Blackbody Radiation ©

by

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Introduction-
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Blackbody radiation is a common phenomenon that probably is familiar to you. When you see stars of different colors, when you observe an electric heating coil on a stove turn red, or when you observe a lightbulb, you are observing blackbody radiation. A blackbody is a device that converts heat into radiant energy. Heating an object to different temperatures causes that object to radiate energy of different wavelengths and therefore, different colors.

In the late 1800's, Stefan showed that the total energy emitted from a blackbody depended on T^4. Wein demonstrated that the wavelength of maximum emittance was inversely proportional to T. Physicists tried to describe the radiation distribution that was produced by a blackbody and thus to explain the observations of Stefan and Wien. Rayleigh and Jeans assumed that an ideal blackbody (one that converts all energy into radiation) contained a large number of electromagnetic oscillators that vibrate with frequency, ν, and these vibrations were thermally activated at temperature, T. They assumed that the energy was equally distributed among all oscillators in the blackbody. Using classical physics, they were able to derive an expression for the radiation density:

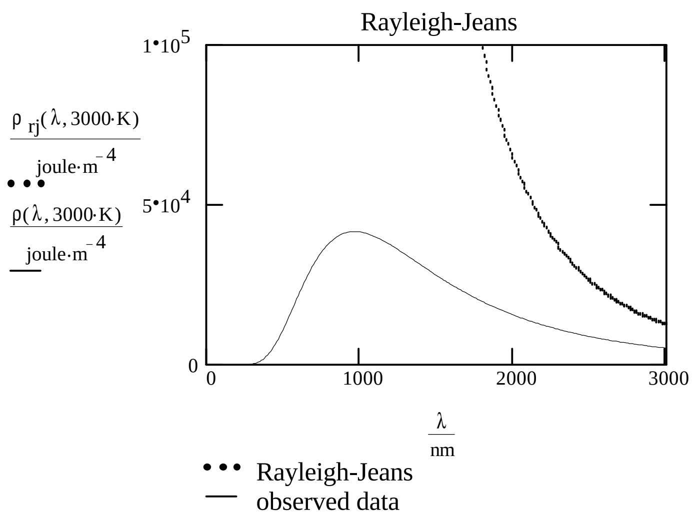
$$\rho_{rj}(\lambda, T) := \frac{8 \cdot \pi \cdot k \cdot T}{\lambda^4}$$

Unfortunately, this expression does not lead to the observed distribution of radiation. In fact, this function blows up at short wavelengths, an effect known as the ultraviolet catastrophe (see first graph). Obviously, no system can have infinite energy density so the Rayleigh-Jeans theory was incorrect. To see this, define some physical constants and plot the radiation density.

Some constants $k \equiv 1.38 \cdot 10^{-23} \cdot joule \cdot K^{-1}$ $h \equiv 6.62 \cdot 10^{-34} \cdot joule \cdot sec$ $c \equiv 3.00 \cdot 10^{8} \cdot m \cdot sec^{-1}$

$nm := 10^{-9} \cdot m$

wavelength range (can be modified) $\lambda := 100 \cdot 10^{-9} \cdot m, 110 \cdot 10^{-9} \cdot m .. 3000 \cdot 10^{-9} \cdot m$



Max Planck assumed that the energy of the oscillators in the blackbody was not continuous, but quantized. He computed the average energy of the oscillators using this assumption. The energy density, dU in the wavelength range dλ is given by the expression:

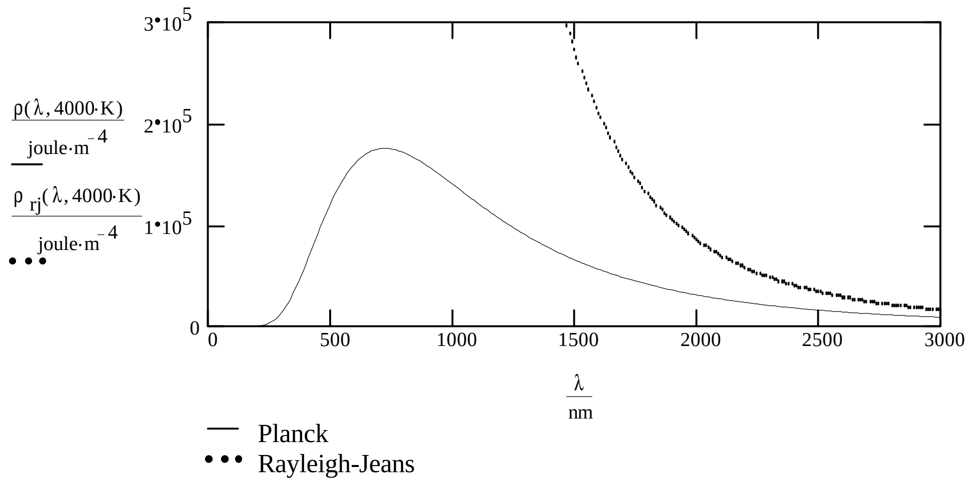
$$dU := \left[\frac{8 \cdot \pi \cdot h \cdot c}{\lambda^5} \cdot \left(\frac{1}{e^{\frac{h \cdot c}{\lambda \cdot k \cdot T}} - 1} \right) \right] \cdot d\lambda$$

The expression in brackets is the Planck distribution and is symbolized by ρ(λ,T). This distribution gives us the radiation density at a given temperature and wavelength.

$$\rho(\lambda, T) := \frac{8 \cdot \pi \cdot h \cdot c}{\lambda^5} \cdot \left(\frac{1}{e^{\frac{h \cdot c}{\lambda \cdot k \cdot T}} - 1} \right)$$

Compare the graph below to the graph above. Note that the Planck distribution is identical to the observed blackbody curve.

Comparison of the Planck and Rayleigh-Jeans functions



Notice that the Planck energy density reaches a maximum and then decays. There is no ultraviolet catastrophe and the predicted curve fits the experimental results.

Exercise 1. In the space below, produce graphs of the energy density at different temperatures (use T > 3000K) and compare them. Rescale the graph to observe the entire curves.
What happens to the position of the maximum as you change the temperature?
Are your results consistent with Wien's law?

Exercise 2. Consider the sun which has a temperature of 10000K.
What color might it appear?
What would be the temperature of a star that appears blue?
Estimate the temperature of a yellow flame in a fireplace.

Experimentally, the total emitted radiation of a blackbody is given by Stefan's Law $W = \sigma T^4$
Where σ, the Stefan-Boltzmann constant, has the value, 5.67 * 10^{-8} . We can derive Stefan's Law by calculating the total emitted radiation. The number of photons passing through a given area can be shown from the Kinetic Theory to be (1/4)cN; each photon contributes c/4. Therefore $W_\lambda = c\rho(\lambda)/4$. Integrating the Planck radiation density over all wavelengths yields:

assume

Note: if a problem occurs with the evaluation of the integral please retype the assume command

$$T>0, h>0, k>0, c>0$$

$$\frac{c}{4} \cdot \int_0^\infty \frac{8 \cdot \pi \cdot h \cdot c}{\lambda^5} \cdot \left[\frac{1}{e^{\left(\frac{h \cdot c}{\lambda \cdot k \cdot T}\right)} - 1} \right] d\lambda \rightarrow \frac{2}{\left(15 \cdot c^2\right)} \cdot \frac{\pi^5}{h^3} \cdot k^4 \cdot T^4$$

Note that the integral gives the T^4 dependence and the numerical result is equal to the Stefan-Boltzmann constant

$$\frac{2}{\left(15 \cdot c^2\right)} \cdot \frac{\pi^5}{h^3} \cdot k^4 = 5.667 \cdot 10^{-8} \cdot m^{-2} \cdot K^{-4} \cdot watt$$

the Stefan-Boltzmann constant

So we can calculate the total emittance of a blackbody at any temperature by using Stefan's Law.

$$T := 3000 \cdot K \qquad \sigma := 5.67 \cdot 10^{-8} \cdot watt \cdot m^{-2} \cdot K^{-4}$$

$$W := \sigma \cdot T^4 \qquad W = 4.593 \cdot 10^{6} \cdot m^{-2} \cdot watt$$

Exercise 3. Calculate the total emittance at 4000K and 10000K.

Next we can ask what fraction of the total radiation density occurs in a given wavelength interval? Assume a temperature of 5000K.

First calculate the total density from Stefan's Law......

$$T := 5000 \cdot K$$

$$W := \sigma \cdot T^4 \cdot K^4 \qquad W = 3.544 \cdot 10^7 \cdot m^{-2} \cdot K^4 \cdot watt$$

Then calculate the density in the region of interest, say, 200 nm to 600 nm

$$\int_{200 \cdot nm}^{600 \cdot nm} \frac{8 \cdot \pi \cdot h \cdot c^2}{4 \cdot \lambda^5} \cdot \left[\frac{1}{e^{\left(\frac{h \cdot c}{\lambda \cdot k \cdot T}\right)} - 1} \right] d\lambda = 9.661 \cdot 10^6 \cdot m^{-2} \cdot watt$$

so the percentage is $\dfrac{.966 \cdot 10^7}{3.544 \cdot 10^7} = 27.257 \cdot \%$

Exercise 4. Solar radiation reaching the earth has a 'color temperature' of 3200K. Calculate the fraction of energy in the visible, UV, and IR regions. The color temperature of an object is the temperature to which a blackbody would be heated to produce the correct radiation density. Consider the visible range to be 350 to 750nm, UV to be 180 to 350nm, and IR to be 750 to 10^6 nm.

Exercise 5. Using the equation for $\rho(\lambda, T)$, derive Wein's law. Remember that you are finding the maximum in the function. The resulting equation cannot be solved analytically, so you will have to use successive approximations to obtain the result.

$$\rho(\lambda, T) := \frac{8\cdot\pi\cdot h\cdot c}{\lambda^5}\cdot\left[\frac{1}{e^{\left(\frac{h\cdot c}{\lambda\cdot k\cdot T}\right)} - 1}\right]$$

This equation permits us to plot points that are equivalent to experimental points in the graph to the left. Planck derived this equation as we mention below the graph.