PTC®

Parametric Equations with Mathcad Prime™
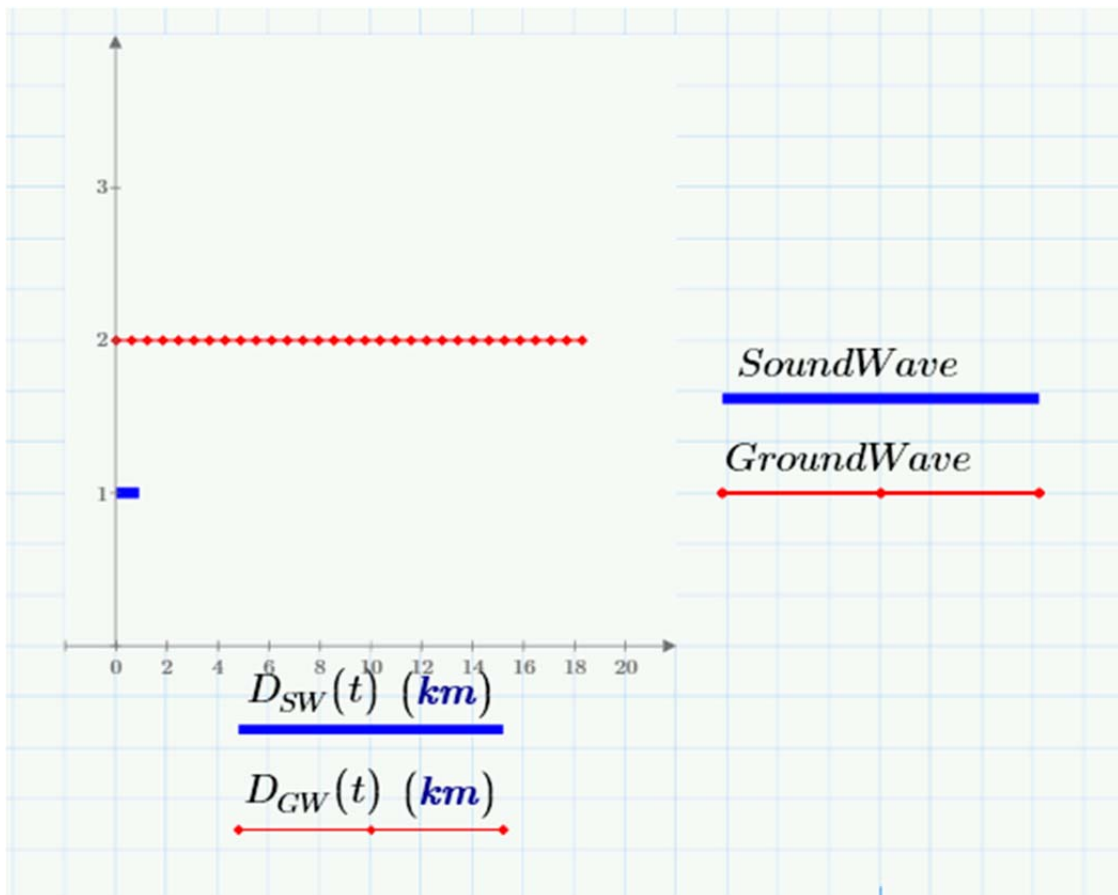
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Acknowledgments

The authors gratefully acknowledge the support of the Mathcad Business Unit and the PTC Academic Program.

Questions or Corrections

Please direct inquiries or questions regarding the contents of this tutorial to the Mathcad Academic Program at MathcadEducation@ptc.com. Suggestions for improvement or further development will be gladly accepted.

Using Parametric Models

Parametric equations are a useful way of modeling many real world situations because they often allow problem solvers to present a richer, more concrete description of the problem context. In this tutorial, you will learn to use Mathcad Prime to generate parametric equations to model the following problem situation and answer questions related to the context.

African elephants are capable of generating seismic waves that travel as far as 32 km when they stomp their feet. Some researchers believe that when elephants stomp their feet, they are communicating with other elephants over long distances.

An elephant stomp generates both airborne sound waves and seismic vibrations in the ground. Sound waves travel through the air at approximately 0.3 km/s. Seismic waves close to the Earth's crust travel at a speed of 6.1 km/s.

(Problem adapted from Discovering Advanced Algebra, Key Curriculum Press, p. 431)
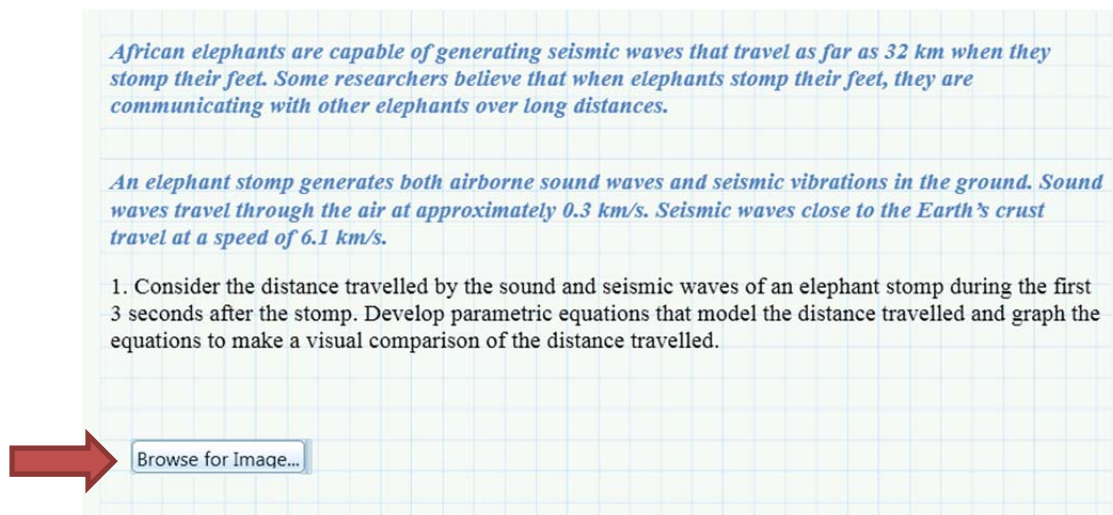
Tutorial Activities

1. Consider the distance travelled by the sound and seismic waves of an elephant stomp during the first 3 seconds after the stomp. Develop parametric equations that model the distance travelled and graph the equations to make a visual comparison of the distance travelled.

2. Use your equations to determine the distance travelled by the seismic wave created by an elephant stomp in 3 seconds. Then, determine how long it would take the corresponding sound wave to travel the same distance.

3. Suppose that you are a researcher studying elephant communication in Africa. Your equipment detects the sound wave in the air from an elephant stomp 10 seconds after detecting the seismic vibration in the ground. Use this information to determine your distance from the elephant that generated the waves.

The pages that follow contain step-by-step instructions for completing each of the tutorial activities.

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Follow these step-by-step instructions to learn how to use Mathcad Prime to create models using parametric equations:

1. We will begin by opening up a Mathcad Prime (.mxcd) document containing the problem description. Open up the **Elephant Problem Statement.mcdx** file from the Using Parametric Models Tutorial Folder.

2. Now we will insert an image to illustrate the problem. Place your cursor on the Mathcad worksheet so that it is beneath the Text Block containing the problem statement. Then, click the Math tab to make sure that the Math Ribbon is displayed. On the left edge of the Math Ribbon, find the Image icon and left-click on it. A **Browse for Image…** button will appear as shown below.

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1. Consider the distance travelled by the sound and seismic waves of an elephant stomp during the first 3 seconds after the stomp. Develop parametric equations that model the distance travelled and graph the equations to make a visual comparison of the distance travelled.

Browse for Image...

3. Left-click on the **Browse for Image…** button and navigate to find the location of the file **Elephant.jpg** in the Tutorial Folder. Click Open and the image of the Elephant will appear in the Mathcad worksheet. You can resize the image by dragging the midpoint of the right or bottom edges of the image region.

4. Left-Click in the space to the right of the image of the Elephant to place the cursor on the worksheet. We will now place a Text Box in this region. Left-Click on the **Text Box** icon on the Math Ribbon. Type the text "**Given Information**" in the Text Box. You can use your mouse to place and resize the Text Box after you enter the text.

Given Information

All regions on a Mathcad worksheet are floating so they can be resized and moved using mouse select and drag operations.

5. Now we will create **Math Regions** stating the information given in the problem statement. We will use Mathcad **assignment statements** so that each value can be used by referencing the variable name later in the worksheet. First, we will create a **Range Variable** describing the time frame for the problem. We will use the unit seconds (*s*) in the definition of the Range Variable.

Range Variables

Range Variables are a special variable type defined by a starting value, an increment, and maximum value for a range of numbers.

The syntax for typing a range variable is:

VarN:Start,NextValue…Maximum

Example:

Days:1,2…7 will define a range of values from 1 to 7 counting by ones.

On the Mathcad worksheet **Left-Click** in the space below the text Given Information

Type: **t:0s,**

Mathcad Prime will provide placeholders for the next and last values in the Range

In the first placeholder type: **0.1s**

In the second placeholder type: **3s <Enter>**

The correctly defined range variable will appear like this:

Given Information

$t := 0\ s, 0.1\ s .. 3\ s$

By default, units appear in blue in Mathcad Prime.
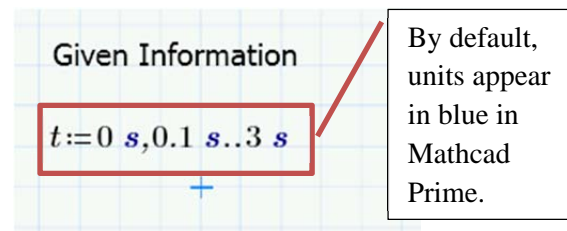
6. We will now complete two standard variable assignments in Mathcad. The symbol for assignment in Mathcad Prime is ":=", which has the keystroke ":". Complete the keystrokes below to assign the number 1 to the variable Soundwave and the number 2 to the variable Groundwave.

SoundWave:1<Enter>

GroundWave:2<Enter>

Given Information

$t := 0\ s, 0.1\ s .. 3\ s$

$SoundWave := 1$
$GroundWave := 2$
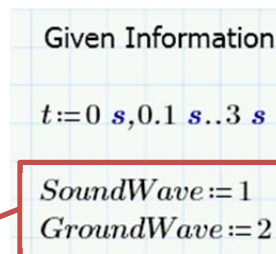
Variable names may not contain spaces. When correctly formatted, they appear in italics.

7. Next we will define two functions describing the motion of the Airborne and Ground waves produced by the elephant stomp. We will use the Range Variable, t, in the function definitions. We will also use subscripts in the function name definitions. Here is an image of the function definitions:

Function name with subscript standing for distance travelled by the sound wave.

When typing in a Math Region, click on the **Subscript** icon on the Math Ribbon to type a subscript as part of a variable or function name. Or you can type **<Ctrl>-** as shown below

Velocity with units km/second

$$D_{SW}(t) := 0.3 \frac{km}{s} \cdot t$$

$$D_{GW}(t) := 6.1 \frac{km}{s} \cdot t$$

The range variable t is stated as a parameter of the function and used in the definition of the function.

Keystrokes:

D<ctrl>-SW(t<right arrow>:0.3km/s<spacebar><spacebar><spacebar>*t<enter>

D<ctrl>-GW(t<rightarrow>:6.1km/s<spacebar><spacebar><spacebar>*t<enter>

Notes:

Mathcad uses standard math notation to express mathematical expressions and formulas. The <spacebar> and the <rightarrow> and <leftarrow> keys are important tools for formatting.

<spacebar> group selects items within an expression by cycling through the terms as you hit the <spacebar> one or more times

<rightarrow> moves the insertion point to the right

<leftarrow> moves the insertion point to the left

Your Mathcad Worksheet should appear something like this:

1. Consider the distance travelled by the sound and seismic waves of an elephant stomp during the first 3 seconds after the stomp. Develop parametric equations that model the distance travelled and graph the equations to make a visual comparison of the distance travelled.
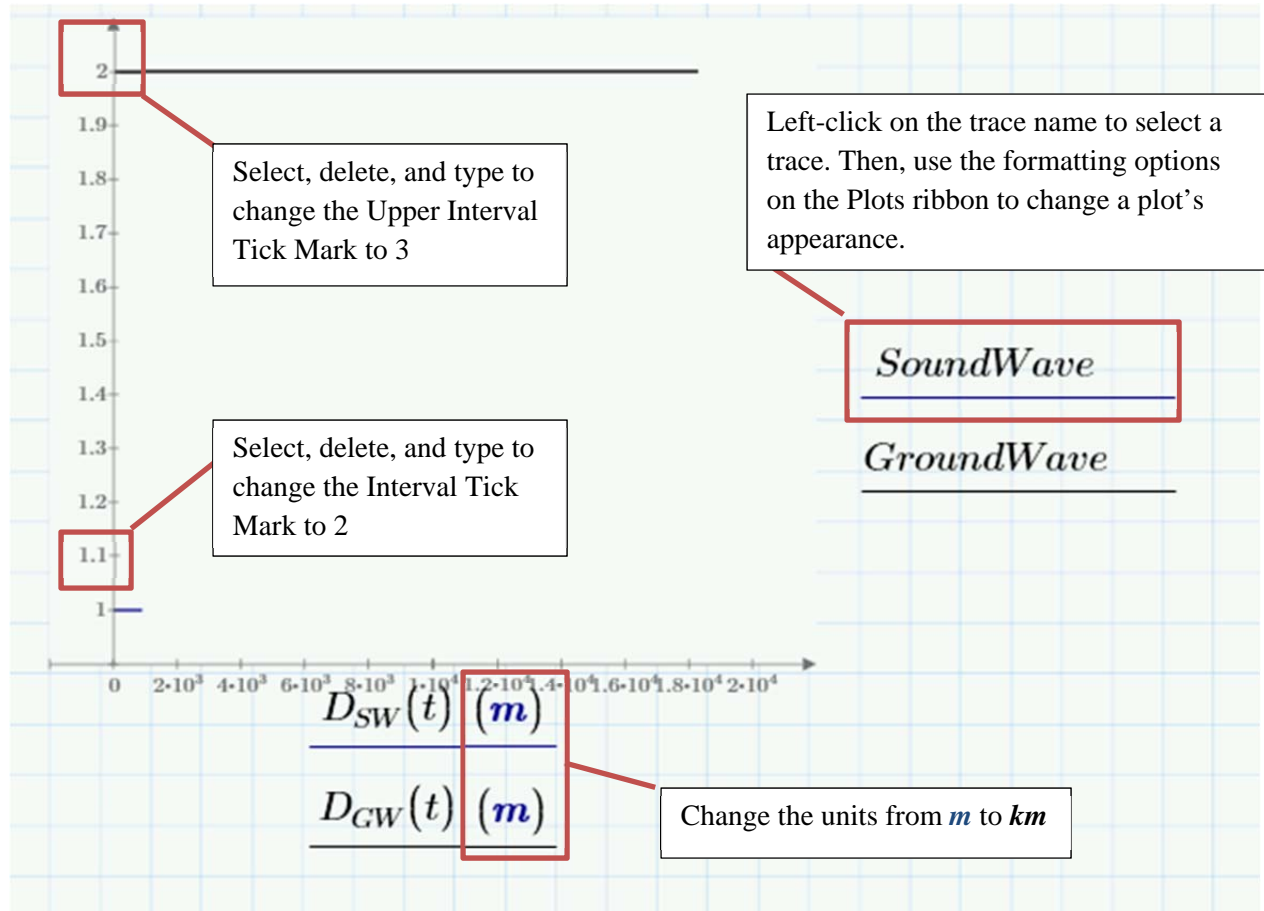

Given Information

$$t := 0 \ s, 0.1 \ s .. 3 \ s$$

$SoundWave := 1$

$GroundWave := 2$

$$D_{SW}(t) := 0.3 \ \frac{km}{s} \cdot t$$

$$D_{GW}(t) := 6.1 \ \frac{km}{s} \cdot t$$

8. We can now use the definitions we have created to graph the two parametric functions D_{SW} and D_{GW}. These functions describe the distance travelled by the Sound Wave and the Ground Wave, respectively. We will use the SoundWave and GroundWave variables as "dummy variables" to give clear labels on our plot. Follow these instructions to create the plot below:

- o **Left-Click on the Plots tab to call up the Plots Ribbon**
- o **Left-Click on the Insert Plot Icon**
- o **Left-Click on X-Y Plot**
- o A plot will appear with placeholders on the right side (y-axis) and the bottom (x-axis) of the graph. We will enter the "dummy variables" on the right and the function names on the bottom.
- o **Click in the right-side placeholder.** Type the variable name: **SoundWave**
- o With the right-side placeholder highlighted, click the **Add Trace** icon on the Plots Ribbon
- o In the new placeholder, type the variable name: **GroundWave**

- o **Left-Click in the placeholder at the bottom of the plot**. Type: **D<ctrl>-SW(t<enter>**
- o With the bottom placeholder highlighted, click the **Add Trace** icon on the Plots Ribbon
- o **Left-Click in the placeholder at the bottom of the plot**. Type: **D<ctrl>-GW(t<enter>**
- o **Left-Click outside of the Graph Region to see the plot**. Use your mouse to resize and relocate the graph as necessary.

Here is my initial plot illustrating the distance travelled by each wave in 3 seconds:

Select, delete, and type to change the Upper Interval Tick Mark to 3

Left-click on the trace name to select a trace. Then, use the formatting options on the Plots ribbon to change a plot's appearance.

SoundWave

GroundWave

Select, delete, and type to change the Interval Tick Mark to 2

$$\underline{D_{SW}(t)}\ \underline{(m)}$$

$$\underline{D_{GW}(t)}\ \underline{(m)}$$
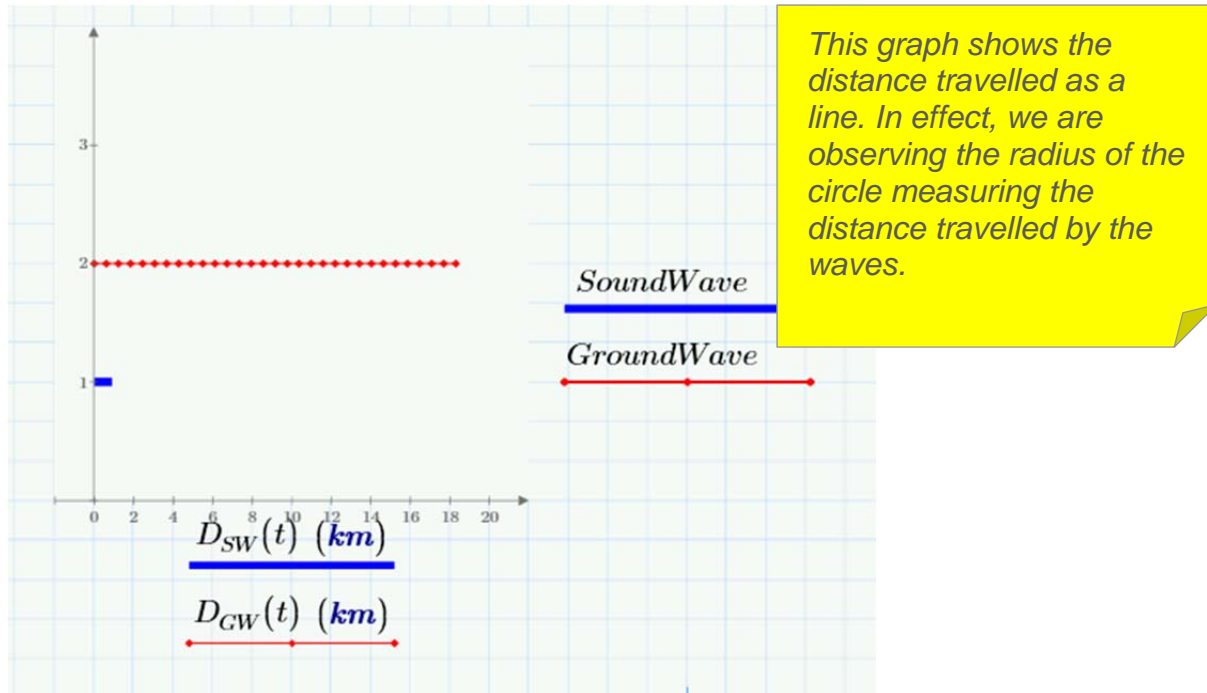
Change the units from *m* to *km*

Mathcad XY Plots
- Mathcad auto-formats the plot based on the values assigned to the variables
- A plot can be rescaled by changing maximum and increment values on the plot axes
- The Plots Ribbon contains formatting tools for changing the appearance of the plot

9. Follow these steps to change the formatting of the plot:

o Change the Upper Limit tick mark on the y-axis to 3 to set the maximum value

o Change the Interval Tick Mark to 2 to make the increment 1 unit

o Change the units on the x-axis from meters to kilometers by deleting the unit *m* for each function and replacing it with *km*. Note how the changes in units on the x-axis.

o Change the formatting of the GroundWave plot to Red and add a symbol

o Change the thickness of the SoundWave Plot

10. The plot enables us to compare the distance traveled by each wave in 3 seconds. On this Plot the "dummy variables" are effectively labels for the position of each wave. The data that we are interested in to answer our questions should be read off the x-axis. We can see that the ground wave is travelling much farther than the sound wave in 3 seconds.

This graph shows the distance travelled as a line. In effect, we are observing the radius of the circle measuring the distance travelled by the waves.

11. The second task for this tutorial is to use the equations for D_{SW} and D_{GW} to determine exactly how far the ground wave travels in 3 seconds, and, how long it will take the sound wave to travel the same distance. We will use this exercise to demonstrate the functions of Mathcad's different equality symbols.

Type the following :

D<ctrl>-GW(3s=

This is an evaluation statement resulting in:

$$D_{GW}(3\ s) = 18.3\ km$$

D<ctrl>-SW(t)<ctrl>=18.3km<enter>

This is an equivalence statement resulting in:

$$D_{SW}(t) = 18.3\ km$$

Compare these two statements:

$$D_{GW}(3\ s) = 18.3\ km$$

$$D_{SW}(t) = 18.3\ km$$

The first line is an evaluation statement. Mathcad has evaluated the function D_{GW} for the given input, 3 seconds determining that the function is equal to 18.3 km at this point in time. The second line uses Mathcad's Boolean Equivalence symbol "=" that has the keystroke "<ctrl>=". In this case we are stating that we need the value of the variable t when the value of $D_{SW}(t)$ is the same as 18.3 km.

Now type this Mathcad Boolean statement:

0.3km/s<spacebar> <spacebar><spacebar>*t<ctrl>=18.3km<enter>

$$0.3\ \frac{km}{s} \cdot t = 18.3\ km$$

We now have an algebraic equation (with units) that can be solved for the unknown quantity t.

We will now use an Assignment Statement with inline evaluation to calculate the answer.

Type:

t<ctrl>-1:18.3km<spacebar> <spacebar>/0.3km/s=

To produce:

$$t_1 := \frac{18.3\ km}{0.3\ \frac{km}{s}} = 61\ s$$

We call this inline evaluation because we are using both Mathcad's assignment symbol ":=" and the evaluation symbol "=" in the same math region. Note that the use of the subscript t_1 is important in this case because we have already defined t as a range variable. If we had simply used t in this assignment statement, we would have changed the definition of t from this point onward in the worksheet.

12. Since we now know the answer to the second part of the tutorial task, let's take advantage of Mathcad's text capabilities to briefly state our answer. Create a Text Box on your worksheet containing the text: **"It takes the sound wave 61 seconds to travel as far as the ground wave travels in 3 seconds."**

Your work in steps 11 & 12 should have produced a section of your worksheet appearing something like this:

$$D_{GW}(3 \ s) = 18.3 \ \textbf{km}$$

b. It takes the sound wave 61 seconds to travel as far as the ground wave travels in 3 seconds.

$$D_{SW}(t) = 18.3 \ km$$

$$0.3 \ \frac{km}{s} \cdot t = 18.3 \ km$$

$$+$$

$$t_1 := \frac{18.3 \ km}{0.3 \ \frac{km}{s}} = 61 \ \textbf{s}$$

Notice that in this calculation we included the units for each value and Mathcad gave us the correct units on the result.

13. Our final task is to use Mathcad to determine your distance from an elephant. The information that we have is that the sound wave reaches your location 10 seconds after ground wave. We can express this using a Boolean statement:

$$D_{SW}(t+10) = D_{GW}(t)$$

This statement says that the value of D_{GW}(t seconds) is the same as the value of D_{SW}(t+10 seconds).

Next, we want to convert this statement to an algebraic equation that we can solve:

$$0.3 \ (t+10) = 6.1 \cdot t$$

We are using Mathcad Boolean statements to document the development of our thinking as we develop a solution.

Here is the rest of my solution. Can you reproduce each step on your Mathcad worksheet?

$$0.3 \ t + 3 = 6.1 \ t$$

$$3 = 5.8 \ t$$

More Boolean Statements

$$t_2 := \frac{3}{5.8} = 0.517$$

The elephant is approximately 3.15 km from the researcher.

$$t_2 := 0.517 \ s$$

$$D_{GW}(t_2) = 3.154 \ \textbf{km}$$

Inline evaluation followed by a re-assignment to add units to the variable, t_2.

$$D_{SW}(t_2 + 10 \ s) = 3.155 \ \textbf{km}$$

Add the Text Box last.

Calculate the distance travelled by each wave.

Summary

1. In this tutorial you have learned:
   - ✓ To add an image to a document
   - ✓ To define and use a range variable
   - ✓ To define parametric functions
   - ✓ To use Mathcad's units in variable definitions, function definitions, and evaluation statements
   - ✓ To plot parametric functions
   - ✓ To add traces to a plot
   - ✓ To use Mathcad's Boolean equivalence, evaluation, and assignment equal signs to perform step-by-step algebraic solutions
   - ✓ To use subscripts in naming variables and functions
   - ✓ To combine text, images, graphics, and symbolic algebra in a single worksheet

2. At this point you should have a complete Mathcad worksheet. Please make note of the following:
   - o It is possible to use your mouse to drag and resize all of the regions on your worksheet to improve the formatting.
   - o If you edit any of the numerical values in assignment statements (e.g., extend the range variable t to 4 seconds), then the worksheet will update wherever t is used (e.g., the graph in this case).
   - o Editing numerical values in Boolean equality statements will not result in changes elsewhere in the worksheet.

3. For more assistance in learning to use Mathcad Prime, consider the following:
   - o The Tutorials provided with Mathcad Prime that are accessible from the Getting Started Tab
   - o Free tutorials available from PTC University's Learning Exchange: LEX.ptc.com
   - o Video tutorials available from the Mathcad Academic YouTube Channel: www.youtube.com/MathcadAcademic

Great job completing this tutorial!