

# Creating Gears and Splines: 3 Methods for Generating True Involute Curves

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# Background and Objectives

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- Many methods have been presented by PTC and others to help users develop involute curves for the generation of spline and gear teeth. However, these methods are sometimes confusing, and not all are completely accurate.
- The methods in this presentation expound and improve on current formulae commonly used.
- After developing the equations, I will suggest some “good modeling” practices for given situations.
- At the end, I will perform several live demos of involute creation involving the methods presented within.

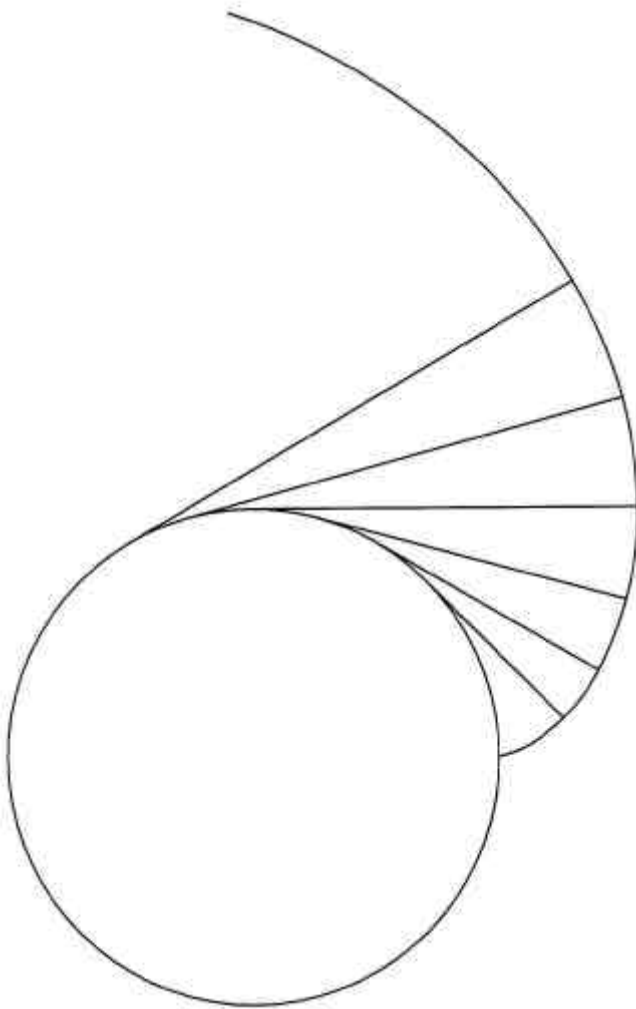
# What is an Involute Curve?

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- An Involute is described as the path of a point on a straight line, called the generatrix, as it rolls along a convex base curve (the evolute).
- The Involute Curve is most often used as the basis for the profile of a spline or gear tooth.

# Generating the Involute Curve

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Imagine a cylinder and a piece of string.

Wrap the string tightly around the cylinder.

Pull the string tight while unwinding it from the cylinder.

Trace the end of the string as it is unwrapped – the result is the involute curve.

# Involute Tooth Profile Terminology

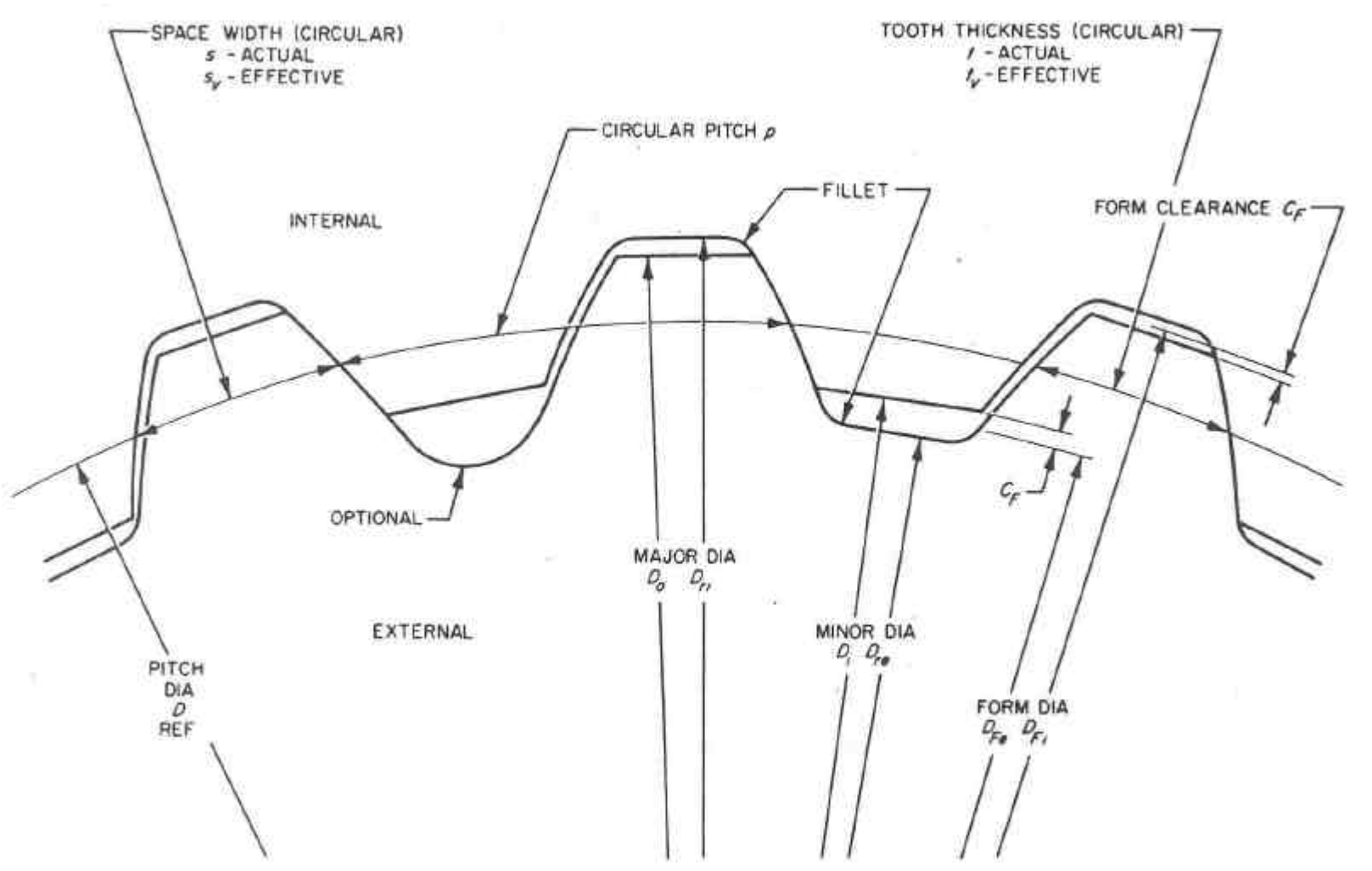


Figure courtesy of ANSI B-92.1-1996, pg. 9, © Society of Automotive Engineers, 1996.

# Involute Tooth Profile Specifications

## DRAWING DATA

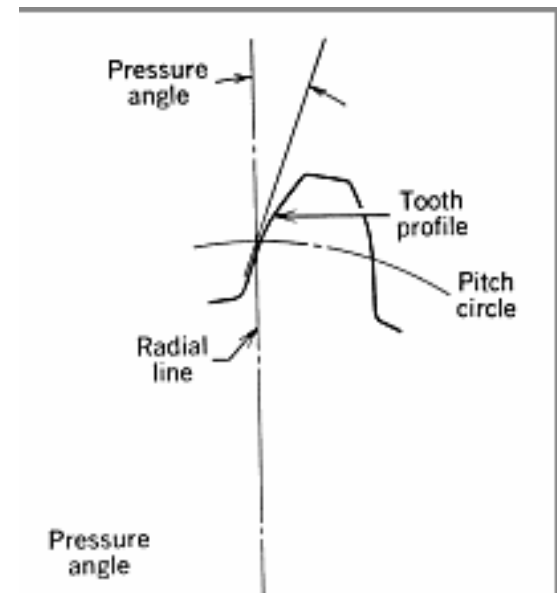
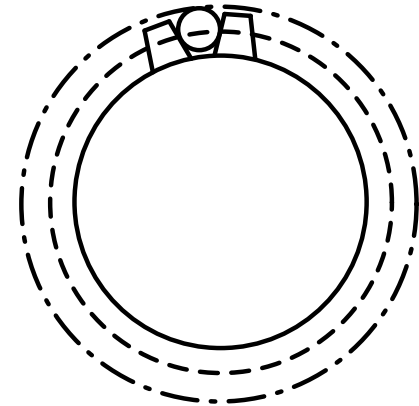
INTERNAL INVOLUTE SPLINE DATA		EXTERNAL INVOLUTE SPLINE DATA	
FLAT ROOT SIDE FIT		FLAT ROOT SIDE FIT	
NUMBER OF TEETH	xx	NUMBER OF TEETH	xx
SPLINE PITCH	xx/xx	SPLINE PITCH	xx/xx
PRESSURE ANGLE	30°	PRESSURE ANGLE	30°
BASE DIAMETER	x.xxxxxx REF	BASE DIAMETER	x.xxxxxx REF
PITCH DIAMETER	x.xxxxxx REF	PITCH DIAMETER	x.xxxxxx REF
MAJOR DIAMETER	x.xxx MAX	MAJOR DIAMETER	x.xxx/x.xxx
FORM DIAMETER	x.xxx	FORM DIAMETER	x.xxx
MINOR DIAMETER	x.xxx/x.xxx	MINOR DIAMETER	x.xxx MIN
CIRCULAR SPACE WIDTH		CIRCULAR TOOTH THICKNESS	
MAX ACTUAL	x.xxxx	MAX EFFECTIVE	x.xxxx
MIN EFFECTIVE	x.xxxx	MIN ACTUAL	x.xxxx
The following information may be added as required:		The following information may be added as required:	
MAX MEASUREMENT BETWEEN PINS	x.xxxx REF	MIN MEASUREMENT OVER PINS	x.xxxx REF
PIN DIAMETER	x.xxxx	PIN DIAMETER	x.xxxx

Tables courtesy of ANSI B-92.1-1996, pg. 9, © Society of Automotive Engineers, 1996.

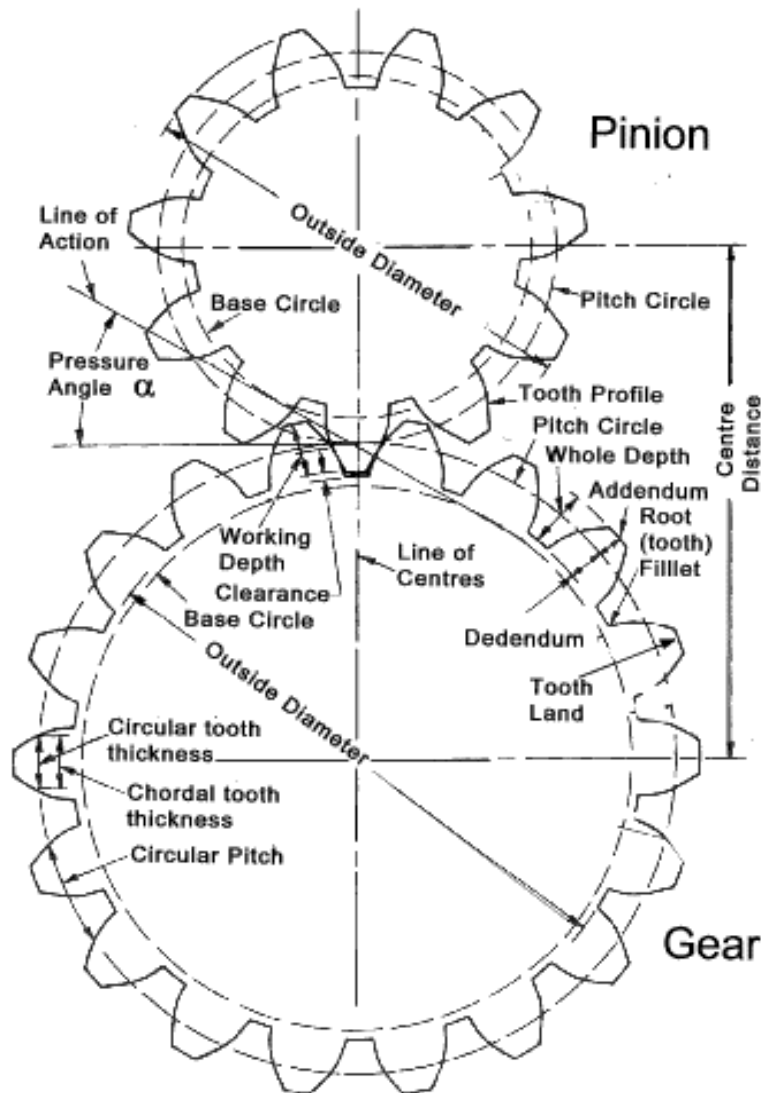
# More Involute Profile Terminology

- Pin Diameter and Diameter over/between pins: Place a circular measurement object tangent to the teeth at the pitch diameter. Measure over the pins for an external tooth, and between pins for an internal tooth

- Pressure Angle: the angle between a line tangent to an involute at the pitch diameter and a radial line through the point of tangency.



# Reasons to Use an Involute Curve



Reasons of Importance:

1. Conjugate action is independent of changes in the center distance.
2. The form of the basic rack tooth is straight-sided, and therefore is relatively simple. Thus, it can be accurately made. As a cutting tool, it imparts high accuracy to the cut gear or spline tooth.
3. One cutter can generate all gear or spline tooth numbers of the same pitch.

Image courtesy of Roy Beardmore,  
[http://www.roymech.co.uk/Useful\\_Tables/Drive/Gears.html](http://www.roymech.co.uk/Useful_Tables/Drive/Gears.html)

# How Do I Model the Involute Curve in Pro/Engineer?

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- The easiest way is to create the profile of the tooth (the Involute Curve) by using a datum curve by equation. This can be done in both Cartesian ( $X, Y, Z$ ) and Cylindrical ( $R, \theta, Z$ ) coordinates.
- Another more difficult method involves a Variable Section Sweep surface feature created by equation. This is done in Cylindrical coordinates only.

# Overall Procedure for the Involute Tooth Profile Geometry Creation

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## Steps to completion

- Set up parameters for key variables!
- Create basic geometry in support of the spline or gear tooth
- Define the Tooth Profile with the Involute Datum Curve or Variable Section Sweep (VSS) Surface
- Create the Tooth solid feature with a cut or protrusion
  - Design vs. Manufacturing intent
  - May need helical curves for VSS generation of helical gear teeth
- Pattern the Tooth around the centerline axis

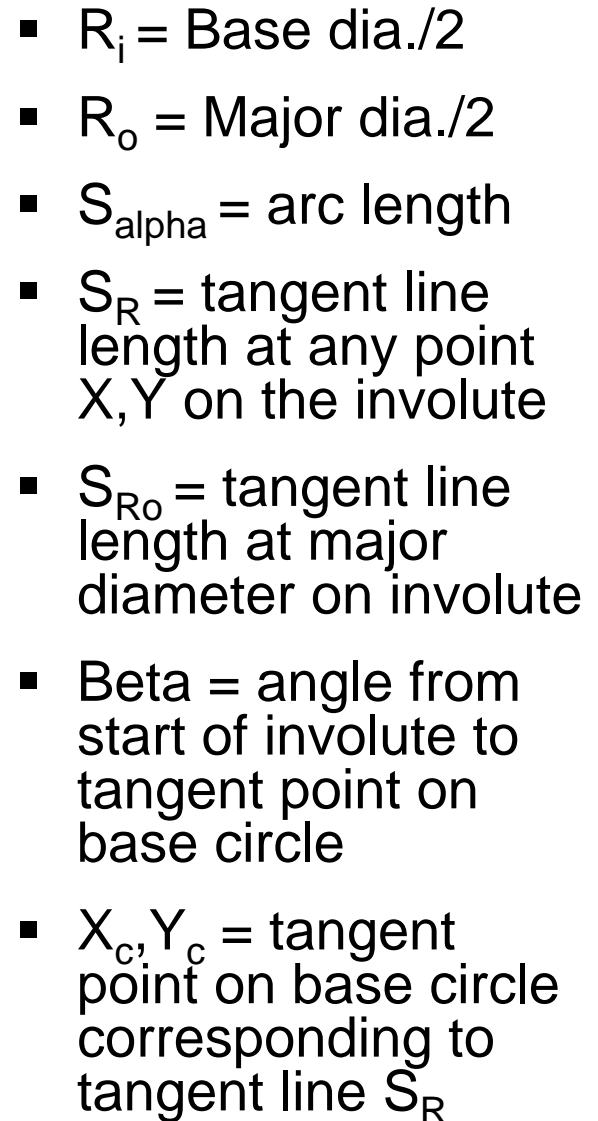
# Choosing the Appropriate Method

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## What to use, and when ...

- Use the Cylindrical coordinate method if you desire the easiest and most versatile method of involute creation, or if you have to use polar coordinates.
- Use Cartesian Coordinates if you have to have the equations in terms of  $X, Y, Z$  only.
- Use the Variable Section Sweep method only if you don't know/care about the major dia., and you are creating a straight (non-helical) tooth surface.

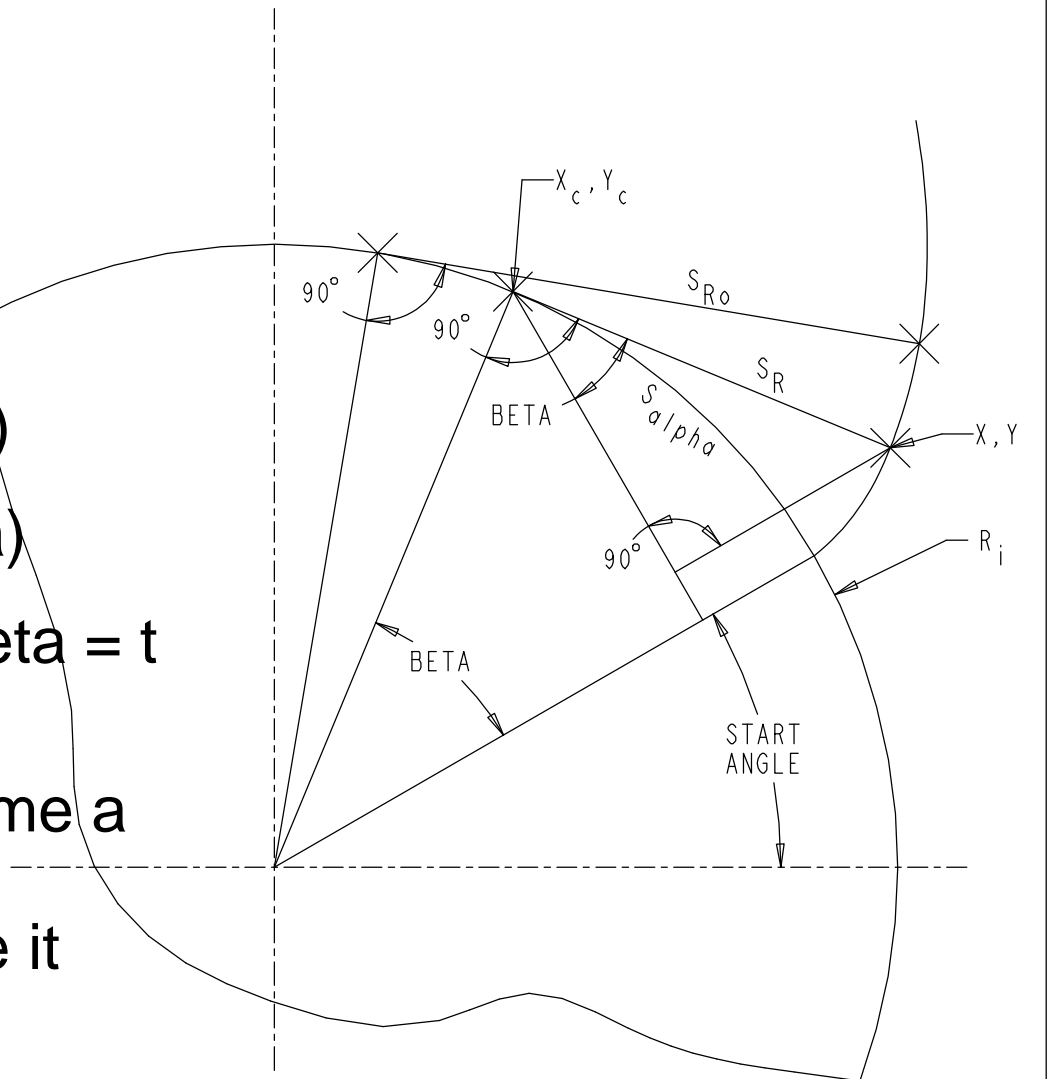
# Cartesian Coordinates



# Deriving the Involute Curve Equations

# Cartesian Coordinates

- $X_c = R_i \cdot \cos(\text{Beta})$
- $Y_c = R_i \cdot \sin(\text{Beta})$
- $S_{\text{alpha}} = S_R = R_i \cdot \text{Beta}$
- $X_R = X_c + S_R \cdot \sin(\text{Beta})$
- $Y_R = Y_c - S_R \cdot \cos(\text{Beta})$
- $R_o = \sqrt{X_R^2 + Y_R^2}$  for  $\text{Beta} = t$
- For now, we will assume a start angle of  $0^\circ$  for simplicity, and remove it from the formulae.



# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- Substituting  $R_i \cdot \cos(\text{Beta})$  for  $X_c$  and simplifying:

$$X_R = X_c + S_R \cdot \sin(\text{Beta})$$

$$X_R = R_i \cdot \cos(\text{Beta}) + \text{Beta} \cdot R_i \cdot \sin(\text{Beta})$$

$$X_R = R_i \cdot [\cos(\text{Beta}) + \text{Beta} \cdot \sin(\text{Beta})]$$

- Substituting  $R_i \cdot \sin(\text{Beta})$  for  $Y_c$  and simplifying:

$$Y_R = Y_c - S_R \cdot \cos(\text{Beta})$$

$$Y_R = R_i \cdot \sin(\text{Beta}) - \text{Beta} \cdot R_i \cdot \cos(\text{Beta})$$

$$Y_R = R_i \cdot [\sin(\text{Beta}) - \text{Beta} \cdot \cos(\text{Beta})]$$

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- Substituting for  $X_R$ ,  $Y_R$ , and simplifying:

$$R_o = v(X_R^2 + Y_R^2)$$

$$R_o = v[(R_i^*(\cos(\text{Beta}) + \text{Beta}*\sin(\text{Beta}))^2 + (R_i^*(\sin(\text{Beta}) - \text{Beta}*\cos(\text{Beta}))^2]$$

$$R_o = v[(R_i^*\cos(\text{Beta}) + R_i^*\text{Beta}*\sin(\text{Beta}))^2 + (R_i^*\sin(\text{Beta}) - R_i^*\text{Beta}*\cos(\text{Beta}))^2]$$

$$R_o = v[R_i^{2*}\cos^2(\text{Beta}) + 2*R_i^{2*}\text{Beta}*\cos(\text{Beta})*\sin(\text{Beta}) + R_i^{2*}\text{Beta}^2*\sin^2(\text{Beta}) + R_i^{2*}\sin^2(\text{Beta}) - 2*R_i^{2*}\sin(\text{Beta})*\cos(\text{Beta}) + R_i^{2*}\text{Beta}^2*\cos^2(\text{Beta})]$$

$$R_o = v[R_i^{2*}(\sin^2(\text{Beta}) + \cos^2(\text{Beta})) + R_i^{2*}\text{Beta}^2*(\sin^2(\text{Beta}) + \cos^2(\text{Beta})) + R_i^2(2*\text{Beta}*\cos(\text{Beta})\sin(\text{Beta}) - 2*\text{Beta}(\cos(\text{Beta})\sin(\text{Beta})))]$$

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- Substituting for  $X_R$ ,  $Y_R$ , and simplifying, cont.:

$$R_o = v[R_i^{2*}(\sin^2(\text{Beta}) + \cos^2(\text{Beta})) + R_i^{2*}\text{Beta}^{2*}(\sin^2(\text{Beta}) + \cos^2(\text{Beta})) + R_i^{2*}(2*\text{Beta}*\cos(\text{Beta})\sin(\text{Beta}) - 2*\text{Beta}(\cos(\text{Beta})\sin(\text{Beta})))]$$

$$R_o = v[R_i^{2*}(1)+R_i^{2*}\text{Beta}^{2*}(1) + (0)]$$

$$R_o = v[R_i^{2*}(1+\text{Beta}^2)]$$

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- Squaring  $R_o$  gives us:

$$R_o^2 = \{v[R_i^2(1+Beta^2)]\}^2$$

$$R_o^2 = R_i^2(1+Beta^2)$$

- Solving the above equation for Beta gives:

$$R_o^2 / R_i^2 = 1+Beta^2$$

$$(R_o^2 / R_i^2) - 1 = Beta^2$$

$$v[(R_o^2 / R_i^2) - 1] = Beta$$

$$\text{or, } Beta = v[(R_o^2 / R_i^2) - 1]$$

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- We need to define a term, alpha, in terms of  $R_i$  and  $R_o$ , so that we can solve the parametric equation for the creation of the datum curve.
- We need to evaluate Beta over its full range (from  $R_i$  to  $R_o$ ) to derive the involute curve, so we multiply by  $t$  in the equation ( $t$  varies linearly from 0 to 1):

$$\alpha = t \cdot \beta$$

$$\alpha = t \cdot \sqrt{(R_o^2 / R_i^2) - 1}$$

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- We need the parametric equations for X and Y in terms of  $R_i$  and alpha. We will use  $X_c$  and  $Y_c$  as the basis, substituting alpha for Beta:

$$X = R_i * [\cos(\alpha * (360/2 * p)) + (\alpha * \sin(\alpha * (360/2 * p)))]$$

$$Y = R_i * [\sin(\alpha * (360/2 * p)) - (\alpha * \cos(\alpha * (360/2 * p)))]$$

(Note that we have converted the angles from radians to degrees by multiplying them by  $360/2 * p$ )

Finally,  $Z = 0$  (since we wish to create a 2-D planar curve!)

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- So, the relation equations used in the creation of the involute profile datum curve will be:

solve

$$\alpha = t \cdot v[(R_o^2 / R_i^2) - 1]$$

for alpha

$$X = R_i [\cos(\alpha \cdot (360/2 \cdot p)) + (\alpha \cdot \sin(\alpha \cdot (360/2 \cdot p)))]$$

$$Y = R_i [\sin(\alpha \cdot (360/2 \cdot p)) - (\alpha \cdot \cos(\alpha \cdot (360/2 \cdot p)))]$$

$$Z = 0$$

Remember that all variables (alpha, Ro, Ri) must be predefined. Since we don't know alpha yet, just preset it initially to a value of 1.

# Deriving the Involute Curve Equations

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## Cartesian Coordinates

- If we include a start angle of some value other than  $0^\circ$ , the equations become:

solve

$$\alpha = t \cdot v[(R_o^2 / R_i^2) - 1]$$

for  $\alpha$

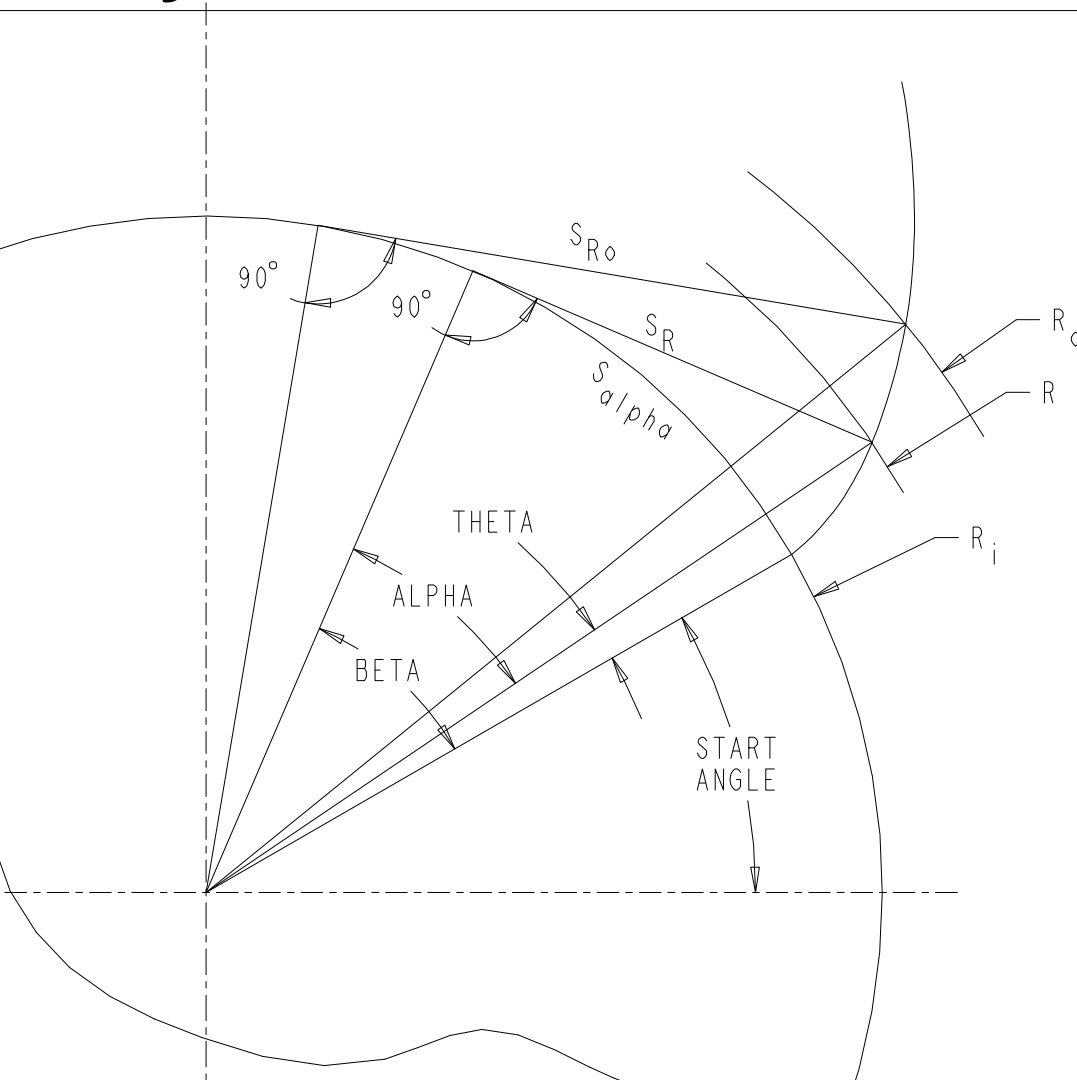
$$X = R_i [\cos(\text{Start Angle} + \alpha \cdot (360/2 \cdot p)) + (\alpha \cdot \sin(\text{Start Angle} + \alpha \cdot (360/2 \cdot p)))]$$

$$Y = R_i [\sin(\text{Start Angle} + \alpha \cdot (360/2 \cdot p)) - (\alpha \cdot \cos(\text{Start Angle} + \alpha \cdot (360/2 \cdot p)))]$$

$$Z = 0$$

# Deriving the Involute Curve Equations -- Terms

## Cylindrical Coordinates

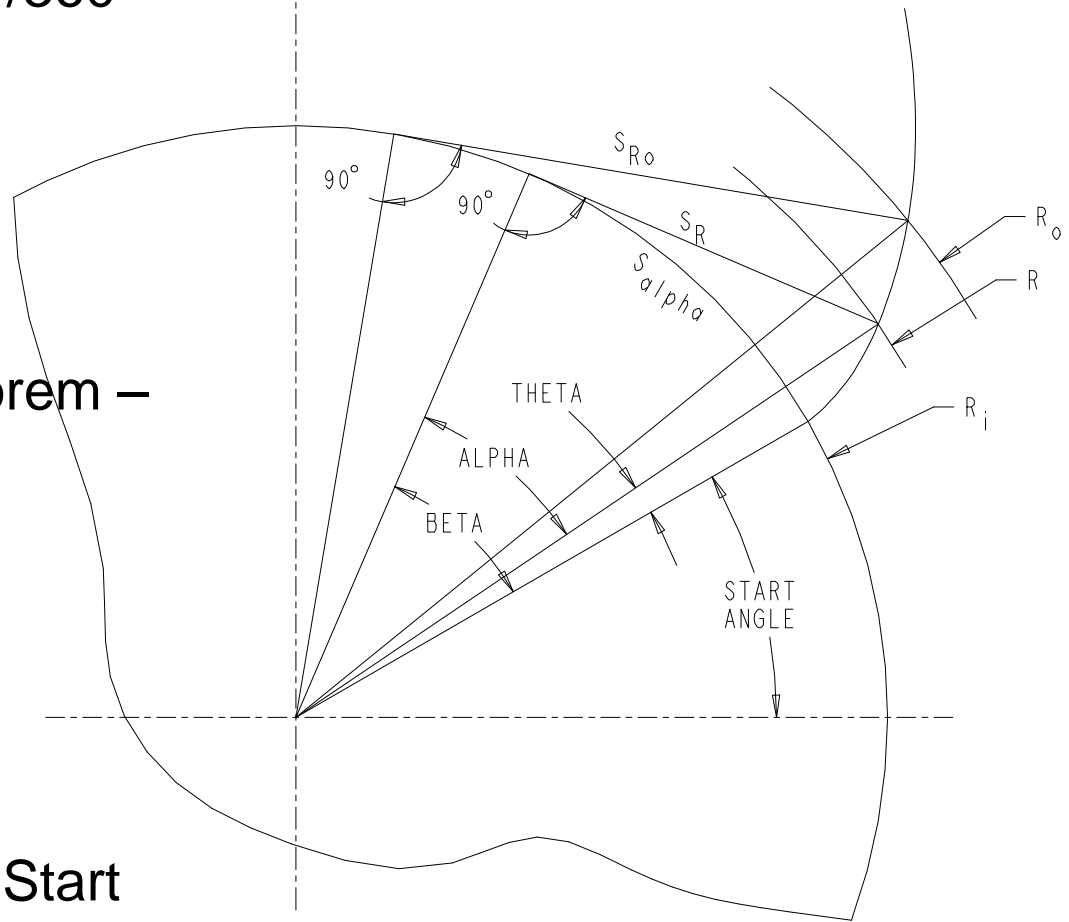


- $R_i$  = Base dia./2
- $R_o$  = Major dia./2
- $R$  = Radius to any point on the involute curve
- $S_{\alpha}$  = arc length from start of the involute to the tangent point
- $S_R$  = tangent line length at any point X, Y on the involute
- $S_{R_o}$  = tangent line length at major diameter on involute
- Beta = angle from start of involute to tangent point on base circle
- Theta = angle from start of involute to any point on the involute between  $R_o$  and  $R_i$
- Alpha = angle from a point on the involute to the tangent point on base circle

# Deriving the Involute Curve Equations

## Cylindrical Coordinates

- $S_{\alpha} = S_R = 2 \cdot \pi \cdot R_i \cdot \text{Beta} / 360$
- $\text{Theta} = \text{Beta} - \text{Alpha}$
- $\text{Alpha} = \text{atan}(S_{\alpha} / R_i) = \text{atan}(S_R / R_i)$
- $S_R = S_{R0} \cdot t$
- By the Pythagorean Theorem –  
 $R = \sqrt{S_R^2 + R_i^2}$
- By observation and Pythagorean Theorem –  
 $S_{R0} = \sqrt{R_o^2 - R_i^2}$
- Again, we are setting the Start Angle to  $0^\circ$  for simplicity.



# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- Substituting  $S_R$  into the equation for  $R$ :

$$R = v[(v(R_o^2 - R_i^2)*t)^2 + R_i^2]$$

- Solving the equations for Alpha and Beta, substituting  $(S_{R0}*t)$  for  $S_R$ :

$$S_R = 2*\pi*R_i*\text{Beta} / 360$$

$$\text{Beta} = (S_{R0}*t*360) / (2*\pi*R_i)$$

and:

$$\text{Alpha} = \text{atan}(S_R/R_i)$$

$$\text{Alpha} = \text{atan} (S_{R0}*t /R_i)$$

# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- Substituting for  $S_{R_o}$  in the equations for Alpha and Beta:

$$\text{Beta} = (v[(R_o^2 - R_i^2)] * t * 360) / (R_i^2 * p)$$

$$\text{Alpha} = \text{atan}((v(R_o^2 - R_i^2) * t) / R_i)$$

# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- Substituting for Beta and Alpha in the equation for Theta:

$$\text{Theta} = (v[(R_o^2 - R_i^2)] * t * 360) / (R_i^2 * p) - \text{atan}((v(R_o^2 - R_i^2) * t) / R_i)$$

- As in the case for the equations for Cartesian involute curves, we still want the curve to be 2-D and planar, so:

$$Z = 0$$

# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- We need to make the equations parametric based on  $R_o$  and  $R_i$  and  $t$  (which varies linearly from 0 to 1), so we create a variable “Gamma”, similar to the alpha term in the Cartesian Coordinate equations:

$$\text{Gamma} = [v(R_o^2 - R_i^2)] * t$$

- Substituting Gamma into the equations for  $R$  and  $\text{Theta}$  gives us:

$$R = v(\text{Gamma}^2 + R_i^2)$$

$$\text{Theta} = \text{Gamma} * 360 / (R_i^2 * \pi) - \text{atan}(\text{Gamma} / R_i)$$

# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- So, the relation equations used in the creation of the involute profile datum curve will be:

solve

$$\text{Gamma} = [v(R_o^2 - R_i^2)] * t$$

for Gamma

$$R = v(\text{Gamma}^2 + R_i^2)$$

$$\text{Theta} = \text{Gamma} * 360 / (R_i^2 * p) - \text{atan}(\text{Gamma} / R_i)$$

$$Z = 0$$

# Deriving the Involute Curve Equations

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## Cylindrical Coordinates

- Note: to account for a start angle  $\neq 0$ , use:

$$\text{Theta} = \text{start angle} + [\text{Gamma} * 360 / (R_i^2 * p) - \text{atan}(\text{Gamma} / R_i)]$$

- Remember to predefine Gamma (preset =1), Ro, and Ri before solving the relations !!! Setting them up as parameters makes your life easier in the long run!

# Another Method for Involute Curve Creation

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## Using the Variable Section Sweep Feature

- Create a cylindrical protrusion with OD = major dia. or minor dia. (depending on whether we are extruding or cutting the feature)
- Create a datum curve at the pitch dia. with CL's at angles Alpha (X-axis to 1<sup>st</sup> endpt. of curve) and Beta (angle between curve endpoints)
- Create a projected datum curve on the back surface of the protrusion
- Begin the process of creating a Variable Section Sweep

# Another Method for Involute Curve Creation

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## Using the Variable Section Sweep Feature

- In the dashboard under references, select the spine trajectory by picking the first datum curve.
- Select the x-dir trajectory by picking the projected curve
- Go to the sketcher, choose the CL Axis of the cylinder and the front and rear surfaces of the cylinder as your sketching references.
- Sketch a line between the front and rear surfaces, parallel to the CL Axis.
- Dimension the line to the CL Axis
- Make sure you choose the option to be a variable section. This allows you to use relations (and input the involute equations!)

# Another Method for Involute Curve Creation

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## Using the Variable Section Sweep Feature

- Similar to the Cylindrical coordinate equations for involute curve creation, the equations for the creation of a Variable Section Sweep uses the trajpar function (instead of the variable t) that varies from 0 to 1. Also, since the VSS runs from the front datum curve to a projected curve (in the Z-dir.), there is no need for an equation to define Z.

Solve

$$\text{Gamma} * (360/2 * \pi) - \text{atan}(\text{Gamma}) = \text{trajpar} * \text{Beta}$$

for Gamma

$$R = R_i * [v1 + (\text{Gamma}^2)]$$

# Another Method for Involute Curve Creation

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## Using the Variable Section Sweep Feature

- Note: Because we are varying the angle (Beta) rather than the radius, we end up solving for Gamma. This equation is similar to the equation for cylindrical coordinate involute curves where we solve for Theta. The radius,  $R$ , is dependent on the angle rather than the outside diameter/radius, hence the use of Gamma instead of  $R_o$ .

# Works Consulted

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- ANSI Standard B92.1-1996, The Society of Automotive Engineers, ©1996.
- CADQuest: Involute Gear Design Tutorial ([www.cadquest.com](http://www.cadquest.com))
- PTC Knowledgebase, Suggested Techniques:
  - Sugg. Tech. for the Creation of an Involute Gear Cutting (3 Methods)
  - Sugg. Tech. for Creating a Cylindrical Gear with Helical Teeth
  - Sugg. Tech. for Creating an Involute Curve
- Machinery's Handbook, Industrial Press Inc., New York, ©1992, 24<sup>th</sup> ed., pp. 1787-2065
- Mechanical Engineering Design, Shigley and Mishke, McGraw Hill Inc., ©1989, 5<sup>th</sup> ed., pp. 527-584
- [Http://www.roymech.co.uk/Useful\\_Tables/Drive/Gears.html](http://www.roymech.co.uk/Useful_Tables/Drive/Gears.html) (by Roy Beardmore, ©2004)

# Live Demos!!!

(Time Permitting!!)

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1. Cylindrical Coordinate Method for a Standard External Spline Tooth
2. Variable Section Sweep Method for a Standard External Spline Tooth
3. Helical and Worm Gear Creation  
Suggestions

The logo features the text "PTCuser" in a white, bold, sans-serif font, with "PTC" in all caps and "user" in a lowercase script font. A small, dark, tilted square is positioned behind the "user" portion of the text. The entire logo is set against a dark circular background with a bright, glowing light source in the upper left corner, creating a lens flare effect.

**PTC***user*

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