20 bolt as an example.

According to military standard FED-STD-H28, the shear area is approximated within 5% by the formula

$$A_{TS} := \frac{3}{4} \cdot \pi \cdot E_{nmax} \cdot L_e$$
$$A_{TS} = 0.121 \text{ in}^2$$

Rearranging, one can estimate the minimum thread engagement required to ensure that the bolt fails rather than the nut threads.

$$L_e := \frac{S_{st} \cdot (2 \cdot A_s)}{S_{nt} \cdot \left( \frac{3}{4} \cdot \pi \cdot E_{nmax} \right)}$$

Where  $A_s$  is the stress area (computed in the bolt strength section)

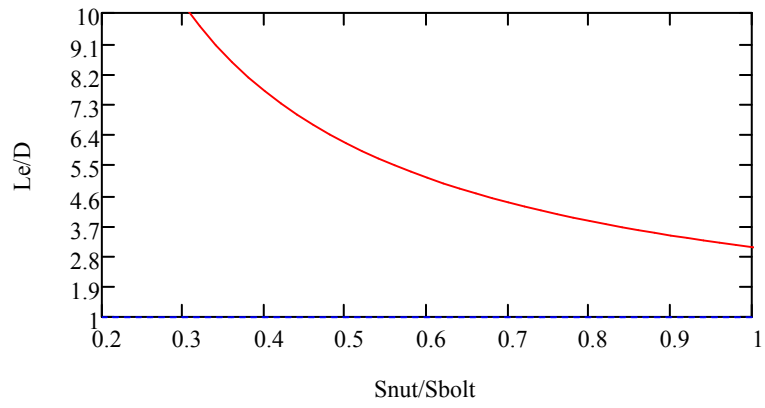
$$A_s = 0.24 \text{ in} \cdot \text{in}$$

$$L_e = 1.582 \text{ in}$$

$$\frac{L_e}{E_{nmax}} = 6.156$$

The weaker the nut material, the more threads must be engaged.

$$S_{st} := \frac{1}{5} \cdot S_{st}, \frac{1.1}{5} \cdot S_{st} \dots 2 \cdot S_{st}$$



# Stiffnes

## S

### Stiffness of the Bolt

Using the stress vs length graph from the strength section above, total deflection of the bolt under load can be estimated as:

$$\Delta L_{\text{bolt}} := \int_0^{L_{\text{head}}+L_{\text{body}}+L_{\text{thread}}} \frac{\sigma_{\text{bolt}}(l)}{E} dl \quad E = 3 \times 10^7 \text{ psi}$$

$$\Delta L_{\text{bolt}} = 2.32 \times 10^{-5} \text{ in}$$

This means that the spring constant of the bolt (and the nut) is:

$$K_{\text{bolt}} := \frac{F_p}{\Delta L_{\text{bolt}}} \quad K_{\text{bolt}} = 2.155 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

## Stiffness of the Joint Material

$$E_{\text{joint}} := 30 \cdot 10^6 \cdot \text{psi}$$

$$T := L_{\text{grip}}$$

Thickness of the joint material

The area of an equivalent cylinder of material that is placed in compression as the bolt is loaded in tension is computed below. This model assumes:

- 1) Elastic material behavior.
- 2) Concentric joint - The bolt goes through the center of the joint material.
- 3) The load is applied along the joint axis.

$$D_j := 1.5 \cdot \text{in}$$

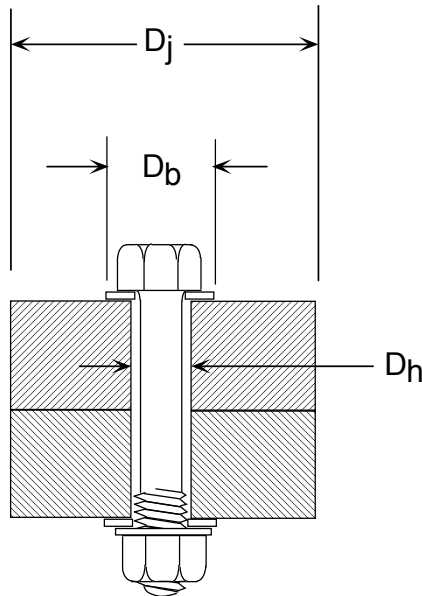
Outside diameter of the joint material

$$D_b := 1.5 \cdot D$$

Nominal diameter of the bolt head (or washer)

$$D_h := 1.01 \cdot D$$

Diameter of the hole the bolt goes through



If the thickness of the upper and lower joint layers are equal:

$$A_c(D_j) := \begin{cases} \frac{\pi}{4} \cdot (D_j^2 - D_h^2) & \text{if } D_b \geq D_j \\ \frac{\pi}{4} \cdot (D_b^2 - D_h^2) + \frac{\pi}{8} \cdot \left( \frac{D_j}{D_b} - 1 \right) \cdot \left( \frac{D_b \cdot T}{5} + \frac{T^2}{100} \right) & \text{if } D_b < D_j \leq 3 \cdot D_b \\ \frac{\pi}{4} \cdot \left[ \left( D_b + \frac{T}{10} \right)^2 - D_h^2 \right] & \text{otherwise} \end{cases}$$

Thanks to Alan Duke, Technical Director of Goodrich Fuel and Utility Systems for correcting an error in a previous version.

Bickford (pg. 111) also indicates that the two last cases apply only when  $T < 8D$ .

He doesn't say what to do if  $T > 8D$ .

Finally, the stiffness of the joint material in compression is given by:

$$K_{Jc} := \frac{E_{\text{joint}} \cdot A_c(D_j)}{T} \qquad K_{Jc} = 5.407 \times 10^6 \frac{\text{lbf}}{\text{in}} \qquad K_{\text{bolt}} = 2.155 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

$$K_j := K_{Jc} \qquad \text{If the joint is concentric.}$$





$s := 1 \cdot \text{in}$	Distance the bolt is off center.
$a := 2 \cdot \text{in}$	Distance the load is off center
$R_G := 0.209 \cdot D_j$	Radius of gyration if joint is rectangular viewed down the bolt. $D_j$ is the length of the shorter side.
$R_G := \frac{D_j}{2}$	Radius of gyration if joint is circular viewed down the bolt.
$t := 3 \cdot \text{in}$	Distance between bolts
$T_{\min} := .2 \cdot \text{in}$	Thickness of the thinner joint cross section.
$W := 5 \cdot \text{in}$	Total width of the joint

$$b := \begin{cases} t & \text{if } t \leq (D_b + T_{\min}) \\ D_b + T_{\min} & \text{otherwise} \end{cases}$$

$$A_j := \begin{cases} b \cdot W & \text{if } W \leq D_b + T_{\min} \\ b \cdot (D_b + T_{\min}) & \text{otherwise} \end{cases}$$

If the load is **on center** with the bolt (ie.  $s=a$ ):

$$K_j := \frac{1}{\frac{1}{K_{Jc}} \left( 1 + \frac{s^2 \cdot A_c}{R_G^2 \cdot A_j} \right)}$$

If the load is **off center** with the bolt:

$$K_j := \frac{1}{\frac{1}{K_{Jc}} \left( 1 + \frac{s \cdot a \cdot A_c}{R_G^2 \cdot A_j} \right)}$$

$$K_j = 5.407 \times 10^6 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

## Gaskets

If the joint contains a gasket, the gasket stiffness may dominate the stiffness of the joint. Gasket material stiffness values are tabulated in Bickford pp.121-2. Use these values with care as gasket stiffness is often highly non-linear and hysteretic.

$$A_g := 0.5 \cdot \text{in}^2$$

Area of the gasket viewed looking down the bolt

$$K_g := 35 \cdot \frac{\text{MPa}}{\text{mm}} \cdot A_g$$

Compressed asbestos, 0.125 mm thick.

$$K_g := 10^{100} \cdot \frac{\text{lb}_f}{\text{in}}$$

If there is no gasket

## Total Joint Stiffness

Individual component stiffnesses behave as springs in series. Therefore they are combined by inverse sum of inverses (as if they were resistances in parallel).

$$K_{\text{bolt}} = 2.155 \times 10^6 \frac{\text{lb}_f}{\text{in}}$$

$$K_g = 1 \times 10^{100} \frac{\text{lb}_f}{\text{in}}$$

$$K_j = 5.407 \times 10^6 \frac{\text{lb}_f}{\text{in}}$$

$$K_{\text{washer}} := 10 \cdot K_{\text{bolt}}$$

$$K_{\text{joint}} := \frac{1}{\frac{1}{K_{\text{bolt}}} + \frac{1}{K_{\text{washer}}} + \frac{1}{K_j} + \frac{1}{K_g}}$$

$$K_{\text{joint}} = 1.438 \times 10^6 \frac{\text{lb}_f}{\text{in}}$$

## Slocum's Method

Slocum offers an alternative to the methods given

above. According to Slocum, if the bolt produces a 45 deg cone of influence:

$$K_{\text{flange\_comp}} := \frac{\pi \cdot E_{\text{joint}} \cdot D_h}{\ln \left[ \frac{(D_h - D_b - 2 \cdot L_{\text{grip}}) \cdot (D_h + D_b)}{(D_h + D_b + 2 \cdot L_{\text{grip}}) \cdot (D_h - D_b)} \right]}$$

$$K_{\text{flange\_comp}} = 4.143 \times 10^7 \frac{\text{lbf}}{\text{in}}$$

$$\frac{K_{\text{flange\_comp}}}{K_j} = 7.663$$

$\nu := 0.3$  Poisson's Ratio of joint material

$$K_{\text{flange\_shear}} := \frac{\pi \cdot L_{\text{grip}} \cdot E_{\text{joint}}}{(1 + \nu) \cdot \ln(2)}$$

$$K_{\text{flange\_shear}} = 3.088 \times 10^8 \frac{\text{lbf}}{\text{in}}$$

$$\frac{K_{\text{flange\_shear}}}{K_j} = 57.122$$

$$E_{\text{nut}} := E_{\text{steel}}$$

$$K_{\text{bed\_shear}} := \frac{\pi \cdot D \cdot E_{\text{nut}}}{(1 + \nu) \cdot \ln(2)}$$

$$K_{\text{bed\_shear}} = 6.589 \times 10^7 \frac{\text{lbf}}{\text{in}}$$

$$\frac{K_{\text{bed\_shear}}}{K_j} = 12.186$$

$$E_{\text{bolt}} := E_{\text{steel}}$$

$$K_{\text{bolt}} := \frac{\pi \cdot E_{\text{bolt}} \cdot D^2}{4 \cdot \left( \frac{D}{2} + L_{\text{grip}} \right)}$$

$$\frac{K_{\text{bolt}}}{K_{\text{joint}}} = 1.989$$

$$K_{\text{sum}} := \frac{1}{\frac{1}{K_{\text{flange\_comp}}} + \frac{1}{K_{\text{flange\_shear}}} + \frac{1}{K_{\text{bed\_shear}}} + \frac{1}{K_{\text{bolt}}}}$$

$$K_{\text{interface}} := 5 \cdot K_{\text{sum}}$$

Stiffness of the interface between the two joint material faces (e.g., the bed and the rail)

A typical value. This usually must be determined empirically.

$$K_{\text{one_bolted_joint}} := \frac{1}{\frac{1}{K_{\text{interface}}} + \frac{1}{K_{\text{sum}}}}$$

Comparing Slocum's result to Bickford's which is a fairly good agreement (mostly because the bolt stiffness is in good agreement and tends to dominate).

$$\frac{K_{\text{one_bolted_joint}}}{K_{\text{joint}}} = 1.478$$

For a system including a part bolted to a bed at many points:

$$K_{\text{part}} := 10^6 \cdot \frac{\text{lbf}}{\text{in}} \quad N_{\text{bolts}} := 8$$

$$K_{\text{system}} := \frac{1}{\frac{1}{K_{\text{part}}} + \frac{N_{\text{bolts}}}{K_{\text{one_bolted_joint}}}}$$

One can vary the number of bolts and bolt diameter to find different bolted joint designs with the same stiffness.

## Tightening Bolted Joints

### Calculating torque (T<sub>in</sub>) required to generate a desired preload level

$F_p := F_y$       Desired preload in the bolt. Equals F<sub>y</sub> as defined in strength section above if tightening to yield.

$p = 0.079 \text{ in}$       Thread pitch

$\mu_t := 0.1$       Coefficient of friction between the nut and the bolt threads

$r_t := \frac{D + E_{Smin}}{4}$       Effective contact radius of the threads

$\beta := 30 \cdot \text{deg}$       Half angle of the threads.

$\mu_n := 0.1$       Coefficient of friction between the face of the nut and the upper surface of the joint (or the washer).

$r_n := 0.6 \cdot D$       Effective contact radius of the contact between the nut and joint surface.

$$T_{in} := F_p \cdot \left( \frac{p}{2 \cdot \pi} + \frac{\mu_t \cdot r_t}{\cos(\beta)} + \mu_n \cdot r_n \right) \quad T_{in} = 1.014 \times 10^3 \text{ in} \cdot \text{lbf}$$

The first term is inclined plane action.  
The second is thread friction.  
The third is friction acting on the face of the nut.

One can instead rely on an experimental constant, the nut factor (K<sub>nut</sub>) that combines all the terms above. K<sub>nut</sub> values are tabulated in Bickford (pp. 141-143).

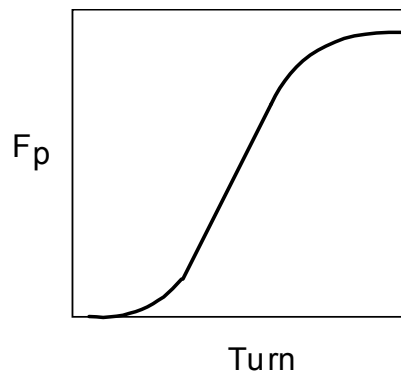
$K_{nut} := 0.2$       steel on steel

$$T_{in} := F_p \cdot K_{nut} \cdot D \quad T_{in} = 1.72 \times 10^3 \text{ in} \cdot \text{lbf}$$

You can see there is a reasonable agreement between the two estimates.

## Setting preload with turn angle

Often, preload can be set more accurately by controlling the number of turns rather than input torque. Typical preload vs turn behavior is depicted in the figure below. The behavior is soft at first as the threads embed. Then there is a linear portion of the curve. As the bolt begins to yield, the behavior becomes non-linear again.



To estimate the needed turn angle:

$$F_p = 1.365 \times 10^4 \text{ lbf}$$

$$K_{\text{bolt}} = 2.861 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

$$K_j = 5.407 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

$$\Theta_R := F_p \cdot \frac{360}{p} \cdot \left( \frac{K_{\text{bolt}} + K_j}{K_{\text{bolt}} \cdot K_j} \right)$$

$$\Theta_R = 1.911 \times 10^3 \text{ deg}$$

Turn angle to apply a preload of  $F_p$ .

# Behavior of Joints in Service

## Externally Applied Forces

$$K_{\text{bolt}} = 2.861 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

Stiffness of the bolt

$$K_j = 5.407 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

Stiffness of the joint materials

$$K_{\text{joint}} = 1.438 \times 10^6 \frac{\text{lbf}}{\text{in}}$$

Stiffness of the whole joint

$$F_p := 75\% \cdot F_y$$

$$F_p = 1.024 \times 10^4 \text{ lbf}$$

Preload Force

$$OL_{\text{bolt}} := \frac{F_p}{K_{\text{bolt}}}$$

$$OL_{\text{bolt}} = 3.579 \times 10^{-3} \text{ in}$$

Extension of the bolt due to preload.

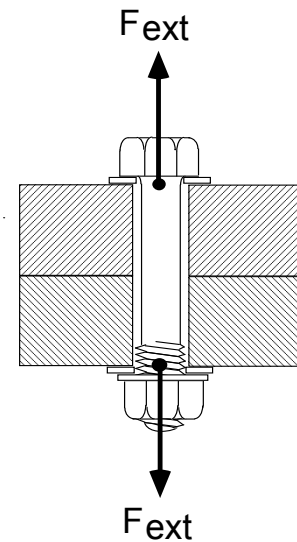
$$OL_j := \frac{F_p}{K_j}$$

$$OL_j = 1.894 \times 10^{-3} \text{ in}$$

Compression of the joint due to preload.

Externally applied force. Applies tension to the base of the bolt head and nut.

$$F_{\text{ext}} := 8000 \cdot \text{lbf}$$





$$\Delta F_j := F_{\text{ext}} \left( \frac{K_j}{K_j + K_{\text{bolt}}} \right) \quad \Delta F_j = 5.232 \times 10^3 \text{ lbf}$$

$$\Delta L_j := \frac{\Delta F_j}{K_j} \quad \Delta L_{\text{bolt}} := \Delta L_j$$

$$\Delta F_{\text{bolt}} := K_{\text{bolt}} \cdot \Delta L_{\text{bolt}} \quad \Delta F_{\text{bolt}} = 2.768 \times 10^3 \text{ lbf}$$

$$F(L) := \begin{cases} K_{\text{bolt}} \cdot L & \text{if } L \leq OL_{\text{bolt}} + \Delta L_{\text{bolt}} \\ F_p - K_j \cdot (L - OL_{\text{bolt}}) & \text{otherwise} \end{cases}$$

$$F_{\text{crit}} := F_p + K_{\text{bolt}} \cdot OL_j \quad F_{\text{crit}} = 1.566 \times 10^4 \text{ lbf} \quad \text{The force at which the clamping force goes to zero.}$$

If the applied force > Fcrit then all of the additional applied force is borne by the bolt alone. This is critical because while there is some clamping force the ratio additional load seen by the bolt due to applied load is:

$$\frac{\Delta F_{\text{bolt}}}{F_{\text{ext}}} = 0.346$$

This will be a low number especially if the joint alone is much stiffer than the bolt ( $K_j \gg K_{\text{bolt}}$ ).

Since external loads are significantly attenuated by this effect, to maximize fatigue life preloads should be set high enough to ensure that Fcrit is not exceeded in service.

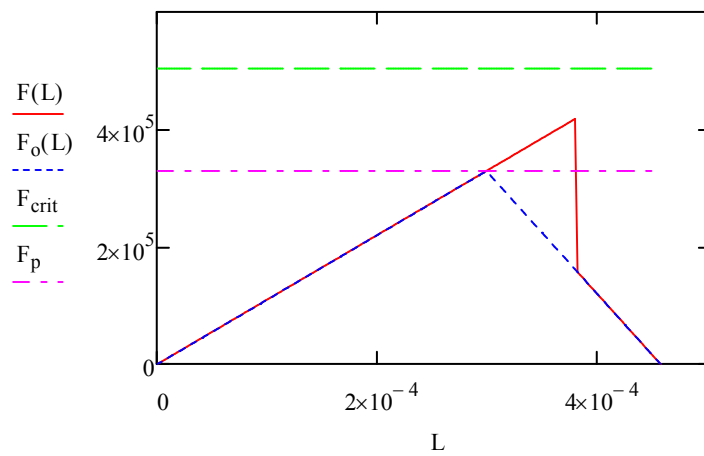
$$\frac{F_{\text{crit}}}{F_p} = 1.529$$

Also note that the critical load of the joint is always higher than the bolt preload. The ratio is higher when the joint is less stiff compared to the bolt. This can be understood better by studying the joint diagram below.

## The Joint Diagram

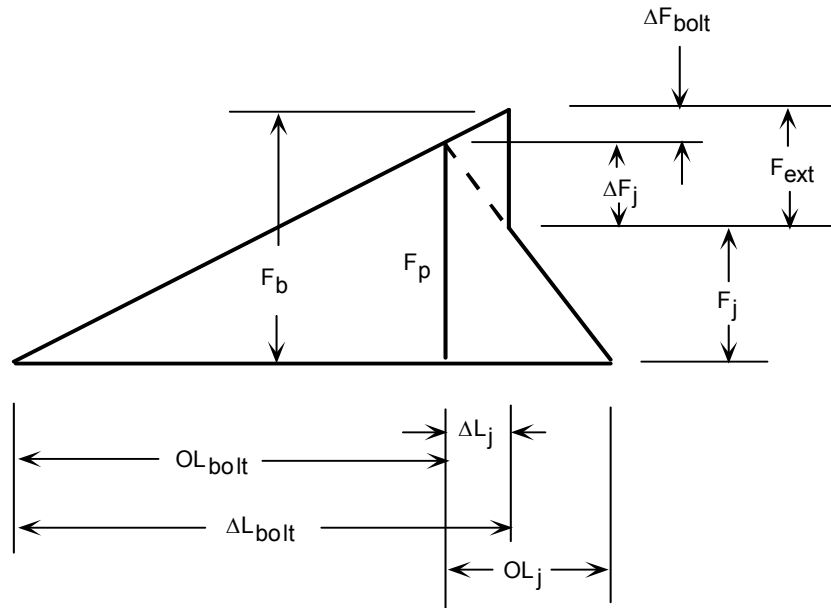
$$F_o(L) := \begin{cases} K_{\text{bolt}} \cdot L & \text{if } L \leq OL_{\text{bolt}} \\ F_p - K_j \cdot (L - OL_{\text{bolt}}) & \text{otherwise} \end{cases}$$

$$L_j := 0, \frac{OL_{\text{bolt}} + OL_j}{200} \dots OL_{\text{bolt}} + OL_j$$



To better understand this diagram, see the labeled figure below.

This figure is helpful for understanding the behavior of bolted joints under applied loads. Bickford explains the concept well on pp.354-360. This sheet allows one to see how the figure applies to different bolted joint geometries.



## Tension in a bolt due to differential thermal expansion

$$\begin{array}{lll} \alpha_{\text{bolt}} := 6 \cdot 10^{-6} & \text{per deg} & \text{Carbon} \\ & \text{F} & \text{steel} \\ \alpha_j := 13 \cdot 10^{-6} & \text{per deg} & \text{Aluminu} \\ & \text{F} & \text{m} \\ \Delta T := 10 & \text{deg F} & \end{array}$$

$$\Delta L_{\text{bolt}} := \alpha_{\text{bolt}} \cdot \Delta T \cdot L_{\text{grip}}$$

Thermal expansion of the bolt and joint

$$\Delta L_j := \alpha_j \cdot \Delta T \cdot L_{\text{grip}}$$

$$F_T := \frac{K_{\text{bolt}} \cdot K_j}{K_{\text{bolt}} + K_j} \cdot (\Delta L_j - \Delta L_{\text{bolt}})$$

$$F_T = 386.725 \text{ lbf}$$

For steel / aluminum combination.

$$\frac{F_T}{F_p} = 0.038 \quad \text{Fraction of preload.}$$

## Stress Corrosion Cracking

$C := 1.5$       Shape factor (1.5 for threads)

$$\sigma_{\max} := \sigma_{\text{bolt}} \left( L_{\text{head}} + L_{\text{body}} + \frac{L_{\text{thread}}}{2} \right)$$

Stress at point of interest (in this case the threaded portion of the bolt).

$a := .001 \cdot \text{in}$       Crack depth

$$K_{\text{ISCC}} := C \cdot \sigma_{\max} \cdot \sqrt{\pi \cdot a} \quad \text{Threshold stress intensity factor for SCC}$$

KISCC is material dependant and must be tabulated. See Bickford pp. 560.

If stress exceeds KISCC, then crack growth will be accelerated by corrosion.



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