PLANETARY GEARS: GEAR RATIO CALCULATOR

Designate the number of teeth on the sun gear, planet gear and planet carrier as N_S, N_P and N_R respectively. The angular velocity of the sun gear, planet gear and planet carrier is designated as ω_S, ω_P and ω_C respectively, when the ring gear is stationary, and has a velocity of 0.

	Stationary Ring
Sun	ω_S
Planet	ω_P
Carrier	ω_C
Ring	0

Table 1

The difficulty arises on account of not being able to predict the carrier velocity directly using standard gearing ratios since the planet carrier along with planet gear is a moving axis system.

So, we can use standard gear relations, only if the planet carrier is rendered stationary. In order to do this, subtract the angular velocity of planet carrier, ω_C from each element of the epicyclic assembly as shown below:

	Stationary Planet Carrier
Sun	$\omega_S - \omega_C$
Planet	$\omega_P - \omega_C$
Carrier	0
Ring	$-\omega_C$

Table 2

Likewise, for the configuration where the sun gear is stationary, its angular velocity ω_S is to be subtracted from table showing initial assignment to obtain velocity of corresponding gear or element.

	Stationary Sun
Sun	0
Planet	$\omega_P - \omega_S$
Carrier	$\omega_C - \omega_S$
Ring	$-\omega_S$

Table 3

The three tables above show the angular velocity for each element based on which configuration is used (or which part is made stationary) viz. the Ring gear, or the Planet Carrier or the Sun Gear.

Next for the configuration where the Planet Carrier is held stationary, and if we rotate the sun gear by one rotation in say counterclockwise direction, the planet gear would rotate by $\frac{N_S}{N_P}$ rotations in clockwise direction. Designating counter-clockwise rotations as positive and clockwise rotations as negative.

Similarly, one rotation of planet gear in anticlockwise direction would produce $\frac{N_R}{N_P}$ rotation of ring gear in anticlockwise direction.

Angular Speed of Gear 1 x number of teeth of Gear 1 = Angular Speed of Gear 2 x number of teeth of Gear 2

If we consider sun gear as input gear, in order to find gear ratio, its angular velocity can be considered as 1, meaning we can designate $\omega_S - \omega_C = 1$.

These 3 equations can be solved to get ω_S, ω_P and ω_C, the angular velocities when ring gear is fixed.

$$\omega\left(N_S, N_P, N_R\right) := \begin{bmatrix} \frac{\left(\omega_S - \omega_C\right)}{\omega_P - \omega_C} = \frac{-N_P}{N_S} \\ \frac{\left(\omega_P - \omega_C\right)}{-\omega_C} = \frac{N_R}{N_P} \\ \omega_S - \omega_C = 1 \end{bmatrix} \xrightarrow{solve, \omega_S, \omega_P, \omega_C} \begin{bmatrix} \frac{N_S + N_R}{N_R} & \frac{\left(-N_R + N_P\right) \cdot N_S}{N_P \cdot N_R} & \frac{N_S}{N_R} \end{bmatrix}$$

$$\omega_S\left(N_S, N_P, N_R\right) := \frac{N_S + N_R}{N_R}$$

$$\omega_P\left(N_S, N_P, N_R\right) := \frac{\left(-N_R + N_P\right) \cdot N_S}{N_P \cdot N_R}$$

$$\omega_C\left(N_S, N_P, N_R\right) := \frac{N_S}{N_R}$$

These three equations can be plugged in the relevant tables to get the angular velocities of each gear and for each of the three configurations.

Next we define the input values of number of teeth on sun and planet gear. The number of teeth on ring gear is derived from these two values

$$N_S := 22$$

$$N_P := 18$$

$$N_R := N_S + 2 \cdot N_P$$

The table below shows the angular velocities of each of the planetary gear element for each of the 3 configurations. Each row represents a particular configuration of the planetary geartrain. So the first row shows velocities of all the gear elements when the Ring gear is held stationary and so on.

Stationary	*Sun*	*Planet*	*Carrier*	*Ring*
"Stationary Ring"	$\omega_S\left(N_S, N_P, N_R\right)$	$\omega_P\left(N_S, N_P, N_R\right)$	$\omega_C\left(N_S, N_P, N_R\right)$	0
"Stationary Carrier"	$\omega_S\left(N_S, N_P, N_R\right) - \omega_C\left(N_S, N_P, N_R\right)$	$\omega_P\left(N_S, N_P, N_R\right) - \omega_C\left(N_S, N_P, N_R\right)$	0	$-\omega_C\left(N_S, N_P, N_R\right)$
"Stationary Sun"	0	$\omega_P\left(N_S, N_P, N_R\right) - \omega_S\left(N_S, N_P, N_R\right)$	$\omega_C\left(N_S, N_P, N_R\right) - \omega_S\left(N_S, N_P, N_R\right)$	$-\omega_S\left(N_S, N_P, N_R\right)$

The table above shows angular velocities. But since we want to find gear ratios or number of turns executed by each gear for the required 3 configurations, we divide the entire row with the angular velocity corresponding to the input gear.

So when the Ring gear is held stationary, and the sun gear is input gear, so we want to find the number of turns executed by the planet and the planet carrier for one rotation of sun gear.

For this we divide the entire row with the velocity of sun gear in that row which in this configuration is ω_S.

Likewise when the Planet carrier is held stationary, and the sun gear is input gear, we divide each of the velocities in the entire corresponding row by $\omega_S - \omega_C$ because that is the velocity of sun gear in that configuration.

Finally, in the third configuration, the sun gear is held stationary and the planet carrier is the input. So in this case we divide all velocities in the third row by the velocity of planet carrier that is $\omega_C - \omega_S$. In this way our table is updated and looks as given below:

Stationary	*Sun*	*Planet*	*Carrier*	*Ring*
"Stationary Ring"	1	$\dfrac{\omega_P(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R)}$	$\dfrac{\omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R)}$	0
"Stationary Carrier"	1	$\dfrac{\omega_P(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)}$	0	$\dfrac{-\omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)}$
"Stationary Sun"	0	$\dfrac{\omega_P(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)}{\omega_C(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)}$	1	$\dfrac{-\omega_S(N_S, N_P, N_R)}{\omega_C(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)}$

The above table gives us the number of turns of each of the gears and the planet carrier for the 3 scenarios or configurations.

Enter the number of teeth on the sun and planet gears and select the desired configuration in the combo box below to see the gearing ratios.

$Config := $ Stationary Gear: Carrier

$$Soln := \left\| \begin{array}{l} \text{if } Config = 1 \\ \quad \left\| \begin{array}{l} sun \leftarrow 1 \\ planet \leftarrow \dfrac{\omega_P(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R)} \\ carrier \leftarrow \dfrac{\omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R)} \\ ring \leftarrow 0 \\ [sun \ \ planet \ \ carrier \ \ ring] \end{array} \right. \\ \text{else if } Config = 2 \\ \quad \left\| \begin{array}{l} sun \leftarrow 1 \\ planet \leftarrow \dfrac{\omega_P(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)} \\ carrier \leftarrow 0 \\ ring \leftarrow \dfrac{-\omega_C(N_S, N_P, N_R)}{\omega_S(N_S, N_P, N_R) - \omega_C(N_S, N_P, N_R)} \\ [sun \ \ planet \ \ carrier \ \ ring] \end{array} \right. \\ \text{else} \\ \quad \left\| \begin{array}{l} sun \leftarrow 0 \\ planet \leftarrow \dfrac{\omega_P(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)}{\omega_C(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)} \\ carrier \leftarrow 1 \\ ring \leftarrow \dfrac{-\omega_S(N_S, N_P, N_R)}{\omega_C(N_S, N_P, N_R) - \omega_S(N_S, N_P, N_R)} \\ [sun \ \ planet \ \ carrier \ \ ring] \end{array} \right. \end{array} \right.$$

$$REV_{SUN} := Soln^{\langle 0 \rangle} \rightarrow [1]$$

$$REV_{PLANET} := Soln^{\langle 1 \rangle} \rightarrow \left[-\frac{11}{9} \right]$$

$$REV_{CARRIER} := Soln^{\langle 2 \rangle} \rightarrow [0]$$

$$REV_{RING} := Soln^{\langle 3 \rangle} \rightarrow \left[-\frac{11}{29} \right]$$