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Shaft Bearing Speed

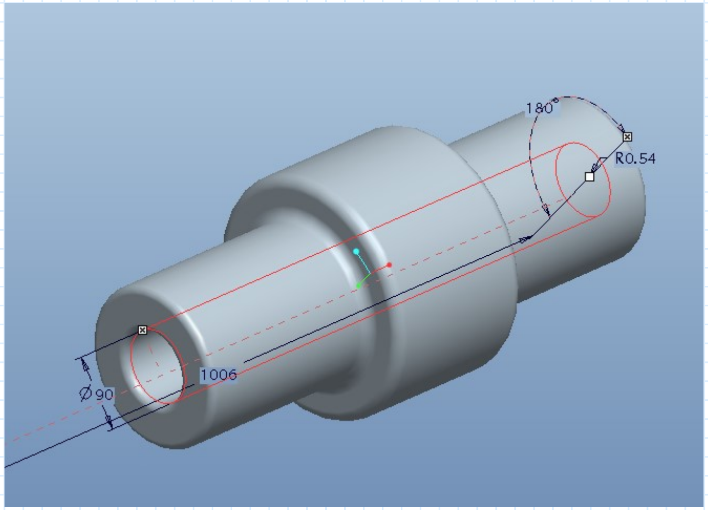
08/17/2021

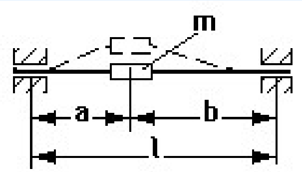
Critical Speed of a Shaft Bearing

Problem: Calculate the critical shaft speed of a shaft bearing with asymmetrical load, as shown in the figure below.

Given: Young's Modulus $E := 2.1\ GPa$

CAD Part





Define your variables:

Outer Diameter $D := 14\ mm$

Inner Diameter $d := 8\ mm$

Mass $M := 0.15\ kg$ $a := 5\ mm$ $b := 5\ mm$

Shaft Length $l := a + b = 10\ mm$

Solution :

Second Moment of Area $I := \frac{\pi}{64} \cdot \left(D^4 - d^4\right) = \left(1.68468 \cdot 10^3\right)\ mm^4$

Stiffness (cq) of 2-bearing shaft, asymmetrical load $c_q := \frac{3 \cdot E \cdot I \cdot l}{a^2 \cdot b^2} = 169.82\ \frac{kg}{ms^2}$

Critical speed (nc) of shaft $n_c := \sqrt{\frac{c_q}{M}} = 321302.872\ rpm$

Math formatting tips:

- **put cursor in whitespace to set default preferences for formatting numerical results**

Label tips:

- **Label styles control how different math elements should APPEAR (Math formatting tab)**
- **Labels control how different elements are USED (set label from Math tab)**

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The final speed of the shaft is $n_c = 321302.872\ rpm$ so this means that.....

$$n_c := \sqrt{\frac{c_q}{M}} \xrightarrow{explicit, ALL} \sqrt{\frac{\frac{169.8156492971427 \cdot kg}{ms^2}}{0.15\ kg}} = 321302.872\ rpm$$

Tips:

- **Use math in text to automatically link/update results in documentation/ explanations**
- **Use keyword "explicit" for variable substition**

$R_{min} := 250000\ rpm$ set a threshold / limit for the rpm

Evaluate shaft speed while iterating over multiple diameter combinations:

$$
\begin{aligned}
n_c(D,d) := \; & \text{for } i \in 0..\text{rows}(D)-1 \\
& \quad \text{for } j \in 0..\text{rows}(d)-1 \\
& \quad\quad I_{i,j} \leftarrow \left(\frac{\pi}{64}\cdot\left(\left(D_i\right)^4-\left(d_j\right)^4\right)\right) \\
& \quad\quad C_{q_{i,j}} \leftarrow \frac{3\cdot E\cdot I_{i,j}\cdot l}{a^2\cdot b^2} \\
& \quad\quad n_{c_{i,j}} \leftarrow \sqrt{\frac{C_{q_{i,j}}}{M}} \\
& \quad\quad check_{i,j} \leftarrow \text{if}\left(n_{c_{i,j}} > R_{min}, \text{“Pass”}, \text{“Fail”}\right) \\
& \begin{bmatrix} n_c \\ check \end{bmatrix}
\end{aligned}
$$

Programming tips:

- **Use "rows" and "cols" functions to auto adjust for changes to data inputs**
- **Use matrix index operator to create output vectors and arrays within the program**
- **store multiple outputs in an nx1 vector on last line**

*Create a spec table to input variants of test cases, in this case arbitrary-sized vectors for **D** and **d**.*

D (mm)	d (mm)
13	6
12	7
11	8
10	9

D --> outer diameter
d --> inner diameter

Input table tips:

- **Column-centric or row-centric approach**
- **Column-centric approach to iterate through inputs (assess all combinations of inputs) with for loops**

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Critical shaft speeds for all combinations of diameters:

$$n_c(D,d)_0 = \begin{bmatrix} 286381 & 280517 & 271277 & 257248 \\ 241818 & 234845 & 223727 & 206492 \\ 200355 & 191881 & 178100 & 155900 \\ 161808 & 151189 & 133264 & 101708 \end{bmatrix} rpm$$

Pass/Fail results for all combinations of diameters:

$$n_c(D,d)_1 = \begin{bmatrix} \text{"Pass"} & \text{"Pass"} & \text{"Pass"} & \text{"Pass"} \\ \text{"Fail"} & \text{"Fail"} & \text{"Fail"} & \text{"Fail"} \\ \text{"Fail"} & \text{"Fail"} & \text{"Fail"} & \text{"Fail"} \\ \text{"Fail"} & \text{"Fail"} & \text{"Fail"} & \text{"Fail"} \end{bmatrix}$$

$$FreqTable := \text{stack}\left(\text{augment}\left(\text{"D\\d [mm]"}, \overrightarrow{\text{concat}\left(\text{num2str}\left(\frac{d}{mm}\right), \text{"mm"}\right)}^{T}\right), \text{augment}\left(\frac{D}{mm}, \frac{n_c(D,d)_0}{rpm}\right)\right)$$

FreqTable =

"D\d [mm]"	"6mm"	"7mm"	"8mm"	"9mm"
13	286380.68	280517.42	271277.38	257248.18
12	241818.26	234845.06	223726.6	206491.65
11	200355.03	191880.87	178100.14	155900.19
10	161808.16	151188.65	133264.4	101708.45

frequencies are in RPM

$$PassFailTable := \text{stack}\left(\text{augment}\left(\text{"D\\d [mm]"}, \overrightarrow{\text{concat}\left(\text{num2str}\left(\frac{d}{mm}\right), \text{"mm"}\right)}^{T}\right), \text{augment}\left(\frac{D}{mm}, n_c(D,d)_1\right)\right)$$

PassFailTable =

"D\d [mm]"	"6mm"	"7mm"	"8mm"	"9mm"
13	"Pass"	"Pass"	"Pass"	"Pass"
12	"Fail"	"Fail"	"Fail"	"Fail"
11	"Fail"	"Fail"	"Fail"	"Fail"
10	"Fail"	"Fail"	"Fail"	"Fail"

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Create aesthetic table with Excel for presenting and sharing results data:

Shaft Bearing Speeds and Pass/Fail Results Conditional Formatting

	6mm	7mm	8mm	9mm		6mm	7mm	8mm	9mm
13mm	286381	280517	271277	257248		Pass	Pass	Pass	Pass
12mm	241818	234845	223727	206492		Fail	Fail	Fail	Fail
11mm	200355	191881	178100	155900		Fail	Fail	Fail	Fail
10mm	161808	151189	133264	101708		Fail	Fail	Fail	Fail

Excel Component tip:

- **Use Excel for conditional formatting and for displaying tabular data with full control over how you want to format it**
- **Activate Excel by double clicking on the component: edit your Excel, close Excel, view updates in Mathcad**

APPENDIX: Repeat the above exercise using *functions*

Given: Young's Modulus $E := 2.1\ GPa$

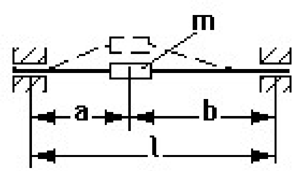
Define your variables:

Outer Diameter $D := 10\ mm$

Inner Diameter $d := 5\ mm$

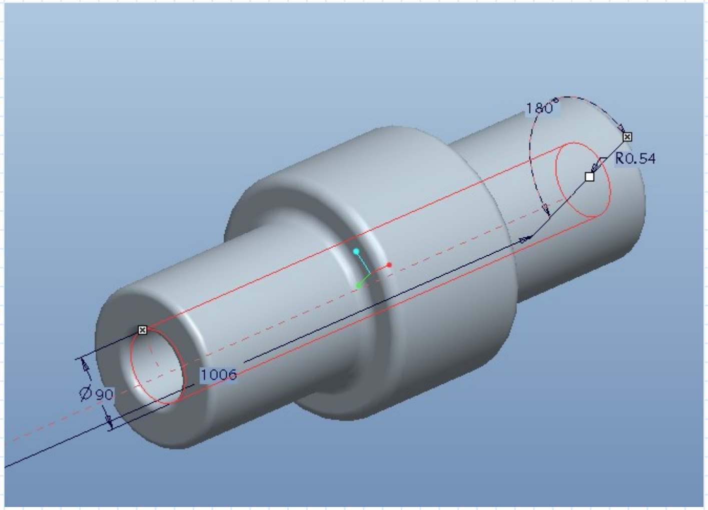
Mass $M := 0.15\ kg$

Shaft Length $l := a + b$



$a := 5\ mm$ $b := 5\ mm$

Creo Part



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Solution :

Second Moment of Area $I(D,d) := \frac{\pi}{64} \cdot \left(D^4 - d^4\right)$

Stiffness cq of 2-bearing shaft, asymmetrical load $c_q(D,d) := \frac{3 \cdot E \cdot I(D,d) \cdot l}{a^2 \cdot b^2}$

Critical speed nc of shaft $n_c(D,d) := \sqrt{\frac{c_q(D,d)}{M}}$

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QUICK PLOT (Plots tab):

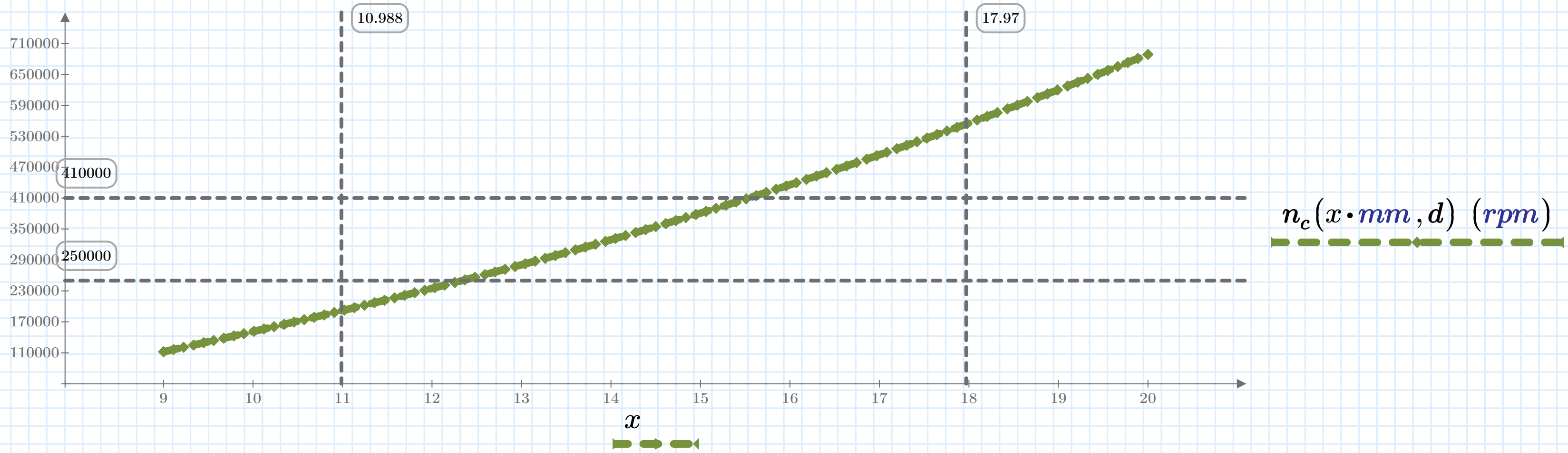
$speed\_max := 410000$

Quick Plots tips:

- **Use markers to mark thresholds, boundary conditions, operating bandwidth**
- **Control marker placement with variables (or with your mouse)**
- **In this case, a combo box is used to select the inner diameter for the plot**
- **marker values must be unitless values!**

Shaft Speeds as a function of varying Outer Diameter for a *Given Inner Diameter*

$d :=$ ID: 7



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